An Elegant Method of Analysis for the BJT Amplifiers using Floating Admittance Matrix

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Abstract—The Floating Admittance Matrix (FAM) is an elegant, neat, illustrative, and simplified technique for analyzing all configurations of the BJT amplifiers, starting with the maneuvering of the FAM of the phase-splitter circuit. The conventional analysis method requires a small-signal equivalent circuit, and then conventional tools, either KCL, KVL, or Thevenin, Norton, etc., are used for the analysis. The researcher has to guess which conventional tool suites better than the other for any particular circuit, whether active or passive. The proposed technique is equally ell useful for all circuits. In the FAM method, once the device matrix is known rest of the circuit can be embedded in it by inspection. The sum property of this matrix provides a check to know whether FAM has been written correctly to proceed further.

Keywords—BJT, CB, CE, CC, Floating Admittance Matrix (FAM), hybrid-model, hybrid- π model, T-model.

I. NTRODUCTION

There are several methods of mathematical modelling available in the literature based on the equivalent circuit approach. Chirlian [1] suggested a very general approach; wherein any three-terminal devices can fit in. Sedra [2]-[6] have provided unique approaches to the equivalent circuit of the BJTs. We have to select the proper mathematical modelling scheme under given a constraint to achieve the best result with ease. The nullor [7]-[8] and admittance matrix [9] methods have been used in the symbolic form extensively in the past. We have suggested an elegant mathematical modelling approach for both active devices and passive circuits, called the floating admittance matrix model. As the word spelt floating, it does not have any reference terminal in the analysis and design of any circuit, whether active or passive or mixed of the two. The floating admittance matrix (FAM) model has been developed for the BJT to demonstrate the simplicity of the method over other conventional techniques. The outcome of the developed mathematical models has been used for any amplifier configuration that corroborates the result obtained in the available literature, overriding in the simplicity.

The background of the article forms the basis for arriving at the solution of circuits containing two or more than two BJTs or even MOSFETs and BJTs in different configurations. It is likely that one can make mistakes in putting the smallsignal equivalent of the entire complicated circuit, which can lead to an erroneous solution. Additionally, a researcher must be able to guess for a particular circuit which conventional tools KCL, KVL, Thevenin, and Norton etc., becomes suitable for arriving at the solution with ease. Solution of even passive circuits such as twin-T, bridge-T, and lattice networks run into pages using conventional tools. On the contrary, the same FAM is used for all types of circuits, whether simple or complicated, taking the help of matrix partitioning in the case of complicated circuits.

The history of the development of FAM using hybrid small-signal equivalent circuits of the devices and its applications in deriving all types of transfer and self-node functions of any amplifier configuration dates back to 1978. However, we developed the FAM of the active device using recent trends in the small-signal model of the devices i.e. T- and hybrid- π , and used it in arriving at the solutions of all

types of transfer functions of different configurations of the amplifier, with ease, in this article.

We aim to derive all types of transfer functions of all BJT configurations, starting with a properly biased ac circuit of a phase-splitter amplifier shown in Fig.1.

Before we start the analysis of any configurations of the BJT amplifier, we develop the floating admittance matrix in resistance (r_e) , in hybrid- π , and in hybrid (h_e) models of the BJT.



Fig. 1 BJT Phase-Splitter Amplifier

Among these models, the hybrid- π model is versatile. For this reason, we take up the development of the floating admittance matrix starting with T-model, then in r_{π} and finally in h_e model. To indicate the superiority of the floating admittance matrix approach over the convention method of analysis, we derive all types of transfer functions of the amplifiers for a comparison.

II MODEL DEVELOPMENT OF BJT

A. FAM of BJT in T– Model

Figure 2 (a) simplifies as Fig. 2 (b) with forward-biased dynamic resistance of the base-emitter junction diode along with a controlled current source βi_b [1]-[6].



Fig. 2 Common emitter configuration of the BJT

The input and output side of the BJT in Fig. 2 (b) has to be isolated for the ease in the analysis. The isolation of input side to output in Fig. 2 (b) is achieved using the generator splitting theorem as in Fig. 3.



Fig. 3 Splitting the current source

The currents at the base point are the same before and after splitting the current source in Fig. 3. The algebraic sum of current at the base point $= i_b + \beta i_b = (1 + \beta)i_b$; which is equal to the emitter current i_e that flows through the dynamic resistance of the diode r_e and then the controlled source βi_b is left open, resulting Fig. 4.



Fig. 4 Input side and output sides isolated

The input resistance is seen through the base terminal of the CE configuration in Fig. 4 results as;

$$v_{be} = r_e i_e = r_e (1+\beta) i_b$$

$$r_{in(CE)} = \frac{v_{be}}{i_b} = \frac{r_e (1+\beta) i_b}{i_b} = (1+\beta) r_e = r_\pi$$
(1)

The input resistance in CB configuration is obtained by looking into the input of the amplifier through the emitter terminal and is given as;

$$r_{in(CB)} = \frac{v_{eb}}{i_e} = r_e \tag{2}$$

The reverse-biased collector-base junction offers very large output resistance r_o , obtained using the Early voltage equation. This r_o is included in Fig. 4 to complete the large-signal equivalent circuit of the BJT. The derivation of the floating admittance matrix of the BJT assumes three digits 1, 2, and 3, assigning it to the base, the collector, and the emitter terminals, respectively. Hence, all terminal voltages and currents are expressed using these three digits in further analysis. Form Fig. 4,

$$\begin{split} i_{b} &= i_{1} = \frac{1}{(1+\beta)r_{e}}v_{1} + (0)v_{2} - \frac{1}{(1+\beta)r_{e}}v_{3} \\ &= \frac{g_{e}}{(1+\beta)}v_{1} + (0)v_{2} - \frac{g_{e}}{(1+\beta)}v_{3} \qquad (3) \\ i_{c} &= i_{2} = \beta i_{b} + g_{0}(v_{ce}) = \beta i_{1} + g_{0}(v_{c} - v_{e}) \\ &= \beta \left\{ \frac{g_{e}}{(1+\beta)}v_{1} - \frac{g_{e}}{(1+\beta)}v_{3} \right\} + g_{0}(v_{2} - v_{3}) \\ i_{c} &= i_{2} = \frac{\beta g_{e}}{1+\beta}v_{1} - \frac{\beta g_{e}}{1+\beta}v_{3} + g_{0}(v_{2} - v_{3}) \\ &= g_{e}v_{1} + g_{0}v_{2} - g_{e}v_{3} + g_{0}(-v_{3}) \\ i_{2} &= i_{c} = g_{e}v_{1} + g_{0}v_{2} - \{g_{e} + g_{0}\}v_{3} \qquad (4) \end{split}$$

 $\begin{array}{l} \text{Applying KCL, } i_{1} + i_{2} + i_{3} = 0, \\ i_{3} = -i_{1} - i_{2} \\ = -\frac{g_{e}}{(1+\beta)}v_{1} + \frac{g_{e}}{(1+\beta)}v_{3} - g_{e}v_{1} - g_{0}v_{2} + \{g_{e} + g_{0}\}v_{3} \\ i_{3} = -\left\{g_{e} + \frac{g_{e}}{1+\beta}\right\}v_{1} - g_{0}v_{2} + \left\{g_{e} + \frac{g_{e}}{1+\beta} + g_{0}\right\}v_{3} \tag{5}$

Arranging Eq (3) to Eq (5) in the matrix form yields the floating admittance matrix [10]-[14] of BJT in $g_e = 1/r_e$ model as;

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \frac{g_e}{1+\beta} & 0 & -\frac{g_e}{1+\beta} \\ g_e & g_o & -g_e - g_o \\ \frac{g_e}{1+\beta} - g_e & -g_o & \frac{g_e}{1+\beta} + g_e + g_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$
(6)

The simplified floating admittance matrix of the BJT in r_e

model is;

$$\begin{bmatrix} g'_{e} & 0 & -g'_{e} \\ g_{e} & g_{o} & -g_{e} - g_{o} \\ g'_{e} - g_{e} & -g_{o} & g'_{e} + g_{e} + g_{o} \end{bmatrix}$$
(7)
here $r_{e} = \frac{v_{be}}{i_{e}} = \frac{\frac{v_{be}}{i_{c}}}{\frac{i_{e}}{i_{c}}} = \left(\frac{v_{be}}{i_{c}}\right) \left(\frac{i_{c}}{i_{e}}\right) = \frac{\alpha}{g_{m}},$
 $g'_{e} = \frac{g_{e}}{(1+\beta)}, g_{e} = \frac{1}{r_{e}}, g_{0} = \frac{1}{r_{0}}$ (8)

B. FAM OF BJT IN r_{π} MODEL

Fig. 5 is the simplified circuit of **hybrid**- π model of the BJT [2]-[3]. The floating admittance matrix of hybrid- π can be obtained by analyzing Fig. 5.

From Fig. 5,

$$\begin{split} &i_1 = g_\pi v_\pi = g_\pi (v_1 - v_3) = g_\pi v_1 + (0) v_2 - g_\pi v_3 \quad (9) \\ &i_2 = g_m v_\pi + g_o (v_2 - v_3) = g_m (v_1 - v_3) + g_o (v_2 - v_3) \\ &= g_m v_1 + g_o v_2 - (g_m + g_o) v_3 \quad (10) \\ &i_3 = -g_\pi v_1 + g_\pi v_3 - g_m v_1 - g_o v_2 + (g_m + g_o) v_3 \\ &= -(g_\pi + g_m) v_1 - g_o v_2 + (g_\pi + g_m + g_o) v_3 \quad (11) \end{split}$$

Arranging (9)-(11) yields the matrix of hybrid- π as;

$$\begin{bmatrix} i_1\\i_2\\i_3 \end{bmatrix} = \begin{bmatrix} g_{\pi} & 0 & -g_{\pi}\\g_m & g_o & -g_m - g_o\\-g_{\pi} - g_m & -g_o & g_{\pi} + g_m + g_o \end{bmatrix} \begin{bmatrix} v_1\\v_2\\v_3 \end{bmatrix}$$
(12)



Fig.5 Hybrid- π equivalent circuit

C. FAM OF BJT IN H-PARAMETER MODEL

For the development of the floating admittance matrix of the BJT in the h-parameter model [1]-[5], Fig. 6 is drawn and from this figure,

$$\begin{aligned} v_1 - v_3 &= h_i i_1 - h_r (v_2 - v_3) \\ i_1 &= \frac{v_1}{h_i} - \frac{v_3}{h_i} + \frac{h_r}{h_i} v_2 - \frac{h_r}{h_i} v_3 \\ &= g_i v_1 - g_i v_3 + g_4 h_F v_2 - g_4 h_F v_3 \\ i_1 &= g_i v_1 + (0) v_2 - g_i v_3 \end{aligned}$$
 (13)



Fig. 6 h-parameter model of the BJT

$$\begin{split} i_{2} &= h_{f}i_{1} + g_{o}(v_{2} - v_{3}) \\ &= h_{f}g_{i}v_{1} - h_{f}g_{i}v_{3} + g_{o}(v_{2} - v_{3}) \\ &= g_{m}v_{1} - g_{m}v_{3} + g_{o}(v_{2} - v_{3}) \\ i_{2} &= g_{m}v_{1} + g_{o}v_{2} - (g_{m} + g_{o})v_{3} \qquad (14) \\ i_{3} &= -i_{1} - i_{2} \\ &= -(g_{i} + g_{m})v_{1} - g_{o}v_{2} + (g_{i} + g_{m} + g_{o})v_{3} \qquad (15) \\ \text{Arranging Eqs [13]-[15] yields the floating admittance matrix as} \\ \begin{bmatrix} i_{1} \\ i_{2} \\ i_{3} \end{bmatrix} = \begin{bmatrix} g_{i} & 0 & -g_{i} \\ g_{m} & g_{o} & -g_{m} - g_{o} \\ -g_{i} - g_{m} & -g_{o} & g_{i} + g_{m} + g_{o} \end{bmatrix} \begin{bmatrix} v_{1} \\ v_{2} \\ v_{3} \end{bmatrix} \qquad (16) \end{split}$$

The floating admittance matrix of BJT in r_e , r_{π} , and h_e Models are available in Eqs (7), (12), and (16), respectively.

III. BJT PHASE SPLITTER AMPLIFIER

To analyze the BJT phase-splitter circuit of Fig. 1, the floating admittance matrix of the BJT in the hybrid- π model of Eq (12) is used as it is versatile. The overall floating admittance matrix of active and passive elements present in Fig. 1 results [10]-[17] as;

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ g_{\pi} + g_{s} + G_{B} & 0 & -g_{\pi} & -g_{s} - G_{B} \\ g_{m} & g_{o} + G_{L} & -g_{m} - g_{o} & -G_{L} \\ -g_{\pi} - g_{m} & -g_{o} & g_{\pi} + g_{m} + g_{o} + G_{E} & -G_{E} \\ -g_{s} - G_{B} & -G_{L} & -G_{E} & g_{s} + G_{B} + G_{E} + G_{L} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$
where, $g_{\pi} = \frac{i_{b}}{v_{be}}$ and
 $g_{e} = \frac{(1+\beta)i_{b}}{v_{eb}} = (1+\beta)\frac{i_{b}}{v_{eb}} = (1+\beta)g_{\pi}$, and $r_{\pi} = (1+\beta)r_{e}$,
 $G_{L} = G_{C} + G_{L1}$

IV. COMMON EMITTER AMPLIFIER

To obtain the floating admittance matrix of the common emitter amplifier from the floating admittance matrix of the phase-splitter circuit of Eq (17), its 3rd row and 3rd column are added to its 4th row and 4th column. Now, deleting the original 3rd row and 3rd column and assigning the 4th row and 4th column as 3rd row and 3rd column yields the matrix;

$$\begin{bmatrix} 1 & 2 & 3 \\ g_{\pi} + g_{s} + G_{B} & 0 & -g_{\pi} - g_{s} - G_{B} \\ g_{m} & g_{o} + G_{L} & -g_{m} - g_{o} - G_{L} \\ -g_{\pi} - g_{m} - g_{s} - G_{R} & -g_{o} - G_{L} & g_{\pi} + g_{m} + g_{o} + g_{s} + G_{R} + G_{L} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
(18)

This Eq (18) is the floating admittance matrix of Common Emitter Amplifier in hybrid- π model shown in Fig. 7.

The voltage gain between its output terminals 2 & 3 and input terminals 1 & 3 of a common emitter amplifier in Fig. 7 results [10]-[17] as;

$$A_{\nu}|_{13}^{23} = sgn(2-3)sgn(1-3)(-1)^{9}\frac{|Y_{23}^{13}|}{|Y_{13}^{13}|}$$
(19)



(28)

Fig. 7 Common Emitter Amplifier

$$|Y_{23}^{13}| = g_m, |Y_{13}^{13}| = g_0 + G_L$$
$$A_v|_{13}^{23} = -\frac{g_m}{g_0 + G_L} = -g_m(r_0||R_L) \cong -g_m R_L$$
(20)

The input resistance between terminals 1 & 3 of a common emitter amplifier in Fig. 7 results [10]-[13], [16]-[17]] as;

$$R_{i(13)} = \frac{|Y_{13}^{18}|}{|Y_{3}^{2}|}_{g_{s}=0}$$
(21)
$$|Y_{13}^{13}| = g_{0} + G_{L}$$
$$|Y_{3}^{3}|_{g_{s}=0} = \begin{vmatrix} g_{\pi} + G_{B} & 0 \\ g_{m} & g_{0} + G_{L} \end{vmatrix}$$
$$= (g_{\pi} + G_{B})(g_{0} + G_{L}) = (g_{\pi} + G_{B})(g_{0} + G_{L})$$
$$R_{i(13)} = \frac{|Y_{13}^{13}|}{|Y_{2}^{2}|}_{g_{s}=0} = \frac{g_{0} + G_{L}}{(g_{\pi} + G_{B})(g_{0} + G_{L})} = r_{\pi} \parallel R_{B} \cong r_{\pi}$$
(22)

The output resistance between terminals 2 & 3 of common emitter amplifier in Fig. 7 results [10]-[17] as;

$$R_{O(23)} = \frac{|Y_{23}^{23}|}{|Y_{3}^{3}|}_{G_{L}=0}$$

$$|Y_{23}^{23}| = g_{\pi} + g_{s} + G_{B}$$

$$|Y_{3}^{3}|_{G_{L}=0} = (g_{\pi} + g_{s} + G_{B})g_{O}$$

$$R_{O(23)} = \frac{g_{\pi} + g_{s} + G_{B}}{(g_{\pi} + g_{s} + G_{B})g_{O}} = \frac{1}{g_{O}} = r_{O}$$
(23)
(23)

The current gain between output terminals 2 & 3 and input terminals 1 & 3 of common emitter amplifier in Fig. 7 results [10]-[12] as;

$$A_{i}|_{13}^{23} = sgn(2-3)sgn(1-3)(-1)^{9} \frac{|Y_{23}^{13}|}{|Y_{3}^{3}|} G_{L} \quad (25)$$

$$A_{i}|_{13}^{23} = -\frac{|Y_{23}^{13}|}{|Y_{3}^{3}|} G_{L} = -\frac{g_{m}}{(g_{\pi}+g_{s}+G_{B})(g_{0}+G_{L})} G_{L} \quad = -g_{m} \left(\frac{1}{g_{\pi}}\right) = -\beta \quad (26)$$

The power gain between output terminals 2 & 3 and input terminals 1 & 3 of a common emitter amplifier in Fig. 7 results [10]-[11] as;

$$A_P|_{13}^{23} = A_v|_{13}^{23} x A_i|_{13}^{23} = (-g_m R_L)(-\beta)$$

= $\beta g_m R_L$ (27)

V. COMMON COLLECTOR AMPLIFIER

To derive the floating admittance matrix of the common collector amplifier from the floating admittance matrix of the phase-splitter circuit of (17), its 2^{nd} row and 2^{nd} column are added to the 4^{th} row and 4^{th} column. Now, the original 2^{nd} row and 2^{nd} column are deleted after assigning 4^{th} row, and 4^{th} column as 2^{nd} row and 2^{nd} column yields the matrix as;

$$\begin{bmatrix} 1 & 3 & 2 \\ g_{\pi} + g_{s} + G_{B} & -g_{\pi} & -g_{s} - G_{B} \\ -g_{\pi} - g_{m} & g_{\pi} + g_{m} + g_{o} + G_{E} & -g_{o} - G_{E} \\ g_{m} - g_{s} - G_{R} & -g_{m} - g_{o} - G_{R} & g_{n} + g_{s} + G_{R} + G_{R} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{aligned} A_{\nu}|_{12}^{32} &= sgn(3-2)sgn(1-2)(-1)^{8} \frac{|Y_{32}^{*}|}{|Y_{12}^{*}|} \\ |Y_{32}^{12}| &= -g_{\pi} - g_{m} = -g_{\pi}(1+\beta) \\ |Y_{12}^{12}| &= g_{\pi} + g_{m} + g_{o} + G_{E} \cong g_{\pi} + g_{m} + G_{E} \end{aligned}$$

$$\begin{aligned} A_{\nu}|_{12}^{32} &= -\frac{-(1+\beta)g_{\pi}}{(1+\beta)g_{\pi}+G_{E}} \frac{R_{E}}{R_{E}+\frac{r_{\pi}}{(1+\beta)}} = \frac{R_{E}}{R_{E}+\frac{(1+\beta)r_{e}}{(1+\beta)}} \\ &= \frac{R_{E}}{R_{E}+r_{o}} \cong 1 \end{aligned}$$
(30)

Equation (30) defines the voltage gain of the emitter follower, i.e., common collector amplifier.

The current gain between output terminals 3 & 2 and input terminals 1 & 2 of a common collector amplifier in Fig. 8 results [10]-[11] as;

$$A_i|_{12}^{32} = sgn(3-2)sgn(1-2)(-1)^{8}\frac{|Y_{32}^{*}|}{|Y_2^{*}|}G_E \quad (31)$$



Fig. 8 Common Collector Amplifier

$$\begin{aligned} |Y_{32}^{12}| &= -g_{\pi} - g_{m} = -(1+\beta)g_{\pi} \\ |Y_{2}^{2}| &= \begin{vmatrix} g_{\pi} & -g_{\pi} \\ -g_{\pi} - g_{m} & g_{\pi} + g_{m} + g_{o} + G_{E} \end{vmatrix} \\ &= \begin{vmatrix} g_{\pi} & 0 \\ -g_{\pi} - g_{m} & g_{o} + G_{E} \end{vmatrix} = g_{\pi}G_{E} \\ A_{i}|_{12}^{32} &= -\frac{-(1+\beta)g_{\pi}}{g_{\pi}G_{E}}G_{E} = 1+\beta \end{aligned}$$
(32)

Equation Eq (32) is the ideal current gain of the emitter follower circuit.

The input resistance between terminals 1 & 2 in Fig. 8 using Eq (28) results [10]-[13], [16]-[17]] as;

$$R_{i(12)} = \frac{|Y_{12}^{12}|}{|Y_{2}^{2}|}_{g_{s}=0}$$
(33)
$$|Y_{12}^{12}| = (1+\beta)g_{\pi} + g_{0} + G_{E}$$

$$|Y_2^2|_{g_s=0} = \begin{vmatrix} g_{\pi} & -g_{\pi} \\ -g_{\pi} - g_m & g_{\pi} + g_m + g_o + G_E \end{vmatrix}$$
$$= \begin{vmatrix} g_{\pi} & 0 \\ -g_{\pi} - g_m & g_o + G_E \end{vmatrix} = g_{\pi}(g_0 + G_E)$$

$$R_{i(12)} = \frac{(1+\beta)g_{\pi} + g_0 + G_E}{g_{\pi}(g_0 + G_E)} = \frac{(1+\beta)g_{\pi} + G_E}{g_{\pi}G_E}$$
$$= r_{\pi} + (1+\beta)R_E$$
(34)

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The output resistance between terminals 3 & 2 of a common collector amplifier in Fig. 8 results [10]-[13], [16]-[17]] as;

$$R_{O(32)} = \frac{|Y_{32}^{32}|}{|Y_{2}^{2}|}_{G_{E}=0}$$
(35)
$$|Y_{32}^{32}| = g_{\pi} + g_{s} + G_{B} = g_{\pi} + g_{s}$$

$$|Y_{2}^{2}|_{G_{E}=0} = \begin{vmatrix} g_{\pi} + g_{s} & -g_{\pi} \\ -g_{\pi} - g_{m} & g_{\pi} + g_{m} + g_{0} \end{vmatrix}$$
$$= \begin{vmatrix} g_{\pi} + g_{s} & g_{s} \\ -g_{\pi} - g_{m} & g_{0} \end{vmatrix} = (g_{\pi} + g_{s})g_{0} + (1 + \beta)g_{\pi}g_{s}$$

$$\begin{split} R_{O(32)} &= \frac{g_{\pi} + g_{s}}{(g_{\pi} + g_{s})g_{0} + (1+\beta)g_{\pi}g_{s}} \\ G_{O(32)} &= \frac{1}{R_{O(32)}} = \frac{(g_{\pi} + g_{s})g_{0} + (1+\beta)g_{\pi}g_{s}}{g_{\pi} + g_{s}} \\ &= g_{0} + \frac{(1+\beta)g_{\pi}g_{s}}{g_{\pi} + g_{s}} = g_{0} + \frac{(1+\beta)}{r_{\pi} + r_{s}} \\ R_{O(32)} &= \frac{r_{\pi} + r_{s}}{1+\beta} \end{split}$$
(36)

Power gain = $A_{\nu}|_{12}^{32} x A_i|_{12}^{32} = \left(\frac{R_E}{R_E + r_e}\right) (1 + \beta)$ (37)

VI. COMMON BASE AMPLIFIER

To derive the circuit and floating admittance matrix of a common base amplifier in hybrid- π model from the circuit of a phase-splitter shown in Fig. 1 and its (17), its 1st row and 1st column are added to the 4th row and 4th column. Now, the original 1st row and 1st column are deleted, and 4th row and 4th column are assigned 1st row and 1st column with $G_E = G_E + g_s$ yields;

$$\begin{bmatrix} 2 & 3 & 1 \\ g_{o} + G_{L} & -g_{m} - g_{o} & g_{m} - G_{L} \\ -g_{o} & g_{m} + g_{\pi} + g_{o} + g_{s} + G_{E} & -g_{m} - g_{\pi} - g_{s} - G_{E} \\ -G_{L} & -g_{\pi} - g_{s} - G_{E} & g_{\pi} + g_{s} + G_{E} + G_{L} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$
(38)

The circuit of Fig. 9 of a common base amplifier results from (38). The BJT base terminal is common to both input and output sides in Fig. 9, and it is called a common base amplifier.



Fig. 9 Common Base Amplifier

The voltage gain between its output terminals 2 & 1 and input terminals 3 & 1 of a common base amplifier in Fig. 9 results [10]-[14] as;

$$A_{\nu}|_{31}^{21} = sgn(2-1)sgn(3-1)(-1)^{7} \frac{|Y_{21}^{31}|}{|Y_{31}^{31}|}$$
(39)
$$|Y_{21}^{31}| = -g_m - g_0, |Y_{31}^{31}| = g_0 + G_L$$

$$A_{\nu}|_{31}^{21} = -\frac{-g_m - g_0}{g_0 + G_L} = \frac{g_m}{G_L} = g_m R_L$$
(40)

The current gain between its output terminals 2 & 1 and input terminals 3 & 1 of a common base amplifier in Fig. 9 using (38) results [10]-[11] as;

$$\begin{aligned} A_{i}|_{31}^{21} &= sgn(2-1)sgn(3-1)(-1)^{7} \frac{|Y_{21}^{21}|}{|Y_{1}^{1}|} G_{L} \quad (41) \\ |Y_{1}^{1}| &= \begin{vmatrix} g_{0} + G_{L} & -g_{m} - g_{0} \\ -g_{\pi} & (1+\beta)g_{\pi} + g_{0} + g_{s} \end{vmatrix} \\ &= (1+\beta)g_{\pi}G_{L} \\ A_{i}|_{31}^{21} &= -\frac{-\beta g_{\pi} - g_{\Theta}}{(1+\beta)g_{\pi}G_{L}} G_{L} = \frac{\beta}{(1+\beta)} = \alpha \end{aligned}$$

The input resistance between terminals 3 & 1 of a common base amplifier in Fig. 9 results [10]-[13], [16]-[17]] as;

$$R_{i} = R_{i(31)} = \frac{|r_{31}^{31}|}{|r_{1}^{1}|}_{g_{s}=0}$$

$$|Y_{31}^{31}| = g_{0} + G_{L} \qquad (43)$$

$$|Y_{1}^{1}|_{g_{s}=0} = \begin{vmatrix} g_{0} + G_{L} & -\beta g_{\pi} - g_{0} \\ -g_{0} & (1+\beta) g_{\pi} + g_{0} + g_{s} + G_{E} \end{vmatrix}$$

$$= \{(1+\beta)g_{\pi} + G_{E}\}G_{L} = \{r_{\pi} + (1+\beta)R_{E}\}G_{L}g_{\pi}G_{E}$$

$$R_{i} = R_{i(31)} = \frac{g_{0} + G_{L}}{\{r_{\pi} + (1+\beta)R_{E}\}G_{L}g_{\pi}G_{E}}$$
$$= \frac{r_{\pi}}{(1+\beta)} = \frac{(1+\beta)r_{e}}{(1+\beta)} = r_{e} = \frac{1}{g_{m}}$$
(44)

The output resistance between terminals 2 & 1 of a common base amplifier in Fig. 9 results [10]-[13], [16]-[17]] as;

$$R_{O(21)} = \frac{|Y_{21}^{21}|}{|Y_1^1|}_{G_L = 0} \tag{45}$$

$$|Y_{21}^{21}| = (1+\beta)g_{\pi} + g_0 + g_s + G_E$$

= { $r_{\pi} + (1+\beta)R_E$ } $g_{\pi}G_E$

$$\begin{aligned} |Y_1^{-}|_{G_L=0} &= \begin{vmatrix} -g_0 & g_m + g_\pi + g_0 + g_s + G_E \end{vmatrix} \\ &= \begin{vmatrix} g_0 & -g_m - g_\pi \\ 0 & g_\pi + g_s + G_E \end{vmatrix} \end{aligned}$$

$$R_{O} = R_{O(21)} = \frac{\{r_{\pi} + (1+\beta)R_{E}\}g_{\pi}G_{E}}{g_{O}(g_{\pi}+G_{E})} = \frac{\{r_{\pi} + (1+\beta)R_{E}\}}{g_{O}(r_{\pi}+R_{E})}$$
$$= \frac{r_{\pi} + (1+\beta)R_{E}}{g_{O}R_{E}} = \frac{(1+\beta)R_{E}}{g_{O}R_{E}} = (1+\beta)r_{O}$$
(46)

If we look at the output resistance through R'_L , then

$$R_{O(21)} = (1+\beta)r_0 \mid \mid R_C \cong R_C \tag{47}$$

Power gain =
$$A_p = A_v x A_i = g_m \alpha R_L$$
 (48)

VII. CONCLUSIONS

The floating admittance matrix method of analysis is very simple. Anybody with little knowledge of electronics and the matrix maneuvering process can quickly analyze the circuit. Additionally, the floating admittance matrix's zero-sum property provides a self-check before further analyzing the circuit. The method does not require a small-signal equivalent circuit. All transfer functions of any circuit use only cofactors ratios so that calculation can be accurate without any approximation.

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