New method calculating water residence time for trihalomethane in a water supply network

Teruji Sekozawa, Kazuaki Masuda, and Tomohiro Murata

***Abstract*—**We present a valuation method for the purpose of including information systems in divestment value, something not considered heretofore, to reveal the potential value of information systems in corporate management. . We propose a method for calculating water residence time at each node of a complicated water supply network. Trihalomethanes, which are carcinogenic, are known to be formed in tap water in water supply networks, and there are calls for measures to maintain the safety of water supplies. Trihalomethane formation is known to be associated with the time chlorine, which is injected as a disinfectant at water purification plants, is in contact with organic substances trihalomethane precursors present in raw water.

In this paper, because of its necessity for determining the state of trihalomethane formation, we propose a new solution algorithm for analytically determining water residence time in a water supply network in order to ascertain the time as represented by residence time that chlorine and organic matter are together, which is what determines trihalomethane formation. However, because the volumetric flow rate and velocity of water flowing in a water supply network fluctuate over time, the water residence time in the network cannot be obtained simply by determining the paths leading to a node in the network and adding up the pipe flow times along those paths. To get around this problem, we propose a method for calculating flow time and volume, path by path, for the flow of water from a supply point as water purification plant or distribution reservoir in a water supply network in unit time intervals.

***Keywords*—**Trihalomethanes, Residence time, Large-scale water supply networks, Flow time, Pipe network analysis, Water supply safety

I. Introduction

In this paper, we propose a method for calculating water residence time in a complicated water supply network with a pipe flow rate that fluctuates with time.

Water supplies are operated according to the important mission of providing water, a necessity of life, steadily and safely [1][2]. With recent worsening of the quality of raw water and increase in consumers' safety awareness, water suppliers' interest in safety, in particular, has greatly increased, and there is demand for more sophisticated quality control of tap water [3][4]. Recent studies have reported that trihalomethanes, which are carcinogens, are formed in tap water in water supply networks, and so they have become the subject of calls for measures to maintain the safety of water supplies [5]. Amount of trihalomethane formation is known to be associated with the time chlorine, which is injected as a disinfectant at water purification plants, is in contact with organic substances (trihalomethane precursors) present in raw water, and determining water residence time in a water supply network is critical for taking steps to reduce trihalomethane formation [6][7].

Conventionally, water suppliers obtain water residence time in a water supply network by measuring factors such as chlorine concentration and electric conductivity over a predetermined period at two positions, located upstream and downstream relative to each other, in a network and comparing the obtained time-series data. For example, as shown in Fig. 1, the electric conductivity time-series data at the distribution reservoir, an upstream position, and at a certain node in the network, a downstream position, are compared, and the difference in the times when similar electric conductivity was obtained gives the flow time between the two positions, that is, the water residence time in the network at the downstream position [8]. However, for this method, measurement using a sensor is always required, and it is impossible to apply this method to calculate water residence times for an entire network covering a wide area. Also, for a node at which water arrives from multiple distribution reservoirs, similar time-series data cannot be obtained because the above-described comparison method cannot be used. In addition, as will be described later, we have been unable to find in the literature an analytic method for determining water residence time in a network using the results of a pipe network analysis. In order to solve these problems, we propose in this study an analytic method for determining water residence time at each node of a complicated water supply network.

In Section Ⅱ, we clarify the problems that need to be solved in order to calculate water residence time in a water supply network. In Section Ⅲ, we propose an analytical method for determining water residence time at each node of a network. In Section Ⅳ, we present the results of a simulation of our proposed method and discuss the results.

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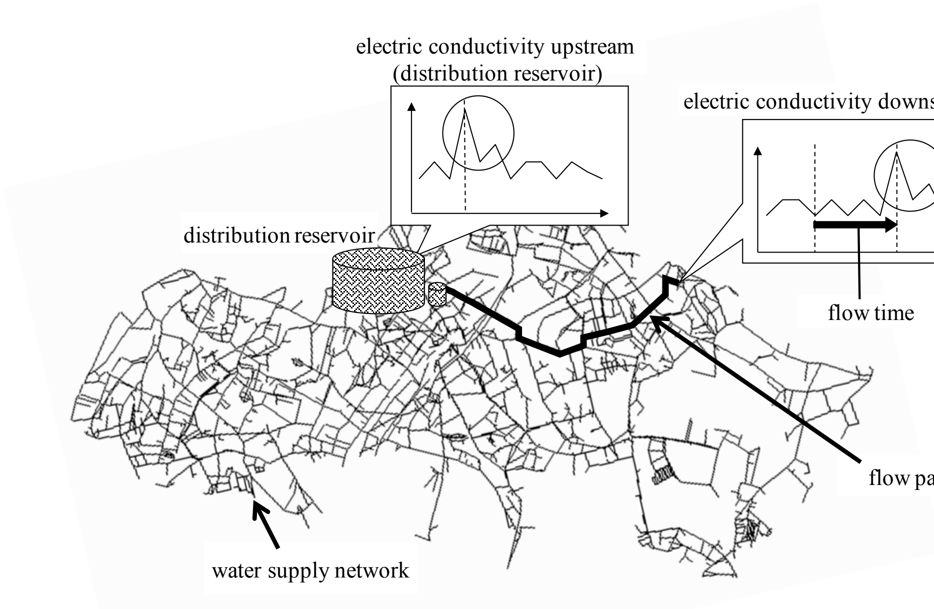


Fig. 1 Typical method for measuring the flow time of water

II. Motivation And Problems Of Calculating Residence Time

In this section, we will explain the motivation for calculating water residence time in a water supply network and clarify the problems faced.

A. Background and the motivation for calculating residence time

Recently, the quality of water in rivers and lakes, which are water supply sources, has been deteriorating because of, for example, the increasingly substantial influx of household waste water. This makes the injection of chlorine as a disinfectant necessary. However, recent reports have revealed that chloroform ($CHCl_3$), a carcinogen, is formed when river water is treated with chlorine. Other trihalomethanes, chemical compounds in which three of the four hydrogen atoms of a methane (CH_4) molecule have been replaced by halogen atoms, are also known to be carcinogenic. Chloroform is the most common trihalomethane formed in tap water, but in general, trihalomethanes are formed in tap water as a result of the chemical reactions between organic substances (trihalomethane precursors) present in raw water and chlorine injected for prechlorination (for the improvement of the efficiency of filtration and the removal of ammoniacal nitrogen) and postchlorination (for disinfection) at water purification plants. Because a variety of organic trihalomethane precursors exist, a complete picture of the chemical reactions involved is still unknown. However, it has been reported that concentration of precursors in tap water, concentration of chlorine, pH, and contact time between chlorine and precursors are typically related. Figure 2 shows the mechanism of trihalomethane formation.

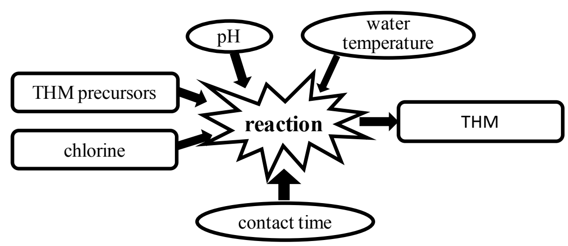


Fig. 2 Mechanism of trihalomethane formation

Japan's Water Supply Act specifies the minimum concentration of chlorine in tap water at users' faucets for the purpose of disinfection, and the governments of other countries regulate chlorine concentration at the time of injection. Thus, despite the risk of forming carcinogens, reducing the amount of chlorine injected at water purification plants is difficult. A mere reduction in the amount of chlorine injected may result in a fall in chlorine concentrations in unexpected areas. Because of this, there is demand for methods for predicting trihalomethane formation over a whole water supply network [9] and for determining the water residence time at each node of a network. Figure 3 shows a basic model for predicting trihalomethane formation in a water supply network. The proposed method for calculating water residence time in a network forms the first step of the basic model shown in Fig. 3 and will be described in detail in the following sections.

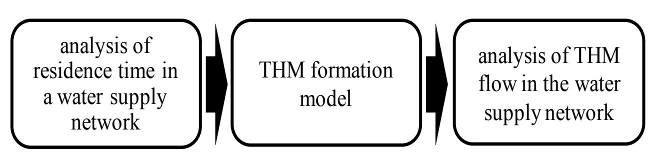


Fig. 3 Basic model for predicting trihalomethane Formation

We briefly describe the second and third steps here. In the second step, "THM formation model", the amount of trihalomethanes formed at each node is calculated on the basis of water residence times obtained through analysis of all nodes of the water supply network, chlorine concentrations, water temperatures, and pH values. The chemical reaction modeling for trihalomethane formation is represented by

$$[\mathrm{THM}] = k\,(\mathrm{pH} - a)\,[\mathrm{TOC}]\,[\mathrm{Cl_2}]^m\, t^n \tag{1}$$

where [THM], [TOC], and $[Cl_2]$ are the concentrations of trihalomethanes, trihalomethane precursors, and chlorine, respectively, and t is the contact time between the trihalomethane precursors and chlorine. The parameter values are $k = 8.3 \times 10^{-4}$, $a = 2.8$, $m = 0.25$, and $n = 0.36$.

In the third step, "analysis of THM flow in the water supply network", the flow of trihalomethanes formed as determined by the above-described trihalomethane formation model at the nodes of the network is calculated, using information on volumetric flow rate and velocity for the network. Once formed, trihalomethanes do not degrade in pipes. Because the concentration gradient of trihalomethanes is small and the diffusion rate is sufficiently small relative to the velocities, trihalomethane flows can be calculated by applying the following advection equation, obtained by omitting the diffusion term from the diffusion equation, to all the pipes:

$$\frac{\partial c}{\partial t} = -v\frac{\partial c}{\partial x} + q \tag{2}$$

where c is trihalomethane concentration, v is velocity, t is time, x is position measured along the length of the pipe, and q is the amount of trihalomethanes formed in the pipe.

The proposed method for calculating residence time in a water supply network is used to calculate contact time between trihalomethane precursors and chlorine in the network, that is, water residence time at each node, which is the information required to predict trihalomethane formation.

B. *Challenges for calculating residence time*

We now describe the challenges involved in proposing a method for calculating water residence time at each node of a water supply network.

A water supply network usually consists of thousands or tens of thousands of pipes and includes a large number of diverging and merging points. Thus, the water supplied by a water purification plant or distribution reservoir usually travels along complicated paths to reach end-users, and in the process, water from a variety of paths is mixed. For example, in the case of the simple network consisting of six nodes shown in Fig. 4, there are six paths for water to travel from the leftmost node, node S, to the rightmost node, node 5, when the pipe flow directions are as shown in the upper diagram of Fig. 4. The six paths are shown in the balloon in Fig. 4. The paths are represented by listing the node numbers in the order the water passes through them; this notation will be used hereafter.

The time it takes water to flow between nodes can be determined by solving the pipe network analysis model represented by Eqs. (3) and (4) below [10].

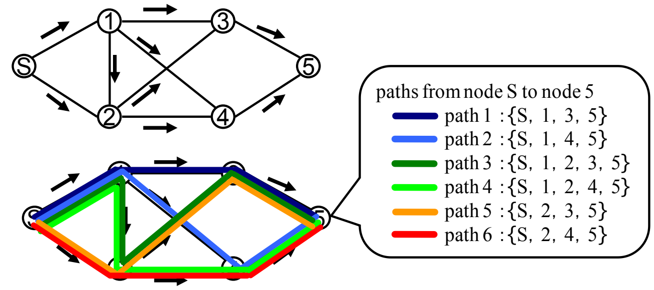


Fig. 4 Water flow paths in a water supply network consisting of six nodes

The set of nodes that serve as supply points for the water supply network (e.g., distribution reservoirs) is represented by N_{in} and the set of the other nodes by N, the volumetric flow rate in pipe j is x_j, the inflow at node i (equivalent to the total amount of water supplied by the distribution reservoir) is w_i, and the outflow (demand) at node i is y_i. The set of pipes having node i as their starting point and the set of pipes having node i as their end point are represented by $A^+(i)$ and $A^-(i)$, respectively. Then, the flow balance equation is

$$\sum_{j\in A^-(i)} x_j - \sum_{j\in A^+(i)} x_j = \begin{cases} -w_i & (i \in N_{\mathrm{in}}) \\ y_i & (i \in N) \end{cases} \tag{3}$$

In the following equation, the unit of the volumetric flow rate is in meters cubed per second and pressure refers to the pressure head in units of meters. The pressure at node i is P_i, the start and end points of pipe j are $s(j)$ and $e(j)$, respectively, and the resistance of pipe j is R_j. Then for a set of pipes B, the pressure balance equation is

$$P_{s(j)} - P_{e(j)} = R_j \left|x_j\right|^{\alpha-1} \cdot x_j \quad (j \in B) \tag{4}$$

Using the Hazen–Williams equation, the pipe resistance R_j is

$$R_j = 10.666\, C_j^{-1.85} D_j^{-4.87} L_j, \tag{5}$$

$$\alpha = 1.85 \tag{6}$$

where C_j, D_j, and L_j represent the velocity coefficient, diameter, and length of pipe j, respectively. When the pipe is equipped with a valve or pump, a second term that represents the decrease or increase in pressure caused by the valve or pump is added to the right-hand side of Eq. (4)

In Eq. (3), the units of x_j, the volumetric flow rate in pipe j, is meters cubed per seconds. The water flow time in pipe j is t_j, and is given by

$$t_j = \frac{L_j \cdot A_j}{x_j} \tag{7}$$

where A_j is the cross-sectional area of the pipe. Therefore, if the pipe flow rate in a water supply network is constant, the arrival time can be calculated by adding the water flow times, for example in the case of path 1 in Fig. 4 ({S, 1, 3, 5}), the water flow times from node S to node 1, node 1 to node 3, and node 3 to node 5. However, the conventional measuring method described in Section I has shown that it takes water several to several tens of hours to travel through a network before it reaches users, and the volumetric flow rate in the pipes is expected to fluctuate greatly over this period. The first challenge to overcome for a method for calculating water residence time in a network is to calculate while taking into account this fluctuation in pipe flow rates.

As shown in Fig. 1, a water supply network usually consists of thousands of both nodes and pipes. As can be easily imagined, in such a network, it is extremely difficult to list all the paths to a node as was done in Fig. 4. Thus, the second challenge is to calculate residence time in a network within an acceptable timescale. We consider an acceptable timescale to be roughly a few to several tens of minutes, which is based on the somewhat vague and subjective judgment that the timescale should fall within the range that the expected users of our proposed method, staff at water suppliers, can comfortably accommodate when calculating during their working hours.

III. Concept Behind And Solution For Analytically Determining Residence Time

In this section, we will describe the concept behind and method for calculating water residence time at each node of a water supply network.

A. *Concept behind residence time calculation*

We will describe a model for the proposed method of calculating residence time in a water supply network and solution algorithm.

It takes water several to several tens of hours to flow through a water supply network, in addition to which, the pipe flow rate and velocity in the network fluctuate over time. Because the number of pipes that make up a network is large, unsteady flow analysis, which is described by partial differential equations, is difficult to apply. Hence, in the method we propose, we set a unit time and assume that the flow remains steady over one unit time. For the analysis of a steady flow in a water supply network, pipe network analysis using minimum cost flow calculus is used [10]. For the steady flow, within a unit time interval, at time T_k, the downstream nodes at which flows from all nodes have arrived, and the arrival times and arrival flow rates via each path are determined. If there are multiple water flows arriving at a node via identical paths, then, at the start of the time T_k calculation, the flows calculated from the downstream nodes are deleted. Next, the same procedure is repeated replacing T_k with T_{k+1}. At the initial time T_0, flows reaching each node are unknown, and so flows at a water supply point (a water purification plant or distribution reservoir) for the network are calculated starting from an arrival time of zero and arrival flow rate equal to the total amount of water supply, and the flows that arrive within a unit time are determined sequentially. At time T_1 and thereafter, the flows determined from the flows reaching the nodes at previous time-steps are used, and the abovementioned determinations of the next set of downstream nodes and the path-by-path arrival times and arrival flow rates are calculated. This describes the basic algorithm for our method for calculating residence time and is illustrated in Fig. 5.

We will describe more specifically the basic algorithm shown in Fig. 5 using the pipe network shown in Fig. 6 as an example. In Fig. 6, node S is the water supply point (a water purification plant or distribution reservoir) for the water supply network. In this example, all the water flows from node S and is supplied to nodes 1, 2, 3, 4, and 5 (from which it is consumed).

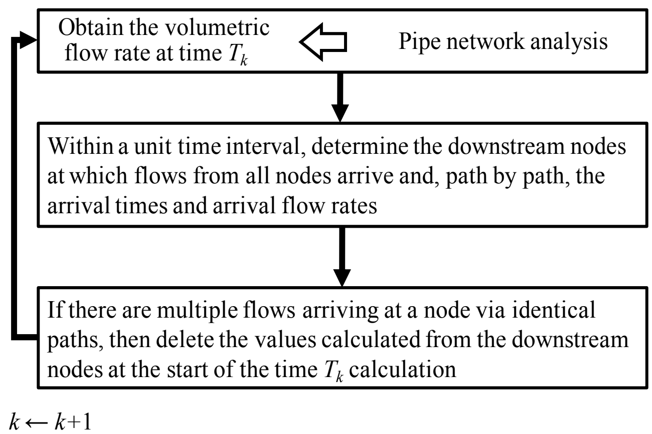


Fig. 5 Basic algorithm for residence time calculation

The total amount of water supply from node S is x_S. The volumetric flow rate and flow time of the pipe (i, j) connecting nodes i and j are represented by x_{ij} and t_{ij}, respectively. The x_{ij} and t_{ij} are obtained by pipe network analysis and assumed to be known. The volumetric flow rate x_{ij} and the flow time t_{ij} are time dependent.

In our proposed method for calculating the residence time, at time T_0, only the flow from node S is paid to attention. The outflow from node S is distributed to nodes 1 and 2 in the ratio x_{S1}:x_{S2}, and it takes arrival times t_{S1} and t_{S2} to reach nodes 1 and 2, respectively.

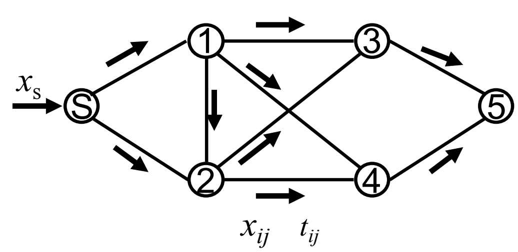


Fig. 6: Example of a water supply network

Usually, there are multiple paths leading to a node i, so that the m-th path leading to node i is represented by p_i^m, and the flow that reaches node i via path p_i^m is represented by $Q_i(p_i^m)$ and called the "flow". Flow $Q_i(p_i^m)$ has two attributes, arrival flow rate and arrival time. The arrival flow rate of flow $Q_i(p_i^m)$

is represented by $Q_i(p_i^m)_x$, and the arrival time is represented by $Q_i(p_i^m)_t$. For convenience, we imagine that node S has an inflow with volumetric flow rate x_S. This is expressed as

$$Q_S(p_S^1) = (Q_S(p_S^1)_t, Q_S(p_S^1)_x) = (0, x_S) \quad (8)$$

$$p_S^1 = \{S\} \quad (9)$$

and flow $Q_1(p_1^1)$ leading to node 1 is expressed as

$$Q_1(p_1^1) = (Q_1(p_1^1)_t, Q_1(p_1^1)_x) \quad (10)$$

$$Q_1(p_1^1)_x = Q_1(\{S,1\})_x = Q_S(p_S^1)_x \cdot \zeta_S^1 \quad (11)$$

$$Q_1(p_1^1)_t = Q(\{S,1\})_t = Q_S(p_S^1)_t + t_{S1} \quad (12)$$

where ζ_i^j is the fraction of the total inflow to node i which outflows to node j. If we represent the total inflow to node i as q_i, then $\zeta_i^j = x_{ij}/q_j$. If node S has no node demand then $\zeta_S^1 = x_{S1}/(x_{S1} + x_{S2})$.

Flow $Q_2(p_2^1)$ leading to node 2 can be determined likewise, and the processing for node S at time T_0 is completed. Next, a similar procedure is applied to determine $Q_2(\{S, 1, 2\})$, $Q_3(\{S, 1, 3\})$, $Q_4(\{S, 1, 4\})$, $Q_3(\{S, 2, 3\})$, and $Q_4(\{S, 2, 4\})$ for flows $Q_1(p_1^1)$ and $Q_2(p_2^1)$ determined above. This is repeated within the unit time $\Delta T = T_1 - T_0$, i.e., for all flows satisfying

$$Q_i(p_i^m)_t \le \Delta T \quad (13)$$

In the example water supply network given in Fig. 6, when water reaches all the nodes within a unit time, a total of 16 flows, shown in Table 1, are determined. Equation (13) applies to the conditions at time T_0, and the following Eq. applies at an arbitrary time T_k :

$$Q_i^k(p_i^m)_t - Q_j^{k-1}(p_j^n)_t \le \Delta T \quad (14)$$

where Q_i^k and Q_i^{k-1} are flows under the conditions at times T_k and T_{k-1}, respectively, and j is the starting node of flow Q_i^k determined under the conditions at time T_k.

Table 1. All the flows determined

Flow to node S	$Q_S(\{S\})$
Flow to node 1	$Q_1(\{S, 1\})$
Flow to node 2	$Q_2(\{S, 2\})$, $Q_2(\{S, 1, 2\})$
Flow to node 3	$Q_3(\{S, 1, 3\})$, $Q_3(\{S, 2, 3\})$, $Q_3(\{S, 1, 2, 3\}$
Flow to node 4	$Q_4(\{S, 1, 4\})$, $Q_4(\{S, 2, 4\})$, $Q_4(\{S, 1, 2, 4\}$
Flow to node 5	$Q_5(\{S, 1, 3, 5\})$, $Q_5(\{S, 2, 3, 5\})$, $Q_5(\{S, 1, 2, 3, 5\}$ $Q_5(\{S, 1, 4, 5\})$, $Q_5(\{S, 2, 4, 5\})$, $Q_5(\{S, 1, 2, 4, 5\}$

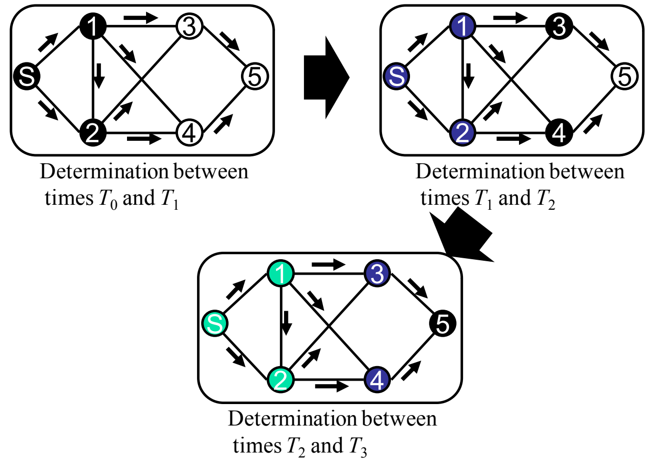


Fig. 7 Sequence of determinations of flows at nodes in our method for calculating residence time

In the calculation for determining flows at time T_1, the 16 flows determined at time T_0, shown in Table 1, form the initial state. That is, at time T_0 the flow was calculated on the basis of the flow only from node S, but at time T_1, all the nodes S, 1, 2, 3, and 4 are taken as points from which to determine flows to the downstream nodes, where node 5 is omitted because it does not have any downstream nodes. Under the conditions at time T_0, if the flows do not reach all the nodes, the flows that reached the furthest nodes will form the initial state at time T_1, and then after a number of further calculations at unit time intervals, the most downstream node (node 5) will be reached. Figure 7 shows an illustration of flows being generated by calculations at unit time intervals.

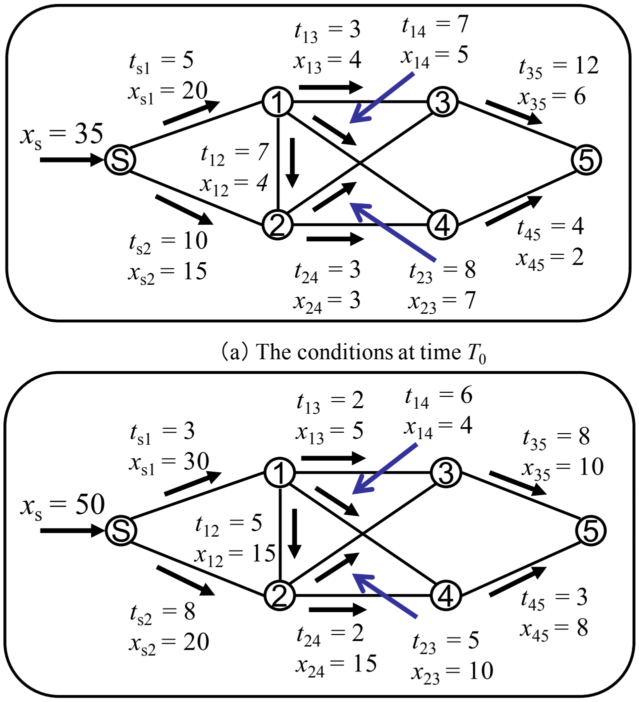
At this stage, the flows $Q_i(p_i^m)$ calculated with different nodes as their initial state may overlap, in which case, at the start of the time T_k calculation, the flows calculated from the downstream nodes are deleted. Also, because the pipe flow rates at times T_k and T_{k+1} are different, at the time of transition from T_k to T_{k+1}, the flow balance equation is satisfied by the following steps:

$$Q_i(\{S,\ldots,j,i\})_x \leftarrow Q_i(\{S,\ldots,j,i\})_x \times \frac{x_{ji}^{k+1}}{x_{ji}^{k}} \quad (15)$$

$$Q_S(\{S\})_x \leftarrow q_S^{k+1}, \quad (16)$$

where x_{ji}^k and x_{ji}^{k+1} are the volumetric flow rates for pipe (i, j) at times T_k and T_{k+1}, respectively, and q_S^{k+1} is the total outflow from node S at time T_{k+1}.

The example in Fig. 8 shows the pipe flow rates and flow times under conditions at time T_0 and those at T_1 in the water supply network of Fig. 6.



(b) The conditions at time T_1

Fig. 8 Examples of volumetric flow rates and flow times at two different times

Table 2. Flows under the conditions at times T_0 and T_1

Flows under the conditions at time T_0	Flows under the conditions at time T_1
$Q_S(\{S\}) = (0, 35)$	$Q_S(\{S\}) = (0, 50)$ * $Q_1(\{S, 1\}) = (3, 30)$ * $Q_2(\{S, 1, 2\}) = (8, 15)$ * $Q_2(\{S, 2\}) = (8, 20)$ * $Q_3(\{S, 1, 3\}) = (5, 5)$ * $Q_4(\{S, 1, 4\}) = (9, 4)$ * $Q_4(\{S, 1, 2, 4\}) = (10, 6.43)$ * $Q_4(\{S, 2, 4\}) = (10, 8.57)$
$Q_1(\{S, 1\}) = (5, 20)$	§ $Q_1(\{S, 1\}) = (5, 30)$ § $Q_2(\{S, 1, 2\}) = (10, 15)$ $Q_3(\{S, 1, 2, 3\}) = (15, 4.29)$ § $Q_3(\{S, 1, 3\}) = (7, 5)$ § $Q_4(\{S, 1, 2, 4\}) = (12, 6.43)$ § $Q_4(\{S, 1, 4\}) = (11, 4)$ $Q_5(\{S, 1, 2, 4, 5\}) = (15, 2.71)$ * $Q_5(\{S, 1, 3, 5\}) = (15, 3.33)$ $Q_5(\{S, 1, 4, 5\}) = (14, 1.68)$
$Q_2(\{S, 2\}) = (10, 15)$	§ $Q_2(\{S, 2\}) = (10, 20)$ $Q_3(\{S, 2, 3\}) = (15, 5.71)$ § $Q_4(\{S, 2, 4\}) = (12, 8.57)$ $Q_5(\{S, 2, 4, 5\}) = (15, 3.61)$
$Q_3(\{S, 1, 3\}) = (8, 4)$	§ $Q_3(\{S, 1, 3\}) = (8, 5)$ § $Q_5(\{S, 1, 3, 5\}) = (16, 3.33)$

With a unit time of 10, the conditions at time T_0 last for time 10 before transition to the conditions at time T_1. The flows at each node under the conditions at time T_0 are shown on the left-hand side of Table 2. Because the unit time is 10, only four flows are determined. Next, we assume that the conditions of the network have shifted to the conditions at time T_1. Then the flows under the conditions at time T_0 have a flow attribute, their volumetric flow rate, modified according to Eqs. (15) and (16), and with the thus obtained flows $Q_S(\{S\})$, $Q_1(\{S, 1\})$, $Q_2(\{S, 2\})$, and $Q_3(\{S, 1, 3\})$ as the initial state, the flows arriving downstream are determined. On the right-hand side of Table 2, the corresponding flows under the conditions at time T_1, which are derived from the flows at each node, are listed. Those marked with * or § in the list of flows under the conditions at T_1 overlap and, at the start of the time T_1 calculation, those which were calculated from downstream nodes, that is, those marked with a § in Table 2, are deleted.

The above is the basic procedure for calculating residence time in a water supply network using our method. However, when the flow time along a pipe is always longer than the time a volumetric flow rate remains unchanged, a time-step is finished before an upstream flow reaches a downstream node. For example, in the case of $t_{35} = 12$ in Fig. 8(a), because the unit time is 10 in these examples, the above algorithm never enables a flow to reach node 5 while the conditions at time T_0 last.

In order to solve this problem, when, at a certain node, the generation of a new flow further downstream according to Eq. (15) is halted, the information on the status of the progression downstream of the flow at the node in question is retained and used for generating a flow under the conditions at the subsequent time T_{k+1}.

More specifically, when it is determined that a flow that has reached a certain node will be unable to reach the next downstream node within the remaining time of the conditions at time T_k, the information of the fraction of the connecting pipe traversed (in the remaining time) and the time required are retained. Fig. 9 illustrates the retention of the information about the flow's progression downstream. As an example, under the conditions at time T_k in Fig. 9, we assume that the unit time is 10 and that flow $Q_i(p_i) = (6, 10)$ is present at node i . The time component to extend the flow $Q_i(p_i)$ to node j is $Q_i(p_i + \{j\})_t = Q_i(p_i)_t + t_{ij} = 22 > 10$; thus, the flow at node j cannot be generated. However, because $Q_i(p_i)_t = 6$, the flow that reaches node $\underline{i}$ is able to travel along the pipe ($\underline{i}$, j) for the time $10 - Q_i(p_i)_t = 4$. The definitions of the variables used below are the following:

$b_i^j(Q_i(p_i))$ is the information on the progression of flow $Q_i(p_i)$ at node i to the next downstream node j; the fraction of the pipe (i, j) traversed, and the time taken are retained as its components.

$b_i^j(Q_i(p_i))_t$ is the time component of $b_i^j(Q_i(p_i))$.
$b_i^j(Q_i(p_i))_{ra}$ is the fraction component of $b_i^j(Q_i(p_i))$.

Using the definitions given above, when it is determined that the flow $Q_i(p_i)$ in Fig. 9 will be unable to generate a new flow at node j, the following information on its progression is created and retained:

$$b_i^{\,j}(Q_i(p_i))_t = 10 - Q_i(p_i)_t = 4 \tag{17}$$

$$b_i^{\,j}(Q_i(p_i))_{ra} = b_i^{\,j}(Q_i(p_i))_t / t_{ij} = 0.25 \tag{18}$$

The above indicates that flow $Q_i(p_i)$, which has reached node i, will have traversed 0.25 of the pipe (i, j) when the conditions at time T_k end, and it will require time 4 to reach that point. Next, we assume that flow $Q_i(p_i)$, while retaining the above information on its progression, transits from the conditions at time T_k to the conditions at T_{k+1}. Because the flow time along the pipe (i, j) under the conditions at time T_{k+1} is $t_{ij} = 12$ and the conditions at time T_{k+1} last for time 10, water is unable to reach node j from node i. However, because the information on the progression of flow $Q_i(p_i)$, that is, $b_i^j(Q_i(p_i))$, is retained, the time component of the new flow $Q_j(p_j)$ at node j, the next node downstream from node i, is created according to

$$Q_j(p_j)_t = Q_i(p_i)_t + b_i^{\,j}(Q_i(p_i))_t + (1 - b_i^{\,j}(Q_i(p_i))_{ra}) \times t_{ij} = 19 < 20 \tag{19}$$

where $p_j = p_i + \{j\}$. This enables the flow that has reached node i to be extended to the next downstream node j. Also, if for example, we assume that the flow time along the pipe (i, j) under the conditions at time T_{k+1} is $t_{ij} = 20$, then flow $Q_i(p_i)$ does not reach node j before the conditions at time T_{k+1} end. However, because the flow only travels along the pipe (i, j) for time 10, the information on its progression, $b_i^j(Q_i(p_i))$, is updated as follows:

$$b_i^{\,j}(Q_i(p_i))_t \leftarrow b_i^{\,j}(Q_i(p_i))_t + 10 = 4 \tag{20}$$

$$b_i^{\,j}(Q_i(p_i))_{ra} = b_i^{\,j}(Q_i(p_i))_{ra} + \frac{10}{t_{ij}} = 0.75 \tag{21}$$

and the above information on its progression is passed down to the conditions at the subsequent time T_{k+2} and the calculation is continued.

The above is the model and solution algorithm for our method for calculating residence time. The algorithm presupposes a network of digraphs, and if the network has a closed loop, the calculation cannot be completed. Therefore, to apply the above algorithm, no closed loop can exist in the network. Usually, there are no closed loops in water supply networks because water physically flows down from where the hydraulic head is higher to where it is lower. However, when a network is equipped with pumps, these can raise the hydraulic head and a closed loop may therefore be formed. Accordingly, when our method for calculating residence time is applied to a network equipped with pumps, it needs to be confirmed in advance that no closed loops exist.

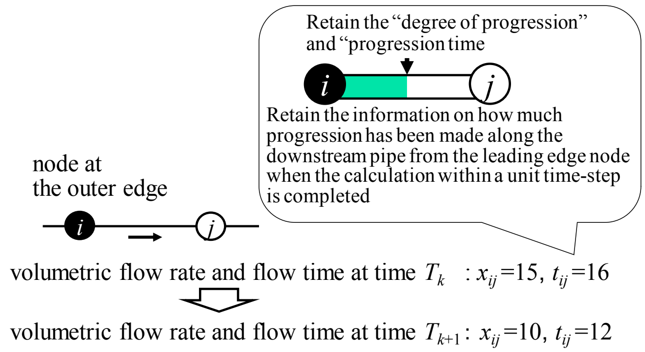


Fig. 9 Example of the retention of the information about the flow's progression downstream

B. Method to reduce the amount of calculation needed to calculate residence time

In this section, we will describe a method for reducing the amount of calculation needed to calculate residence time using our proposed method.

In a complicated water supply network that has a large number of diverging and merging points, it is easy to imagine that a large amount of calculation will be required to determine all the possible paths from node S to each of the other nodes according to the method of analysis described in Section Ⅲ.*A*.

To generate a flow according that method, Eqs. (11) and (12) are used to determine the flow at the next downstream node from the flow that has reached the node in question. Of these two equations, Eq. (11) is the one associated with the volumetric flow rate of the flow, where ζ_i^j is the fraction of the total inflow to node i that outflows to node j, and so it should be 1 or less. Thus, the flows determined according to the method described in previous paragraph become smaller as they progress further toward the end of the water supply network, being multiplied by ζ_i^j. In contrast, trihalomethane formation, which is the practical motivation for this study's calculation method, depends on the water residence time, but, in Eq. (1), the trihalomethane concentration is reportedly proportional to contact time to the power of 0.36. Thus, it is assumed that as time passes, the amount of trihalomethanes formed becomes more dependent on the volumetric flow rate of the flow reaching the node. This means that the amount of calculation can be reduced by omitting flows on the basis of their volumetric flow rate. Also, the deterioration in the precision of the method for analyzing the residence time due to the exclusion of flows having small volumetric flow rates can be evaluated using the volumetric flow rates of the excluded flows.

Exclusion of flows with small volumetric flow rates can be implemented by deleting flows that have a volumetric flow rate $Q_i(p_i^k)_x$, along path k leading to node i, less than the total volumetric flow rate that reaches node i, Q_i, multiplied by a parameter r, which sets the volumetric flow rates that are to be excluded from the calculation:

$$Q_i(p_i^k)_x < r \cdot Q_i \tag{22}$$

We call such flows branch flows. The parameter r is less than 1, and the smaller it is, the smaller the flows whose calculation is excluded from the residence time calculation. This exclusion reduces the amount of calculation.

Lastly, Fig. 10 is a flow chart summarizing the proposed method for calculating residence time in a water supply network described in Section Ⅲ.

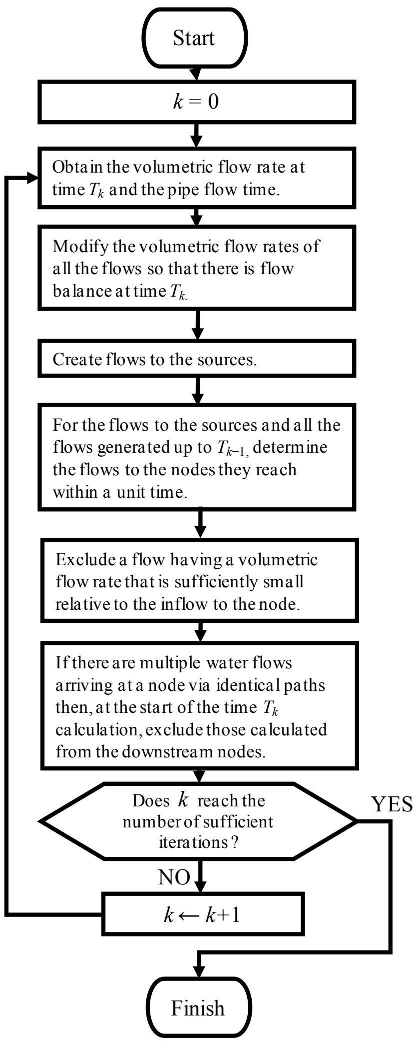


Fig. 10 Flow chart of our method for calculating residence time in a water supply network

Ⅳ. Simulation Results And Discussion

In this section, we will present and discuss the results of a simulation using the proposed method.

We applied our method for calculating water residence times to a water supply network. This water supply network comprises six distribution reservoirs serving as input sources, 2421 nodes, and 3043 pipes. The mean and maximum numbers of pipes connected to a node are 2.3 and 8, respectively.

In the simulation, we set the unit time to be 3 hours and analyzed residence time for 24 unit times, corresponding to 72 hours or 3 days, on the basis of the total demand shown in Fig. 6. We chose 72 hours as the period of the simulation because, according to the description in Section Ⅲ, flows in our method are determined sequentially from node S(distribution reservoir), and thus a few-hour simulation was deemed unable to generate flows that reach all the end nodes in the real water supply network, whereas a 72-hour simulation was judged to be able to generate flows that reach all the end nodes, and the number of flows would be stable after that time. The parameter r, which sets the volumetric flow rates to be excluded, was set to 0.01 in the simulation.

Figure 7 shows the distribution of the flows with respect to the arrival time. The upper and lower graphs in Fig. 7 follow one another in time but have different vertical-axis scales. Figure 7 shows that 91% of the flows arrived within 8.5 hours and that 99% of the flows arrived within 18 hours, that is, most flows arrived within a dozen or so hours

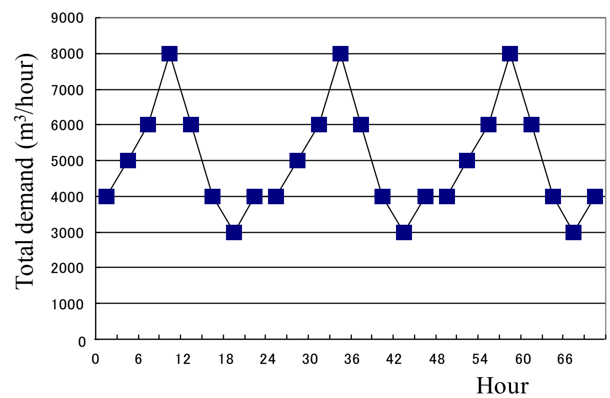


Fig. 6 Fluctuation in total demand input data

Table 2. Results for the method for calculating residence time in a pipe network

Total number of flows (sum of the number of flows at the previous nodes)	63,125 (flows)
Mean number of flows per node	26.01 (flows/node)
Largest number of flows at a node	251 (flows)
Mean path length of a flow	25.85 (pipes/flow)
Longest path of a flow	62 (pipes/flow)
Longest arrival time of a flow	58 hours
Mean exclusion rate	0.81%

The simulation took 38 seconds to calculate residence times for the 24 three-hour unit time steps (72 hours) described above. The computer used for the simulation had an Intel Pentium4, 3.20GHz CPU and 1.0 GB memory. To do a more realistic and faithful analysis, reducing the time unit to less than 3 hours and increasing the number of steps may be considered, in which case the calculation should also be completed within a practically acceptable timescale because the time required for calculation using our proposed method is, in principle, proportional to the number of steps.

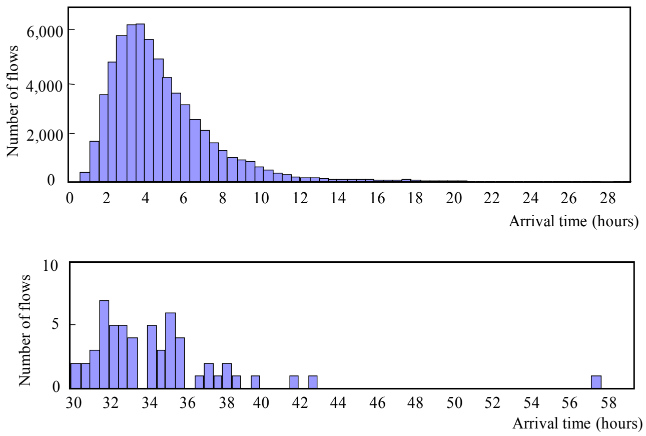


Fig. 7 Distribution of flows with respect to arrival time

V. Conclusions

In this paper, we proposed a method for calculating water residence time at each node of a water supply network. We identified the challenge facing the implementation to take into account pipe flow rates that fluctuate over time.

The challenge arises because the number of pipes that make up a water supply network is large, and therefore, unsteady flow analysis, which is described by partial differential equations, is difficult to apply, making the determination of all residence times impossible. Also, the volumetric flow rate and velocity of water in a water supply network fluctuate over time. Thus, water residence time in a network of pipes cannot be obtained only by determining the paths leading to a node in the water supply network and adding up the pipe flow times along the paths. The algorithm used in the proposed method calculates the fluctuating pipe flow rates by determining, path by path, the flow times and flow rates per unit time interval. The algorithm has the following basic structure: first, the flow is assumed to be steady during a unit time and minimum cost flow calculus is used for the pipe network analysis. Within the subsequent unit time, the downstream nodes to which flows from all nodes arrive, and the arrival times and arrival flow rates via each path are determined. If there are multiple water flows arriving at a node via identical paths, then the flows calculated from the downstream nodes are deleted. Proceeding in unit time intervals, the above procedure is repeated.

Also, it took a PC with ordinary performance 38 seconds to calculate residence times in a realistic large-scale pipe network over a period of 72 hours (3 days), which confirmed that the computation time of the proposed method is within a practically acceptable timescale range for the anticipated application conditions under which the method would be used by operations manager at water suppliers. Thus, the results indicate that our method for calculating residence time in a water supply network is practical and effective.

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