Legendre polynomials Active contour method for image segmentation

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Abstract— In this paper, we present a novel region based active contour method for image segmentation in the presence of intensity inhomogeneity. We formulate our model based on the SBGFRLS model and the Legendre polynomials. Instead of using the average intensity of the region we represent the regions by a linear combination of a set of Legendre basis functions, which lead to deal with intensity inhomogeneities in the segmentation.
In addition, to avoid re-initialization keep the level set function smooth in the evolution process we regularize it using Gaussian filtering.
Experimental results on synthetic and real-images show that our method is more robust to initialization, faster and more accurate than the well-known piecewise smooth model.

Keywords— Active contour, intensity inhomogeneity, level set method, segmentation.

I. Introduction

Existing level set methods for image segmentation can be categorized into two major classes: edge-based models [1], [2], [3], [4], [5] and region-based models [6], [7], [8], [9], [10], [11].

Edge-based models can be applied to images with intensity inhomogeneities because these models do not assume homogeneity of image intensities. However, these types of methods are in general quite sensitive to the initialization and they are not able to achieve a desirable segmentation result for an image with weak boundaries. Most of region-based models [6], [8] and [9] are based on the assumption of intensity homogeneity and they guide the curve evolution using a certain region descriptor. However, it is very difficult to define a region descriptor for images with intensity inhomogeneities.

In the last decade some region-based level set methods have been proposed based on a general piecewise smooth (PS) formulation originally proposed by Mumford and Shah [13]. Among them: local binary fitting (LBF) model [11], the region-scalable fitting (RSF) model [7], local intensity clustering (LIC) method [12], LRB method [13], local region model (LRM) [14], and edge driven level set method [15], …etc. These methods are able to segment images with intensity inhomogeneities. However, these methods are computationally too expensive and are quite sensitive to the initialization of the contour [11], which greatly limits their utilities.

In this paper, we propose a novel region based active contour method for image segmentation in the presence of intensity inhomogeneity. By exploiting the Legendre polynomials we formulate our model based on the SBGFRLS model [11]. Instead of using the average intensity of the region we represent the regions by a linear combination of a set of Legendre basis functions, which lead to segment noisy image and image with intensity inhomogeneities.

In addition, to avoid re-initialization keep the level set function smooth in the evolution process we regularize it using Gaussian filtering. Extensive evaluations on synthetic and real images are performed, including comparisons with the related works such as CV [6] and LGD [17] models.

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II. Methode

The Chan–Vese model is the curve evolution implementation of a piecewise-constant case of the Mumford–Shah model [18]. The general form for the Mumford–Shah energy functional can be written as:

$$E_{MS}(u,C) = \int_{\Omega} |u_0(x,y) - u(x,y)|^2 dxdy + \mu \int_{\Omega\setminus C} |\nabla u(x,y)|^2 dxdy + \nu . lengh(C). \quad (1)$$

where μ and ν are positive constants, Ω denotes the image domain, the segmenting curve $C \subset \Omega$.

The Chan–Vese (C-V) model solves the minimization of (1) by minimizing the following energy functional:

$$E_{CV}(c_1,c_2,C) = \mu . length(C) + \nu . area\big(inside(C)\big) + \lambda_1 \int_{inside(C)} |I(x) - c_1|^2 dx + \lambda_2 \int_{outside(C)} |I(x) - c_2|^2 dx , \quad x \in \Omega \quad (2)$$

Where c_1 and c_2 are two constants related to the global properties of the image contents which approximate the image intensities inside and outside the contour C, respectively.

$$\begin{cases} C = \{x \in \Omega : \phi(x) = 0\}, \\ inside(C) = \{x \in \Omega : \phi(x) > 0\}, \\ outside(C) = \{x \in \Omega : \phi(x) < 0\}. \end{cases} \quad (3)$$

λ_1 and λ_2 control the image data driven force inside and outside the contour, respectively, generally, they are taken to be $\lambda_1 > 0,\ \lambda_2 > 0$.

In [11] Zhang et al. proposed a novel active contour model with selective local or global segmentation SBGFRLS, in which the uses a new region-based signed pressure function (spf) defined as follows:

$$spf(I(x,y)) = \frac{I(x,y) - \frac{c_1 - c_2}{2}}{max\left(I(x,y) - \frac{c_1 - c_2}{2}\right)} \quad (4)$$

The spf function modulates the signs of the pressure force inside and outside the region of interest so that the contour shrinks when outside the object, or expands when inside the object. In addition, they used a Gaussian filtering process to further regularize the level set function. The level set formulation of the SBGFRLS model was written as follows:

$$\frac{\partial \phi}{\partial t} = spf(I(x)).\alpha.|\nabla \phi| \quad (5)$$

where α is the scale parameter which controls the speed of level set update, it should be tuned according to image.

For the SBGFRLS model, the energy is based on the difference between each pixel and the average intensity of the region.

In our model, the *spf* function (4) can be reformulated and generalized by replacing the scalars c_1 and c_2 by two smooth functions $c_1^m(x)$ and $c_2^m(x)$ presented in [19] as follow:

$$c_1^m(x) = \sum_k \alpha_k \mathfrak{p}_k(x) \ \text{ and } \ c_2^m(x) = \sum_k \beta_k \mathfrak{p}_k(x) \quad (6)$$

$\mathfrak{p}_k$ is a multidimensional Legendre polynomial, The 2-D polynomial is computed as:

$\mathfrak{p}_k(x,y) = p_k(x)p_k(y), \quad x = (x,y) \in \Omega \subset [-1,1]^2$

$p_k$is a one dimensional Legendre polynomial of degree k defined as:

$$p_k(x) = \frac{1}{2^k}\sum_{i=0}^{k}\binom{k}{i}(x-1)^{k-i}(x+1)^i \quad (7)$$

for the 2D case, we would represent the regions by a linear combination of a set of $(m+1)^2$ 2D Legendre basis functions.

Let $\mathbb{P}(x) = (\mathfrak{p}_0(x), \dots, \mathfrak{p}_N(x))^T$ as the vector of Legendre polynomials. $A = (\alpha_0, \dots, \alpha_N)^T$ and $B = (\beta_0, \dots, \beta_N)^T$ are the coefficient vectors for the two regions. $N = (m+1)^2$ is the total number of basis functions. We can formulate the level set function of our model as follows:

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[I(x) - \frac{\widehat{A}^T\mathbb{P}(x) + \widehat{B}^T\mathbb{P}(x)}{2}\right] \quad (8)$$

The optimal A and B ($\widehat{A}$ and $\widehat{B}$) are obtained as:

$$\begin{cases} \widehat{A} = [K + \lambda_1 \mathbb{I}]^{-1}P \\ \widehat{B} = [L + \lambda_2 \mathbb{I}]^{-1}Q \end{cases} \quad (9)$$

where

$$P = \int_\Omega \mathbb{P}(x)I(x)m_1(x)dx$$

$$Q = \int_\Omega \mathbb{P}(x)I(x)m_1(x)dx \quad (10)$$

$$m_1(x) = H_\varepsilon(x) \text{ and } m_2(x) = (1 - H_\varepsilon(x))$$

$H_\varepsilon(x)$ denotes the regularized version of the Heaviside function [11].

And the Gramian matrices [K] and [L] are obtained as [16]:

$$[K]_{i,j} = \langle\sqrt{m_1(x)}\mathfrak{p}_i(x), \sqrt{m_1(x)}\mathfrak{p}_j(x)\rangle$$

$$[L]_{i,j} = \langle\sqrt{m_2(x)}\mathfrak{p}_i(x), \sqrt{m_2(x)}\mathfrak{p}_j(x)\rangle \quad (11)$$

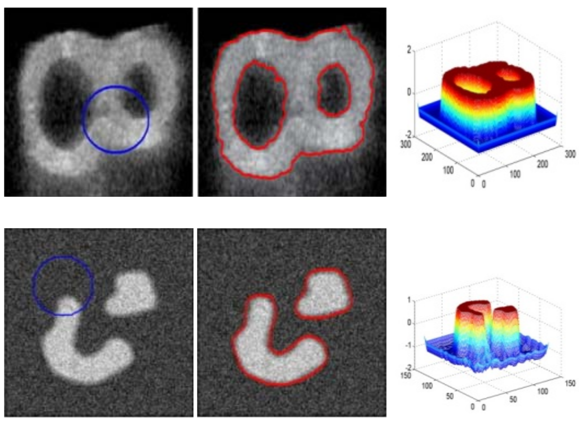
$0 \le i, \quad j \le N, \quad \langle,\rangle$: the inner product operator

III. Experimental Results

In order to regularize the level set function, we use a Gaussian filtering process as follows [11]:

$$\phi^{n+1} = K_\sigma * \phi^n \quad (12)$$

We validate the proposed model by various experiments on several challenging images (synthetic and real images) which demonstrate the effectiveness and robustness of our model compared to some of the well known models.



A B C

Fig. 1 Segmentation results of noisy images

A: the initial contour, B: our model C: the level set function

Firstly we test our proposed model on noisy images Fig. 1 shows the segmentation process of two synthetic images using the proposed model. The sizes of two images are 286×286 pixels and 128×128 pixels from top to bottom. The first column is the initial contour. For our model and in both the two images the evolving curves stop on the true boundary of each shape after only 2 iterations, and takes only 0.064 s for the image in the top row and 3 iterations, in only 0.019 s for the image in the bottom row.

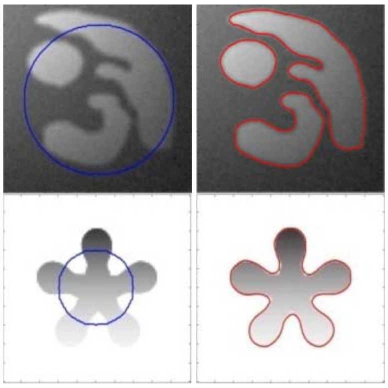


Fig. 2 Results of our method for images with intensity inhomogeneity

Fig. 2 presents the results of our method for tow images corrupted by intensity inhomogeneity. Our method successfully extracts the object boundaries for these two images in just 7 iterations taking only 0.037 s for the first image and 11 iterations and takes 0.029 s for the second image.

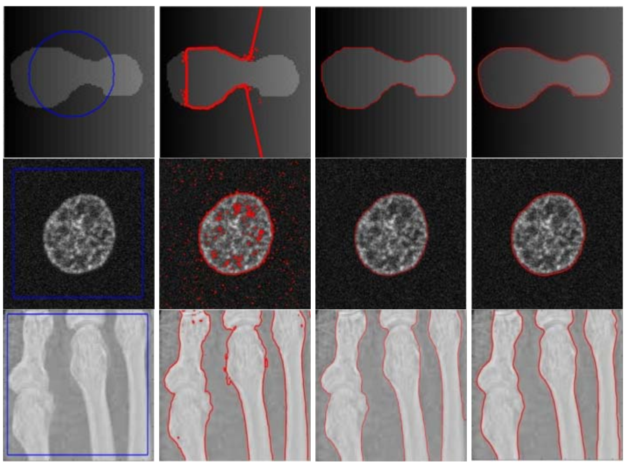


Fig.3 Comparison of our model with LGD model and C-V model.

In this experiment we use the traditional C-V model and the LGD model in the comparison. Fig. 3 shows the segmentation results of three images, a synthetic image with intensity inhomogeneity, a nucleus fluorescence micrograph image in the second row and an X ray image of bones in the third row. The left column of Fig.3 shows the initial contours, the results of the C-V model in the second column, the results of LGD model in the third column and the results of the our model in the last column. For the first image our method accurately detects the boundaries of the objects, whereas the traditional C-V model fails to segment the object. Both the LGD model and our model have the same results for the second image where the C-V model failed. The results for the third image show that our method is more accurate than the LGD model and the C-V model. This experiment shows the performance of the proposed model over the LGD model and the C-V model in terms of accuracy, computational time and number of iterations as shown in table 1.

Table 1 CPU time for the images in Fig. 3

	Image 2 88×88 pixels		Image 2 128×128 pixels		Image 3 128×128 pixels	
	Iterations	Time (s)	Iterations	Time (s)	Iterations	Time (s)
Our model	9	0.037	3	0.037	1	0.013
LGD model	60	1.42	570	16.42	670	22.84
C-V model	600	17.86	2700	138.59	849	13.00

IV. Conclusion

Using Legendre polynomials for region intensity approximation, we formulate a novel level set method for segmenting images with intensity inhomogeneity. Our method is insensitive to different initializations of the level set function, making it useful for automatic applications; In addition, experimental results on both synthetic and real images demonstrated that the proposed model is robust, efficient and faster than the well-known model.

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