# Different Approach to Thermal Modeling of Transformers - a comparison of methods

Vlado Madžarević, Izudin Kapetanović, Majda Tešanović, Mensur Kasumović

Abstract— This paper presents a thermal models to simulate a thermal behaviour of different type of transformers. Heat disipated is always problem in transformers, esspecialy in large power transformers.

Coupled physical and mathematical models would assist in the development of a system that was both accurate and simple to implement. Material proporties, the geometry of the model, heat transfer coefficients for each surfaces are introduced as the input values. For the accurate results of the temperature distributions, the exact values of heat transfer coefficients are required. However this can be managed by solving the flow field equation by using any means, i.e. numerical methods, analytical methods.

The performances of the models are compared to the analytically determined performance of transformer or to the experimentally determined performance of transformer and the results obtained are in a good enough agreement with open literature. The thermal models based on finite element analysis are shown to be more accurate then the model based on equivalent electrical circuit.

Finite element software, as CAD tool or other application modes for fundamental physics provide several benefits: reduction of the costs for designing a new device, reduction of number of prototype, reduction of price, simplification of the manufacturing process, increase technical performance, ….

***Keywords***— hot-spot temperature, numerical calculation, thermal modeling, thermal field, transformer

## I. INTRODUCTION

The electrical devices are complicated mechanisms with specific interconnected elements and parts, and various physical phenomena that describe the real system behavior.

In many cases demands for their proper operation force reconsideration conventional design methods to include complexity of non-linear materials, composite geometry structures, operation at transient and steady-state conditions. The prediction and determination of the thermal phenomena in the metal parts of transformers is a very important step in the process of designing equipment. Transformer faults always cause internal damage. The basic criterion that limits transformer loading and its lifetime is partially determined by the ability of the transformer to dissipate internally generated heat into the surrounding area.

Therefore, it is of great importance to predict the thermal behavior of a transformer under normal load. Also, creating a transformer model is very important for the process of monitoring transformer operation. This paper provides an overview of research, development and application of various computational methods for numerical calculation of thermal field of air core transformers and oil immersed power transformer.

The identification of transformers according to the cooling method consist the following information:

- type of internal cooling medium in contact with the windings
- identification of the circulation mechanism for internal cooling medium and external cooling medium
- identification of the circulation mechanism for external cooling medium [2]. is coded as ONAN (Oil Natural Air Natural). A four cooling methods exist: ONAF (Oil Natural Air Forced), OFAN (Oil Forced Air Natural) (OFAF Oil Forced Air Forced) and OFWF (Oil Forced Water Forced).

Transformer design must take into account numerous performance parameters and technical constraints.

Computational methods and engineering models proposed for transformer analysis and the accurate prediction of their characteristics can be categorized into four main groups:

1. Numerical techniques that consist some of the most widely used tools for transformer simulation. Among the proposed techniques of this group, the Finite Element Method (FEM) is the most prevalent one.
2. Stochastic methods including Artificial Intelligence (AI) techniques, such as Genetic Algorithms (GA), which have seen increased usage in the transformer design area over the last few years.
3. Versions of the transformer equivalent circuit. Its use is still common in the manufacturing industry, due to its simplicity and its ability to provide reliable results, especially in cases of standardized geometries.

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4. Experimental methods, combining data provided by measurements with analytical or other methods, in order to provide efficient models for the accurate representation of certain transformer characteristics.

Laboratory air-core transformer data were used for the numerical determination of the distribution of the electromagnetic and temperature fields. The results obtained from the numerical calculation in the next chapter have been compared to the results obtained from laboratory measurements.
The air core transformer data are as follows:

- type DP/0-9896,
- nominal power 2,4 kVA
- nominal voltage, high voltage winding (HV) 500 V
- nominal voltage, low voltage winding (LV)380 V
- frequency 50 Hz
- nominal primary current 3 A
- nominal secondary current 3,6 A

Oil immersed transformer data were used for the numerical determination of the distribution of the temperature field. The results obtained from the numerical calculation in the next chapter have been compared to the results obtained from analytical calculation.
The oil immersed transformer data are as follows:

- type TM–6300/35
- nominal power 6300 kVA
- nom.voltage,HVwinding 35±(2X2,5%)KV
- nominal voltage, LV winding 10,5 KV
- frequency 50 Hz
- Windings losses Pk = 46 500 W
- Iron losses Px = 7600 W
- Short circuit voltage $u_k = 7{,}5\%$
- Open circuit current $i_0 = 0{,}6$ %

## II. MATHEMATICAL MODELS OF THERMAL FIELD

### *A. Model I*

The sources of electromagnetic and thermal fields are currents that flow through transformer windings, i.e. joule losses that occur in consequence of the current flowing through conductors, i.e. transformer windings [2].
A thermal field is described by the following equation:

$$\nabla(\lambda \nabla T) - \rho c \frac{\partial T}{\partial t} + q_{\mathrm{v}} = 0,$$
(1)

and represents the differential equation of non-steady state heat transfer, in which:

$T$ – the sought function of temperature distribution in space and time [K]
$c$ – the specific heat capacity [J/kg·K]
$\rho$ – the specific material density [kg/m$^3$]
$\lambda$ – the coefficient of thermal conductivity [W/m·K]
$q_{\mathrm{v}}$ – heat generation of the eventual heat source at the observed point [J]
$t$ – time [s],

where the above are functions of space and temperature.

Heat exchange among the surfaces of the conductor, core, oil and ambient air are given in the equation (2) :

$$-\lambda \frac{\partial T}{\partial t} = \alpha\left(T_{\mathrm{p}} - T_{\mathrm{f}}\right).$$
(2)

For the solution of differential equations with the given initial and final conditions, the finite element method (FEM) was used. FEM is an approximate procedure. By applying this method, the problem of solving the partial differential equation of heat transfer is reduced to the solution of a system of simultaneous linear equations. The region within which the problem is solved is divided into a finite number of elements. The temperatures of the element nodes are obtained as solutions, while the temperatures within the elements are approximated using the values of the element nodes.

### *B. Model II*

Transformer oil pumped through the coils performs the necessary cooling. The oil has a viscosity and density that vary with temperature, so heating affects the fluid-flow pattern. The model in this paper simulates the steady-state temperature distribution in the transformer by modeling both the conduction-convection problem and the non-isothermal flow field.
The model uses two stationary application modes to simulate the problem: Weakly Compressible Navier-Stokes and General Heat Transfer. It simulates the momentum transport and mass conservation with the Weakly Compressible Navier-Stokes equations that describe the fluid velocity **u**, and the pressure field $p$. In this case, the density ρ, and the viscosity η, are temperature dependent [5]:

$$\begin{aligned} &\rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \left[-p\mathbf{I} + \eta\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}\right) - (2\eta/3)(\nabla \cdot \mathbf{u})\mathbf{I}\right] + \rho \mathbf{g} \\ &\nabla \cdot (\rho \mathbf{u}) = 0 \end{aligned} \qquad (3)$$

Variations in density result in buoyancy forces, expressed as ρ**g**, and a continuity equation for the total mass, as expressed in the previous equations.
The General Heat Transfer application mode is based on a general energy balance:

$$\nabla \cdot (-k\nabla \mathrm{T}) = \mathrm{Q} - \rho \mathrm{C_p} \mathbf{u} \cdot \nabla \mathrm{T} \qquad (4)$$

k - thermal conductivity
$C_p$ - (temperature-dependent) specific heat capacity
Q - heating power per unit volume.

### *C. Model III*

The top oil thermal model is based on the equivalent thermal circuit shown in figure 3. A simple RC circuit is employed to predict the top oil temperature [1]. In the thermal model all transformer losses are represented by a current source injecting heat into the system. The capacitances are combined as one lumped capacitance. The thermal resistance is represented by a non-linear term. The differential equation for the first equivalent circuit on fig.3 is:

$$q_{Tot} = C_{th-oil}\frac{d\Theta_{oil}}{dt} + \frac{1}{R_{th-oil}}\left[\Theta_{oil} - \Theta_{A}\right]^{1/n} \tag{5}$$

$q_{Tot}$ - heat generated by total losses, W
Cth-oil - oil thermal capacitance, Wmin /$^0$ C
Rth-oil - thermal resistance C/W
Θoil - the top oil tempetature, $^0$C

The differential equation for the second equivalent circuit on fig.3 is:

$$q_{w} = C_{th-H}\frac{d\Theta_{H}}{dt} + \frac{1}{R_{th-H}}\left[\Theta_{H} - \Theta_{oil}\right]^{1/m} \tag{6}$$

$q_W$ - heat generated by the losses at hot spot location, W
Cth-oil - oil thermal capacitance at hot spot location
Rth-oil - thermal resistance at hot spot location $^0$C/W
Θoil - the top oil tempetature, $^0$C

In table 1 is given data about geomtry of air core transformer. In table 2 is given data about geomtry of oil immersed transformer.

Figure 1 shows geometry of transformer.

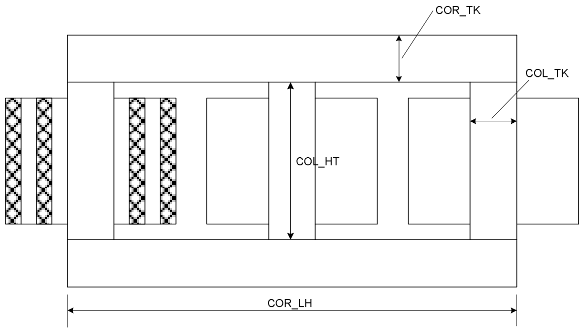


Figure 1. Transformer geometry

Table 1: Air core transformer geometry data

| Parameter | Description | Value [mm] |
|---|---|---|
| COL_HT | Leg height | 145 |
| COR_TK | Top and bottom yolk thickness | 50 |
| COL_TK | Leg thickness | 30 |
| COR_LH | Core leg | 250 |
| INS_TK | Insulation thickness | 7 |
| C1_TK | Thickness of Winding 1 | 7,5 |
| C2_TK | Thickness of Winding 2 | 7,5 |
| COIL_HT | Winding height | 145 |
| R_INT | Internal diameter of the calculation domains | 2000 |
| R_EXT | External diameter of the calculation domains | 2300 |

Table 2: Oil immersed transformer geometry data

| Parameter | Description | Value in cm |
|---|---|---|
| COL_HT | Height of columns | 143 |
| COR_TK | Thickness of uperr and lower part of the core | 34.84 |
| COL_TK | Thickness of the column | 34.84 |
| COR_LH | Lenght of the core | 168.84 |
| INS_TK1 | Thickness of insulator | 1.75 |
| INS_TK2 | Thickness of insulator | 2.7 |
| C1_TK | Coil 1 thickness | 4.86 |
| C2_TK | Coil 2 thickness | 5.27 |
| COIL_HT | Coil height | 123 |
| R_INT | Inner diametter of calculation domen | 222x258 |

## III. DIFFERENT APPROACH FOR NUMERICAL CALCULATION OF TEMPERATURE FIELD OF POWER TRANSFORMERS

Presented models of air core and oil immersed power transformer are intended to provide essential information about the status of a transformer.

They provided information about important thermal data for prognosis, simulation and analysis of the transformer operation.

Sources of electromagnetic and temperature field are currents in the coils, Joules losses which are consequence of current flow through transformer coils.

Numerical calculation of temperature field is realised with three methods: finite element method with CAD software package, finite element method using multi-physics application that involves heat transfer and fluid flow and using thermal-electrical analogy.
Results of these methods are shown on following figures.

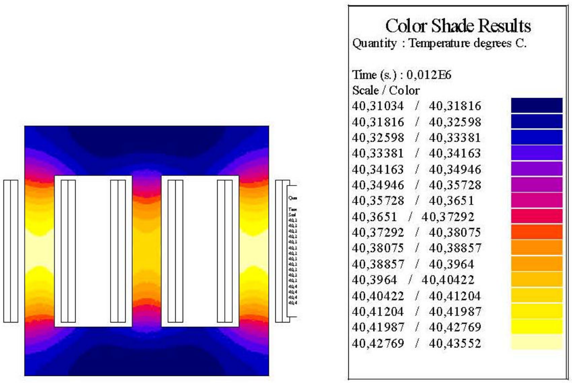


Figure 2. Temperature distribution in the core of the three-phase air-core transformer

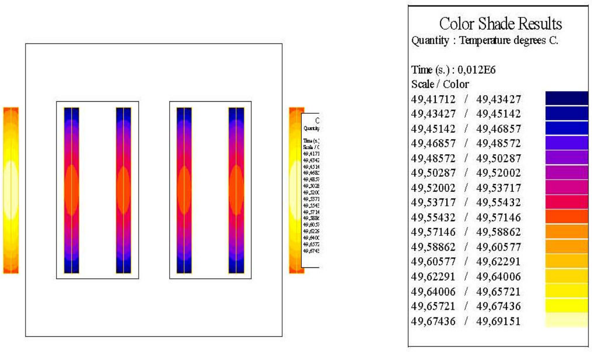


Figure 3. Temperature distribution in the windings of the three-phase air-core transformer

In Figures 4 and 5, charts are presented of the changes in temperature during a time period of 12 000 s at a point in the center of the winding and core.

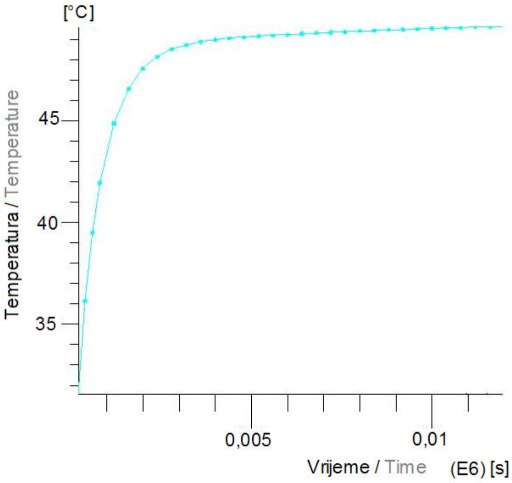


Figure 4. Temperature at a point in the center of the winding (coordinates: -146,524, -1,254), after 12 000 seconds

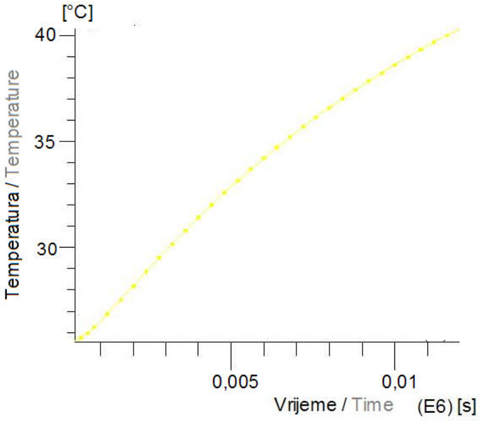


Figure 5. Temperature at a point in the center of the core, (coordinates: -1,568, 128,012), after 12 000 seconds

Since this concerns an air-core transformer, temperature measurements can be performed with a thermometer.

Table 3: Comparision of results of measurement and numerical calculation

| Value | Measurement results | Numerical calculation | Absolute error | Relative error % |
|---|---|---|---|---|
| Max. temperature of winding ($^0$C) | 49 | 49,69 | 0,69 | 1,408 |
| Max. temperature of core ($^0$C) | 40,4 | 41 | 0,6 | 1,485 |

For these transformers, the difference between the average winding temperature and the ambient temperature is not great.

Measurement was performed at the hottest accessible spot of the winding.
The temperature of the external part of the iron core for dry transformers may also be measured by a thermometer.
The temperature of the cooling agent, in this case the temperature of the ambient air, was measured at a distance of 1 to 2 m from the transformer, at half of its height.

The thermometers were protected from air flow and radiation.

The measurement results are presented in Figures 6 and 7.

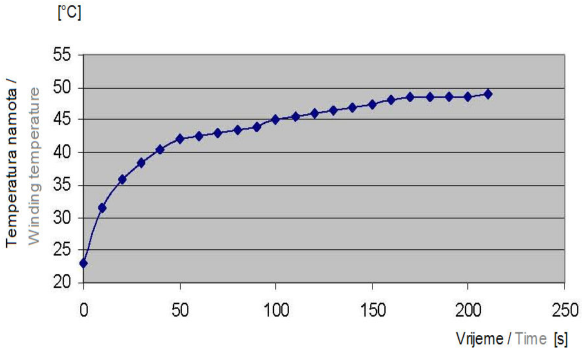


Figure 6. Changes in winding temperature over time

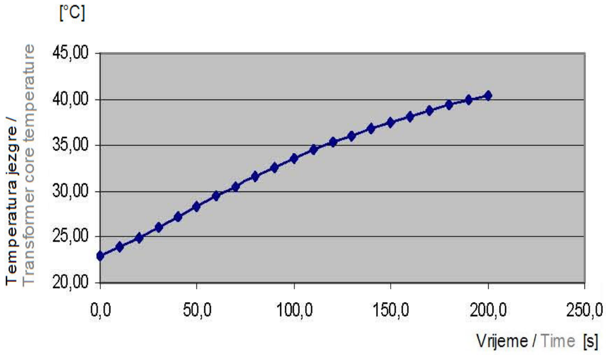
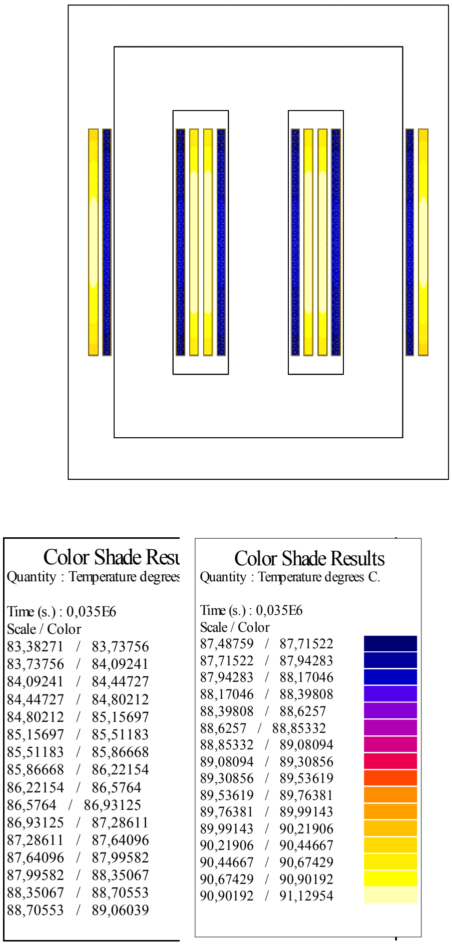


Figure 7. Changes in core temperature over time

Temperature distribution in the coils of all three phases of oil immersed transformer is shown on fig.8.

The most warm up parts of power transformer are coils, LV coil with maximum temperature 91.12 $^0$C, fig.8.


Color Shade Resu
Quantity : Temperature degrees

Time (s.) : 0,035E6
Scale / Color
83,38271 / 83,73756
83,73756 / 84,09241
84,09241 / 84,44727
84,44727 / 84,80212
84,80212 / 85,15697
85,15697 / 85,51183
85,51183 / 85,86668
85,86668 / 86,22154
86,22154 / 86,5764
86,5764 / 86,93125
86,93125 / 87,28611
87,28611 / 87,64096
87,64096 / 87,99582
87,99582 / 88,35067
88,35067 / 88,70553
88,70553 / 89,06039

Color Shade Results
Quantity : Temperature degrees C.

Time (s.) : 0,035E6
Scale / Color
87,48759 / 87,71522
87,71522 / 87,94283
87,94283 / 88,17046
88,17046 / 88,39808
88,39808 / 88,6257
88,6257 / 88,85332
88,85332 / 89,08094
89,08094 / 89,30856
89,30856 / 89,53619
89,53619 / 89,76381
89,76381 / 89,99143
89,99143 / 90,21906
90,21906 / 90,44667
90,44667 / 90,67429
90,67429 / 90,90192
90,90192 / 91,12954

Figure 8. Temperature distribution in the coils of all three phases of power transformer during 35 000 sec

Table 4. Results of analytical and numerical calculation for realised oil immersed transformer model

| Value | Analitical calculation | Numerical calculation | Absolute error | Relative error |
|---|---|---|---|---|
| Iron losses (W) | 7948 | 7963 | 15,33 | 0,19 |
| Rated current HV (A) | 104 | 106,28 | 2,28 | 2,19 |
| Rated current LV (A) | 346 | 352,71 | 6,71 | 1,94 |
| Max.temperature of winding ($^0$C) | 89 | 93,733 | 4,733 | 4,98 |
| Open circuit current (A) | 0,67 | 0,63 | 0,04 | 5,97 |

Using thermal-electrical analogy [1], RC model of transformer is realised by PSPICE software package, figures 9.

Average temperature on the oil surface as a result of simulation is 65 $^0$C. Hot-spot temperature as result of simulation is 100 $^0$C.

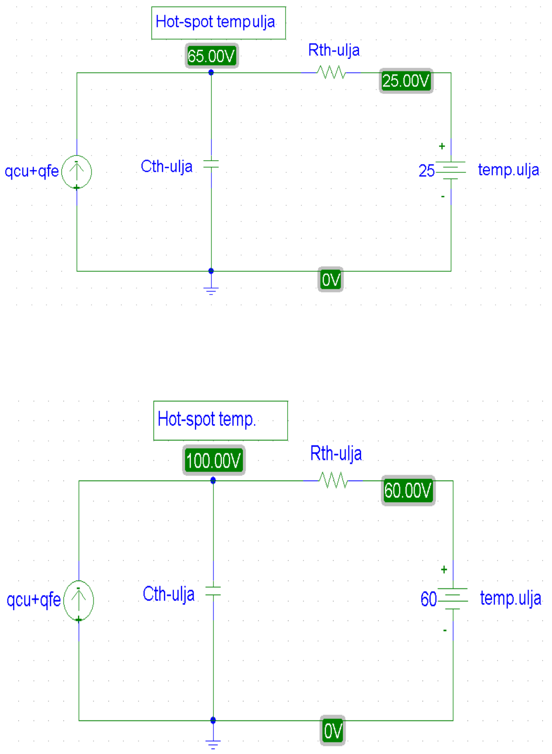


Figure 9. PSPICE model for calculation of hot-spot temperature

Temperature distribution and fluid flow in the one part of oil immersed transformer cross section is shown on fig.10.

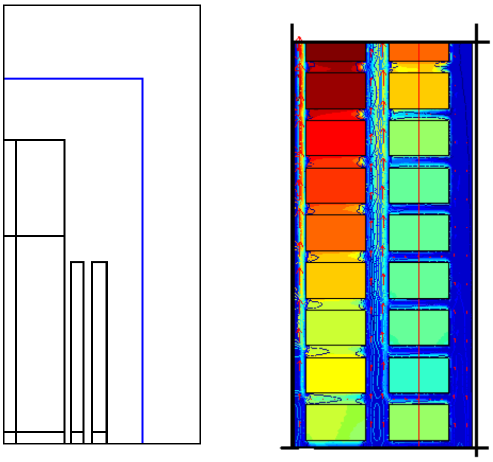


Figure 10. Temperature distribution and fluid flow in the one part of oil immersed transformer cross section

For the non-isothermal model from figure 11, the maximum temperature (at the hot spot) is 88.5 °C, occurring at the top inner coil, and the isothermal-flow model predicts a somewhat higher temperature 91 0C.

The differences between the models are caused by different fluid flows, which are affected by the temperature change[5,7].

Table is shown results of analytical and numerical calculation for realised oil immersed transformer model.

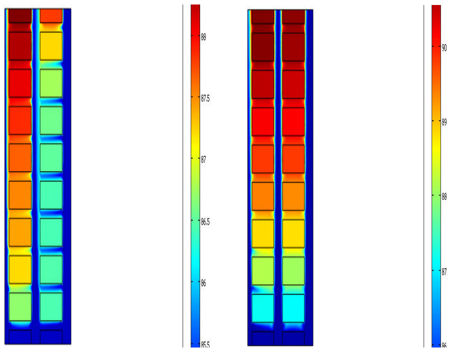


Figure 11. Temperature distribution in the transformer cross section

Table 5. Results of analytical and numerical calculation for realised oil immersed transformer model

| Value | Analitical calculation | Numerical calculation | Absolute error | Relative error |
|---|---|---|---|---|
| Max. temperature of winding ($^0$C) | 89 | 88,5 (91) | 0,5 (2) | 0,56 (2,25) |

In following section the results are presented for transformer winding simulations of three cases representing the three different cooling modes (ON, OF).

The disc winding is modeled using a two-dimensional axi-symmetric set-up. The effect of the cooling modes (ON, OF) on the flow and temperature distributions is accounted for by specifying appropriate values for the oil velocity through the inlet of the winding.

For each of the cases in two simulations were performed; one without internal buoyancy (setting $g = 0$ in the model) and one with buoyancy ($g = 9.81$).

For Case I representing the ON cooling mode (modeled through a lower oil inlet velocity) it is shown in Figure 12b that the calculated hot-spot temperature decreases with almost three degrees.

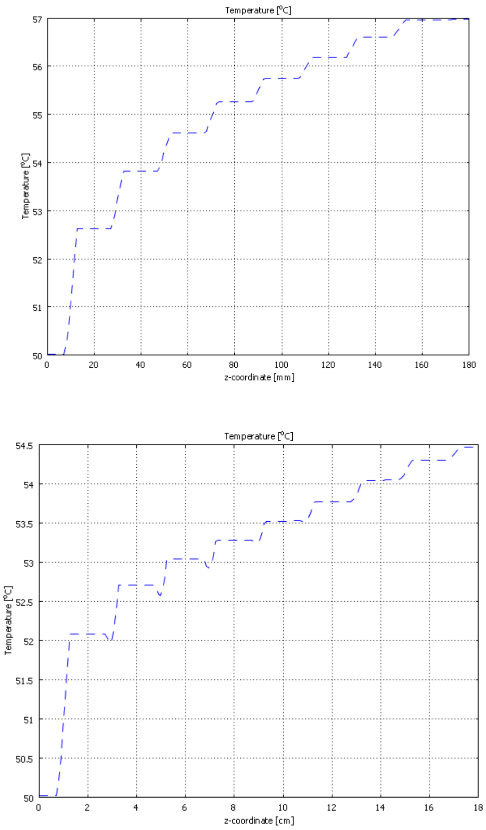
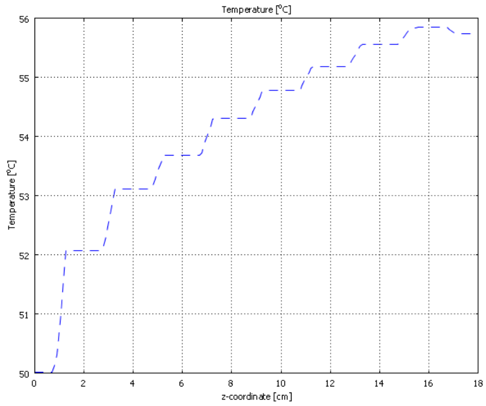


Figure 12. Maximum disc temperatures
a) without bouyancy force
b) with bouyancy force

Case II representing an OF cooling mode.

The results for Case II reveal a flow pattern in each section that is typical for flow dominated by forced convection.



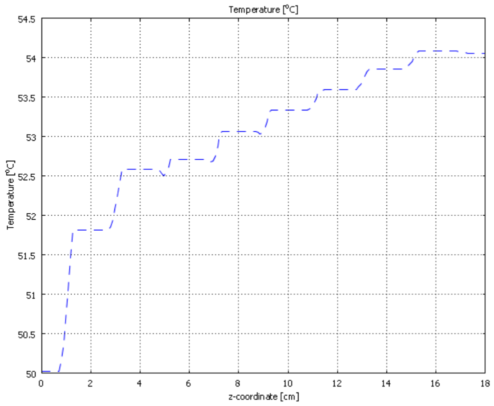


Figure 13. Maximum disc temperatures
a) without bouyancy force
b) with bouyancy force

## IV. CONCLUSION

This is particularly practical from the economic point of view since in this manner, i.e. by using suitable software programs (e.g. COMSOL, FLUX2D/3D) it is possible to model of any device whatsoever, including a transformer, and in this way reduce the need for expensive experimental measurements and repairs.

By analysis of results of temperature field distribution in air core and oil immersed power transformer cross section, also and characteristics of temperature changes in particular points during calculation period, can be concluded:

- Temperature of surrounding air is 25 $^0$C,
- The most warm up parts of transformer are coils, and then core and oil .

Internal buoyancy and hot streak formation are shown to play an important role in defining the oil flow and temperature distributions in a transformer disc winding [6].
The precision of thermal models is dependent upon the exactitude of the parameters [8,9].

In order to improve power transformer reliability, a special focus has been carried out on insulating oils and flow of fluid. The most common used oil is mineral oil because of low price and its good properties. Natural esters and vegetable oils could be very good substitute oils, because of their good properties such as safety against a fire, environmental friendliness and improved transformer performance.
Results accuracy of numerical and analytical calculation is very good. This shows importance of development of these numerical calculations for practical problems of different natures.
This types of calculations are very practical, by application of adequate software model of any kind of machines, including all types of transformers can be realised.
This is very practical by economic reasons; expensive laboratory experiments, measurements and repairs are reduced.

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