Identification of Series DC Motors Based on the Strecj Method

Jesús U. Liceaga-Castro, Irma I. Siller-Alcalá, Roberto A. Alcántara-Ramírez, and Jorge M. Jaimes-Ponce

***Abstract*—**Based on the Strecj method a procedure for the identification of Series DC motors is presented. It requires only the measurement of the rotors speed and current consumption. The Strecj methodology is applied to obtain the parameters of a non-linear model without magnetic saturation of a particular Series DC motor. To prove the validity of the proposal here presented comparisons between real time responses of the series DC motor with those of the identified nonlinear model are presented.

***Keywords*—** Electric Machine; Identification; Linear Systems; Series DC Motor; Strecj method

I. Introduction

MODELS, in specific accurate models, are a very important factor on the analysis and design of any control system. In many cases appropriate models are available usually in the form of linear or nonlinear differential equations. Also, models are normally obtained by analyzing the physical properties of the phenomena. However, still remains the problem of determining the value of the physical parameters of these models in relation to a particular process or phenomena.

Although one of the most important objectives of any control system is to compensate or minimize parameter uncertainties or variations, it is also true that better models facilitate the controller design resulting in control systems with improved performance and robustness. In order to fulfill this requirement models have to be validated by comparing its responses with those of the real process. In doing so, it is also well known that models –linear or nonlinear- are most of time valid around an operating point or a region of operation.

In the case of electric machines, in specific electric motors, normally the suppliers do not provide all the parameter information that allows obtaining accurate models. Therefore, in order to generate a model for a specific motor it is necessary to implement an identification strategy.

In the case of linear systems and some nonlinear models the algorithms based on Least Square are the most popular methodologies to estimate or identify these parameters. Nonetheless, a necessary condition to apply this algorithm is that the model must be linear at its parameters, [1]. Another identification approach is the Strecj algorithm, successfully applied in [2] for the identification of the asynchronous machine, which is based on the transient response analysis to a step input of the process. In the case of linear first and second order systems this procedure renders good models by estimating its poles and steady state gain. In some cases, it is also possible to estimate the physical parameters of the process. Based on this strategy it is possible to estimate the parameters of the nonlinear model without magnetic saturation of Series DC motors.

Series DC motors, as well as series universal motors, are a kind electric motors with one voltage supply and the field winding connected in series with the rotor winding. This series connection results in a motor with very high starting torque. However, torque decreases as the speed builds up due to an increment of the back or counter electromotive force EMF. This is why series DC motors have poor speed regulation. That is, increasing the motors load tends to slow its speed which in turns reduces the back EMF and increases the torque to accommodate the load. A limitation of these motors is that the sense of rotation is fixed for most of their applications. In order to change the direction of torque and rotation, it is necessary to change the polarity of the current flow.

Despite the fact series DC motors generate high torques with very low current consumption and small physical dimensions they are commonly used open loop for short periods of time. This is mainly, as mention above, because they have poor speed regulation. Nonetheless, this kind of motors can be fully exploited if good closed loop controllers are designed. However, in general many control strategies to this kind of motors are based or depend on dynamic cancellations [3]-[7] requiring accurate models.

Therefore, the objective of this article is to prove that the Strecj method can be used to estimate or identify the parameters of the nonlinear model without saturation of Series DC motors. Because the Strecj method is based on the analysis of the step response, transient and steady state responses, of the process an advantage of this strategy over other approaches in the identification of Series DC Motors is that only the measurement of the current consumption and the speed of the motor's rotor are required. That is, it results in a simple an efficient strategy for the identification of Series DC motors.

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The paper is organized as follows: In Section II a description of the nonlinear model of Series DC motors without saturation is given. Section III, using the Strecj algorithm, presents the identification of a Series DC Motor together with a description of the experimental setup. In Section IV a linearization of the identified nonlinear model around an operating point is presented. In order to assess the estimated model comparisons between the time responses of the model and an actual motor are presented in Section V. Finally, conclusions are presented in Section VI.

II. Series DC Motor Model

Series DC motors similar to shunt wound DC motors or compound wound DC motors are self-excited DC motors. They get their name because the field winding is connected internally in series to the armature winding as shown in fig. 1. They are also considered self-excited motors because instead of two separate voltage sources -one for the armature and one for the field winding- they required only one voltage source.

The electric diagram of a series DC motor is shown in fig. 2. Based on the electric diagram of figure 2 the differential equations comprising the mechanical and electrical subsystems of the series DC motor are given by:

$$V(t) = R_a i_a(t) + R_f i_f(t) + L_a \frac{d}{dt} i_a(t) + L_f \frac{d}{dt} i_f(t) + E_a \quad (1)$$

$$T_e(t) = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \quad (2)$$

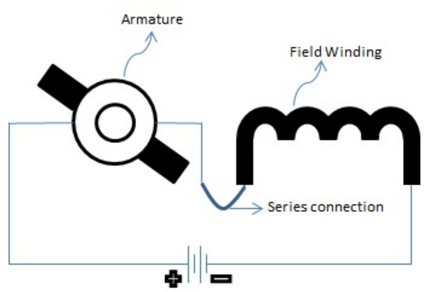

Fig. 1 Series connection of a DC motor

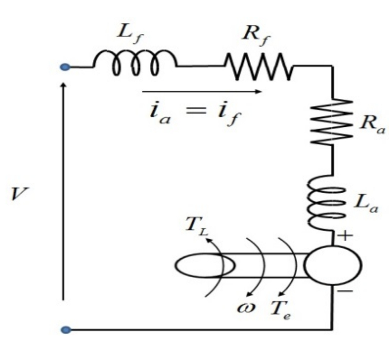

Fig. 2 Electric diagram of a series DC motor

As $i_a(t) = i_f(t)$, (1) and (2) reduce to:

$$V(t) = \left(R_a + R_f\right) i(t) + \left(L_a + L_f\right)\frac{d}{dt} i(t) + E_a \quad (3)$$

$$T_e(t) = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \quad (4)$$

Where, $\omega(t)$ is the rotors speed, E_a represents the counter electromotive force *EMF*, $T_L(t)$ is the load torque, $i(t) = i_a(t) = i_f(t)$ is the current, b is the friction coefficient, J is the rotor's inertia and $T_e(t)$ is the electromagnetic torque produced by the motor.

The *EMF* E_a and $T_e(t)$ depend both on the air-gap flux Φ, that is:

$$E_a(t) = \omega(t)\,\Phi(i) \quad (5)$$

$$T_e(t) = i(t)\,\Phi(i) \quad (6)$$

The flux $\Phi(i)$ is a function of the current $i(t)$ so (1)-(4) are non-linear. Also, it is common practice to approximate the flux $\Phi(i)$ by a linear relation when the magnetic saturation is neglected, that is:

$$\Phi(i) = k_0 i(t) \quad (7)$$

Where k_0 is the mutual inductance between the armature and field coils.

Finally, the nonlinear model of a series DC motor without saturation is given by:

$$V(t) = Ri(t) + L\frac{d}{dt} i(t) + \omega(t) i(t) k_0 \quad (8)$$

$$i^2(t) k_0 = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \quad (9)$$

Where $R = R_a + R_f$ and $L = L_a + L_f$

III. Setup Experiment and Model Identification

The identification of the monophasic universal-motor Koblenz model HC8825M110 with nominal maximum speed and power of 24000 RPM and 0.815HP is presented.

In particular, these machines must operate loaded in order to avoid damage. Hence, a steel disc load was added as shown in fig. 3. This extra load is considered as a part of the rotors inertia. From (8) and (9) it is clear that it is possible to estimate the motors parameters by analyzing the responses of the current and angular velocity of the rotor to step input voltages. This is performed using the set up experiment depicted in fig. 4.

The experimental setup consists of a voltage source with a maximum voltage of 50 volts and a maximum current of 3 Amp, the series DC motor, and a power driver with a USB communication. The power driver was designed considering the electric demand of the motor. Hence, the power driver is based on the MOSFET 12N65, which operates with a maximum current of 12Amp and a maximum input voltage of 650 Volts. An equally important element is the ACS711LC circuit used to monitor the power consumed by the motor.

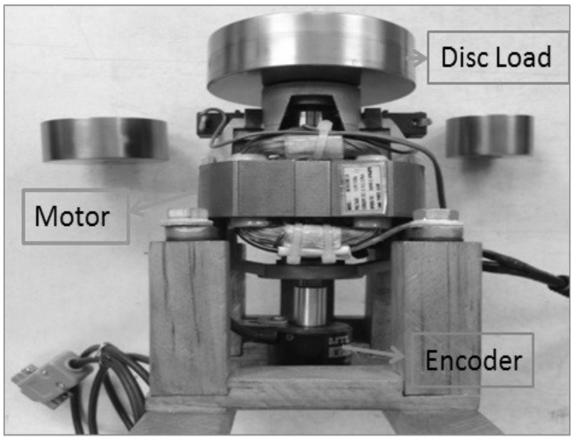

Fig. 3 Series DC motor

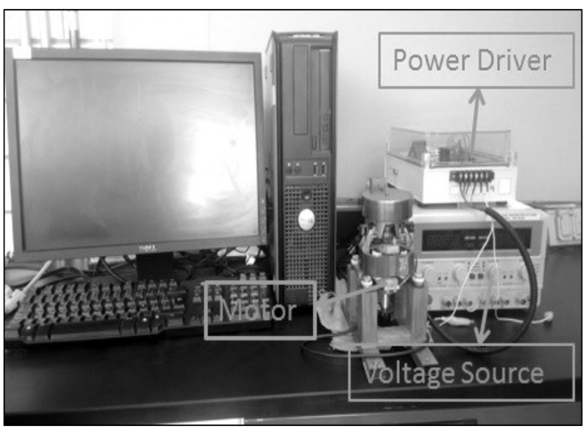

Fig. 4 Experimental Set Up

Inductive loads together with pulsed excitation signals generate reverse currents that may damage switching elements such as MOSFETs. Although the MOSFETs used in the implementation have an internal protection diode, two transistors FR307 of rapid recovery were added in order for additional protection, as shown in fig. 5.

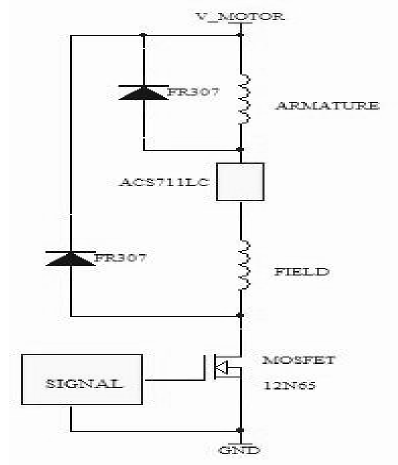

Fig. 5 Power driver

A. Strecj Method

As mention above the Strecj method is based on the analysis of the step response of the process, that is:

Let the first order transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b}{s+a} \tag{10}$$

With $a \in \mathbb{R}^+$ and $b \in \mathbb{R}$.

If input $U(s)$ is assumed $U(s) = \frac{A}{s}$; that is, a step signal with amplitude A. The response $Y(s)$ results, after the application of partial fraction expansion, in:

$$Y(s) = B\left(\frac{1}{s} - \frac{1}{s+a}\right) \tag{11}$$

With a time domain response:

$$y(t) = B\left(1 - e^{-at}\right) \tag{12}$$

From the classical control theory it is well known that the steady state time t_{ss} and the steady state gain K_G of $G(s)$ are given, respectively, by:

$$t_s = \frac{4}{a} \quad \text{and} \quad K_G = \frac{B}{A} = \frac{b}{a} \tag{13}$$

These relations are shown graphically in fig.6

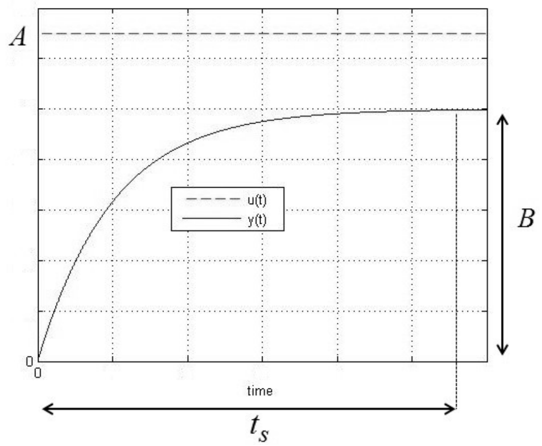

Fig. 6 Step response analysis for Strecj method

Therefore, by obtaining the relation of amplitudes of the steady state response and the input signal, and measuring the steady state time it is possible to identify the gain and pole of the first transfer function (10).

B. Electric Subsystem Identification

From (8), it is clear that if the rotor shaft is fully locked, then $\omega(t) = 0 \ \forall t$, reducing the electric subsystem to a simple RL circuit with a differential equation given by:

$$V(t) = Ri(t) + L\frac{d}{dt}i(t) \tag{14}$$

With a transfer function $G_E(s)$ given by:

$$G_E(s) = \frac{I(s)}{V(s)} = \frac{1}{Ls + R} \tag{15}$$

From (13) the steady state gain and the steady state time of $G_E(s)$ are $K_{G_E} = 1/R$ and, $t_{ss_E} = 4L/R$ respectively. This is in fact the Strecj method for the identification of first order systems without delay.

With the rotor shaft locked, the current response $i(t)$, obtained by feeding a step input voltage $V(t)$ from 0 to 25 volts is shown in fig. 7. The current variation is approximately 1.2 Amp so the steady state gain is $K_{G_E} = 0.048$, hence $R = 20.833\Omega$. Also, from Fig. 7, the steady state time of the current response is $t_{ss_E} = 0.03\,\text{sec}$. Thus, the inductance is $L = 156.24mH$.

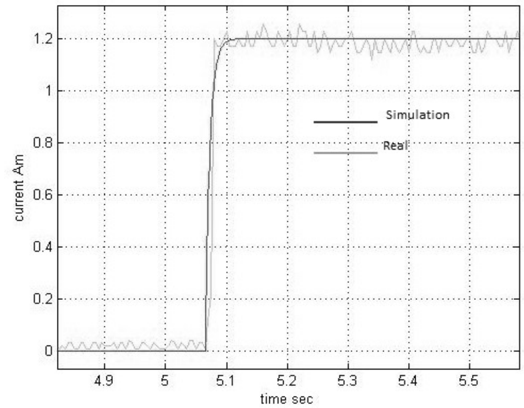

Fig. 7 Current step response (rotor shaft locked)

The mutual inductance k_0 can be estimated measuring the steady state responses of the current $i(t)$ and the speed of the rotor $\omega(t)$ to a step input voltage $V(t)$.

It should be noted that under this condition $\frac{d}{dt}i(t) \approx 0$.

Rearranging (8):

$$k_0 = \frac{V(t) - Ri(t)}{\omega(t)i(t)} \tag{16}$$

In fig. 8 and 9, the current $i(t)$ and rotor speed, $\omega(t)$, responses to a step input voltage from 0 to 25 volts are shown. From the steady state responses, the previously estimated resistance R, and (16), the mutual inductance value is: $k_0 = 0.17554$ N-m/Wb-A.

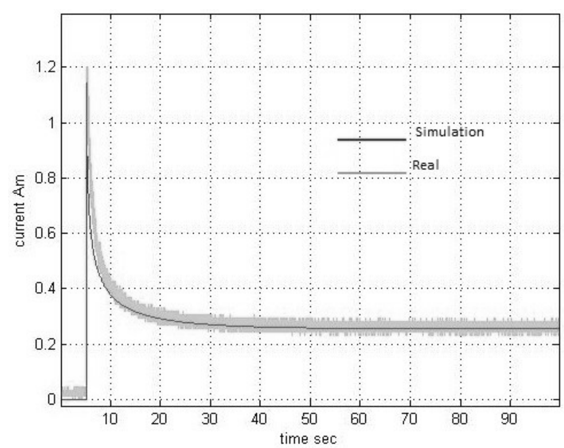

Fig. 8 Current step response

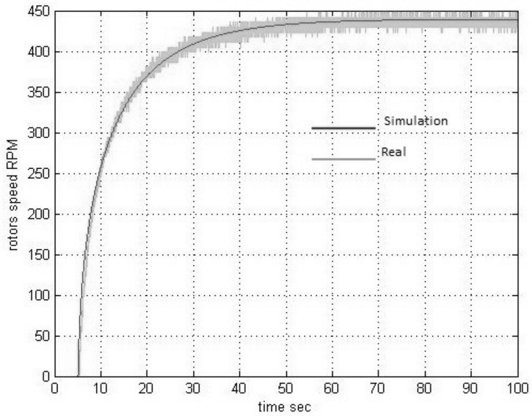

Fig. 9 Rotors speed step response

C. Mechanical Subsystem Identification

From (4), the mechanical subsystem without torque load, $T_L(t) = 0$, is described by:

$$T_e(t) = b\omega(t) + J\frac{d}{dt}\omega(t) \qquad (17)$$

With a transfer function

$$G_M(s) = \frac{\omega(s)}{T_e(s)} = \frac{1}{Js + b} \qquad (18)$$

Similar to the electrical subsystem identification, the steady state gain and the steady state time for the mechanical subsystem are $K_{G_M} = 1/b$ and $t_{ss_M} = 4J/b$, respectively. It should be noted that the electromagnetic input torque $T_e(t) = k_0 i^2(t)$ can be obtained by measuring the current $i(t)$ together with the estimated mutual inductance $k_0 = 0.17554$.

Fig. 8 shows that the steady state variation of the current $i(t)$ is 0.255. Thus, from (6) and (7) the variation of the electromagnetic torque $T_E(t)$ is 0.01141. On the other hand, from fig. 9, the steady state variation of the speed of the rotor is 439.82*RPM*. Hence, the gain of the mechanical subsystem is $K_{G_M} = 1/b = 38531.73$, resulting in a friction coefficient $b = 0.000026$ N-m/Wb-A .

Estimating the rotor inertia J requires measuring the steady state time t_{ss_M} of the velocity $\omega(t)$ to a step input. However, fig. 8 clearly indicates that the magnetic torque input cannot be assumed as a step input. Nevertheless, the inertia J was initially estimated using the relation $J = t_{ss_M} b/4$. Further adjustments via trial and error were carried out finding that $J = 0.0006206$ Kg-m^2 was closer to the actual response of the system.

The résumé of the estimated parameters of the series DC motors is shown in Table I.

TABLE I. ESTIMATED PARAMETERS

$k_0 = 0.17554$ N-m/Wb-A
$R = 20.833\Omega$
$L = 156.24mH$
$b = 0.000026$ N-m/Wb-A
$J = 0.0006206$ Kg-m^2

The Simulink program of the Series DC motor nonlinear model without magnetic saturation is shown in fig. 10.

IV. Model Linearization

A linear approximation of (8) and (9) around any equilibrium point may represent a better model for certain control objectives. In this sense, (8) and (9) are rearranged as follows:

$$\frac{d}{dt}i(t) = -\frac{R}{L}i(t) - \frac{k_0}{L}\omega(t)i(t) + \frac{1}{L}V(t) \qquad (19)$$

$$\frac{d}{dt}\omega(t) = -\frac{b}{J}\omega(t) - \frac{1}{J}T_L(t) + \frac{k_0}{J}i^2(t) \qquad (20)$$

Defining

$$\begin{array}{llll} a_1 =: \frac{k_0}{J}; & b_1 =: \frac{R}{L} & & \\ a_2 =: \frac{b}{J}; & b_2 =: \frac{k_0}{L} & \text{and} & \begin{array}{l} x_1 =: \omega \\ x_2 =: i \end{array} \\ a_3 =: \frac{1}{J}; & b_3 =: \frac{1}{L} & & \end{array} \qquad (21)$$

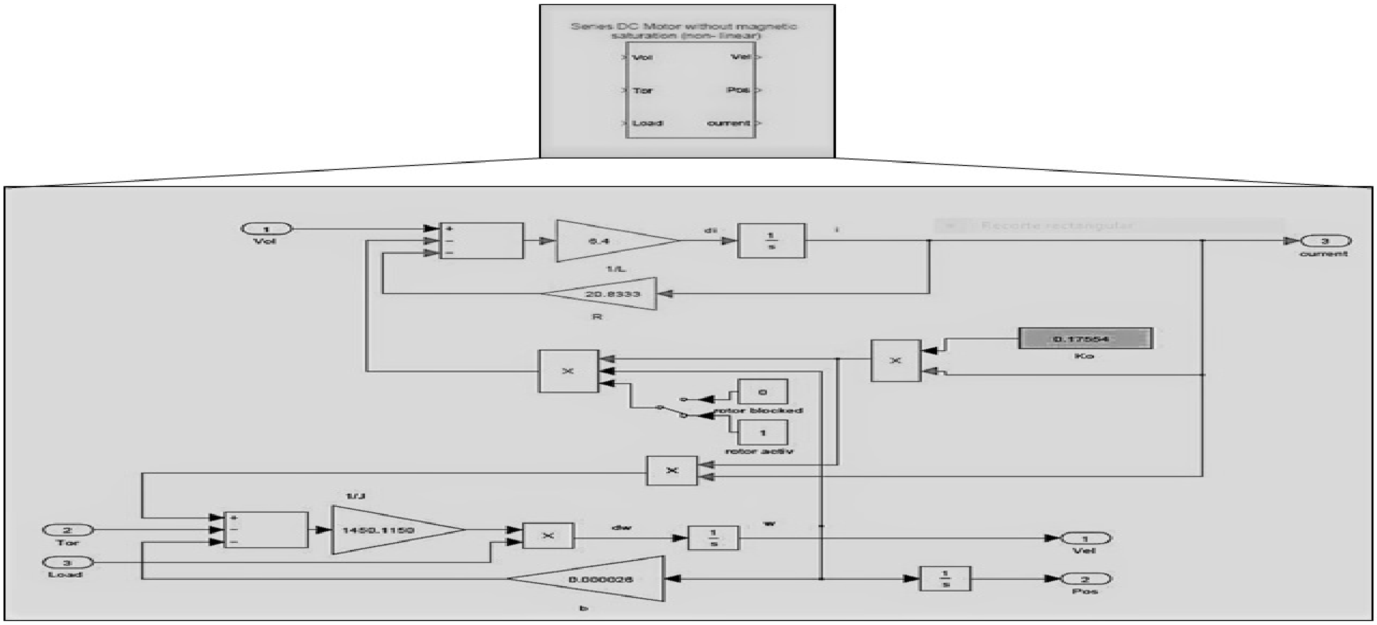

Fig. 10 Simulink program of a Series DC motor without magnetic saturation

The nonlinear state space representation of the series DC motor is given by:

$$\begin{aligned}\dot{x}_1 &= a_1 x_2^2 - a_2 x_1 - a_3 T_L \\ \dot{x}_2 &= -b_1 x_2 - b_2 x_1 x_2 + b_3 V\end{aligned} \tag{22}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_1 x_2^2 - a_2 x_1 - a_3 T_L \\ -b_1 x_2 - b_2 x_1 x_2 + b_3 V \end{bmatrix} = f(x,u) \tag{23}$$

The equilibrium point $\left(x_1^0, x_2^0\right)$ of (23) is given by:

$$x_2^0 = \sqrt{\frac{a_2 x_1^0 + a_3 T_L}{a_1}}; \qquad V = \frac{x_2^0\left(b_1 + b_2 x_1^0\right)}{b_3} \tag{24}$$

The linear approximation of (23) around the equilibrium point (24) is given by:

$$\dot{x} = Ax + Bu;\ y = Cx \tag{25}$$

Where

$$\begin{aligned} A &= \left.\frac{\delta f(x,u)}{\delta x}\right|_{x_1^0, x_2^0} = \begin{bmatrix} -a_2 & 2a_1 x_2^0 \\ -b_2 x_2^0 & -\left(b_1 + b_2 x_1^0\right) \end{bmatrix}; \\ B &= \left.\frac{\delta f(x,u)}{\delta u}\right|_{x_1^0, x_2^0} = \begin{bmatrix} -a_3 & 0 \\ 0 & b_3 \end{bmatrix}; \\ C &= [1, 0] \end{aligned} \tag{26}$$

With

$$u = \begin{bmatrix} T_L & V \end{bmatrix}^T,\quad y = \omega(t) \tag{27}$$

If the load torque is assumed zero $T_L = 0$

$$x_2^0 = \sqrt{\frac{a_2 x_1^0}{a_1}}; \qquad V = \frac{x_2^0\left(b_1 + b_2 x_1^0\right)}{b_3}; \quad B = \begin{bmatrix} 0 \\ b_3 \end{bmatrix} \tag{28}$$

These calculations lead to a model, which can be considered as a system with one input, voltage $V(t)$, subjected to a torque perturbation.

The transfer function $G(s)$ associated to the state space representation (25) around the equilibrium point (28) with $x_1^0 = 439.82\ RPM$ is:

$$G(s) = \frac{\omega(s)}{V(s)} = C\left(sI - A\right)^{-1} B = \frac{924.1}{s^2 + 627.5s + 67.69} \tag{29}$$

The poles of the transfer function (29) are $\{-627.4257, -0.1079\}$ so the series DC motor is stable and over damped. It is also possible to distinguish the two typical modes of a DC motor from the poles: $\left(s + 0.1079\right)$ representing the slow dynamic of the mechanical subsystem and $(s+627.4257)$ the fast dynamic of the electrical subsystem.

In [8] was shown that linear models may be suitable for regulation and tracking control objectives.

V. Real Time Model Validation

To validate the nonlinear model the responses of the actual series DC motor and the identified nonlinear model to different input voltages were compared.

In fig. 8 and 9 the step responses of the current and rotors speed to a step input voltage from 0 to 25 volts are shown. From these figures it is clear that the model matched the actual motors response.

Also, to verify the validity of the model in a range of input voltage variation and frequency a sinusoidal input voltage is applied.

The input voltage is given by:

$$V(t) = 25 + 5\sin(0.05t) \tag{30}$$

The frequency of the input signal $V(t)$ takes into account the slow mode of the mechanical subsystem. The responses of the current and rotors speed to the sinusoidal input voltage of equation (30) are shown in fig. 11 and 12, respectively. From these plots it is clear that the estimated model responses are comparable to the actual motor responses.

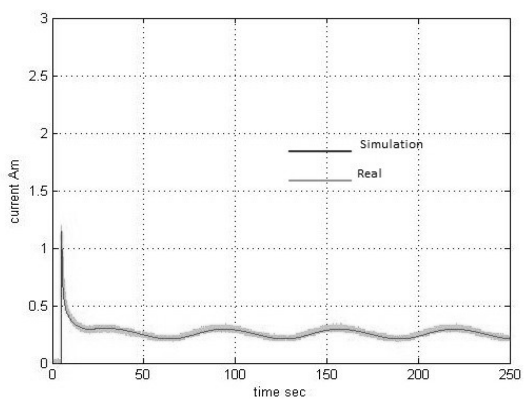

Fig. 11 Rotors current response to a sinusoidal input (actual motor and nonlinear simulation)

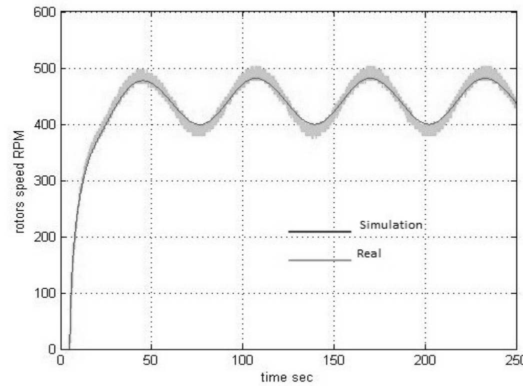

Fig. 12 Rotors speed response to a sinusoidal input (actual motor and nonlinear simulation)

In order to validate the accuracy of the transfer function (29) it is necessary to compare its response to those of the nonlinear model and the actual motor. However, the response of the actual motor to a step input voltage from 0 to 25 volts in fig. 9 shows that the motor has a steady state gain of 17.593. Meanwhile, the linear model of (30) has a steady state gain of 13.652, meaning that the linear model has a loss of gain.

To compensate the loss of gain of (30), it is adjusted resulting in:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{1190.865}{s^2+627.5s+67.69} \tag{31}$$

In fig. 13 the responses of the actual motor, nonlinear model and the linear approximation (31) to a step input voltage from 0 to 26 volts are shown.

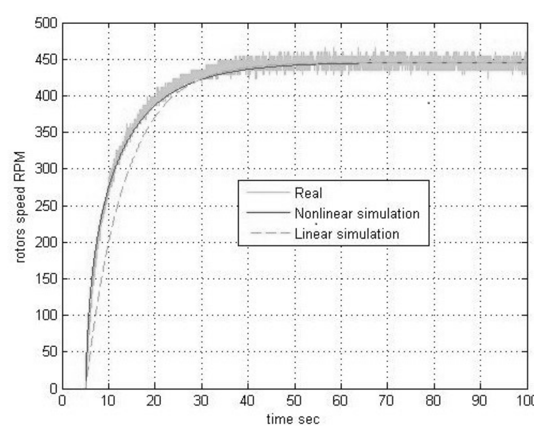

Fig. 13 Rotors speed step response (actual motor, nonlinear and linear simulations)

Fig. 13 shows small differences between the actual motor and the linear approximation (31) during the transient response, Finally, in fig. 14 the responses of the actual Series DC motors, the nonlinear model (7)-(8) and the linear model (31) to a sinusoidal input voltage $V(t) = 25 + 2.5\sin(0.05t)$ shows that a Series DC motor can be accurately approximated around an equilibrium point using linear approximations in the form of transfer functions.

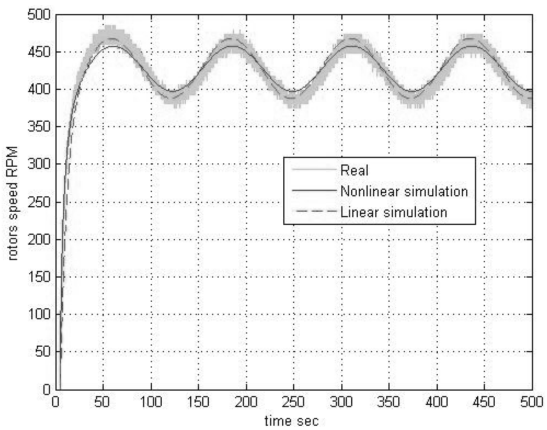

Fig. 14 Rotors speed response to a sinusoidal input (actual motor, nonlinear and linear simulations)

VI. Conclusions

The modeling, without magnetic saturation, and estimation of a series DC motor was obtained. The estimation was performed using the Strecj method which is based on the analysis of transient responses to step inputs of the process. Therefore, in the case of Series DC motors only the transient responses of the current and rotor speed are required resulting in a simple and easy procedure to implement. The resulting nonlinear model matched the actual response of a real Series DC motor. Also, a linearization of the estimated nonlinear model in an equilibrium point is presented. This linear model, which clearly identifies the slow mode of the mechanical subsystem and the fast mode of the electrical subsystem, presents a loss in the steady state gain due to the linearization process itself. This loss of gain was easily compensated resulting in a linear model from which its step responses matched, with small differences during the transient response, the actual response of the motor. Therefore, it can be also conclude that linear models in the form of transfer functions can be used in the analysis and design of control systems for Series DC motors.

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