Nonlinear Tracking Control of an Underactuated Overhead Crane

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Abstract—**This paper proposes a trajectory tracking controller for a two degree of freedom (2-DOF) overhead crane. First, a dynamic model of the crane suitable for feedback control is developed using robotic methodology. A desired trajectory for the trolley motion is generated using a reference differential equation. The proposed control law is based on collocated partial feedback linearization combined with trajectory tracking and linear feedback control which achieves local asymptotic stability of the closed-loop system. Simulation results illustrate the effectiveness of the proposed controller.**

Keywords—overhaed crane; dynamic modeling; nonlinear feadback control; perturbed systems

I. INTRODUCTION

In the last decades, the overhead cranes have been widely used for transportation in many industrial applications and become an interesting issue from automatic control point of view. The goal is to transport the payload quickly and in the same time to reduce the rope swing angle. Recently, different techniques been proposed for the design of Linear Quadratic [1, 2], adaptive [3], nonlinear coupling control [4] controllers for overhead cranes. The overhead cranes belong to the class of underactuated mechanical systems, which have fewer control inputs than degrees of freedom. One of the complexities of these systems is that they are not feedback linearizable. Due to the positive definiteness of the inertia matrix of this class of systems, the so-called collocated partial feedback linearization property [5] holds, which refers to the control that linearizes the equations associated with the actuated degrees of freedom of the system. Available control design methods mainly include approximate linearization [6] and saturation control [7].

In this paper, we propose a simplified control strategy based on collocated partial feedback linearization of the dynamic model combined with trajectory tracking and linear feedback control law, which achieves local asymptotic stability of the closed loop system. The organization of the paper is as follows: In Section II, a dynamic model of the crane suitable for feedback control applications is derived. The Problem formulation is given in Section III. In Section IV, a control law is designed. Section V contains simulation results. Conclusions are presented in Section VI.

II. DYNAMIC MODEL

A schematic view of an overhead crane is shown in Fig. 1. In order to derive a dynamic model suitable for control applications, we make the following assumptions: the payload mass is considered as a point-mass, and the mass and stiffness of the hoisting rope are neglected. The system has two degree-of-freedom and the associate generalized coordinates are

$$q = \begin{bmatrix} d & \theta \end{bmatrix}^T \in \Re^2 \qquad (1)$$

where d is the displacement of the trolley and θ is the swing angle of the load, (Fig. 1).

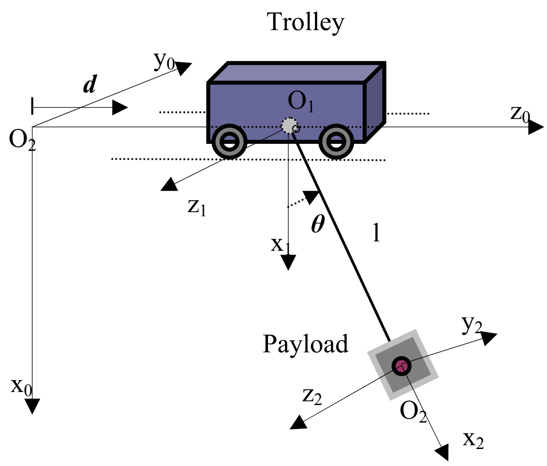

Fig. 1. Schematic of the overhead crane

We use the Denavit-Hartenberg convention [8] for the description of the crane kinematics. An inertial coordinate system $O_0x_0y_0z_0$ is assigned in the work space where the z_0 axis is in direction of the trolley displacement. The z_1 axis of a moving together with the trolley coordinate frame $O_1x_1y_1z_1$ is the axis of revolution of the rope. The z_2 axis of the coordinate frame $O_2x_2y_2z_2$ which is attached to the payload is parallel to z_1. The link parameters are given in Table I,

TABLE I
PARAMETERS FOR A 2-DOFS OVERHEAD CRANE

Parameters	Link 1	Link 2
a_i [m]	0	l
d_i [m]	d=var	0
α_i [rad]	$\pi/2$	0
θ_i [rad]	0	$\theta = var$

where the four quantities a_i, d_i, α_i, and θ_i are parameters of link i and joint i, (i = 1,2).

The corresponding transformation matrices which define the relative position and orientation between the adjacent coordinate systems are

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & l\cos\theta \\ \sin\theta & \cos\theta & 0 & l\sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

Using the transformation matrices (2), the coordinates of point O_2 ($\begin{bmatrix} x_{O_2} & y_{O_2} & z_{O_2} & 1 \end{bmatrix}^T$) with respect to $O_0x_0y_0z_0$ are obtained as follows

$$\begin{bmatrix} x_{O_2} \\ y_{O_2} \\ z_{O2} \\ 1 \end{bmatrix} = A_1 A_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l\cos\theta \\ 0 \\ d + l\sin\theta \\ 1 \end{bmatrix}, \quad (3)$$

where $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$ are the coordinates of O_2 with respect to $O_2x_2y_2z_2$. From (3), it follows that

$$\begin{aligned} x_{O_2} &= l\cos\theta \\ z_{O_2} &= d + l\sin\theta \end{aligned}. \quad (4)$$

Differentiating (4) with respect to time, for the projections of the velocity (with respect to $O_0x_0y_0z_0$) of O_2 it follows

$$\begin{aligned} \dot{x}_{O_2} &= -l\dot{\theta}\sin\theta \\ \dot{z}_{O_2} &= \dot{d} + l\dot{\theta}\cos\theta \end{aligned}. \quad (5)$$

The dynamic equations of motion of the crane are derived using Lagrange formalism

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i, \ i = 1,2 \quad (6)$$

where the Lagrangian L represents the difference between the kinetic and potential energy of the system ($L = T - U$), Q_i are the generalized forces associated with the generalized coordinates, and the generalized coordinates of the system q are given by (1).

The kinetic energy of the system is a sum of the kinetic energies of the trolley (T_1) and payload (T_2) and is determined as follows

$$\begin{aligned} T = T_1 + T_2 &= \frac{1}{2}M\dot{d}^2 + \frac{1}{2}m\left(\dot{x}_{O_2}{}^2 + \dot{z}_{O_2}{}^2\right) \\ &= \frac{1}{2}(M+m)\dot{d}^2 + \frac{1}{2}m(2l\cos\theta\dot{d}\dot{\theta} + l^2\dot{\theta}^2) \end{aligned} \quad (7)$$

where M and m are the mass of the trolley and the load, respectively, l is the length of the rope, and $\dot{x}_{O_2}$ and $\dot{z}_{O_2}$ are given by (5).

The potential energy of the system is given as

$$U = -mgl\cos\theta. \quad (8)$$

Using Eqs. (5), (6), (7) and (8), the dynamic equation of motion of the overhead crane are obtained in the form

$$D\ddot{q} + C\dot{q} + G = Q \quad (9)$$

where

$$D = \begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & ml^2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -ml\sin\theta\dot{\theta} \\ 0 & 0 \end{bmatrix},$$
$$G = \begin{bmatrix} 0 \\ mgl\sin\theta \end{bmatrix}, \quad Q = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad (10)$$

where F is the control force acting on the trolley.

Remark 1: It should be noted that the matrix D is positive definite and the matrix $1/2\dot{D} - C$ is skew-symmetric. □

III. PROBLEM STATEMENT

In this paper, we consider the problem of position control of the overhead crane. The goal is to transport the payload quickly with high precision, and in the same time to reduce the swing angle which does not exceed 5^0 through the entire trajectory of the trolley. The desired trajectory for the trolley motion is proposed in the integral form

$$z^d(t) = z_0^d\left[1 - (1+\rho t)e^{-\rho t}\right] \quad (11)$$

where z_0^d is the desired distance which has to be traveled by the trolley, and ρ is a double root of a desired linear differential equation describing the trolley motion, (a larger ρ leads to a faster motion of the trolley and as a consequence, a bigger swing of the payload).

We make the following change of coordinate

$$z_e = d - z^d . \tag{12}$$

and input

$$w = u - \ddot{z}^d \tag{13}$$

where, due to the collocated partial feedback linearization property, u is obtained in the form

$$u = \frac{1}{M + m\sin^2\theta}\left[F + m\sin\theta(l\dot{\theta}^2 + g\cos\theta)\right]. \tag{14}$$

Finally, using (12)-(14), after some work, the dynamic equations (9) for the crane can be written in error coordinate form as

$$\begin{aligned} \ddot{z}_e &= w \\ \ddot{\theta} &= -\frac{1}{l}\left[(w + \ddot{z}^d)\cos\theta + g\sin\theta\right] \end{aligned} . \tag{15}$$

We assume that $e = \begin{bmatrix} z_e & \dot{z}_e & \theta & \dot{\theta} \end{bmatrix}^T \in \Re^4$ are measurable. Given the crane dynamics in error coordinates described by (15), the control objective is to asymptotically regulate $z_e(t)$ to zero (transportation of the payload) and minimize the swing angle $\theta(t)$ of the payload.

IV. Feedback Control Design

The overhead crane is an underactuated single-input (w) two-output (z_e, θ) system. The control problem consists in finding a feedback control law for the system (13) such that

$$\lim_{t\to\infty}\left(z_e(t)\right) = 0 \quad \text{and} \quad \lim_{t\to\infty}\left(\theta(t)\right) = 0 . \tag{16}$$

Consider the linear control

$$w = -k_1 z_e - k_2 \dot{z}_e + k_3 l\theta + k_4 l\dot{\theta} \tag{17}$$

where k_i, ($i = 1,2,3,4$) are positive gains.
The resulting closed-loop system becomes

$$\begin{aligned} \ddot{z}_e &= -k_1 z_e - k_2 \dot{z}_e + k_3 l\theta + k_4 l\dot{\theta} \\ \ddot{\theta} &= -\frac{1}{l}\left(\left(\left(-k_1 z_e - k_2 \dot{z}_e + k_3 l\theta + k_4 l\dot{\theta}\right) + \ddot{z}^d\right)\cos\theta + g\sin\theta\right) \end{aligned} . \tag{18}$$

Denoting $\dot{z}_e = v_{ze}$ and $\dot{\theta} = \omega$, for small swing angles, the tangent linearization of Eqs. (18) about $\theta = 0$ can be written in state-space form as

$$\dot{x} = f(x) + g(t,x) \tag{19}$$

where

$$x = \begin{bmatrix} z_e & v_{ze} & \theta & \omega \end{bmatrix}^T \tag{20}$$

$$f(x) = \begin{bmatrix} v_{ze} \\ -k_1 z_e - k_2 v_{ze} + k_3 l\theta + k_4 l\omega \\ \omega \\ \frac{k_1}{l} z_e + \frac{k_2}{l} v_{ze} - \left(k_3 + \frac{g}{l}\right)\theta - k_4\omega \end{bmatrix}; \quad g(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{l}\ddot{z}^d \end{bmatrix}. \tag{21}$$

The system (19) can be viewed as a perturbation of the nominal system $\dot{x} = f(x)$, which has exponentially stable equilibrium point at the origin $x = 0$, and $\ddot{z}^d(t)$ is a uniformly bounded disturbance that satisfies $\left|\ddot{z}^d\right| \le \frac{z_0^d \rho^2}{l} = \delta$. Furthermore, $\ddot{z}^d(t) \to 0$ as $t \to \infty$. Using Lemma 4.9, [9, p. 208], for the perturbed system (17), it can be shown that $x(t) \to 0$ as $t \to \infty$. Based on Lyapunov's linearization (indirect) method [9], one can be concluded that the corresponding nonlinear system is locally asymptotically stable.

V. Simulation Results

Several simulations using MATLAB were carried out in order to illustrate the performance of the proposed controller. The desired trajectory of the trolley is given by (9) where the desired distance to travel is $z_0^d = 7m$ and $\rho = 0.5$. The overhead crane is tested with a mass of 200kg and 300kg for the trolley and the payload, respectively. The length of the rope was chosen to be $l = 5m$.

In the first simulation, from Fig. 2, we can see the evolution in time of the swing angle θ during the displacement of the trolley.

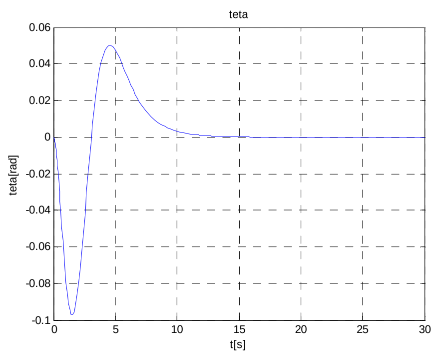

Fig. 2. Time history of the swing angle of the payload

Fig. 3, presents the evolution in time of the movement of the trolley d according to desired trajectory $z^d(t)$. The results of the simulations confirm the validity of the proposed controller.

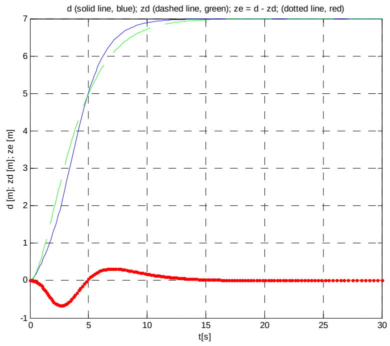

Fig. 3. Time history of the trolley displacement (blue solid line), desired trajectory (green dashed line), and tracking error (red dotted line)

From Fig. 3, that trajectory of the trolley displacement is close to the reference trajectory, and converges asymptotically to the desired position, in accordance with the theoretical result obtained in Section IV.

VI. Conclusion

In this paper, a trajectory tracking controller for a 2-DOF overhead crane has been proposed. A dynamic model of the crane was developed using robotic methodology. A desired trajectory for the trolley motion was generated using a reference differential equation. The proposed control law was based on collocated partial feedback linearization combined with trajectory tracking and linear feedback control and achieved local asymptotic stability of the closed-loop system. Simulation results were carried out and confirmed the effectiveness of the proposed controller.

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