Design of an Observer PI for an Asynchronous Machine without Mechanical Sensor

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***Abstract*—This document approaches the observation of the asynchronous machines state without mechanical sensor. The approach suggested rests on the nonlinear transformation of the machine and the estimate of state is carried out using an observer of the Proportionnel-Integral (PI) known for its properties of robustness. Conditions sufficient to guarantee the convergence of the error in estimation are given in a form equation candidate of LYAPUNOV.**

I. INTRODUCTION

Nowadays the engineering largely call upon the estimate of the state of the asynchronous machines without senseless speed. Indeed, the complete knowledge of the state of an asynchronous machine is often necessary for the development of a law of control or to the installation of a strategy of monitoring or diagnosis. However the state of this system is in general only partially available and the input signals and of out-ut are in practice the only sizes accessible by measurement. The most widespread solution to mitigate this problem consists in coupling the machine an auxiliary system, called ***state-observer*** or ***state-reconstructor***.

The observer provides an estimate of the state of the system starting from its model and measurements of its inputs and out-put. The observer classically used, within the framework of linear systems, is known as with *gain* ***P****roportionnel* **(P)** or of [2]. It is however well-known that the estimate of state provided by this type of observer is degraded considerably if the model of the system in question is known little about or so of the unknown disturbances act on the output or the state of the system.

In order to improve the estimate of state with respect to parametric uncertainties and/or the disturbances, an observer with gain **P**roportionnel-**I**ntegral **(PI)** can be used. Indeed, the observer **PI**, proposed in [3] and since largely studied in the literature, makes it possible to integrate a certain degree of robustness in the estimate of state thanks to the integral action in [4].

The question of the synthesis of the observers **P** or **PI**, of a full or reduced nature for the linear systems, is tackled in [4], [5]. On the other hand, the estimate of state for the nonlinear systems, although having been the subject of many research tasks, remains a subject of topicality.

The solution suggested in this document to estimate the state of a nonlinear system rests on the approach of *nonlinear transformations*.

The plan of the article is the following. Initially we present the model of state of the asynchronous machine without mechanical sensor where we make a non-linear transformation to obtain the pyramidal form; in continuation the study of observation of the states of the asynchronous machine with the speed measured and without sensor speed, after we let us give the design of observer **PI**. To the end, one gives the results of simulations with the conclusion.

II. NONLINEAR MODEL OF THE ASYNCHRONOUS MACHINE

The model of the machine that we adopt rests on the following assumptions [[6], [7]]:

- The perfect symmetry of the machine.
- The absence of saturation and losses in the magnetic circuit.
- The effect of skin negligible.
- The machine is supplied by a sinusoidal system and three-phase tensions.
- The thickness of the air-gap is uniform and the effect of notch is negligible.
- Induction in the air-gap is with sinusoidal distribution.

Its vector of state is composed of the components of the stator currents and rotor flux. This representation is very much used. It is made up of an electromagnetic part and another mechanics, as follows:

$$\left\{\begin{array}{lcl} \dot{i} & = & -\gamma i + KF(\Omega)\psi + \frac{1}{\sigma L_s}v_s \\ \dot{\psi} & = & \frac{M}{T_r}i - F(\Omega)\psi \\ \dot{\Omega} & = & \frac{pM}{JL_r}i^T\mathcal{J}\psi - \frac{f_v}{J}\Omega - \frac{T_l}{J} \\ \dot{T_l} & = & 0 \end{array}\right. \tag{1}$$

where $i = [i_{s\alpha} \;\; i_{s\beta}]^T$, $\psi = [\psi_{r\alpha} \;\; \psi_{r\beta}]^T$, $v_s = [v_{s\alpha} \;\; v_{s\beta}]^T$, are respectively the currents stator, rotor fluxes and the voltages; Ω and T_l respectively denote the rotating speed and the load torque; $F(\Omega) = \frac{1}{T_r}\mathcal{I} - p\Omega\mathcal{J}$, $\mathcal{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the matrix of identity and $\mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$; J is moment of inertia of the rotor; p is the pair number of the pole.

Parameters T_r, σ, K and γ are defined as follows:

$$T_r \triangleq \frac{L_r}{R_r} \quad ; \quad \sigma \triangleq 1 - \frac{M^2}{L_s L_r} \quad ;$$
$$K \triangleq \frac{M}{\sigma L_s L_r} \quad ; \quad \gamma \triangleq \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}$$

Only the stator currents and stator tensions are measurable. For lighting more, one to introduce the following notations:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ avec } x_1 = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}, x_2 = \begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix},$$
$$x_3 = \begin{bmatrix} \Omega \\ T_l \end{bmatrix}$$

The system (1) can be written under the following condensed form:

$$\begin{cases} \dot{x} & = & f(x, v_s) \\ y & = & x_1 \end{cases} \tag{2}$$

where

$$f(x, v_s) = \begin{bmatrix} -\gamma x_1 + KF(\Omega)x_2 + \frac{1}{\sigma L_s} v_s \\ \frac{M}{T_r} x_1 - F(\Omega)x_2 \\ \frac{pM}{JL_r} x_1^T \mathcal{J} x_2 - \frac{f_v}{J}\Omega - \frac{T_l}{J} \\ 0 \end{bmatrix}$$

with

$$F(\Omega) = \frac{1}{T_r}\mathcal{I} - p\Omega\mathcal{J} = \begin{bmatrix} \frac{1}{T_r} & p\Omega \\ -p\Omega & \frac{1}{T_r} \end{bmatrix} \tag{3}$$

Now, will concede the change of the variables according to:

$$\begin{array}{rcl} \Phi : \mathbb{R}^6 & \to & \mathbb{R}^6 \\ x & \mapsto & z = \Phi(x) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \Phi_1(x) \\ \Phi_2(x) \\ \Phi_3(x) \end{bmatrix} \end{array}$$

Where z_1, z_2 et z_3 are defined as follows:

$$\begin{cases} z_1 & = & x_1 \\ z_2 & = & KF(\Omega)x_2 = K\left(\frac{1}{T_r}\mathcal{I} - p\Omega\mathcal{J}\right)x_2 \\ z_3 & = & -pK\mathcal{J}\dot{\Omega}x_2 = -pK\mathcal{J}\left(\frac{pM}{JL_r}x_1^T\mathcal{J}x_2 \right. \\ & & \left. -\frac{f_v}{J}\Omega - \frac{T_l}{J}\right)x_2 \end{cases} \tag{4}$$

One can prove that the above state transformation puts the system (1) under the following form:

$$\begin{cases} \dot{z}_1 & = & z_2 + \varphi_1(v_s, z_1) \\ \dot{z}_2 & = & z_3 + \varphi_2(z_1, z_2) \\ \dot{z}_3 & = & \varphi_3(z) \\ y & = & Cz = z_1 \end{cases} \tag{5}$$

$\varphi_k \in \mathbb{R}^2$, $k = 1, 2, 3$ are :

$$\begin{cases} \varphi_1(v_s, z_1) & = & -\gamma z_1 + \frac{1}{\sigma L_s} v_s \\ \varphi_2(z_1, z_2) & = & F(\Omega)\left(-z_2 + \frac{KM}{T_r} z_1\right) \\ \varphi_3(z) & \triangleq & \frac{\partial \Phi_3(x)}{\partial x_1}\dot{x}_1 + \frac{\partial \Phi_3(x)}{\partial x_2}\dot{x}_2 + \frac{\partial \Phi_3(x)}{\partial x_3}\dot{x}_3 \\ & \triangleq & -pK\mathcal{J}\left(\frac{pM}{JL_r}\frac{\partial h}{\partial i}\psi\right)\dot{i} \\ & & -pK\mathcal{J}\left(\frac{pM}{JL_r}\frac{\partial h}{\partial \psi}\psi + \frac{pM}{JL_r}h - \frac{f_v}{J}\Omega \right. \\ & & \left. -\frac{T_l}{J}\right)\dot{\psi} \\ & & + \begin{bmatrix} pKf_v\mathcal{J}\psi & \frac{pK}{J}\mathcal{J}\psi \end{bmatrix}\dot{x}_3 \end{cases}$$

with $h = i^T \mathcal{J} \psi$

who will transform the non-linear system (2) into a local system of *pyramidal* coordinates:

$$\begin{cases} \dot{z} & = & \mathcal{A}z + \varphi(v_s, z) \\ y & = & Cz = z_1 \end{cases} \tag{6}$$

The matrices $\mathcal{A}$ and C are given by:

$$\mathcal{A} = \begin{bmatrix} 0 & \mathcal{I} & 0 \\ 0 & 0 & \mathcal{I} \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} \mathcal{I} & 0 & 0 \end{bmatrix}$$

III. Study of Observability

The system (5) is observable for any input v_s. As a result, the system (1) will be observable in the rank sense on $\mathbb{R}^6$ as soon as the transformation Φ exists and is regular for all x [8]. Indeed, one will show a condition sufficient under which the JACOBIAN of this transformation is of full rank.

That is to say J_Φ the JACOBIAN de Φ. Correspondent with (4), one has

$$J_\Phi = \begin{bmatrix} \mathcal{I} & 0 & 0 \\ 0 & \frac{\partial \Phi_2(x)}{\partial x_2} & \frac{\partial \Phi_2(x)}{\partial x_3} \\ \frac{\partial \Phi_3(x)}{\partial x_1} & \frac{\partial \Phi_3(x)}{\partial x_2} & \frac{\partial \Phi_3(x)}{\partial x_3} \end{bmatrix} \tag{7}$$

It is clear that the matrix $J_\phi(x)$ is of full rank if and only if the following square matrix is also of full rank:

$$G_\Phi(x) = \begin{bmatrix} \frac{\partial \Phi_2(x)}{\partial x_2} & \frac{\partial \Phi_2(x)}{\partial x_3} \\ \frac{\partial \Phi_3(x)}{\partial x_2} & \frac{\partial \Phi_3(x)}{\partial x_3} \end{bmatrix} \triangleq \begin{bmatrix} G_1(x) & G_2(x) \\ G_3(x) & G_4(x) \end{bmatrix} \tag{8}$$

In the following, one will concentrate on the matrix $G_\Phi(x)$ in order to exhibit a sufficient condition under which this matrix, or in an equivalent way J_Φ, is of full rank. For this purpose, one will calculate each input of $G_\Phi(x)$. Still, according to (4), one has:

$$\begin{cases} G_1(x) & = & KF(\Omega) \\ G_2(x) & = & \begin{bmatrix} -Kp\mathcal{J}x_2 & 0 \end{bmatrix} \\ G_3(x) & = & -pK\mathcal{J}\left(\dot{\Omega}\mathcal{I} + \frac{p}{J}\frac{M}{L_r}x_1^T\mathcal{J}x_2\right) \\ G_4(x) & = & pK\mathcal{J}\begin{bmatrix} f_v x_2 & \frac{x_2}{J} \end{bmatrix} \end{cases} \tag{9}$$

It is clear of (3) that $G_1(x)$ is a square invertible matrix for all $x \in \mathbb{R}^6$. By consequence, the matrix $G_\Phi(x)$ can be factorized as follows:

$$\begin{aligned} G_\Phi(x) &= L(x)U(x) \\ L(x) &= \begin{bmatrix} \mathcal{I} & 0 \\ G_3(x)G_1^{-1}(x) & \begin{pmatrix} G_4(x) - G_3(x) \\ G_1^{-1}(x)G_2(x) \end{pmatrix} \end{bmatrix} \\ U(x) &= \begin{bmatrix} G_1(x) & G_2(x) \\ 0 & \mathcal{I} \end{bmatrix} \end{aligned}$$

According the structures of $L(x)$ and $U(x)$, one can deduce that the matrix $G_\Phi(x)$ is of full rank if and only if the triangular lower matrix $L(x)$ is also of full rank. According to its triangular structure, the full rank condition of $L(x)$ is obtained as soon as the matrix:

$$L_2(x) \stackrel{\triangle}{=} G_4(x) - G_3(x)G_1^{-1}(x)G_2(x) \tag{10}$$

is of full rank. In order to check the full rank condition with the matrix $L_2(x)$, we will check the linear dependence of its two columns. Indeed, set $L_2(x) = [\ L_{21}(x) \quad L_{22}(x)\ ]$ and we derive firstly the expression of $L_{21}(x)$ and $L_{22}(x)$. Using (9),one gets:

$$\begin{aligned} L_{21}(x) &= -pK\mathcal{J}\left(\dot{\Omega}F^{-1}(\Omega) + \frac{p}{J}\frac{M}{L_r}x_1^T\mathcal{J}x_2F^{-1}(\Omega)\right) \\ &\quad p\mathcal{J}x_2 \\ &= -pK\mathcal{J}M(x) \\ L_{22}(x) &= Kp\mathcal{J}\frac{x_2}{J} \end{aligned}$$

Now, columns $L_{21}(x)$ and $L_{22}(x)$ are linearly dependant if and only if $M(x)$ and x_2 are so. It is also equivalent to $x^{2T}\mathcal{J}M(x) = 0$. This condition can be expressed under the following more explicit form:

$$x^{2T}\mathcal{J}M(x) = x_2^T\mathcal{J}\dot{x}_2 - \frac{p}{T_r}\frac{\dot{\Omega}}{\left(\frac{1}{T_r}\right)^2 + (p\Omega)^2}x_2^T\mathcal{J}x_2 \tag{11}$$

To recapitulate, the transformation considered will have a full rank (i.e. the system (1) is observable) for all x

$$x_2^T\mathcal{J}\dot{x}_2 - \frac{p}{T_r}\frac{\dot{\Omega}}{\left(\frac{1}{T_r}\right)^2 + (p\Omega)^2}x_2^T\mathcal{J}x_2 \neq 0 \tag{12}$$

The condition (12) can be explained by employing the original variables of motor as follows:

$$\psi^T\mathcal{J}\dot{\psi} - \frac{p}{T_r}\frac{\dot{\Omega}}{\left(\frac{1}{T_r}\right)^2 + (p\Omega)^2}\psi^T\psi \neq 0 \tag{13}$$

The condition (13) can be expressed equivalent forms. Indeed, such equivalent forms will be shown and will comment on them. Let us define the angle of rotor flux as follows: $\zeta = \arctan\frac{\psi_{r\beta}}{\psi_{r\alpha}}$. According to the model (1) of the machine, one has $\dot{\zeta} = -\frac{\psi^T\mathcal{J}\dot{\psi}}{\psi^T\psi}$ and by excluding the uninteresting case from $\psi = 0$, the condition (13) can be explained as follows:

$$\dot{\zeta} \neq \frac{p}{T_r}\frac{\dot{\Omega}}{\left(\frac{1}{T_r}\right)^2 + (p\Omega)^2}\psi^T\psi \tag{14}$$

or in an equivalent way by taking the integral on the two sides:

$$\left(\arctan\frac{\psi_{r\beta}}{\psi_{r\alpha}} - \arctan\left(pT_r\Omega\right)\right) \quad \text{is not constant.} \tag{15}$$

To note that conditions (13) and (14) similar to those are given to [9].

IV. Structure of Observer PI

One wishes to estimate the state of a nonlinear system described by (6). The rebuilding of the state is carried out using an observer of the type **P**roportionnel **I**ntegral **(PI)**. An integral term ξ, resulting from the integration of the out put, is introduced in order to add thereafter the integral action in the equation of the observer. then:

$$\begin{cases} \dot{z} &= \mathcal{A}z + \varphi\left(v_s, z\right) \\ \dot{\xi} &= Cz = z_1 \\ y &= Cz = z_1 \end{cases} \tag{16}$$

where $\xi = \int_0^t y(\tau)d\tau$. The preceding equations can be put in the compact form:

$$\begin{cases} \dot{Z} &= \tilde{\mathcal{A}}Z + C_1\varphi\left(v_s, Z\right) \\ Y_1 &= CC_1^TZ \\ Y_2 &= C_2^TZ \end{cases} \tag{17}$$

with $Z = \begin{bmatrix} z \\ \xi \end{bmatrix}$; $\tilde{\mathcal{A}} = \begin{bmatrix} \mathcal{A} & 0 \\ C & 0 \end{bmatrix}$; $C_1 = \begin{bmatrix} \mathcal{I} \\ 0 \end{bmatrix}$; $C_2 = \begin{bmatrix} 0 \\ \mathcal{I} \end{bmatrix}$.

The rebuilding of the state (17) is carried out using an observer **PI** of the following form:

$$\begin{cases} \dot{\hat{Z}} &= \tilde{\mathcal{A}}\hat{Z} + C_1\varphi\left(v_s, \hat{Z}\right) + K_P\left(Y_1 - \hat{Y}_1\right) \\ &\quad +K_I\left(Y_2 - \hat{Y}_2\right) \\ \hat{Y}_1 &= CC_1^T\hat{Z} \\ \hat{Y}_2 &= C_2^T\hat{Z} \end{cases} \tag{18}$$

Let us notice that in the equation (18) of the observer **PI** appear only the variables accessible by measurement.

V. Synthesis of the Observer PI

The synthesis of the observer consists in seeking matrices K_P and K_I such as the error in estimation defined by:

$$e = Z - \hat{Z} \tag{19}$$

tends towards zero whatever the initial conditions.

By using the expressions of Z and $\hat{Z}$ data respectively by the equations (17) and (18), the dynamics of the error in estimation is then given by:

$$\begin{aligned} \dot{e} &= \tilde{A}e + C_1\left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) - K_pCC_1^Te \\ &\quad -K_IC_2^Te \end{aligned} \tag{20}$$

Hypothesis*: $\varphi(v_s, Z) - \varphi(v_s, \hat{Z})$ is global* LIPSCHITZ *compared to Z i.e. :*

$$\left\|\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right\| \leq k \|e\|$$

$k > 0$: *Constant of* LIPSCHITZ.

Then

$$\begin{aligned} \dot{e} &= \left(\tilde{A} - K_P C C_1^T - K_I C_2^T\right) e \\ &\quad + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \\ &= A_{obs} e + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \end{aligned} \tag{21}$$

with

$$A_{obs} = \tilde{\mathcal{A}} - K_P C C_1^T - K_I C_2^T \tag{22}$$

A condition to guarantee the convergence of the error in estimation (28) is stated in the following lemma :

lemma *: If the hypothesis 1 east checks then the observer (18) is asymptotically convergent. The gains of the observer are given by K_P and K_I.*

Proof: The demonstration of this theorem is carried out by considering a function candidate of LYAPUNOV of the form:

$$V = e^T P e, \quad P > 0 \quad P = P^T \tag{23}$$

The asymptotic convergence of the error of observation is guaranteed if:

$$\exists P = P^T > 0 \quad \Rightarrow \quad V > 0 \text{ et } \dot{V} < 0 \tag{24}$$

The first constraint is satisfied by the choice with the function candidate of LYAPUNOV (32). Conditions ensuring the second constraint must then be established.

The derivative compared to the time of the function (32) is given by:

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} \tag{25}$$

that is to say still by introducing the dynamic equation of the error given by (30) :

$$\begin{aligned} \dot{V} &= \left(A_{obs} e + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right)\right)^T P e \\ &\quad + e^T P \left(A_{obs} e + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right)\right) \\ &= e^T A_{obs}^T P e + \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right)^T C_1 P e \\ &\quad + e^T P A_{obs} e + e^T P C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \\ &= e^T \left(A_{obs}^T P + P A_{obs}\right) e \\ &\quad + 2 e^T P C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \\ &= -2 e^T Q e + 2 e^T P C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \end{aligned} \tag{26}$$

One raises $\dot{V}$ as follows :

$$\begin{aligned} \dot{V} &= -2 \|e\|^2 \lambda_{\min}(Q) + 2k \|e\|^2 \|C_1\| \lambda_{\max}(P) \\ &= -2 \|e\|^2 \lambda_{\min}(Q) + 2k \|e\|^2 \lambda_{\max}(P) \end{aligned} \tag{27}$$

By using the property :

$$\lambda_{\min}(P) \|e\|^2 \leq V \leq \lambda_{\max}(P) \|e\|^2 \Rightarrow \|e\|^2 \geq \frac{V}{\lambda_{\max}(P)}$$

What gives us: $\dot{V} \leq -2 \left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - k\right) V \Rightarrow \dot{V} \leq -\tau V$

The stability exponential of the observer is to guarantee if : $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} > k$

The synthesis of the observer consists in seeking matrices K_P and K_I such as the error in estimation defined by:

$$e = Z - \hat{Z} \tag{28}$$

tends towards zero whatever the initial conditions.

By using the expressions of Z and $\hat{Z}$ data respectively by the equations (17) and (18), the dynamics of the error in estimation is then given by:

$$\begin{aligned} \dot{e} &= \tilde{A} e + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) - K_p C C_1^T e \\ &\quad - K_I C_2^T e \end{aligned} \tag{29}$$

Hypothesis*: $\varphi(v_s, Z) - \varphi(v_s, \hat{Z})$ is global* LIPSCHITZ *compared to Z i.e. :*

$$\left\|\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right\| \leq k \|e\|$$

$k > 0$: *Constant of* LIPSCHITZ.

Then

$$\begin{aligned} \dot{e} &= \left(\tilde{A} - K_P C C_1^T - K_I C_2^T\right) e \\ &\quad + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \\ &= A_{obs} e + C_1 \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \end{aligned} \tag{30}$$

with

$$A_{obs} = \tilde{\mathcal{A}} - K_P C C_1^T - K_I C_2^T \tag{31}$$

A condition to guarantee the convergence of the error in estimation (28) is stated in the following lemma :

lemma *: If the hypothesis 1 east checks then the observer (18) is asymptotically convergent. The gains of the observer are given by K_P and K_I.*

Proof: The demonstration of this theorem is carried out by considering a function candidate of LYAPUNOV of the form:

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The derivative compared to the time of the function (32) is given by:

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} \tag{34}$$

that is to say still by introducing the dynamic equation of the error given by (30) :

$$\begin{aligned}\dot{V} &= \left(A_{obs}e + C_1\left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right)\right)^T Pe \\ &\quad + e^T P\left(A_{obs}e + C_1\left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right)\right) \\ &= e^T A_{obs}^T Pe + \left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right)^T C_1 Pe \\ &\quad + e^T P A_{obs} e + e^T P C_1\left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \\ &= e^T\left(A_{obs}^T P + P A_{obs}\right)e \\ &\quad + 2e^T P C_1\left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \\ &= -2e^T Qe + 2e^T PC_1\left(\varphi(v_s, Z) - \varphi(v_s, \hat{Z})\right) \quad (35)\end{aligned}$$

One raises $\dot{V}$ as follows :

$$\begin{aligned}\dot{V} &= -2\left\|e\right\|^2 \lambda_{\min}(Q) + 2k\left\|e\right\|^2\left\|C_1\right\|\lambda_{\max}(P) \\ &= -2\left\|e\right\|^2 \lambda_{\min}(Q) + 2k\left\|e\right\|^2 \lambda_{\max}(P) \quad (36)\end{aligned}$$

By using the property :

$$\lambda_{\min}(P)\left\|e\right\|^2 \leq V \leq \lambda_{\max}(P)\left\|e\right\|^2 \Rightarrow \left\|e\right\|^2 \geq \frac{V}{\lambda_{\max}(P)}$$

What gives us: $\dot{V} \leq -2\left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - k\right)V \Rightarrow \dot{V} \leq -\tau V$

The stability exponential of the observer is to guarantee if : $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} > k$

VI. Results of Simulation

In order to check the effectiveness of the observer proposed, a simulation was carried out under MATLAB/SIMULINK. The results of simulation are divided into two parts: on the one hand, the observer is tested under nominal conditions, and on the other hand with variations parameters (tests of robustness).

The state vector of the observer is initialized in a functional state $\begin{bmatrix} \hat{i}_{s\alpha} & \hat{i}_{s\beta} & \hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta} & \hat{\Omega} & \hat{T}_l \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 0.2 & 0.2 & 10 & 0.05 \end{bmatrix}^T$; whereas that of the motor to the stopped state $\begin{bmatrix} i_{s\alpha} & i_{s\beta} & \psi_{r\alpha} & \psi_{r\beta} & \Omega & T_l \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, and the results are given for the engine of which a direct starting. These characteristics and these parameters are given in the tables (Tab.I) and (Tab.II).

Simulations of the flux, the speed and torque load in open loop are given by the figures (Fig.1, Fig.2 and Fig.3)in respectively; where their errors are presented by traced (Fig.4 and Fig.5).

The second simulation for the observer in the action **P** watch which the flux error cancels at $0.5s$ (Fig.9), rotate speed at $0.8s$ (Fig.10) and torque load at $1s$ (Fig.11).

Following simulations for tested the observer **PI** with its robustness where $R_s = +20\%$ and $R_r = +10\%$. We notice that the states estimated reach the values simulated that after the transitional stage (Fig.12, Fig.13 and Fig.14).

VII. Conclusion

The observation of state of an asynchronous machine without sensorless speed was approached in this document. The state observer is built starting from a nonlinear transformation of the system. The estimates of state obtained with the observer **PI** and an observer **P** were compared on a simulation example.

Among the points being able to be the subject of later work, one can mention the reduction of the conservatism of the solution suggested by using other approach such as Linear Matrix Inequalities (**LMI**). It can be considered, in a context of diagnosis, to extend the step exposed to the estimate of unknown entries using an observer **PI**.

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VIII. APPENDIX

A. Characteristics and parameters of the machine

The characteristics and the parameters of the asynchronous machine used to test the observer is a cage machine are the following: ([11])

TABLE I. CHARACTERISTICS OF THE MACHINE

Parameter	Value	Unit
Nominal power	1.5	kW
Nominal speed	1430	tmp/mn
Poles pairs	2	
Simple tension	220	V
Nominal intensity	6.1	A
Nominal torque load	10	Nm

TABLE II. THE PARAMETERS OF THE MACHINE

Parameter	Notation	Value	Unit
Stator resistance	R_s	1.47	Ω
Rotor resistance	R_r	0.79	Ω
Stator inductance	L_s	0.105	H
Rotor inductance	L_r	0.094	H
Mutual Inductance	M_{sr}	0.094	H
Frictions coefficient	f_v	0.0029	$Nm/rad/s$
Rotor inertia	J	0.0077	$Kg.m^2$

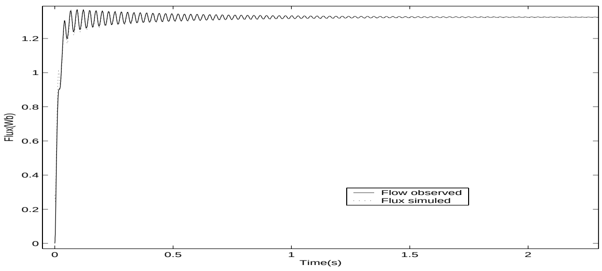

Fig. 1. Flux observed and simulated in open loop.

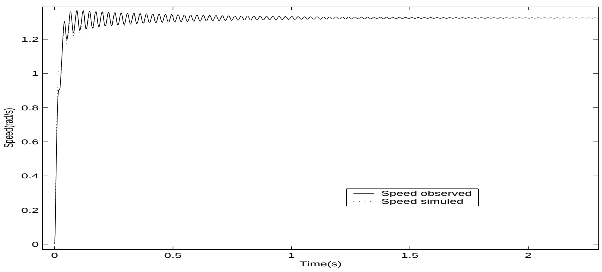

Fig. 2. Speed observed and simulated in open loop.

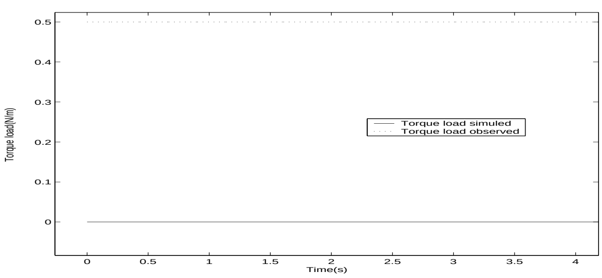

Fig. 3. Couples observed and simulated in open loop.

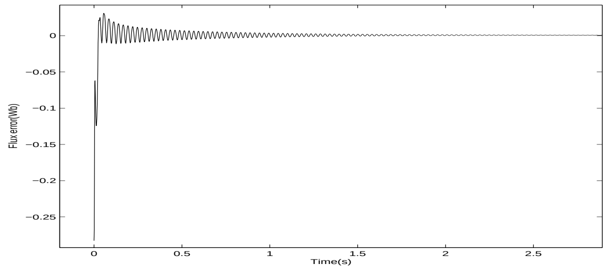

Fig. 4. Flux error in open loop.

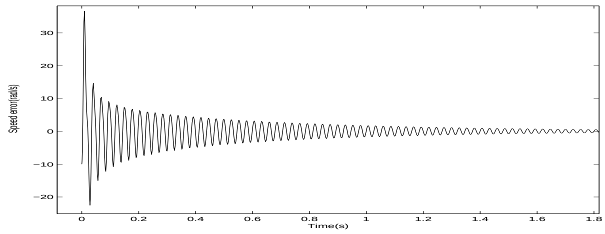

Fig. 5. speed error in open loop.

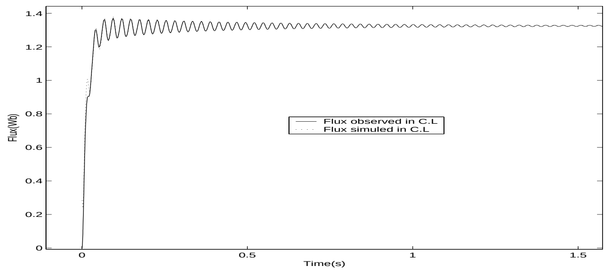

Fig. 6. Flux simulated and observed in C.L for the action P.

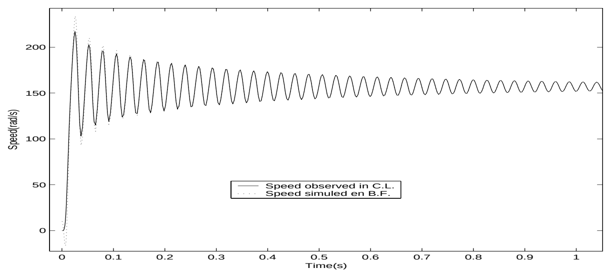

Fig. 7. Speed simulated and observed in C.L for the action P.

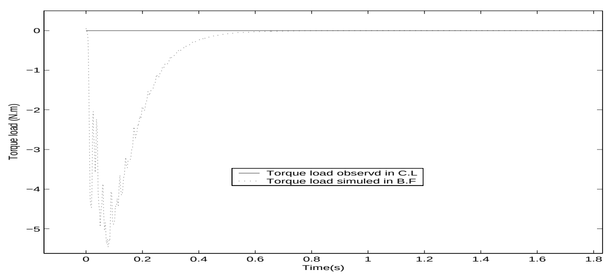

Fig. 8. Torque load simulated and observed in C.L for the action P.

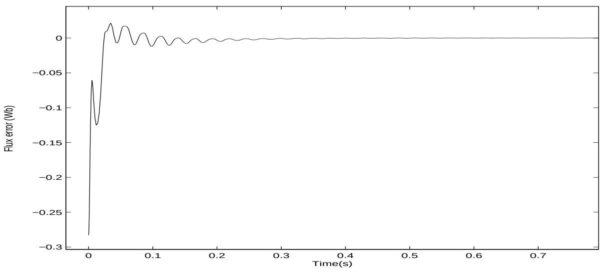

Fig. 9. Flux error in C.L for the action P.

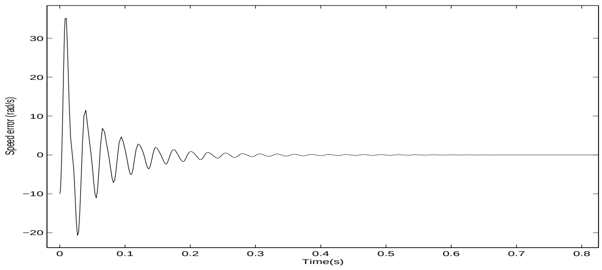

Fig. 10. Speed error in C.L for the action P.

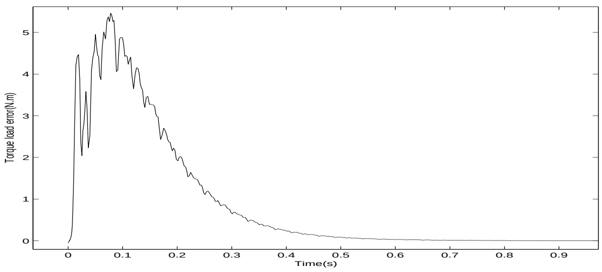

Fig. 11. Torque load error in C.L. for the action P.

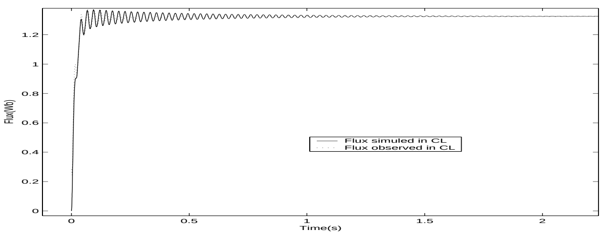

Fig. 12. Flux simulated and observed in C.L for action PI.

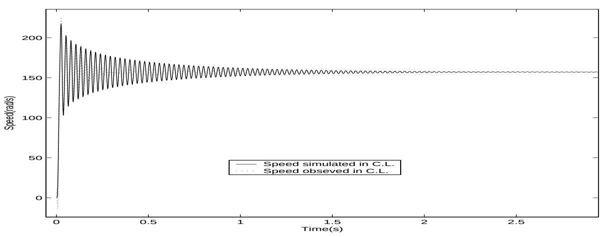

Fig. 13. Speed simulated and observed in C.L. for action PI.

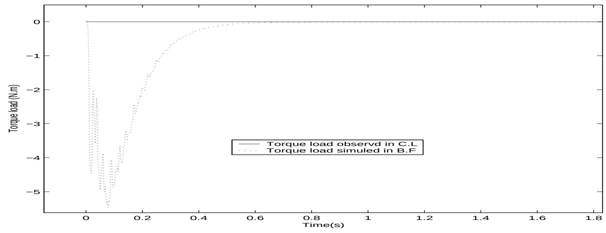

Fig. 14. Torque load simulated and observed in C.L. for action PI.

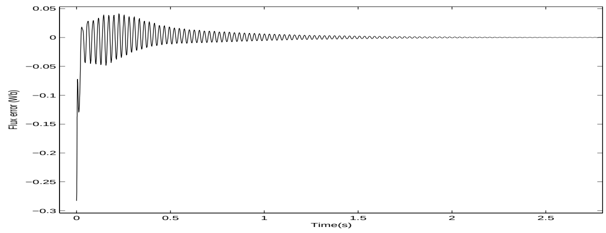

Fig. 15. Flux error in C.L. for action PI.

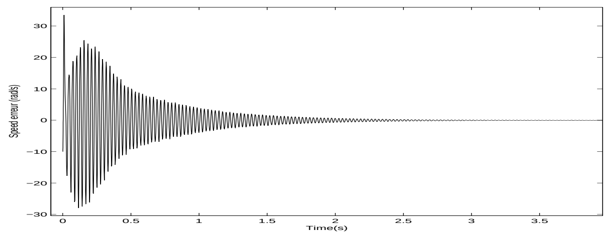

Fig. 16. Speed error in C.L. for action PI.

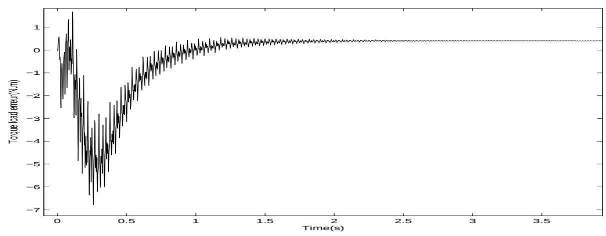

Fig. 17. Torque load error in C.L. for action PI.