Reconfiguration of distribution power system using Evolutionary Algorithm and Branch exchange method for Power Loss Reduction

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Abstract: In this paper, we use an Evolutionary Algorithm (EA) and the branch exchange method to solve the optimal reconfiguration in radial distribution systems for power loss reduction that determine the optimal switches. The EA is a relatively powerful intelligence evolution method for solving optimization problems. It is a population based approach that is inspired from natural behaviour of species. In this paper EA is applied to a realistic distribution system (106 buses) located in the Medea city (Algeria). For the comparison purposes, our method is validated with the classical Branch and Bound (BB) method, widely used by the Distribution Companies. The results confirm the superiority of the EA.

Keywords: Distribution power system, Branch and Bound, Evolutionary Algorithm.

I. INTRODUCTION

In the Distribution Power System (DPS); to reduce the losses and improving the voltage profile the reconfiguration of DPS is an alternative way which doesn't require any investment.

This operation must be considered when planning the operation due to the high variable costs associated with these systems [1].

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The problem of reconfiguration involves the definition of the states (open or closed) of the maneuverable switches attached to certain sections of the distribution network [2]. In the MV PDS these devices include (i) sectionalizing or normally closed (NC) switches and (ii) tie or normally open (NO) switches. This option is used to determine a radial network topology that minimizes losses and voltage deviations [3].

As this problem includes combinatorial variable (0 and 1) with nonlinear objective function and constrained, models of integer nonlinear programming (INLP) are used. These models must consider the integer nature of the problem because the number of possible solutions grows exponentially with the number of discrete variables [4]. Also, the radial and connected topographies of the DPS present additional complexity for the solution techniques.

The heuristic-based methods have been proposed [5–11] in order to reduce the search space associated with reconfiguration problem.

The use of meta-heuristics for INLP problems gives a well exploration of the search space. These algorithms permit the transition between local optima of the feasible region, as well as a more focused search in each subspace. Algorithms based on meta-heuristics, such as Genetic Algorithms [12–16], Simulated Annealing [17,18], Artificial Ant Colony [19] and Tabu Search [20,21], have been used to solve the problem of EDS reconfiguration. With the same purpose in mind, Ref. [22] presents an algorithm based on Artificial Immune Systems to reduce active power losses. In [23], a method based on the bacterial foraging optimization algorithm is proposed for DPS reconfiguration and loss minimization.

Some modifications have been developed to retain the radial structure and reduce the searching requirements. The work in [24] presents a method for the reconfiguration and phase balancing of DPS based on a bacterial foraging approach using a fuzzy multi-objective function. Other family called Determinist techniques can be used to deal with this problem; these techniques are called Branch and Bound which is the extension of the simplex method to INLP.

In this paper, the comparison between GA and B&B algorithm for the problem of DPS reconfiguration will be discussed. Also, an application to a realistic DPS located in Medea, Algeria will be presented.

II. PROBLEM STATEMENT

For a given distribution power system, the solving of the proposed problems is to determine optimal operating schemes according to specified criteria and constraints.

1.1. Distribution Power System Structure

The structure of a network is defined by the constructive and invariant arrangement of all of its components: stations lines, cables etc...

In graphic terms, this structure is determined by:

- A finite set $X = \left\{ x_1, x_2, x_3, \dots, x_N \right\}$ of elements called vertices or buses, where N is the total number of buses.

- A finite set $B = \left\{ b_1, b_2, b_3, \dots, b_M \right\}$ of elements called links or branches, where ***M*** is the total number of branches. Each branch allows to connecting the two components of a pair belonging to a subset of dimension ***M*** and included in the Cartesian product:

$$X \times X = \left\{ \left(x_i, x_j \right) / \; x_i \in X \,\text{and}\, x_j \in X \right\}$$

The sets X and B correspond to the axiomatic definition of a graph, denoted $G\left(X, B\right)$ [24], the orientation of each branch can be a priori any or indefinite (Fig. 1)

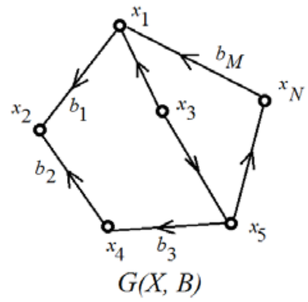

Fig. 1. *Structure of a Distribution Power System*

Network topology, synonym of its operating diagram corresponds to the permanent or temporary assembly of its components.

In graphic terms, this topology is associated with a sub-graph of G , defined by the set X and a set of links L such that $L \subset B$.

The links belonging to L mean that each of them can exchange a flow (current) between the two corresponding nodes. It said, in this case, that the link is in service. For each link in the complementary set, this exchange is interrupt intentionally and it said that the link is out of service.

In the case of distribution power systems, the partial graph is connected. It means that it is possible to achieve, from a given node any other node via a path composed by the elements of L. In addition, the number of branches belonging to L is the total number bus minus 1.

This particular partial graph of G is called a tree, which will be denoted $T\left(X,\ L\right)$.

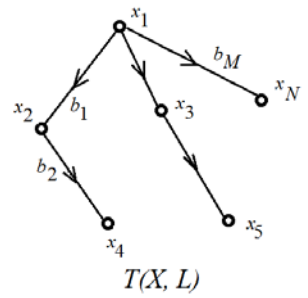

Fig. 2. *Topology of a Distribution Power System*

1.2. General mathematic formulation

The problem to solve is to determine the optimal exploitation schemes in accordance with specified criteria and constraints. In this section, it is proposed to express it mathematically.

For a distribution network defined by its structure, or by the its graph, it is associated to the set B a set Y and a set U such that:

Y set of state variables representing the currents in the branches

U set of decision variables representing the topologic stats of branches; such as:

$$\forall u_k \in U \quad \begin{cases} u_k = 1 & \text{if the branch } b_k \text{ is in service} \\ u_k = 0 & \text{if the branch } b_k \text{ is out of service} \end{cases}$$

Also, it is associated with the set X a set Z such as:

Z set of state variables representing the voltage at the nodes.

According to this notation, an optimization problem can be formulated. Where the components of Y, Z and U should be defined (1).

$$\begin{array}{ll} \min f(Y,Z,U) & \text{objective function} \\ s.t. & \\ g(Y,Z,U) = 0 & \text{Kirchoff's law} \\ h(Y,Z,U) \le 0 & \text{security constraints} \\ s(U) = 1 & \text{radiality} \\ t(U) \le 0 & \text{number of switch} \end{array} \quad (1)$$

1.3. Kirchhoff equations

This constraint laid by the Lemma of Kirchhoff about the currents.

$$\sum_{k=1}^{M} u_k \cdot a_{ik} \cdot I_k = J_i \quad (2)$$

where

$$a_{ik} = \begin{cases} +1 & \text{if } b_k \text{ is oriented to } x_i \\ -1 & \text{if } b_k \text{ is oriented from } x_i \\ 0 & \text{if } x_i \text{ is not an end of } b_k \end{cases}$$

I_k is the current in the branch b_k

J_i is the load current at the bus x_i

This constraint also involves the continuity of service, ie the desire to fully meet the load at any node of the network.

Since the solution of the optimization problem must ansure that the network is radial without isolating any load buses, the second lemma of Kirchhoff about the tensions will be implicitly respected.

1.4. Radiality

This constraint is related to decision variables. It involves the conservation of the radially operation of the network.

$$\sum_{k=1}^{M} u_k = N - 1 \quad (3)$$

To complete the constraint [3]; It is necessary to impose the connectivity of the operating diagram. It can be expressed as follows:

$$\forall x_i, x_j \; \exists (L_i \Delta L_j) \text{ whether } \prod_{b_i \in L_i} u_i \cdot \prod_{b_j \in L_j} u_j = 1 \quad (4)$$

where $L_i \Delta L_j$ is a path connecting the buses x_i and x_j and where u_i and u_j denote the topological states of the branches of L_i and L_j constituting this path (Δ is the symmetric difference).

1.5. Inequality constraints

Security: changing the default, adjusting the template as follows. These constraints imply only the state variables, namely the branches currents and the buses voltages, must not exceed the allowable limits.

For current, the security constraint is expressed for each branch by the following inequality:

$$|I_k| \le u_k \cdot I_k^{\max} \quad (5)$$

where

$I_k^{\max}$ is the thermal limit of current in the branch b_k

I_k is the current in the branch b_k

Operators must ensure voltage as close as possible to the nominal voltage at each consumption point; the maximum tolerated deviation can vary from one company to another. Generally, the absolute value of this difference varies between 5 and 10% depending on the normal operation or failure mode of the network. The expression of this constraint is:

$$\left|V_{in} - V_i\right| / V_{in} \cdot 100 \le 5\% \quad (6)$$

V_{in}, V_i are respectively the nominal voltage and the voltage at the bus x_i

Number of switching operations: Due to failure of a branch, operators wish to restore the continuity of service using a backup operating scheme that is as close as possible to the original topology. This desire led to the limitation of the number of switching operations:

$$\sum_{k=1}^{M}\left|u_k - u_k^0\right| \le \Upsilon_{\max} \quad (7)$$

u_k^0 is the initial topology of the branch b_k

$\Upsilon_{\max}$ is the maximum number of allowed switching

1.6. Objective function

Losses: It is inevitable and cause problems for both technical and economic. They can expressed according to the following expression

$$f_R = \sum_{k=1}^{M} u_k \cdot R_k \cdot I_k^2 \quad (8)$$

R_k is the resistance of the branch b_k

I_k is the current of the branch b_k

Load balancing: The load balancing results from the desire to exploit the network optimally by dividing current reserves uniformly.

$$f_I = \sum_{k=1}^{M} (I_k / I_k^{\max})^2 \quad (9)$$

Maximum voltage deviation: the aim of this objective is to minimize the maximum voltage deviation

$$f_V = \max\left|V_{in} - V_i\right| / V_{in} \quad i = 1,\ldots,N \quad (10)$$

III. EVOLUTIONARY ALGORITHM (EA)

Here, we descript the use of the EA to solve the problem of the reconfiguration of a DPS. The EA used in this work is inspired from the original algorithm presented by David E. Goldberg in [25].

1.7. Fitness:

Like in GA, when at least one constraint is violated the objective function is set to a very small value:

$$val = \begin{cases} \dfrac{1}{L} & \text{if all constraints are respected} \\ -\infty & \text{othewese} \end{cases} \quad (11)$$

Where L is a composite function of f_R, f_I and f_V (8, 9 and 10).

1.8. coding:

The number of opened lines is equal $N_{off} = M - N + N_{feeder}$, where N_{feeder} is the number of feeders. Hence, N_{off} strings should be used. Each string is coded by a binary way, with a length equal to:

$$\text{fix}(\ln(M)) + 1 \quad (12)$$

Where fix is a round towards zero function.

1.9. Initialization:

The starting population is chosed randomly. However, to ensure the research in the feasible space (especialy the respect of the constraint (4)) the original situation is used.

1.10. Branches exchanging operator $\otimes$

The well known and elementary operation an DPS used by the exploitants is the load ripage, which consist of the balancing a load from a overloading branch to other lessloading branch. This operation is elementary and vital as it preserve the radiability and connectivity of DPS.

Let $s^{(\text{old})}$ is the initial configuration of a DPS, the rippage consist of reaching a new configuration $s^{(\text{new})}$ by closing an opened branch $b_{ON} = \{x_n, x_m\}$ and opning a closed branch $b_{OFF} = \{x_i, x_j\}$ that belongs to the same independent loop. This elementary operation can be samarized by:

$$s^{(new)} = s^{(old)} \otimes (b_{ON}, b_{OFF}) \tag{13}$$

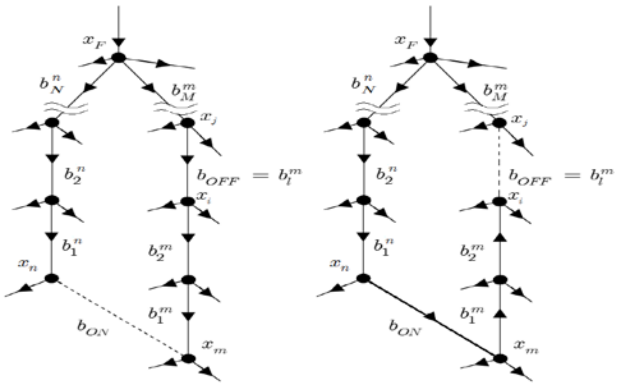

Fig. 3. Branches exchanging $s^{(new)} = s^{(old)} \otimes (b_{ON}, b_{OFF})$

a) configuration $s^{(\text{old})}$

b) configuration $s^{(\text{new})}$

Considering that the load are modelled by a constant current (neglecting the capacitive effect of lines – short lines), only the currents related to the correspondent loop will be changed and consequently only the voltage of the nodes in this loop will be change with the voltage of their aval nodes.

As the three objective functions f_R, f_I and f_V are not in conflict, we interest only in f_R. The new value can be calculated as following:

$$f_R^{(new)} = f_R^{(old)} + \Delta f_R^{(new)} \tag{14}$$

$$\Delta f_R^{(new)} = I_{OFF}^2 \Big(\sum_{b_k \in (L_n \Delta L_m \cup b_{ON})} R_k \Big) + 2I_{OFF} \Big(\sum_{b_k \in L_n} R_k I_k^{(\text{old})} - \sum_{b_k \in L_m} R_k I_k^{(\text{old})} \Big) \tag{15}$$

where

I_{OFF} the current in branch b_{OFF}

R_k the resistance of branch b_k

$f_R^{(\text{new})}$ total Joule's losses at the configuration $s^{(\text{new})}$

$f_R^{(\text{old})}$ total Joule's losses at the configuration $s^{(\text{old})}$

$I_k^{(\text{old})}$ the current of the branch bk b_k at the configuration $s^{(old)}$

x_m the node that is in the side that contains b_{OFF}.

The detail related to this expression can be found in Appendice I.

For the voltage computation, the new values can be evaluated as:

$$V_g^{(new)} = V_g^{(old)} + \Delta V_g^{(new)} \tag{16}$$

where

$$\Delta V_g^{(new)} = \begin{cases} -I_{OFF} \sum_{b_k \in L_g} Z_k & \text{if } L_g \subset L_n \\ +I_{OFF} \sum_{b_k \in L_g} Z_k & \text{if } L_g \subset L_j \\ V_n^{(\text{new})} - V_m^{(\text{old})} + & \text{if } L_g \subset L_m \text{ and } L_j \subset L_g \\ I_{OFF} \Big(\sum_{b_k \in L_g \Delta L_m} Z_k - \sum_{b_k \in L_n} Z_k - Z_{ON} \Big) & \\ V_e^{(\text{new})} - V_e^{(\text{old})} & \text{if } L_e \subset L_g \text{ and} \\ & x_e \in \text{ loop and } x_g \notin \text{ loop} \\ 0 & \text{otherwise} \end{cases}$$

For the current computation, the new values can be evaluated as:

$$I_k^{(new)} = I_k^{(old)} + \Delta I_k^{(new)} \tag{17}$$

where

$$\Delta I_k^{(new)} = \begin{cases} +I_{OFF} & \text{if } b_k \in L_n \cup b_{ON} \\ -I_{OFF} & \text{if } b_k \in L_m \\ 0 & \text{othewise} \end{cases}$$

Thereby, it is faster to calculate the new objective functions, when the exchange operation (mutation or crossover) occurs than at the end of the performance of GA operations, like in the case of all known GA.

1.11. Selection:

To maintaining the elite invidious having a great fitness function, the selection should be competitive. In this paper, the 'roulette wheel selection' scheme is used where each string lodges an area of the wheel that is equal to the string's share of the total fitness.

1.12. Adaptative Mutation

At start the independent loops will identify, each independent loop contains only one open branch. From the open branch, the candidate branches to be closed $b_\alpha = \{x_k, x_m\}$ will be chosen randomly, *i.e.* that have the probability less to P_m . The branch to be opened b_β will be chosen based on the Dynamic Neighbourhood Strategy (DNS); from a set of branches in $L_k \Delta L_m$. if $s^{(\text{old})}$ is the current configuration, $s^{(\text{new})} = s^{(\text{old})} \otimes (b_\alpha, b_\beta)$ is the **first** new configuration that ensure the inequality :

$$f(s^{(\text{new})}) < f(s^{(\text{old})}) \qquad (18)$$

If (18) cannot be realized we take the best b_β , i.e. that minimize as well as possible $f(\cdot)$.

In other hand, we can use the Maximum Neighbourhood Strategy (MNS); the difference from DNS is that all the $L_k \Delta L_m$ branches will be visited.

To speed up the mutation process, the function $f(\cdot)$ will be calculated only in the modified part of the network (see Appendix 1).

Its purpose is to maintain diversity within the population and inhibit premature convergence. Mutation alone induces a random walk through the search (Fig. 4).

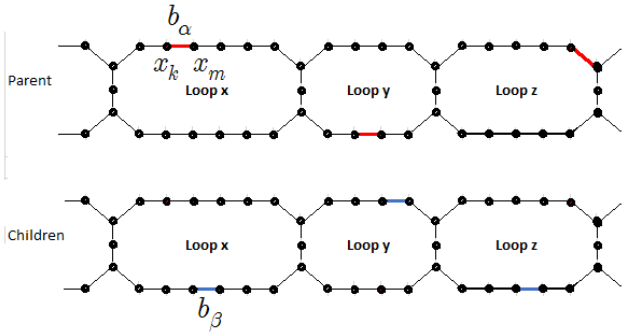

Fig. 4. Mutation in three independent loops (----- open arc in children chromosome, ----- open arc in parent chromosome)

1.13. Adaptative Crossover

The crossover operation is perform on random two invidious (that have the probability less to P_c) from the mating pool and produces two new invidious (children), each will result from one part of the parent string. Mutation gives a technique to generate new information into the knowledge base.

In Figure 4, the crossover with one point is presented, this point is chosen randomly. It is important here to identify the loops to permutated exactly the branches.

For example in Figure 4, if s_1 (where $(b_{1,x}, b_{1,y}, b_{1,z})$ are open) and s_2 (where $(b_{2,x}, b_{2,y}, b_{2,z})$ are open) are the configurations of the parent invidious. Hence, the children invidious will have the following configuration:

$$\begin{aligned} s_1^{'} &= s_1 \otimes (b_{1,y}, b_{2,y}) \otimes (b_{1,z}, b_{2,z}) \\ s_2^{'} &= s_2 \otimes (b_{2,y}, b_{1,y}) \otimes (b_{2,z}, b_{1,z}) \end{aligned} \qquad (19)$$

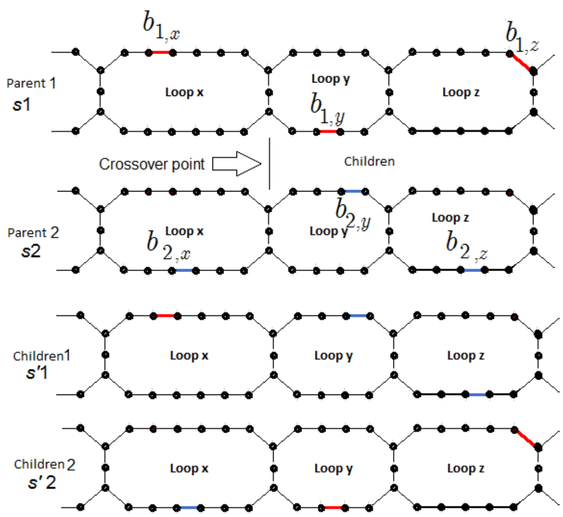

Fig. 5. Crossover with one point in three independent loops (----- open arc in Parent 1 chromosome, ----- open arc in Parent 2 chromosome)

Unlike the simple GA where assessment is done after the crossover and mutation operations, in this work, the evaluation is done along the GA operations. This improves the convergence time of the algorithm.

IV. CASE STUDY

The Distribution power system used is the 106 buses and 03 feeders LV (400 V) network of the R'mali Medea city (Figure 6).

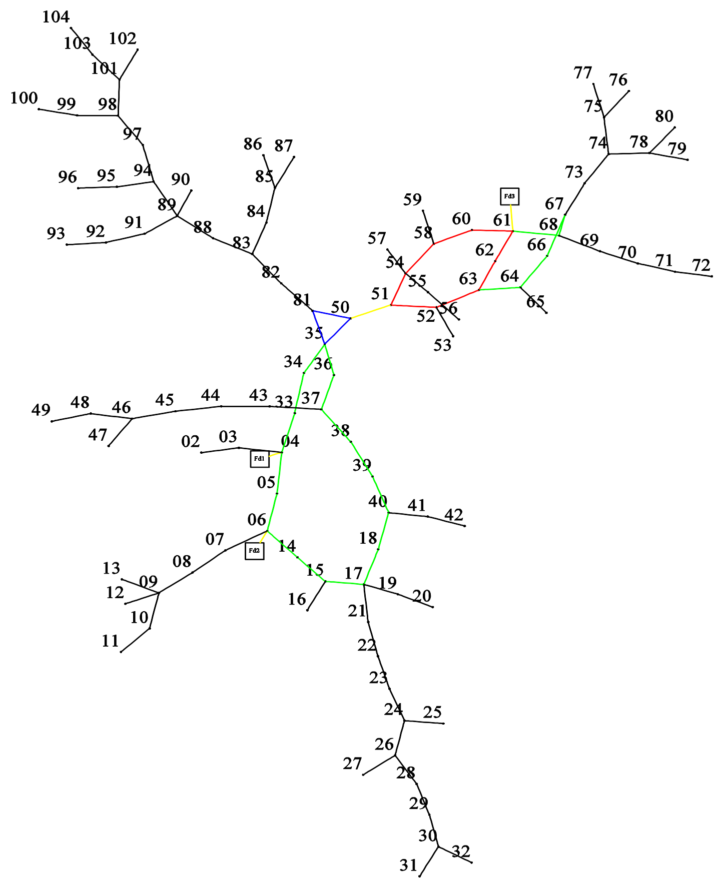

Fig. 6. R'Mali P297 Power system - candidate lines to switch in color

The figure 6 shows that this network contains six dependents loops. Hence, six lines should be switched off. To find these lines, the problem presented in (1) should be resolved, it is clear that it is an Integer Non Linear Programming (INLP) problem. The parameters of the Genetic Algorithms (GA) are described in the Table I.

For the comparison, the classical method BB and is used.

The table 2 confirms the superiority of the EA and EA-DNS then the BB and the classical GA.

Moreover, AGA-DNS is faster than the AGA. In addition, these two methods go to the global optimal solution.

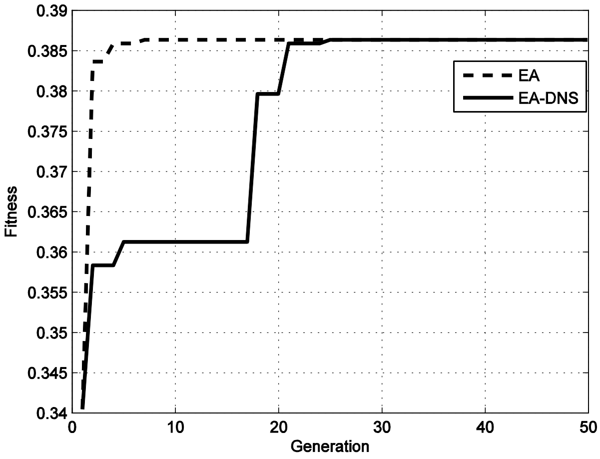

Fig. 7. Comparison EA-DNS and EA

We have evaluated all the C_{35}^{6} solutions and have determine that there are 18640 feasible solutions and the solution obtained with AGA and AGA-DNS is the global salutation

Table 1 EA parameter

Parameter	Values
Population size	20
Maximum generation	50
Crossover probability	0.01
Mutation probability	0.10

It is clear that the three objectives are not conflicting; the optimization of one of these three objectives leads to the optimization of the two others.

The simple GA is very time consuming due to searching on the unfeasible region, whoever EA and EA-DNS can solve this problem.

Table 2 Comparison GA and BB for R'mali DPS

Method	switched off lines	f_R in kW	f_I	f_V
Simple Genetic Algorithms	35-36 52-63 **04-05** **35-50** **50-81** **66-67**	2.7666	1.0765	5.23%
Branch and Bound	51-54 37-38 **04-05** **35-50** **50-81** **66-67**	2.7811	1.0765	5.66%
EA	38-39 35-81 51-52 **04-05** **35-50** **66-67**	2.5883	1.3274	4.92%
EA - DNS	38-39 35-81 51-52 **04-05** **35-50** **66-67**	2.5883	1.3274	4.92%

From figure 7, the AGA-DNS is faster than the AGA this is due to the improvement of the mutation process.

V. CONCLUSION

This paper proposes the application of a met-heuristic optimization technique called Genetic Algorithms (binary coded) for the reconfiguration of the Distribution Power System to minimize the loses, improving the load-ability and the voltage profile. For the comparison the Branch and Bound technique, which is used by the Algerian company, is implemented. To conclude, the GA is clearly superior to B&B.

As the GA is very time consuming; EA and EA-DNS will improve this weakness due to the search tower the feasible space.

REFERENCES

[1] Gonen T. Electric power distribution system engineering. 1st ed. New York: McGraw Hill; 1986.

[2] Sarfi RJ, Salama MMA, Chikhani AY. A survey of the state of the art in distribution system reconfiguration for system loss reduction. Electr Power Syst Res 1994;31(1):61–70.

[3] Kalantar M, Dashti R, Dashti R. Combination of network reconfiguration and capacitor placement for loss reduction in distribution system with based genetic algorithm. In: Proc 41st international universities power engineering conf Newcastle upon Tyne UK, vol. 1; 2006. p. 308–12.

[4] Khator SK, Leung LC. Power distribution planning: a review of models and issues. IEEE Trans Power Syst 1997;12(3):1151–9.

[5] Civanlar S, Grainger JJ, Yin H, Lee SSH. Distribution feeder reconfiguration for loss reduction. IEEE Trans Power Deliver 1988;3(3). p. 1227–3.[6] Baran ME, Wu FF. Network reconfiguration in distribution systems for loss reduction and load balancing. IEEE Trans Power Deliver 1989;4(2):1401–7.

[6] Goswami SK, Basu SK. A new algorithm for the reconfiguration of distribution feeders for loss minimization. IEEE Trans Power Deliver 1992;7(3):1484–91.

[7] Gomes FV, Carneiro Jr S, Pereira JLR, Vinagre MP, Garcia PAN. A new heuristic reconfiguration algorithm for large distribution systems. IEEE Trans Power Syst. 2005;20(3):1373–8.

[8] Gomes FV, Carneiro Jr. S, Pereira JLR, Vinagre MP, Garcia PAN, Oliveira EJ, Araújo LR. A new distribution system reconfiguration approach using optimal power flow technique and sensitivity analysis for loss reduction. In: IEEE Proc Power Engineering Society General Meeting San Francisco, CA, USA, vol. 1(1); 2005. p. 1–5.

[9] Raju GKV, Bijwe PR. An efficient algorithm for loss reconfiguration distribution system based on sensitivity and heuristics. IEEE Trans Power Syst 2008;23(3):1280–7.

[10] Oliveira LW, Oliveira EJ, Carneiro Jr S, Pereira JLR, Costa JS, Silva Jr IC. Optimal reconfiguration and capacitor allocation in radial distribution systems energy losses minimization. Int J Electric Power Energy 2010;32(8):840–8.

[11] Nara K, Shiose A, Kitagawa M, Ishihara T. Implementation of genetic algorithm for distribution systems loss minimum re-configuration. IEEE Trans Syst 1992;7(3):1044–51.

[12] Torres-Jimenez J, Guardado JL, Rivas F, Maximov S, Melgoza E. Reconfiguration of power distribution systems using genetic algorithms and spanning trees Proc electronics, robotics and automotive mechanics conference Cuernavaca morelos Mexico; 210. p. 779–784.

[13] Torres J, Guardado JL, Rivas-Dávalos F, Maximov S, Melgoza E. A genetic algorithm based on the edge window decoder technique to optimize distribution systems reconfiguration. Int J Electric Power Energy 2013, 45(1):28–34.

[14] Mirhoseini S H, Hosseini SM, Ghanbari M, Ahmadi M. A new improved adaptive imperialist competitive algorithm to solve the reconfiguration problem distribution systems for loss reduction and voltage profile improvement Electric Power Energy Syst 2014;55(1):128–43.

[15] Mazza A, Chicco G, Russo A. Optimal multi-objective distribution system reconfiguration with multi criteria decision making-based solution ranking and enhanced genetic operators. Int J Electric Power Energy 2014;54(1):255–67.

[16] Chiang H, Jean-Jumeau R, Optimal network reconfiguration in distribution system. Part 2:

solution algorithms and numerical results. IEEE Trans Deliver 1990;5(3):1568–74.

[17] Zhigang M, Study on distribution network reconfiguration based on genetic simulated annealing algorithm. In: Proc China international conference electricity distribution guangzhou China; 2008. p. 1–7.

[18] Khoa TQD, Phan BTT, Ant colony search-based loss minimum reconfiguration of distribution systems. In: IEEE proc power india conference New Delhi, India; 2006. p. 6.

[19] Mori H, Ogita Y, A parallel tabu search based method for reconfigurations distribution systems. IEEE Proc Summer Power Eng Soc Meeting Seattle USA 2000, 1:73–8.

[20] Mekhamer SF, Abdelaziz AY, Mohammed FM, Badr MAL, A new intelligent optimization technique for distribution systems reconfiguration. In: Proc international middle-east power system conference Aswan Egypt; 2008, 397–401.

[21] Kumar KS, Jayabarathi T. Power system reconfiguration and loss minimization for an distribution systems using bacterial foraging optimization algorithm J Electric Power Energy Syst 2012;36(1):13–7.

[22] Hooshmanda R, Soltanib SH. Simultaneous optimization of phase balancing and reconfiguration in distribution networks using BF–NM algorithm Electric Power Energy Syst 2012;41(1):76–86.

[23] Boardman, J.T.; Meckiff, C.C.,"A Branch and Bound Formulation to an Electricity Distribution Planning Problem", IEEE, Transaction on PAS, Vol. PAS-104, no 8, Aug., 1985, pp. 2112-2117.

[24] Berge, C. "Graphes", Gauthiers-Villars, Paris, 1986.

[25] D. E. Goldberg "Genetic Algorithms in Search, Optimization, and Machine Learning". Addison-Wesley Longman, USA, 1989.

1. Appendices

It is clear that the new value of the power loss $f_{R,k}^{(new)}$ at the branch b_k will be change only in the loop (Fig. 3)

$$f_{R,k}^{(new)} = \begin{cases} R_k(I_k^{(old)} + I_{OFF})^2 & \text{if } b_k \in L_n \\ R_k(I_k^{(old)} - I_{OFF})^2 & \text{if } b_k \in L_m \\ R_k(I_{OFF} - I_{OFF})^2 & \text{if } b_k = b_{OFF} \\ R_k {I_{OFF}}^2 & \text{if } b_k = b_{ON} \\ R_k I_k^{(old)2} & \text{otherwise} \end{cases}$$

Developping this equation

$$f_{R,k}^{(new)} = \begin{cases} R_k I_k^{(old)2} + R_k {I_{OFF}}^2 + 2R_k I_k^{(old)} I_{OFF} & \text{if } b_k \in L_n \\ R_k I_k^{(old)2} + R_k {I_{OFF}}^2 - 2R_k I_k^{(old)} I_{OFF} & \text{if } b_k \in L_m \\ R_k {I_{OFF}}^2 + R_k {I_{OFF}}^2 + 2R_k I_{OFF} I_{OFF} & \text{if } b_k = b_{OFF} \\ 0 + R_k {I_{OFF}}^2 + 0 & \text{if } b_k = b_{ON} \\ R_k I_k^{(old)2} + 0 + 0 & \text{otherwise} \end{cases}$$

The new value of the total loss is

$$f_R^{(new)} = \sum_{k=1}^{M} f_{R,k}^{(new)} = f_R^{(old)} + \sum_{b_k \in L_n \Delta L_m \cup b_{ON}} R_k {I_{OFF}}^2 + 2I_{OFF}\left(\sum_{b_k \in L_n} R_k I_k^{(\text{old})} - \sum_{b_k \in L_m} R_k I_k^{(\text{old})}\right)$$

Now,

$$\Delta f_R^{(new)} = I_{OFF}^2 \Big(\sum_{b_k \in (L_n \Delta L_m \cup b_{ON})} R_k\Big) + 2I_{OFF}\Big(\sum_{b_k \in L_n} R_k I_k^{(\text{old})} - \sum_{b_k \in L_m} R_k I_k^{(\text{old})}\Big)$$

For the voltage; there are five cases :

If $L_g \subset L_n$

$$\begin{aligned} V_g^{(new)} &= V_F - \sum_{b_k \in L_g} Z_k I_k^{(new)} \\ &= V_F - \sum_{b_k \in L_g} Z_k (I_k^{(old)} + I_{OFF}) \\ V_g^{(old)} &= V_F - \sum_{b_k \in L_g} Z_k I_k^{(old)} \end{aligned}$$

Hence ;

$$\begin{aligned} \Delta V_g^{(new)} &= V_g^{(new)} - V_g^{(old)} \\ &= -I_{OFF} \sum_{b_k \in L_g} Z_k \end{aligned}$$

if $L_g \subset L_j$

$$\begin{aligned} V_g^{(new)} &= V_F - \sum_{b_k \in L_g} Z_k I_k^{(new)} \\ &= V_F - \sum_{b_k \in L_g} Z_k (I_k^{(old)} - I_{OFF}) \\ V_g^{(old)} &= V_F - \sum_{b_k \in L_g} Z_k I_k^{(old)} \end{aligned}$$

Hence ;

$$\begin{aligned}\Delta V_g^{(new)} &= V_g^{(new)} - V_g^{(old)} \\ &= +I_{OFF} \sum_{b_k \in L_g} Z_k\end{aligned}$$

If $L_e \subset L_g$ and $x_e \in$ loop and $x_g \notin$ loop

$$\begin{aligned}V_g^{(new)} &= V_e^{(new)} - \sum_{b_k \in L_g \Delta L_e} Z_k I_k^{(new)} \\ V_g^{(old)} &= V_e^{(old)} - \sum_{b_k \in L_g \Delta L_e} Z_k I_k^{(old)}\end{aligned}$$

As in this case; $I_k^{(new)} = I_k^{(old)}$ we will get:

$$\begin{aligned}\Delta V_g^{(new)} &= V_g^{(new)} - V_g^{(old)} \\ &= V_e^{(new)} - V_e^{(old)} \\ &= \Delta V_e^{(new)}\end{aligned}$$

If $L_g \subset L_m$ and $L_j \subset L_g$

$$\begin{aligned}
&\Delta V_g^{(new)} = V_g^{(new)} - V_g^{(old)} \\
&= \left(V_F - \sum_{b_k \in L_g^{(new)}} Z_k I_k^{(new)} \right) - \left(V_F - \sum_{b_k \in L_g^{(old)}} Z_k I_k^{(old)} \right) \\
&= - \sum_{b_k \in L_g^{(new)}} Z_k I_k^{(new)} + \sum_{b_k \in L_g^{(old)}} Z_k I_k^{(old)} \\
&= - \sum_{b_k \in L_{gm}^{(old)}} Z_k I_k^{(new)} - \sum_{b_k \in L_{mn}^{(old)}} Z_k I_k^{(new)} - \sum_{b_k \in L_n^{(old)}} Z_k I_k^{(new)} + \sum_{b_k \in L_g^{(old)}} Z_k I_k^{(new)} \\
&= - \sum_{b_k \in L_{gm}^{(new)}} Z_k (I_k^{(old)} - I_{OFF}) - Z_{ON} I_{OFF} - \sum_{b_k \in L_n^{(new)}} Z_k (I_k^{(old)} + I_{OFF}) + \sum_{b_k \in L_g^{(old)}} Z_k I_k^{(new)} \\
&= - \sum_{b_k \in L_{gm}^{(new)}} Z_k I_k^{(old)} - \sum_{b_k \in L_n^{(new)}} Z_k I_k^{(old)} + \sum_{b_k \in L_g^{(old)}} Z_k I_k^{(old)} + I_{OFF} (\sum_{b_k \in L_{gm}^{(old)}} Z_k - Z_{ON} - \sum_{b_k \in L_n^{(old)}} Z_k) \\
&= V_m^{(old)} - V_g^{(old)} + V_n^{(old)} - V_F^{(old)} + V_F^{(old)} - V_g^{(old)} + I_{OFF} (\sum_{b_k \in L_{gm}^{(old)}} Z_k - Z_{ON} - \sum_{b_k \in L_n^{(old)}} Z_k) \\
&= V_m^{(old)} + V_n^{(old)} + I_{OFF} (\sum_{b_k \in L_g^{(old)} \Delta L_g^{(old)}} Z_k - Z_{ON} - \sum_{b_k \in L_n^{(old)}} Z_k)
\end{aligned}$$

In the last case, there is no change in current and voltages