Modeling the precipitation of bidisperse suspension with allowance for the aggregation of clusters of various fractions

D. Dairabay, V. Golubev, L. Musabekova, B. Korganbayev

***Abstract*—** The paper deals with the mathematical model for describing processes of simultaneous aggregation and sedimentation of dispersed phases from bidisperse suspensions. The submitted model allows to calculate of almost all the main characteristics of the process of bidisperse suspension sedimentation, taking into account aggregation of two fractions and formation of the phase of aggregates clusters. These processes play an important role in many industrial processes and natural phenomena.

***Keywords*—**aggregation, bidisperse suspension, clusters, precipitation.

I. INTRODUCTION

PRECIPITATION of polydisperse suspensions plays a key role in many industrial processes and natural phenomena. Despite the attention of scholars to these processes many aspects in this field remain little studied [1, 2].

With all seeming simplicity and prevalence of sedimentation phenomena their experimental and theoretical study entails many great difficulties [2, 3]. Working out of theoretical models of polydisperse suspensions sedimentation, even in the absence of essential interaction between the particles, is not a trivial problem and its solution is still can't be considered as completed [3, 4].

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It can be shown that a kinetic curve that describes the deposition of polydisperse suspension in the absence of fractions aggregation consists of three linear segments. At the first segment the particles of both fractions will fall simultaneously, the second segment corresponds to the stage when precipitation of the large fraction has been completed,

but deposition of the fine fractions continues. The third segment corresponds to the stage of completion of both fractions deposition.

The Stokes law can be used for calculating the fine particles stationary sedimentation [5, 6, 7]. Terms and conditions of such description applicability can be briefly summarized as follows.

1. The particles are assumed to be spherical shape. Since particles of dispersed phase in suspensions in reality often deviate from spherical shape there is introduced a certain so-called effective radius of the hypothetical spherical particle of the same mass as the real particle and which moves with the same speed.

2. The interaction between the particles is absent. This condition may be performed in the absence of aggregation processes at low concentrations of the dispersed phase. The average distance between the particles should be sufficiently large compared to their sizes so that the motion of particles did not affect the nature and speed of movement of the other particles.

3. The reservoir in which sedimentation occurs should have considerably larger scale compared to the sizes of the particles. Then we can neglect the influence of the walls, as in the vicinity of the walls the sedimentation rate can't be described with the help of Stokes law []8, 9].

4. The particles must be sufficiently hard and smooth to prevent their strain while moving.

5. It is also assumed that there is no slippage between the depositing particles and the fluid, i.e, particles are well wetted by the liquid.

6. The sedimentation rate is not high, because otherwise there is a turbulent flow, and applicability of the Stokes law will be violated.

In the steady regime the deposition of each fraction goes with constant rate determined by the particle fraction size [7].

In the paper we submit the simple model for calculating the kinetics of gravitational sedimentation of bidisperse suspensions in the presence of the mutual aggregation of the particles of fine and coarse fractions.

Thus the second supposition from the above list is not fulfilled. Such processes take place everywhere in natural phenomena and they ought to be specially organized in the industrial water purification systems [5, 6].

D. Dairabay is with the State University of South KazakhstanShymkent, Kazkakhstan, (e-mail: amb_52@mail.ru).

V. Golubev and L. Musabekova are with State University of South Kazakhstan, Shymkent, Kazkakhstan, (e-mail: amb_52@mai.ru, mleyla@bk.ru).

B. Korganbayev is with International Humanitarian-Technical University, Shymkent, Kazakhstan, (e-mail: amb_52@mail.ru).

II. THEORETICAL DETAILS AND DISCUSSION

In our model, we assume that the aggregation of particles of various fractions takes place in the bidisperse suspension [8, 10]. Thus, second assumption from the above list is not fulfilled. The other listed conditions remain correct. This approach is valid for weakly concentrated suspensions, in which there is no influence of one particle fraction on the hydrodynamic conditions of the deposition of the other one [9, 10]. The process of aggregation will be described by the Smoluchowski equation [11, 12, 13].

$$\frac{\partial C_i}{\partial t} = \frac{1}{2}\sum_{j=1}^{i-1} N_{i-j,j} C_{i-j} C_j - C_i \sum_{j=1}^{\infty} N_{i,j} C_j, \tag{1}$$

where C_i is the concentration a cluster of i-th order, $N_{i,j}$ is the aggregation core for cluster of i-th and j-th orders, t is time.

According to the specific problem formulation, the following system of kinetic equations can be written as

$$\frac{d\rho_3}{dt} = a\rho_1\rho_2,$$

$$\frac{d\rho_1}{dt} = -a\rho_1\rho_2,$$

$$\frac{d\rho_2}{dt} = -a\rho_1\rho_2, \tag{2}$$

where ρ_1, ρ_2 are the partial volume densities of two initial fractions of the suspension, $1/m^3$; ρ_3 is the partial volume density of the aggregates fraction, $1/m^3$, a is the kinetic coefficient, m^3/s.

Initial conditions are

$$\rho_1(0) = \rho_1^0, \quad \rho_2(0) = \rho_2^0. \tag{3}$$

Let us introduce the denotations

$$\rho_0 = \rho_1^0 + \rho_2^0, \quad \theta_0 = a\rho_1^0\rho_2^0. \tag{4}$$

Thus, after rearrangement of system (2), we get

$$\frac{d^2\rho_3}{dt^2} = -a\frac{d\rho_3}{dt}(\rho_1 + \rho_2), \tag{5}$$

The following relations are obviously correct

$$\frac{d}{dt}(\rho_1 + \rho_2) = -2\frac{d\rho_3}{dt}, \tag{6}$$

$$\rho_1 + \rho_2 - \rho_0 = -2\rho_3, \tag{7}$$

The kinetic equation for partial density of the phase of clusters aggregates reads

$$\frac{d^2\rho_3}{dt^2} = -a\frac{d\rho_3}{dt}(\rho_0 - 2\rho_3). \tag{8}$$

From (8) it follows

$$\frac{d\rho_3}{dt} = -a\rho_3(\rho_0 - \rho_3) + \theta_0, \tag{9}$$

$$\frac{d\rho_3}{dt} = a(\rho_3 - \rho_1^0)(\rho_3 - \rho_2^0). \tag{10}$$

The solution of (10) reads

$$\frac{1}{\rho_1^0 - \rho_2^0}\ln\frac{\rho_2^0(\rho_3 - \rho_1^0)}{\rho_1^0(\rho_3 - \rho_2^0)} = at. \tag{11}$$

As the result the following expression for the density of aggregates phase has been obtained

$$\rho_3 = \frac{\rho_1^0\rho_2^0\left(1 - \exp\left(a(\rho_1^0 - \rho_2^0)t\right)\right)}{\rho_2^0 - \rho_1^0\exp\left(a(\rho_1^0 - \rho_2^0)t\right)} \tag{12}$$

For $\rho_1^0 = \rho_2^0$ we obtain

$$\rho_3 = \frac{\rho_0^2 at}{2(2 + \rho_0 at)}. \tag{13}$$

Using obtained expressions for the evolution of the fractions densities, the curve of accumulation of the sediment which consists of different fractions including the aggregates phase can be calculated.

In the process of sedimentation just two frontal sections will be formed. They are the front of fine fraction suspension which separates the clear liquid and the dreggy liquid, and the second front which separates the suspension consisting of the two initial fractions and the layer of suspension consisting of three fractions including the aggregates. Further we will ignore the height of the sediment layer, supposing the full height of the container H is large enough, and the initial concentration of a disperse phase is not too great. Let the fine fraction be denoted by index "1", the mixture of two fractions - by index "2", and the mixture of three fractions - by index "3". In accordance with the above assumptions, the deposition rates of the each fraction W_i remain unchanged. Then the movement time of the front of the suspension of the three fractions prior to its complete precipitation with accounting those velocities of frontal planes $W_1 < W_2 < W_3$ is

$$T_2 = \frac{H}{W_2}. \tag{14}$$

The time of full clarification of the suspension reads

$$T_1 = \frac{H}{W_1}. \tag{15}$$

Figures 1 and 2 show some graphs illustrating the evolution of the density of the different fractions in the area of the joint motion of all three fractions of the suspension.

Relative time of the process

1- $a = 0.01$ m³/s; 2- $a = 0.05$ m³/s; 3- $a = 0.1$ m³/s; 4- $a = 0.2$ m³/s

Fig. 1 Evolution of the aggregate clusters concentrations

Relative time of the process
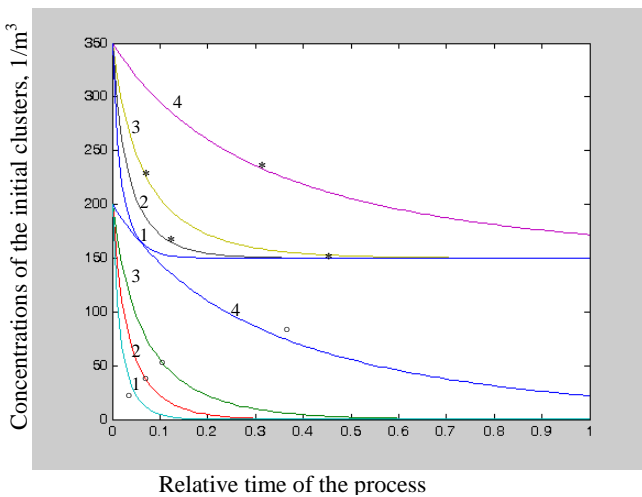
1- $a = 0.01$ m³/s; 2- $a = 0.05$ m³/s; 3- $a = 0.1$ m³/s; 4- $a = 0.2$ m³/s; ∘ - fraction 1; * - fraction 2

Fig. 2 Evolution of the initial clusters concentrations

We can write for the flow of i - th fraction through a cross section of the vessel

$$q_i(t) = \rho_i(t)W_i \tag{16}$$

So the evolution of the specific quantity of the deposit of i - th fraction per unit of the surface of a cylindrical vessel over time $t \leq T_2$ is defined as

$$Q_i(t) = W_i \int_0^t \rho_i(s)ds. \tag{17}$$

From this it follows

$$Q_3 = W_3 \left[\rho_{10}t - \frac{1}{a}\ln\left(\frac{\rho_{20} - \rho_{10}\exp(a(\rho_{10} - \rho_{20})t)}{\rho_{20} - \rho_{10}} \right)\right], \tag{18}$$

$$Q_1 = W_1 \frac{1}{a}\ln\left(\frac{\rho_{20} - \rho_{10}\exp(a(\rho_{10} - \rho_{20})t)}{\rho_{20} - \rho_{10}} \right), \tag{19}$$

$$Q_2 = W_2 \left[(\rho_{20} - \rho_{10})t + \frac{1}{a}\ln\left(\frac{\rho_{20} - \rho_{10}\exp(a(\rho_{10} - \rho_{20})t)}{\rho_{20} - \rho_{10}} \right)\right] \tag{20}$$

After the first period of deposition the turbidity of the first fraction remains still into the zone with height

$$H_1 = (W_2 - W_1)T_2. \tag{21}$$

In this zone there will be a gradient of density of the first fraction. This gradient can be calculated from the following considerations.

Since $\rho_1(t) = \rho_{10} - \rho_3(t)$ then to accounting the time lag between the process of sedimentation fronts can be written
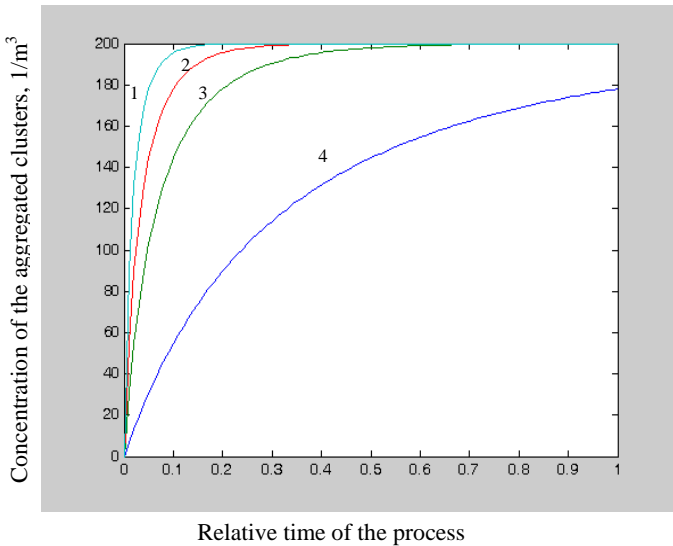
$$t = \frac{z}{W_2 - W_1}, \tag{22}$$

where z is the current vertical coordinate.

Then we get the expression for calculating the density of the fine fraction in the zone between the front sections

$$\rho_1(z) = \rho_{10} - \rho_3\left(z/(W_2 - W_1)\right). \tag{23}$$

Figures 3, 4 depict plots for changing the specific quantities of different fractions in the deposit during the first period of sedimentation T_2, i.e. up to the full sedimentation of the dense part of suspension.
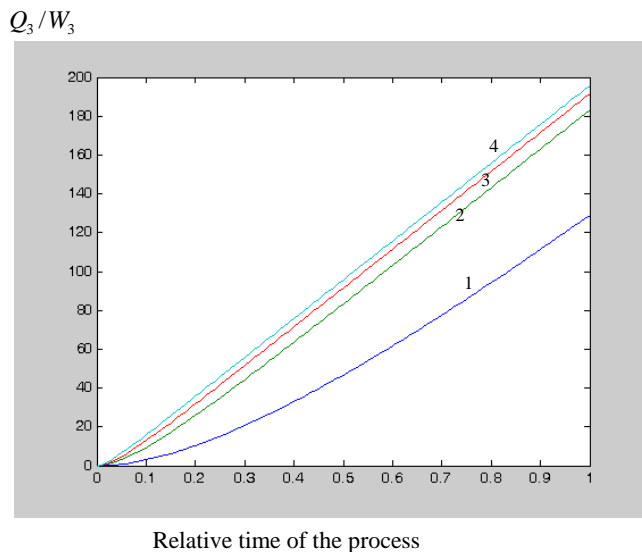
Q_3 / W_3

Relative time of the process

1- $a = 0.01$ m^3/s; 2- $a = 0.05$ m^3/s; 3- $a = 0.1$ m^3/s;
4- $a = 0.2$ m^3/s

Fig. 3 Specific quantity of the sediment of aggregated fractions during the first period of precipitation

$Q_{1,2} / W_{1,2}$

Relative time of the process
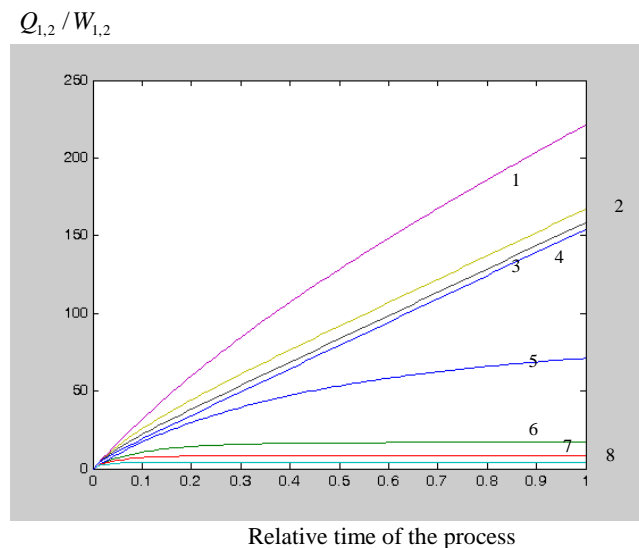
Fraction 1: 1- $a = 0.01$ m^3/s; 2- $a = 0.05$ m^3/s; 3- $a = 0.1$ m^3/s; 4- $a = 0.2$ m^3/s;

fraction 2: 5- $a = 0.01$ m^3/s; 6- $a = 0.05$ m^3/s; 7- $a = 0.1$ m^3/s; 8- $a = 0.2$ m^3/s;

Fig. 4 Specific quantities of the sediments of initial fractions during the first period of precipitation

III. Conclusion

Thus, the present model allows to calculate of almost all the main characteristics of the process of bidisperse suspension sedimentation, taking into account aggregation of fractions. This model can be developed in case of reversible aggregation, i.e., in the presence of partial fragmentation of the dispersed phase clusters. Analysis of the dependencies shows good qualitative agreement with the experimental data and studied regularities of sedimentation [8]. Of course, for reliable practical application this model needs the identification of control parameters for the specific physical and chemical systems.

Although the submitted model has been developed for bidisperse suspensions, the offered and discussed in this paper method for simultaneous accounting the aggregation and sedimentation processes into the volume of industrial apparatuses can be applied to modeling similar processes in the case of polydisperse suspensions. Calculations may become more cumbersome but it is not a principal problem nowadays.

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