PID Controller Tuning Based on the Guardian Map Technique

Tanya Boone, Gerardo Romero, David Lara, Irma Pérez, Aldo Méndez, Alberto Reyna and Jaime Ramos

***Abstract*—Proportional + Integral + Derivative (PID) controllers are the the base of industrial automation today. The key issue in PID tuning is finding the suitable parameters to obtain a good close loop performance. Most of PID controllers are tuned manually and the derivative gain K_d is often turned off because if its difficult to find values that give a robust system. There are different methods for tuning PID controllers, but in this paper will be use de computation of the ultimate gain and period in order to get the suitable parameters selection of the PID controller. To do this, we are going to apply the Guardian Map concept. An application to a particular process will be considered in order to validate the proposed method.**

***Index Terms*—PID Controllers, Control Tuning, Guardian Maps, Ziegler and Nichols Method.**

I. Introduction

THE PID controller is undoubtedly the most popular controller in all industries worlwide. This is due to it's simplicity and good performance. The PID controller calculation involves three separate parameters: proportional, integral and derivative values.The proportional term reacts to current errors, past errors are accounted for by the integral term and the derivative term anticipates future errors by linear extrapolation of the error. Despite all the progress in advanced control, there are substantial opportunities for improving PID control. The tuning of PID controller parameters has been studied by researchers for many years. [2] presents the state of the art of PID control and reflects on its future. In particular, it discusses on specifications, stability, design, applications, and performance of PID control. Also, that paper mentions some other alternatives to PID such as: RST: Discrete-time linear MISO controllers, SFO: State feedback and observers, MPC: Model predictive control. which they have as their main challenge to consider the performance, tuning, ease of use and maintenance. Two types of design techniques are used to design a PID controller for the lower part of the leg, around the knee joint is presented in [4]. The resulting PID controllers are compared by simulating a squat movement and a normal gait. In [9] it is presented a new methodology for PID control tuning by coupling the Gain and Phase Margin method with the Genetic Algorithms in which the micropopulation concept and adaptive mutation probability are applied . A methodology for tuning the PID parameters using fuzzy control techniques are presented in [5], [7] and [8]. Others results related to the topic of evolutionary computation are presented in [6] and [18]. A self-tuning algorithm for the PID parameters, derived from the Lyapunov method and using just-in-time learning (JITL) is presented in [10]. In [14], the Ziegler and Nichols step response method, the Chien-Hrones-Reswick method and the Cohen-Coon method are compared for PID control of a single axis of a XY stage of a 3D surface profiler. In [17] some rules for tuning PID controllers are modified to improve the performance of the closed-loop control system, represented by a reduced model. A new method for PID controller tuning based on Bode's integrals is proposed in [11], where derivatives of phase and amplitude of minimum-phase and stable plant models with respect to the frequency were approximated using these Bode's integrals. Some design method for PID controllers based on optimization related to robust H_∞ control are presented in [12] and [13]. In [16], there was presented an analysis of the robustness of some PID tuning techniques in the space of the controller parameters, in order to obtain robustness with respect to small perturbations in the closed-loop control system.

The performance of a PID controller is mainly determined by the choice of it's parameters. Tuning a PID implies calculating suitable values in order to obtain the desired control performance. Among the conventional PID tuning, the Ziegler and Nichols methods may be the most well known technique. Developed by John G. Ziegler and Nathaniel B. Nichols in 1942, , see [20]. The step response method is based on measurement of the open-loop response, on the other hand, the frequency response method is based on a closed-loop experiment where the system is brought to the stability boundary under proportional control.These rules work quite well for wide range of practical processes. However, as these techniques are experimental, sometimes it could be time consuming and can venture into unstable regions while testing the P controller, which could cause the system to become out of control or even the crash of the control system. Therefore this paper considers the calculation of tuning parameters using the concept of guardian map presented in [15] and using a modified Ziegler and Nichols method presented in [1].

This work is organized as follows: section II presents the problem statement. In section III, some mathematical preliminaries results are discussed. In section IV, the main result is described, where an approach based on the Guardian Map is used to obtain the tuned parameter of a PID controller . In section V, simulation of an particular application are discussed. Finally, in section VI, conclusions are presented.

The authors are with the Department of Electrical and Electronic Engineering, Universidad Autonoma de Tamaulipas, Reynosa, Tamaulipas, 88779 Mexico e-mails: tanyaboone20@gmail.com; gromero@uat.edu.mx

Manuscript received April 19, 2005; revised January 11, 2007.

II. Problem Statement

A. PID Controllers

A PID Controller is defined by three parameters: the proportional gain K_p, integral gain K_i and derivative gain K_d. The typical PID control system can be described as Figure 1.

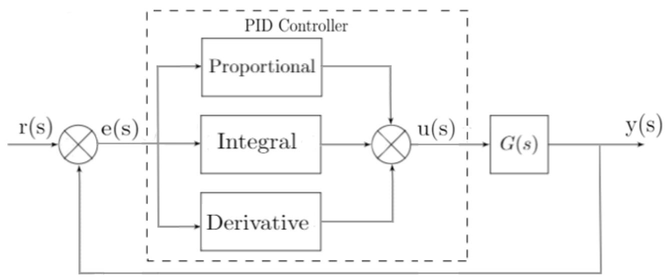

Fig. 1: PID Controller.

where $G(s)$ represents the transfer function of the plant. The output of the PID controller; i.e., the input to the plant, in time-domain is as follows:

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \tag{1}$$

The control signal $e(t)$ represents the tracking error, the difference between the desired input value $r(t)$ and the output signal $y(t)$.
The transfer function of a PID controller is found by taking the Laplace transform of Equation (1).

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \tag{2}$$

and the block diagram of the closed-loop control system is presented in the next figure:

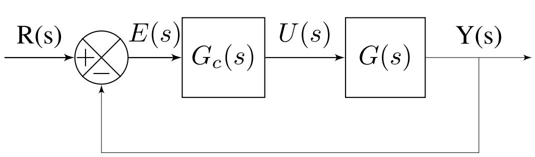

Fig. 2: Closed-loop control system.

The closed-loop control system performs well if the parameters are chosen properly, on the other hand a poor selection of the PID controller parameters cause poor performance and may even lead to instability in the closed-loop control system. Therefore, it is crucial to determine the best values for K_p,K_i,and K_d in order to reach an optimal performance of the control system.

B. Modified Ziegler-Nichols PID tuning method

The design method presented in this paper is based in the Ziegler-Nichols frequency response method which is based on the knowledge of the point on the Nyquist curve of the open-loop transfer function $G_l(s) = G_c(s)G(s)$ where the Nyquist curve intersects the point $(-1, 0)$ of the complex plane. This point is characterized by two parameters: K_u and T_u, which are called the ultimate gain and the ultimate period. To obtain these parameters, in this paper will use the Guardian Map concept, as it will be presented later. The performance of the closed-loop control system will be guaranted selecting the PID controller parameters using the following equations, see [1]:

$$\begin{aligned} K_p &= K_u r_a \cos(\phi_a) && (3)\\ K_i &= \frac{T_u}{\pi}\left(\frac{1+\sin(\phi_a)}{\cos(\phi_a)}\right) && (4)\\ K_i &= \frac{T_u}{4\pi}\left(\frac{1+\sin(\phi_a)}{\cos(\phi_a)}\right) && (5)\end{aligned}$$

where the parameters r_a and ϕ_a are related to the point A, which is shown in the following figure:

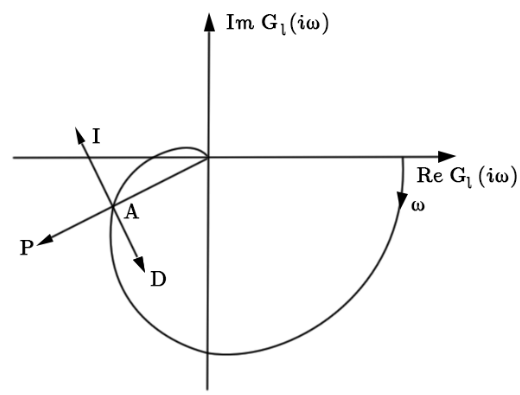

Fig. 3: Influence of PID controller on the Nyquist plot.

where the point A is defined by the following equation:

$$A = G_l(j\omega_u) = r_a \exp(j\phi_a) \tag{6}$$

with $\omega_u = 2\pi/T_u$. It is worth to note that, in this way it is possible to achieve an adequate performance in the closed-loop control system, since through the selection of the PID parameters it is possible to move the point A from Figure 3 and achieve, in this way, a favorable change of the phase (ϕ_m) and gain (G_m) margin of the closed-loop control systems; as we can see in the following figure:

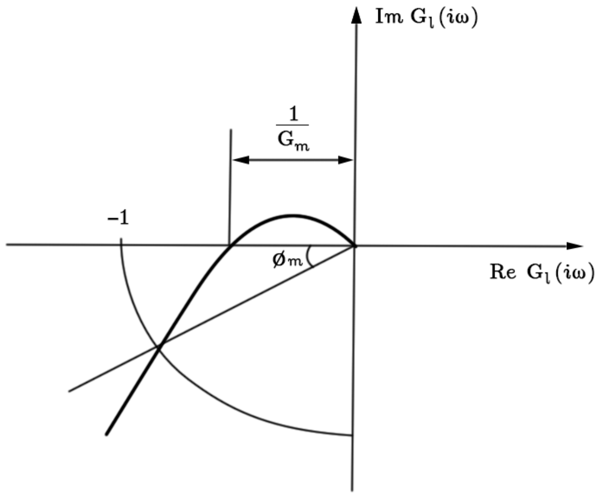

Fig. 4: Phase (ϕ_a) and (G_m) gain margins.

It is important to note that the phase and gain margins are closely related to the stability property, so that a good selection of these parameters, allow to have good condition for filtering noise and disturbance rejection. That is why in this paper we are interested in tuning the PID parameters to achieve good margin and phase gains.

III. Mathematical Preliminaries

The result that it will be presented in this paper is based on the concept of Guardian Map which was presented at [15] and is defined below.

Definition 3.1: Let $\mathcal{X}$ be the set of all the polynomials of degree at most n with real coefficients, and let $\mathcal{S}$ be an open subset of $\mathcal{X}$. Let ν map $\mathcal{X}$ into $\mathbb{C}$, where $\mathbb{C}$ represents the set of complex number. We say that ν guards $\mathcal{S}$ if for all $x \in \bar{\mathcal{S}}$ ($\bar{\mathcal{S}}$ is the closure of $\mathcal{S}$), the equivalence

$$x \in \partial\mathcal{S} \iff \nu(x) = 0 \tag{7}$$

holds. Where $\partial\mathcal{S}$ represents the boundary of $\mathcal{S}$. In this case, we also say that ν is a *Guardian Map* for $\mathcal{S}$. In other words, a Guardian Map is an operator which maps to zero every point of the boundary of their domain set.

In particular we are interested in a particular Guardian Map defined as follows. Considering the following polynomials with real coefficient and roots with negative real part:

$$p(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \tag{8}$$

associated with this polynomial is the following Hurwitz matrix:

$$H[p(s)] = \begin{bmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \cdots & \cdots & 0 \\ a_n & a_{n-2} & a_{n-4} & \cdots & \cdots & \vdots \\ 0 & a_{n-1} & a_{n-3} & a_{n-4} & \cdots & \vdots \\ 0 & a_n & a_{n-2} & a_{n-5} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_0 \end{bmatrix} \tag{9}$$

The set of polynomials $p(s)$ is guarded by the map $\nu : p(s) \mapsto \det(H[p(s)])$ where $\det(H[p(s)])$ represents the determinant of the Hurwitz matrix associated to $p(s)$. The proof of this statement can be performed using the Orlando's formula, see [3]. The previous result implies that the determinant of the Hurwitz matrix associated to $p(s)$ will be equal to zero if and only if, the roots of the polynomials $p(s)$ belongs to the imaginary axis of the complex plane and then it is possible to use this result to obtain the ultimate gain and period.

IV. Main Result

Essentially, if we need to obtain parameters K_u and T_u we need the process to oscillate, leading the system to an unstable boundary. Therefore, this problem can be solved makes the following system swing by increasing the gain K:

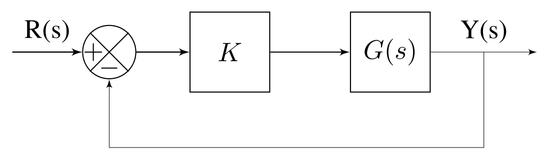

Fig. 5: Closed-loop control system with proportional control action.

where:

$$G(s) = \frac{n(s)}{d(s)} \tag{10}$$

the characteristic equation of the previous system, is given by the following polynomial:

$$p(s, K) = d(s) + Kn(s) \tag{11}$$

we make the assumption that $d(s)$ is a polynomial with all its roots have negative real part. The following result permit to obtain the ultimate gain using the Guardian Map defined in previous section.

Theorem 4.1: Consider the closed-loop control system shown in figure 5 and $G(s)$ defined as in (11), then the ultimate gain K_u can be obtained by:

$$K_u = -\frac{1}{\lambda_{min}\left(H^{-1}[d(s)]H[n(s)]\right)} \tag{12}$$

where λ_{min} represents the minimum negative real eigenvalue of $H^{-1}[d(s)]H[n(s)]$.

Proof

The Hurwitz matrix of the characteristic equation is as follows:

$$\begin{aligned} H[p(s,K)] &= H[d(s) + Kn(s)] \\ &= H[d(s)] + KH[n(s)] \\ &= KH[d(s)]\left(\frac{1}{K}I + H^{-1}[d(s)]H[n(s)]\right) \end{aligned}$$

computing the determinant of the Hurwitz matrix

$$\det(H[p(s,K)]) = \det(H[p(s,K)])\det\left(\frac{1}{K}I + H^{-1}[d(s)]H[n(s)]\right)$$

The term $\det(H[p(s,K)]) \neq 0$, since $d(s)$ is a Hurwitz polynomial. So the only way the determinant of $\det(H[p(s,K)])$ be zero is that the second term be zero and this condition is satisfied if:

$$K = -\frac{1}{\lambda_{min}\left(H^{-1}[d(s)]H[n(s)]\right)}$$

under the consideration that the Hurwitz matrix is a Guardian Map, the above expression represents the ultimate gain K_u.

□

The ultimate period can be obtained solving for ω, the following equation:

$$(s + \omega_u^2)g(s) = p(s, K_u) \quad (13)$$

where $g(s)$ is an unknown polynomial and $T_u = 2\pi/\omega_u$. Once the ultimate gain and period are obtained, then the tuned PID parameters can be computing using the formulas presented in (3)-(5). It is important to note that the performance of the closed-loop control system will be given through the proper definition of the parameters r_a and ϕ_a of (6), which are directly related to the phase and gain margin of the closed-loop control system.

V. Application

The method proposed in this paper will be applied to an Automatic Voltage regulator(AVR) model described on [19], which it is characterized by the following transfer function:

$$G(s) = \frac{0.1s + 10}{0.0004s^4 + 0.045s^3 + 0.555s^2 + 1.41s + 1}$$

considering this transfer function the characteristic equation is as follows:

$$p(s, K) = d(s) + Kn(s)$$

where

$$\begin{aligned} n(s) &= 0.1s + 10 \\ d(s) &= 0.0004s^4 + 0.045s^3 + 0.555s^2 + 1.41s + 1 \end{aligned}$$

and the corresponding Hurwitz matrix are the following:

$$H[d(s)] = \begin{bmatrix} 0.045 & 1.41 & 0 & 0 \\ 0.0004 & 0.555 & 1 & 0 \\ 0 & 0.045 & 1.41 & 0 \\ 0 & 0.0004 & 0.555 & 1 \end{bmatrix}$$

$$H[n(s)] = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

the eigenvalues of $H[d(s)]^{-1}H[n(s)]$ are

$$\lambda = \{0, 10, -0.5517, 0.002\} \quad (14)$$

and then:

$$\lambda_{min} = -0.5517 \quad (15)$$

the ultimate gain is:

$$K_u = -\frac{1}{-0.5517} = 1.8125 \quad (16)$$

To calculate the ultimate period T_u we need to substitute in the following equation the Ku:

$$(s^2 + \omega_u^2)\bar{P}(s) = P_0(s) + KuP_1(s)$$

$$\begin{aligned} (s^2 + \omega_u^2)\bar{P}(s) &= 0.0004s^4 + 0.045s^3 \\ &+ 0.555s^2 + 0.18125s + 18.125 \end{aligned}$$

where:

$$\bar{P}(s) = a_2s^2 + a_1s + a_0$$

solving for ω_u, it is obtained:

$$\omega_u = 5.7147$$

and

$$T_u = \frac{2\pi}{\omega_u} = 1.0995$$

To set up the PID gains Kp, Ki and Kd we can use the formulas shown in (3)-(5). We are going to consider three different cases:

- for $r_a = 0.5$ and $\phi_a = 20$:

$$K_p = 0.8516, K_i = 1.7038, K_d = 0.1064$$

- for $r_a = 0.41$ and $\phi_a = 60$:

$$K_p = 0.3716, K_i = 0.2845, K_d = 0.1213$$

- for $r_a = 0.29$ and $\phi_a = 46$:

$$K_p = 0.3651, K_i = 0.4215, K_d = 0.0791$$

Different simulations were performed considering the parameters obtained previously and the following responses were obtained:

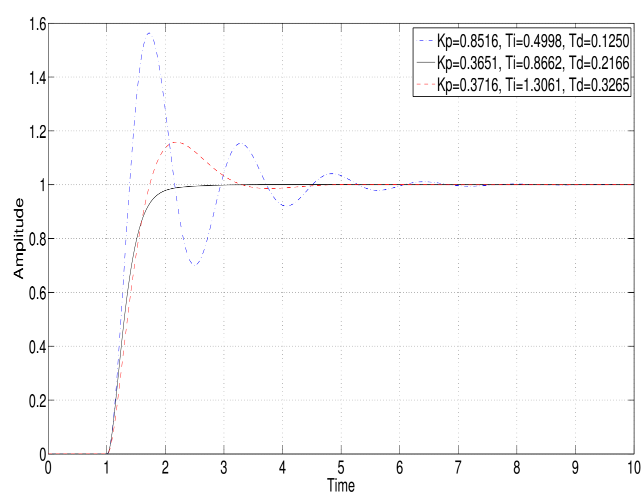

Fig. 6: Closed-loop control system for different values of parameters of the PID controller.

VI. Conclusions

This paper presented a new method for tuning PID controllers. The method used the concept of Guardian Map to calculate the ultimate gain and then, also computing the ultimate period, the tuning parameters for a PID controller were obtained. This methodology will, in future, be able to use different optimization techniques in order to improve the PID controller selection, getting to meet different performance criteria..

References

[1] K.J. Astrom, T. Hagglund, "PID Controllers: Theory, Design, and Tuning", Instrument Society of America, 1995.
[2] K.J. Astrom, T. Hagglund, "The Future of PID Control", Control Engineering Practice, Vol. 9, No. 11, pp. 1163-1175, 2001.
[3] B.R. Barmish, New Tools for Robustness of Linear Systems, Macmillan, 1994.
[4] A. Chavalier, C.M. Ionescu, R. De Keyser, "Model-based vs Auto-tuning Design of PID Controller for Knee Flexion During Guit", Proceeding of the 2014 IEEE International Conference on System, Man, and Cybernetics, San Diego CA, pp. 3878-3883, October 5-8, 2014.
[5] X.G. Duan, H. Deng, H.X. Li, "A Saturation-based Tuning Methods for Fuzzy PID Controller", Vol. 60, No. 11, pp. 5177-5185, 2013.
[6] S. Ganjefar, M. Farahani, "Damping of Subsynchronous Resonance Using Self-tuning PID and Wavelet Neural Network", COMPEL - The international journal for computation and mathematics in electrical and electronic engineering, Vol. 31, No. 4, pp. 1259-1276, 2012.
[7] P. Gil, C. Lucena, A. Cardoso, L. Brito Palma, "Gain Tuning of Fuzzy PID Controllers for MIMO Systems: A Performance-Driven Approach", Vol. 23, No. 4, pp. 757-768, 2014.
[8] J. Guo, G. Wu, S. Guo, "Fuzzy PID Algorithm-based Motion Control for the Spherical Amphibious Robot", Proceedings of the 2015 IEEE International Conference on Mechatronics and Automation, Beijing China, pp. 1583-1588, August 2-5, 2015.
[9] Y. Jaen-Cuellar, R.J. Romero-Troncoso, L. Morales-Velazquez, R. A. Osornio-Rios, "PID-Controller Tuning Optimization with Genetic Algorithms in Servo Systems", International Journal of Advanced Robotics Systems, Vol. 10, No. 324, pp. 1-14, 2013.
[10] Y. Kansha, L. Jiu, M. Chiu, "Self-tuning PID Controllers Based on the Lyapunov Approach", Chemical Engineering Science, Vol. 63, pp. 2732-2740, 2008.
[11] A. Karimi, D. Garcia, R. Longchamp, "PID Controller Tuning Using Bode's Integrals", IEEE Transactions on Control Systems Technology, Vol. 11, No. 6, 2003.
[12] H. Panagopoulos, K.J. Astrom, T. Hagglund, "Design of PID Controllers Based on Constrained Optimization", Proceedings of the American Control Conference, pp. 3858-3862, San Diego CA, June 1999.
[13] H. Panagopoulos, K.J. Astrom, "PID Control Design and H_∞ Loop Shaping", Proceedings of the 1999 IEEE International Conference on Control Applications, pp. 103-108, Kohala Coast-Island of Hawai, Hawai USA, August 22-27, 1999.
[14] R. Sen, C. Pati, S. Dutta, R. Sen, "Comparison Between Three Tuning Methods of PID Control for High Precision Positioning Stage", MAPAN-Journal of Metrology Society of India, Vol. 30, No. 1, pp. 65-70, March 2015.
[15] L. Saydy, A.L. Tits, E.H. Abed, "Guardian Maps and the Generalized Stability of Parametrized Families of Matrices and Polynomials", Mathematics of Control, Signal, and Systems, Vol. 3, No. 4, pp. 345-371, 1990.
[16] G.J. Silva, A. Datta, S.P. Bhattacharyya, "On The Stability and Controller Robustnes of Some Popular PID Tuning Rules", IEEE Transactions on Automatic Control, Vol. 48, No. 9, pp. 1638-1641, 2003.
[17] S. Skogestad, "Simple Analytic Rules for Model Reduction and PID Controller Tuning", Journal of Process Control, Vol. 13, pp. 291-309, 2003.
[18] G. Stimac, S. Braut, R. Zigulic, "Comparative Analysis of PSO Algorithms for PID Controller Tuning", Chinese Journal of Mechanical Engineering, Vol. 27, No. 5, pp. 928-936, 2014.
[19] N. Yadaiah, S. Malladi, "An Optimized Relation Between T_i and T_d in Modified Ziegler Nichols PID Controller Tuning", Proceedings of the 2013 IEEE International Conference on Applications, pp. 1275-1280, Hyderabad India, August 28-30, 2013.
[20] J.B. Ziegler, N.B. Nichols, "Optimum Setting for PID Controllers", Transactions of ASME, Vol. 64, pp. 759-768, 1942.