On stabilizing fractional order time delay systems by first order controllers

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***Abstract*—**In this paper, the problem of stabilizing fractional order time delay systems by first order controller is investigated. The proposed solution is based on finding a set of global stability regions in the controller's parameter space. One of the controller's parameters is fixed and the stability region in the space of the remaining parameters is determined using the D-decomposition method. An illustrative example is given to show the effectiveness of the proposed approach.

***Keywords*—** First order controller, fractional order systems, stability, time delay.

I. Introduction

FRACTIONAL order systems are encountered in many fields such as electrochemistry, material science and biological systems. The dynamic behavior of such systems is represented by differential equations of arbitrary order, not necessarily integer order. Experimental observation and analytic study have demonstrated that standard laws, for instance integer order slopes in the frequency domain or ordinary integer order differential equations in time domain, can not accurately represent these complicated systems. Fractional calculus is a solution to better represent these systems by incorporating non integer order slopes in their frequency responses and hence dealing with poles and zeros of fractional power. Many examples can be found in the literature where fractional order systems are studied and controlled. In [1], the position control of a single link flexible robot is studied using fractional calculus. See [1]-[3] and the references therein for more examples.

Calculating the set of all stabilizing controllers for linear time invariant systems is an essential first step in obtaining optimal controllers. This line of research was further extended by fixing the order and structure of the controllers used [4]-[6]. In fact, most controllers used in industry are fixed structure, low order controllers being proportional (P), proportional derivative (PD), proportional integral (PI), proportional integral derivative (PID), or first order lead lag controllers. Determining the set of all stabilizing controllers with fixed order and structure is an important task and has many advantages, such as avoiding stability checks every time the controller parameters need to be changed. In addition, the set of all stabilizing controllers can be used as a search space for optimal controllers to further satisfy some design criteria.

Many physical systems such as communication systems, power systems or nuclear reactors inherently contain time delay which makes stability analysis and controller design for these systems more difficult. In fact, time delay systems possess an infinite number of roots which makes straightforward stability check almost impossible. Systems with zero time delay are relatively easy to stabilize. An infinitesimally small delay will result in infinitely new roots appearing at infinity. The relative degree of the plant under study plays an important role in deciding whether the system is stabilizable or not. For non proper plants, the new roots are at infinity in the right half plane which makes stabilizing such systems impossible. For strictly proper plants these roots come at infinity on the left half plane which makes stabilizing such systems possible. For this reason there is an extensive literature on stabilizing time delay systems by low order controllers [7]-[19] such as first order controllers in [7] and [17], PI controllers in [10] and PID controllers [14] and [16]-[17] and the references therein.

In the above mentioned studies, the stabilizing question is answered for systems which are described by integer order differential equations and little attention was given to fractional order systems with time delay [20].

In this paper, we determine the set of all stabilizing first order controllers for fractional order time delay systems. The paper is organized as follows. In section II, some preliminary results are given. Section III describes a

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method of determining the set of all stabilizing first order controllers for fractional order time delay systems. In section IV, an illustrative example is given and section v contains some concluding remarks.

II. Preliminaries

In this paper, the stabilizing regions in the parameter space of a first order controller, applied to a fractional order time delay system, are determined. We consider the classical feedback system of Fig. 1,

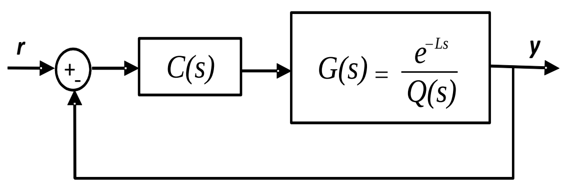

Fig. 1 Classical feedback system

where the system's transfer function is given by

$$G(s)=\frac{e^{-Ls}}{Q(s)} \tag{1}$$

with $L>0$ is the time delay, and $Q(s)$ is a polynomial where the powers of s can take any real number, not necessary integer numbers. $Q(s)$ is given by

$$\begin{aligned}Q(s)&=b_n s^{\beta_n}+b_{n-1}s^{\beta_{n-1}}+\cdots+b_1 s^{\beta_1}+b_0 s^{\beta_0}\\&=\sum_{i=0}^{n} b_i s^{\beta_i}\end{aligned} \tag{2}$$

where $\beta_n>\beta_{n-1}>\cdots>\beta_1>\beta_0\geq 0$ are arbitrary numbers.

Our objective is to determine the set of all first-order controllers given by

$$C(s)=\frac{\alpha_2 s+\alpha_3}{s+\alpha_1} \tag{3}$$

that stabilizes the feedback system of Fig. 1.

The closed-loop characteristic equation is given by

$$\Delta^{*}(s,\alpha_1,\alpha_2,\alpha_3)=(s+\alpha_1)Q(s)+(\alpha_2 s+\alpha_3)e^{-Ls} \tag{4}$$

Replacing $Q(s)$ by the expression given in (2), we obtain the following fractional order characteristic equation

$$\begin{aligned}\Delta^{*}(s,\alpha_1,\alpha_2,\alpha_3)=(s+\alpha_1)(b_n s^{\beta_n}+b_{n-1}s^{\beta_{n-1}}+\cdots+\\ b_1 s^{\beta_1}+b_0 s^{\beta_0})+(\alpha_2 s+\alpha_3)e^{-Ls}\end{aligned} \tag{5}$$

We are seeking values of $(\alpha_1,\alpha_2,\alpha_3)$ for which the quasi-polynomial $\Delta^{*}(s,\alpha_1,\alpha_2,\alpha_3)$ has no roots in the closed right-half of the s-plane. The stability domain S, we are searching in the space of the controller's parameters $(\alpha_1,\alpha_2,\alpha_3)$ will be determined using D-decomposition method [21]-[23]. This method is based on the fact that roots of the quasipolynomial (5) change continuously as the coefficients are changed continuously. As a result, a stable quasipolynomial becomes unstable if and only if at least one of its roots crosses the imaginary axis from the left half complex plane to the right half complex plane. Using this fact we can partition the controller's parameter's space such that each region has a fixed number of roots in the left half plane. Stability can be checked by choosing a point inside the region and applying any classical method.

III. Stabilizing first order controllers

The boundaries of the stability domain are obtained by replacing s by $j\omega$, equating the fractional order characteristic equation (5) to zero and sweeping over values of $\omega\geq 0$.

The real root boundary is obtained by setting $\omega=0$ in $\Delta^{*}(j\omega,\alpha_1,\alpha_2,\alpha_3)$, which turns out to be an equation of straight line given by

$$\alpha_3=-b_0\alpha_1 \tag{6}$$

for $s^{\beta_0}=1$ in (1).

The complex root boundary can be determined by substituting $s=j\omega$ in (5). Note that the term e^{Ls} has no finite roots, as a result the quasi-polynomial $\Delta^{*}(s)$ and $\Delta(s)$ have the same roots, therefore stability of $\Delta(s)$ is equivalent to stability of $\Delta^{*}(s)$. In the rest of the paper, the quasi-polynomial $\Delta(s)$ will be used to study stability of the closed-loop system of Fig. 1 where $\Delta(s)$ is given by

$$\begin{aligned}\Delta(s)&=(s+\alpha_1)Q(s)e^{Ls}+(\alpha_2 s+\alpha_3)\\&=(s+\alpha_1)(\sum_{i=0}^{n} b_i s^{\beta_i})e^{Ls}+(\alpha_2 s+\alpha_3)\end{aligned} \tag{7}$$

Replacing s by $j\omega$, we get

$$\begin{aligned}\Delta(j\omega)=&\left\lfloor\sum_{i=0}^{n} b_i((j\omega)^{\beta_i+1}+\alpha_1(j\omega)^{\beta_i})e^{jL\omega}\right\rfloor\\&+(j\omega\alpha_2+\alpha_3)\end{aligned} \tag{8}$$

the fractional order power of j can be expressed as

$$j^{\beta_i+1} = \cos((\beta_i+1)\frac{\pi}{2}) + j\sin((\beta_i+1)\frac{\pi}{2}) = x_i + jy_i \quad (9)$$

$$j^{\beta_i} = \cos(\beta_i\frac{\pi}{2}) + j\sin(\beta_i\frac{\pi}{2}) = z_i + jt_i \quad (10)$$

Hence, $\Delta(j\omega)$ can be written as

$$\Delta(j\omega) = \sum_{i=0}^{n} b_i\omega^{\beta_i+1}(x_i + jy_i)(\cos(Lw) + j\sin(Lw)) + \alpha_1\sum_{i=0}^{n} b_i\omega^{\beta_i}(z_i + jt_i)(\cos(Lw) + j\sin(Lw)) + (j\omega\alpha_2 + \alpha_3) \quad (10)$$

Let $\Re(\omega,\alpha_1,\alpha_2,\alpha_3)$ and $\Im(\omega,\alpha_1,\alpha_2,\alpha_3)$ denote the real and imaginary parts of the fractional order characteristic equation, respectively. Then, (10) can be written as

$$\Delta(s) = \Re(\omega,\alpha_1,\alpha_2,\alpha_3) + j\Im(\omega,\alpha_1,\alpha_2,\alpha_3) \quad (11)$$

Equating the real and imaginary parts of (11) to zero, we obtain

$$P(\omega) + \alpha_1 R(\omega) + \alpha_3 = 0$$
$$S(\omega) + \alpha_1 T(\omega) + \alpha_2 W(\omega) = 0 \quad (12)$$

Where

$$P(\omega) = \sum_{i=0}^{n} b_i\omega^{\beta_{i+1}}(x_i\cos(L\omega) - y_i\sin(L\omega))$$

$$R(\omega) = \sum_{i=0}^{n} b_i\omega^{\beta_i}(z_i\cos(L\omega) - t_i\sin(L\omega))$$

$$S(\omega) = \sum_{i=0}^{n} b_i\omega^{\beta_{i+1}}(y_i\cos(L\omega) + x_i\sin(L\omega)) \quad (13)$$

$$T(\omega) = \sum_{i=0}^{n} b_i\omega^{\beta_i}(t_i\cos(L\omega) + z_i\sin(L\omega))$$

$$W(\omega) = \omega$$

Solving the system (12), α_2 and α_3 are obtained in terms of α_1 as

$$\alpha_2 = -\frac{R(\omega) + \alpha_1 T(\omega)}{W(\omega)} \quad (14)$$

$$\alpha_3 = -P(\omega) + \alpha_1 R(\omega) \quad (15)$$

These equations draw the curves in the (α_2,α_3) plane for a fixed value of α_1, which represents the complex root boundary.

IV. Illustrative Example

Given the fractional order time delay system studied in [24] with the following transfer function

$$G(s) = \frac{e^{-0.5s}}{s^{1.5}} \quad (16)$$

Our objective is to determine the stability regions which make the closed loop characteristic equation stable. Using (5) the fractional order characteristic equation of the control system is given by

$$\Delta^*(s) = (s^{2.5} + \alpha_1 s^{1.5}) + e^{-0.5s}(\alpha_2 s + \alpha_3) \quad (17)$$

Fixing the value of α_1 and solving for α_2 and α_3 using (14) and (15), we get

$$\alpha_2 = -\frac{1}{\omega}\left[\left(\omega^{2.5}\cos(5\pi/4) + \alpha_1\omega^{1.5}\cos(3\pi/4)\right)\sin(0.5\omega) + \left(\omega^{2.5}\sin(5\pi/4) + \alpha_1\omega^{1.5}\sin(3\pi/4)\right)\cos(0.5\omega)\right] \quad (18)$$

$$\alpha_3 = \left[\left(\omega^{2.5}\sin(5\pi/4) + \alpha_1\omega^{1.5}\sin(3\pi/4)\right)\sin(0.5\omega) - \left(\omega^{2.5}\cos(5\pi/4) + \alpha_1\omega^{1.5}\sin(3\pi/4)\right)\cos(0.5\omega)\right] \quad (19)$$

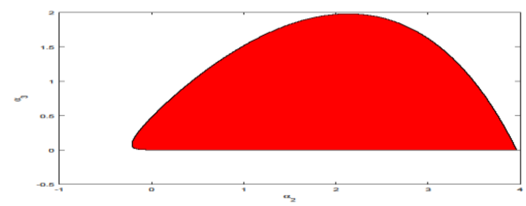

Fig. 2 Stability region in the plane of (α_2,α_3) for $\alpha_1 = 1$

Using (6), we get an equation of a straight line given by $\alpha_3 = 0$. For a fixed value of $\alpha_1 = 1$, we use (18) and (19) to draw the complex root boundary curve as shown in Fig. 2. The shaded region represents the stability region in the (α_2,α_3) plane. Repeating the above procedure for different values of α_1, we obtain the 3D stability regions as shown in Fig. 3

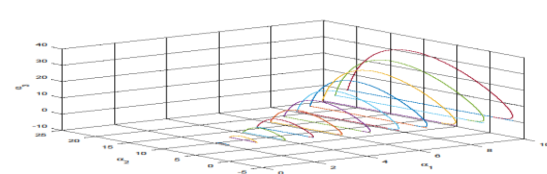

Fig. 3 Stability region in the plane of $(\alpha_1,\alpha_2,\alpha_3)$

V. CONCLUSION

In this paper, the D-decomposition method is used to determine the stability regions of a first order controller applied to fractional order time delay system. The proposed approach is based on fixing one of the controller's parameters and finding the stability regions in the space of the remaining parameters. By sweeping over the first parameter the complete set of stability regions can be determined. Stability being an essential step in any controller design, the proposed method can be used as a first step towards designing optimal controllers for fractional order time delay systems. Stabilizing fractional order time delay systems by fractional order first order controllers is under investigation.

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