Calculation of error inherent physical-mathematical model due to finite number of recorded variables

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***Abstract*—**The physical-mathematical model describing a researched phenomenon with high accuracy, allows designers to understand deeper the developed process, and for manufactures produce the energy-efficient refrigeration and heating equipment. The measurement uncertainties of main variables describing the observed material object should take into account all possible and most influencing factors. One from them is the finiteness of this model that causes the existence of a-priori error. The proposed formula for calculation of this error provides a comparison of its value with the actual experimental measurement error that cannot be done an arbitrarily small. According to the suggested approach, the error of the researched variable, measured in conventional field studies, will always be higher than the error caused by the finite number of dimensional variables of physical-mathematical models. Examples of practical application of the considered concept for mechanics and heat- and mass-transfer processes are discussed.

***Keywords*—**Error analysis, heat- and mass-transfer, information theory, theory of similarity.

I. Introduction

OUR goal is to present that, even at the stage of formulating/formation of a physical-mathematical model (PMM) of the phenomenon, there is an error due to a finite number of variables taken into account. The idea seems obvious. However, the theory of measurements covers only aspects of the *measuring* procedure and data analysis for the value of the variable, which describes the observed phenomenon - from the point of view of the scientist/engineer.

It is assumed by default that the researcher has chosen the most appropriate PMM and the needed number of variables that are investigated, based on her or his experience and knowledge. On another side, a measurement result should contain information quantity about the recorded variable uncertainty which is required for a correct interpretation and judgment in making a decision. In this situation, the question of calculating the model error due to the finiteness of the PMM is ruled out. Second what is the relationship between this error and the actual measurement error of the variable that obtained in the experiment with the use of available instruments?

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This article aims to review an expediency of idea that observed/recorded variables have random nature from a perspective of information complexity in the System of Primary variables (SPV), such as the International system of units (SI). The idea may be challenged, on philosophical or thermodynamic theory of information processes grounds. At the same time, as we will see, in the frame of the suggested approach, you can advance, prior to the field studies of mechanics, heat- and mass-transfer processes, find the minimum value of the *estimated* and *required* experimental error for the confirmation of the eligibility of the chosen model or to redefine it before carrying out the experiment. This error will correspond to the error inherent model and caused only by its finiteness.

II. Background

A. Modeling of Heat- and Mass-processes

The increasing complexity of power equipment, the tightening of the requirements for its operation, the development of improved systems and methods of intensification, control and management make it necessary to carry out systematic studies of the actual designs of heat exchange equipment, the processes and phenomena (hereinafter, material objects - MO) by methods of physical and mathematical modeling. At the same time, requirements are more stringent for the correct interpretation (comprehension) of the results obtained using these methods.

The existence of these two trends is caused by the following reasons:

- Modern heat- and mass-exchange apparatus consists of a large number of interrelated elements, between which there are flows - material, energy and information. These streams are deterministic-stochastic nature, which manifests itself in the imposition of stochastic characteristics of the hydrodynamic motion in MO on the processes of mass- and heat-transfer. Flows are distributed in space and time, and are characterized by a large number of variables. Therefore, the problems of modeling and optimization of power equipment should be decided in close contact with each other, and from the basis of the cybernetic approach to energy issues;
- Mathematical modeling is based on use of abstract, mathematical characteristics of the studied MO and the writing of rigorous relations between these characteristics, which "express" intuitive, imprecise, vague notions about MO, that

are drawn from experience, observation, common sense. Therefore, no physical-mathematical model (PMM), even it may seem very perfect, is adequate to reality;

- Widespread use of computers and creating a new method of research (computer simulation), provide for special requirements to the process of comparing the results of a computer calculations with the studied MO. This is explained by the fact that one and the same MO can be described or studied by different physical models that explaining the same empirical/experimental material by different way. At the same time, one and the same physical model (PM) can be described by different mathematical models (MM);
- At a time of the study of the complicated MO, each stage of computer simulation - choosing PM, mathematical formulation of the problem, its solution, etc. – is often conducted by separate group of scientists and engineers who specialize in a narrow field of science and technology. In this way, there is not decrease, but rather increases the probability that the wrong or not exactly decision will lead to a result that does not have scientific and practical value, for which, however, spent considerable material resources.

Thus, the modeling of MO of energy, heat and refrigerant equipment, consider the appropriateness of the problem that allowing:

- to formulate an approach to assessing the level of detail describing of MO;
- to establish general criteria that would determine the necessary and sufficient conditions for a matching PMM describing MO, the requirements for accuracy or error in the initial data set;
- to quantify the error of MM due to finite number of variables taken into account.

B. Overview of the Related Research

Many books and papers have been written on the problem of matching PMM and the researched/observed/measured physical system.

In what follows, PMM is a framework of ideas and concepts from which a researcher/conscious observer *O* interprets his observations and experimental results. It includes PM and MM. PM interprets MM, including its assumptions and constraints. MM is a set of equations using symbolic representations of quantitative variables in a simplified physical system.

Currently, in most scientific publications, it is presupposed that the achievement of high-precision measurements allows making a judgment on the appropriateness of a completed experience as a criterion legality of the proposed PMM. The eligibility choice of PMM is confirmed in terms of *O* if the theoretical calculated results coincide with the experimental data within the reached known error of measurements.

In turn, the errors occur not only during measurements, but also during synthesis of the theoretical model. In this process, in accordance with the nature of emergence, there are significant errors that arise in the PMM formulation, the computer analysis/numerical computations when developing PM and MM, which are associated with a finite amount of digits of variables in calculations etc.

According to the general theory of information [1], the process of PMM formulation can be called information processing. It includes information construction that is an operation when the information and/or its initial representations about MO are not changed, but new information and/or representations are created. Physicists and engineers obtain information from MO and can develop scientific laws and analyze natural phenomena or engineering processes based only upon this information.

In other words, *O* knows about certain MO only if MO has a name N_{mo} in the mind M_{o} of *O*, and there are some data D_{mo} in M_{o} that represent the properties of MO. It must be emphasized that any *O* is not ideal because, in the opposite case, he/she has to be capable of potentially acquiring infinite knowledge.

Some scientists assume that verification and validation of numerical models of natural systems are impossible. New tools would be applicable to quantifying the uncertainties inherent in calculations and for evaluating the validity of the models [2, 3].

There are specific approaches that are used either to derive additional properties of MO or to analyze matching PMM and MO.

For instance, an information criterion was suggested [4] in order to select the most appropriate model describing the researched MO. The model, chosen according to the smallest value of the information criterion, is “closest" to the unknown reality that generated the data among all the candidate models considered.

Authors of [5] developed a method for estimating the systematic error of a model, and proposed its introduction into a physical experiment for the case of correlated measurements of unequal accuracy. They obtained algorithms for calculating the confidence limits of the systematic error of the mathematical model and also demonstrated their efficiency.

The systematic approach is used for qualitative analysis of the measurement procedure [6]. This procedure is considered as a system containing different elements interacting with each other, including MO, and MMs describing this MO. The traditional analysis of the accuracy of the measurements is supplemented by study of qualitative characteristics such as reliability and complexity of the measurement procedure. MM of the measurement procedure is developed and studied. The qualitative characteristics of MM are also investigated, including the adequacy of the number of used variables.

In [7] there were demonstrated the methods for measuring uncertainty contents in the form of different MMs. Authors discussed and analyzed a class of models in engineering and sciences, taking into account the relationship between input and output variables for a system. These models are built on the basis of knowing the underlying physical laws such as material mechanics, and utilizing constraints such as boundary conditions.

An interesting approach was proposed in the study of quantum gates, which are, in essence, physical devices [8]. Therefore, they are subject to random errors. The reliability of quantum gates is considered from the perspective of information complexity. In turn, the complexity of gate operation is defined in terms of the difference between the entropy of variables associated with the initial and final states of computation. The approach explained that the gate operation can be associated with unbounded entropy, implying an impossibility of implementation under some conditions.

In [9] authors stated that the criterion for choosing the method to estimate the values is not clearly addressed in the Guide to the Expression of Uncertainty in Measurement (GUM). This statement is true if repeated measurements are performed. The two methods recommended in the GUM to estimate the values of a measure are compared. Thus, a certain criterion is formulated for selecting the preferable method based on the calculation of contributions to the acquisition uncertainty.

In research [10], three criteria (robustness, fidelity and prediction-looseness) were used in order to assess the credibility of mathematical or numerical models. It is shown that these criteria are mutually antagonistic. The recommended main strategy is to explore the trade-offs between robustness and uncertainty, fidelity and data, and tightness of predictions.

Thus, there is a sizable body of literature on the methods of development of PMM that describes MO with the maximum possible accuracy. At the same time, nobody has tackled the task of quantifying the conceptual PMM error caused by choosing a finite number of variables taken into account and certain SPV. It is a peculiar channel due to which information is either transmitted to *O* or *O* extracts an information quantity about MO from SPV.

III. Tracing the Idea

De facto, the PMM formulation is based on two guidelines:

1. **Observation is framed by a System of Primary Variables**. *General knowledge of the world is significantly limited by the act of choice of the System of Primary Variables*. It is a set of dimensional (DL) variables, primary and, designed on their basis, secondary [11], which are necessary and sufficient to describe/characterize, the observed MO, as in physical content and quantitatively. The number of DL variables is finite. SI or CGS may be offered as an example of SPV.
2. **Number of variables taken into account in PMM is limited**. *The limits of description of the studied MO are caused due to the choice of class of phenomena (COP) and the number of secondary parameters taken into account in MM*. COP is a set of physical phenomena and processes described by a finite number of primary and secondary variables, which characterize certain specific features of MO with qualitative and quantitative aspects [12]. In electromagnetism, for example, it may be useful to apply SI dimensions of $\boldsymbol{LMTI}$ where $\boldsymbol{L}$–length, $\boldsymbol{M}$–weight, $\boldsymbol{T}$–time, and $\boldsymbol{I}$–powered by electric current. In thermodynamics, the base set of dimensions often includes $\boldsymbol{L}$, $\boldsymbol{M}$, $\boldsymbol{T}$, and $\boldsymbol{\Theta}$–thermodynamic temperature.

If SPV and COP are not given, then the definition of "information about MO" loses its force. Without SPV, the modeling of MO is impossible. You can never get something out of nothing, not even by watching [13]. It is possible to interpret SPV as a base of all accessible knowledge that humans are able to have about their environment at the moment.

In turn, the mathematical theory has a strong structure [1], which is suitable for any area without any restrictions. At the same time, error of limited dimension MM cannot be done an arbitrarily small. It is explained by the fact that this error relates to the validity of each natural or computer-based experiment, and should be a part of the theory of measurements. When this theory is used as PM, it becomes the object of applying both the above restrictions. In physics, this leads to the assumption of the possibility of the existence of certain errors (limited accuracy) before MM is applied.

There are fundamental, objective (e.g., thermodynamic) limits for accuracy during the experimental study. This, in turn, determines the existence of a-priori source of inaccurate knowledge on all MO, the information about which is received and processed by *O*.

Fundamental limits on the maximum precision with which we can determine the physical variables are created by the Heisenberg's uncertainty principle. However, Planck's constant is extremely small, so the uncertainty in the macroscopic measurements is devoid of practical meaning. Uncertainties of position and momentum, which follow from it, lie far beyond the achievable accuracy of the experiments.

Thus, at the information processing stage of the MO modeling, it is appropriate to consider tasks/problems allowing the following: improving the reliability and accuracy of the results of physical and mathematical modeling; reduction in the amount and duration of natural and computer simulations; mathematical formulation of "Life-activity" of MO in the consolidated criteria form; dissemination of the obtained results on similar MO.

All the above can be attributed to the basic task involved in the problem of improving research efficiency, and accelerate its practical implementation.

The purpose of the paper is to calculate an error Δ_{pmm} actually caused only by PMM finiteness (a finite number of DL variables selected from SPV) in order to assess the expedient experiment error in determining the desired dimensionless (DS) field $\boldsymbol{u}$. Field is a set of data recorded on continuous or discrete scales, or, as a special case – a physical field [12]. Consideration of a DS field $\boldsymbol{u}$ is permissible because of its similarity with any DL field, and is motivated by the desire to further generalize the results obtained to different areas of physical applications. The error Δ_{pmm} is due to the fact that the PMM, developed in the study of MO, most often involves a small number of variables taken into account once, or in tens, and in rare cases – in hundreds. This is because of two reasons: the complexity involved in computing using multivariate models and the desire to present the final result in a form convenient for practical use.

IV. Formulation of $\aleph$–Hypothesis

In what follows, we denote Δ_{pmm} the error in determining the DS theoretical field $\boldsymbol{u}$, "embedded" in PMM and caused only by its dimension that is the property of the model to reflect a certain number of characteristics of MO, its external and internal connections (links).

The error $\varDelta_{pmm}$ can be represented as the sum of two terms

$$\varDelta_{pmm} \le \varDelta_{pmm}' + \varDelta_{pmm}'' , \qquad (1)$$

where $\varDelta_{pmm}'$ – error due to COP, which is associated with reduction in the amount of counted primary variables compared with SPV; $\varDelta_{pmm}''$ – error due to the choice of the amount of counted influencing variables within the framework of the set of COP.

The size $\varDelta_{pmm}'$ can be defined as follows (the sequence of reasons at calculation $\varDelta_{pmm}''$ is similar).

We formulate an approach for the introduction of a measure of the information quantity about MO in SPV and the definition of a sequence of actions (algorithm) allowing a measurement of this quantity.

A certain complexity of MO description is offered as a measure of the complexity of the MO model. *O* can decide only the category of the model. Any claim can be made only with respect to the model. Of course, the notion of "complexity" also requires the definition, and there is a possibility of arbitrariness. And yet, the process of cognition of MO as a physical system, in general, is infinite. Thus, the model of this system is a formal structure built according to certain rules, and this design certainly is predictable.

A certain totality (SPV) can be represented by two different ways. Just by listing its elements (the researcher suggests that a set of values is finite), or by specifying a system of rules (algorithm), based on which you can perform such an enumeration (totality is accounted for).

From a practical point of view, the most natural assertion is that the measure of complexity of the totality is the number of elements contained therein. So, one of the simplest ways is to find the magnitude calculated according to the number of elements included in this description. This value is an information quantity measure contained in the description of a physical system.

Let there are $\boldsymbol{x}_1, \boldsymbol{x}_2 ... \boldsymbol{x}_n$ ($\mathbf{n} \in \mathbf{N}$) primary variables, where $\mathbf{N}$ is the set of all natural numbers. Then for secondary variables [11], primary variables are entered into the formula of dimension with exponents' $\boldsymbol{\tau}_1, \boldsymbol{\tau}_{2...}$ $\boldsymbol{\tau}_n \in \mathbf{Q},$ where $\mathbf{Q}$ is the set of all rational numbers. If the set of values $\mathbf{E}_{\tau n}$, which can accept $\boldsymbol{\tau}_n$ in different variants of formulas of dimension for secondary variables, has the top and bottom verges, then $\mathbf{E}_{\tau n}$ is finite [14]. The amount of elements in $\mathbf{E}_{\tau n}$ will make $\boldsymbol{e}_n$. Consideration of a case $\boldsymbol{\tau}_n \in \mathbf{R}$, $\boldsymbol{\tau}_n \in \mathbf{E}_{\tau n}$, $\mathbf{E}_{\tau n} \in \mathbf{R}$, where $\mathbf{R}$ is a totality of all real numbers, seems unauthorized since it may give $\boldsymbol{\tau}_n \in \mathbf{R} \setminus \mathbf{Q}$, $\boldsymbol{\tau}_n$ – an irrational number, which does not make physical sense for a method of generalized variables.

The total amount of variants of dimensions of the physical variables describing an interaction of MO with an environment reaches $\breve{\boldsymbol{G}} = \Pi \boldsymbol{e}_n - 1$, where "-1" corresponds to the occasion when all indexes of primary variables in the formula have zero dimension, and Π means the multiplication of elements $\boldsymbol{e}_n$.

The information quantity from an object depends on its symmetry [15]. The equivalent parts of the symmetrical object $\{\mathbf{E}_{\tau n}\}$ have an identical structure, where $\{\mathbf{E}_{\tau n}\}$ is a totality including elements of $\mathbf{E}_{\tau n}$ totalities. So the real carrier of information content is one of the equivalent parts. Hence, an object can be judged knowing only one of its symmetrical parts, whereas others duplicating it structurally can be regarded as empty information quantities. Then the number $\breve{\boldsymbol{G}}$ can be reduced by $\boldsymbol{\omega}$ times (quantity of equivalent parts in SPV): $\boldsymbol{G} = \breve{\boldsymbol{G}}/\boldsymbol{\omega}$.

According to $\boldsymbol{\pi}$-theorem [11], the number $\boldsymbol{\aleph}$ of DS complexes (similarity criteria) equals the number $\boldsymbol{G}$ of DL physical variables in the chosen SPV, net of $\boldsymbol{\xi}$ primary variables, i.e., $\boldsymbol{\aleph} = \boldsymbol{G}\text{-}\boldsymbol{\xi}$.

Let us consider that each DS complex represents the original readout [16] (outcome/event) through which some information content on DS field $\boldsymbol{u}$ can be obtained. It is supposed that the accounting of readouts (complexes) is equiprobable. Then, there is an uncertainty directly related to $\boldsymbol{\aleph}$. That is, larger the $\boldsymbol{\aleph}$, greater the uncertainty. Its measured numerical value is called algorithmic entropy [1], and may be calculated by the formula:

$$\boldsymbol{H} = \mathbf{k} \cdot \ln\boldsymbol{\aleph} , \qquad (2)$$

where $\mathbf{k}$ – Boltzmann constant.

In this case, algorithmic entropy H($\boldsymbol{\aleph}$) gives a measure of uncertainty of random variable $\boldsymbol{\aleph}$.

While choosing the influencing factors (conscious limitation of amount of variables describing MO), the PMM algorithmic entropy is decreased *a-priori*. It is natural to define the algorithmic entropy change by:

$$\boldsymbol{\varDelta H} = \boldsymbol{H}_{pr} - \boldsymbol{H}_{ps}, \qquad (3)$$

where pr – "*a-priori*", ps - "*a-posteriori*".

It looks natural to assume that, for the passive (mental) selection of influencing variables, the efficiency *Q* [16] of this process is equal to one. This is explained by the fact that only the mental experiment is organized, and the indignation is not brought in real system/structure. It is evident that *O* does not interfere with MO. In other words, an act of observation without space-time interaction with source does not perturb either the source or the data gathered from that source. Then you can write by taking into account (3):

$$\boldsymbol{\varDelta A} = \boldsymbol{Q} \cdot \boldsymbol{\varDelta H} = \boldsymbol{H}_{pr} - \boldsymbol{H}_{ps}, \qquad (4)$$

where $\boldsymbol{\varDelta A}$ – priori information quantity about MO.

Using (2), (4) and imposing symbols: $\boldsymbol{z}'$ – the number of physical DL variables in the selected COP, $\boldsymbol{\beta}'$ – the number of primary physical DL variables in the selected COP, we get:

$$\boldsymbol{\varDelta A}' = \mathbf{k} \cdot \ln\left[\boldsymbol{\aleph}/(\boldsymbol{z}' - \boldsymbol{\beta}')\right], \qquad (5)$$

where $\boldsymbol{\varDelta A}'$ – priori information quantity about MO due to the choice of COP.

There is a necessary condition for the possibility of obtaining information quantity under the supervision of MO. If the range of observation of the MO goal function is not set, it is impossible to determine the information quantity obtained in the study of MO. So, the value $\Delta A'$ is linked to Δ_{pmm}' and S (DS interval of observation/supervision of a field u) by the dependence [16]:

$$\Delta_{pmm}' = S \cdot \exp(-\Delta A'/\mathbf{k}). \quad (6)$$

Substitute (5) in (6):

$$\Delta_{pmm}' = S \cdot (z'-\beta')/\aleph. \quad (7)$$

Following the same reasoning, it can be shown that Δ_{pmm}'' is the formula:

$$\Delta_{pmm}'' = S \cdot (z''-\beta'')/(z'-\beta'), \quad (8)$$

where z'' – the number of physical DL variables recorded in PMM; β'' – the number of primary physical DL variables of the total variable number recorded in PMM.

Then, summarizing Δ_{pmm}' и Δ_{pmm}'', one can estimate the value Δ_{pmm}.

It should be noted that the approval of the required occurrence (same probability) of readout is justified by the purpose of the research: finding the absolute value of Δ_{pmm} stipulated by the level of description of MO. Indeed, any other distribution of readouts gives less information quantity [17], which leads to a larger Δ_{pmm}* in comparison with the Δ_{pmm} calculated at the uniform distribution of readouts.

All the above can be summarized as follows in the form of $\aleph$ –hypothesis:

Let during PMM formulation the chosen System of Primary Variables in the total number of DL physical variables be G, ξ of which are independent dimensions. In the framework of class of phenomena (the total number of DL variables - z', the number of primary variables - β'), there is a DS field u raised in a given range of values S. Then, the DS absolute error of definition of u for a given number of recorded physical DL variables z'', and β'' – the number of recorded primary physical DL variables, can be determined from the relationship:

$$\Delta_{pmm} \le S \cdot [(z' - \beta')/(G - \xi) + (z'' - \beta'')/(z' - \beta')]. \quad (9)$$

Using (9), you can find the minimum error of calculations with the theoretical analysis of the physical phenomena. On other hand, Equation (9) also sets a limit for the advisable increase in measurement accuracy in conducting pilot studies.

Within the above approach, we can find the relation between (z''- β'') and (z'- β'), so that the “relative error” Δ_{pmm}/S [16] is minimal for the specific COP

$$(\Delta_{pmm}/S)'_{z'-\beta'} = [(z'-\beta')/(G-\xi) + (z''-\beta'')/(z'-\beta')]' = = [1/(G-\xi) - (z''-\beta'')/(z'-\beta')^2], \quad (10)$$

$$[1/(G-\xi) - (z''-\beta'')/(z'-\beta')^2] = 0, \quad (11)$$

$$(z'-\beta')^2/(G-\xi) = (z''-\beta''). \quad (12)$$

According to (12), for SI and the chosen COP, for example, ***LMTI***, a least relative error can be reached at (z''- β'')≈6; for ***LMTΘ*** , the number of DS parameters causing a minimum value of Δ_{pmm}/S is about 19 (a detailed explanation of the numerical calculations due to the choice of SPV and COP is presented in Chapter V).

On the basis of (9), the situation described above, can be regarded as an uncertainty principle for the process of the PMM formulation. Namely, any change in the level of detailed description of MO (z''-β''; z'-β') causes a change in the relative error of PMM (Δ_{pmm}/S), and in the accuracy calculation of each variable, which characterizes features of the internal structure of MO or the interaction of MO with the environment.

This raises an interesting question about the relationship/difference between the essence of "detail describing" and "complexity" in the process of creating/formulation of PMM. The goal of modeling is to predict the forecast behavior of a process or system, to clarify concepts, to identify the quantitative relationships between the interacting elements of MO. At the same time, models attempting to reproduce a real situation with a large number of variables taken into account tend to accomplish the opposite. Models aim to expose pertinent relationships between variables, but unnecessary information can conceal these. As such, within the proposed approach and the chosen COP, a “good” model, on one hand, has a relatively low complexity that is calculated according to (12), on another hand, as possible while retaining the details necessary to approach the specific goal function, of which PMM is designed to examine.

When comparing different PMM (according to a value of Δ_{pmm}) describing the same MO, preference should be given to the PMM for which $\Delta_{pmm}/\Delta_{exp}$ is closer to 1, where, DS error Δ_{exp} is the estimated experimental absolute error in the determination of the generalized objective function (similarity criterion). This criterion characterizes the behavior of the investigated MO and is compiled from observable physical variables. The values of variables are measured with certain accuracy.

V. Calculation of ℵ

Let us specify the number of $\aleph = G - \xi$. As an example, choose SI [18] and recommend its list of main primary variables: L–length, M–weight, T–time, I–powered by electric current, Θ–thermodynamic temperature, J–force of light, F–number of substances. The dimension of any secondary variable q can only express a unique combination of the dimensions of main primary variables in different degrees [11]:

$$q \ni L^l \cdot M^m \cdot T^t \cdot I^i \cdot \Theta^{\Theta} \cdot J^j \cdot F^f, \quad (13)$$

where the badge ɔ – means "corresponds to dimension"; l, m... f are integers, and according to [19]:

$$-3 \le l \le +3,\ -1 \le m \le +1,\ -4 \le t \le +4,\ -2 \le i \le +2,\ -4 \le \Theta \le +4,\quad -1 \le j \le +1,\quad -1 \le f \le +1. \quad (14)$$

Then $\hat{G}$ =7 · 3 · 9 · 5 · 9 · 3 · 3-1=76,544. The value $\breve{G}$ includes both required and backward variables (for example, L^{+1}–length, L^{-1}–running length); so the number of options of dimensions may be reduced by ω=2 times, meaning $G = \hat{G}/2$ = 38,272. According to π-theorem, the number $\aleph_{SI}$ of possible DS complexes (criteria) with ξ = 7 main DL variables will be $\aleph_{SI} = G-\xi$=38,265. The numerical value of $\aleph_{SI}$ can only increase with the deepening of knowledge about the material world.

At this moment, numerical value $\aleph_{SI}$ can be calculated by use of a heuristic approach and main fundamental constants (with a relative error of $2.6 \cdot 10^{-7}$), as

$$\aleph_{SI} = ((a_{cos}/\pi^2)/(1+100\cdot \beta^2)) \cdot [m_e/(eV/c^2)] = 38{,}265, \quad (15)$$

where β=1/1,836.152746, $\beta=m_e/m_p$, m_e-electron mass, $9.109381 \cdot 10^{-31}$, [kg], m_p–proton mass, $1.672621 \cdot 10^{-27}$ [kg], c–speed of light, 299 792 458 [$m^1 \cdot s^{-1}$], eV–electron-volt, [$m^2 \cdot kg \cdot s^{-2}$], $eV/c^2 = 0.178266 \cdot 10^{-35}$, [kg], a decimal value of the constant of cosines a_{cos}=0.739085 can be found from the transcendental equation $\cos a_{cos} = a_{cos}$ with any degree of accuracy [20], π=3.141593.

VI. EXAMPLES OF $\aleph$–HYPOTHESIS APPLICATION

A. Mechanics Application

It should be mentioned that a style of the examples below is instructive and didactic that can be explained by a necessity to show features of $\aleph$-hypothesis application.

Consider the motion of a simple pendulum - the ball of mass m, suspended in a gravitational field on a weightless rod of length l. We also assume that the pendulum is moving in the same plane. Let the pendulum is under impact of the friction force R_{fr}, which is, it turn, proportional to the velocity of the sinker v, $R_{fr} = -A\cdot v$, where A- proportionality factor, which is determined by the properties of the medium and the shape of the body. The angle of deviation of the pendulum from the vertical direction is x.

The dependence of the dimensionless amplitude x_{max} of the ball can be represented by the following dimensionless equation [21]:

$$x_{max} = \varphi(a = (A/m)\cdot(l/g)^{1/2},\ p = R_{fr}/(mg)), \quad (16)$$

where, g- acceleration of gravity.

Such transformation shows some similarity laws: the dependence x (for given boundary conditions) is the same for different values of m, l, g, A, if the dimensionless combinations of a and p, composed from them, are the same. The numerical values of these complexes do not have to depend on SPV. The form of these functions can be determined either by solving the equation of motion of the pendulum, or experimental method. This fact allows us to reduce the amount of full investigations on the problem, since it suffices to consider different values of the two parameters instead of four. In other words, the results of a pendulum can be transferred to other simple change of scale.

In addition, at the numerical solution of the dimensionless equations of motion of the pendulum, we usually do not have to deal with the values that differ from each other by many orders of magnitude, while the size of the equations of motion of the pendulum it could well happen with failure choice of units.

According to (9) and (14), for COP_{SI} LMT

$$(\Delta_{pmm}/S)_1 = [(z' - \beta')/(G - \xi) + (z'' - \beta'')/(z' - \beta')] = \\ = 189/38{,}265 + 2/189 = 0.0049 + 0.0106 = 0.0155. \quad (17)$$

If in this model we neglect the effect of friction ($p = R_{fr}/(mg) = 0$), then using (9) and (14), the priori relative error of MM due to its dimension, will be

$$(\Delta_{pmm}/S)_2 = 189/38{,}265 + 1/189 = 0.0049 + 0.0054 = 0.0103, \quad (18)$$

i.e., it reduced by the value 0.0052.

And yet, it is well known that the neglect of friction, on the contrary, increases the error of MM, and this increase is not constant, but depends on the size of the complexes a and p. It is the smaller than less p and the value of a is far from a resonance region.

The apparent contradiction is explained by the fact that, if we ignore friction, MM worse describes the studied MO. Therefore, to obtain reliable experimental data and verify eligibility of the selected MM, it requires increase of the accuracy of the measuring instruments. Then, the DS error Δ_{exp} (the estimated experimental absolute error in the determination of the dimensionless amplitude of sinker x_{max}) will be smaller, and ratio of $\Delta_{pmm}/\Delta_{exp}$ will be closer to 1 (see Section IV). In this case, if the spread of the experimental data in comparison with the results of computer simulation is in the range allowed by the researcher, it can be assumed that the selected MM adequately describes the observed process.

As a second example, we use (9) for the comparison of relative errors of mathematical models describing the same MO, but with different COP. We consider a thin metal plate, moving in a viscous and elastic medium that is under the influence of an external force distributed on one surface of the plate. Suppose that in the first case, the pressure of the mechanical force affects on the plate $P_{mech} = p_o \cdot \exp(-t/\tau)$, where τ- time constant of the process, p_o- initial constant value of P_{mech}. Consequently, MO can be represented as a mechanical system (COP_{PMM}: LMT). In the second case, the magnetic field pressure affects on the side of the plate $P_{mag} = 0.5\cdot\mu\cdot H^2$, $H = h_o \cdot \exp(-t/2\cdot\tau)$, where μ is the magnetic permeability, h_o - initial constant value of the magnetic field. In this case, MO is represented in the form of electro-mechanical system (COP_{PMM}: $LMTI$).

Find out the relationship between the required number of dimensionless complexes in LMT (K_{LMT}) and $LMTI$ (K_{LMTI}), in which the relative errors are equal.

According to (9) and (14), for COP_{SI} - LMT

$$(\Delta_{pmm}/S)_{LMT} = 189/38{,}265 + K_{LMT}/189, \quad (19)$$

for COP_{SI} – ***LMTI***

$$(\Delta_{pmm}/\boldsymbol{S})_{LMTI} = 945/38{,}265 + K_{LMTI}/945. \qquad (20)$$

Equating (19) and (20), we obtain

$$K_{LMTI} \approx 5 \cdot (K_{LMT} - 4). \qquad (21)$$

Obviously, although the compared processes are described by the same equation form, the difference of modeled objects and statements of research problems leads to a difference in the values of the relative errors of MM and to differences in the requirements for verifying the accuracy of the experiments.

Thus, within the proposed approach, to achieve the equal relative errors of mathematical models describing the same MO, but with different COP, requires a distinctive number of dimensionless complexes used in MM.

B. Heat- and Mass-Transfer Application

Unfortunately, the sizable body of publications does not provide enough input data for calculation and verification of the obtained results by (9). For example, in [22]-[24], a similarity theory with non-dimensional numbers/criteria is applied in order to describe the PMMs for different heat- and mass-transfer processes. Researchers have described the test apparatus and procedure in detail. They compared the calculated results of the developed PMM with the measurement results obtained from field experiments. Each author declares "good agreement" or "reasonable accuracy" between numerical predictions NP and experimental results ER. In reality, *no one defines/calculates a generalized absolute error AE of a specific goal function (similarity criterion/consolidated criteria) that describes the "Life-activity" of the researched MO, characterizes the interaction of MO with the environment, and that is compiled from observed physical variables that are measured at certain accuracy. No one compares the difference NP–ER with AE*.

That is why we consider the engineering task [25] as an example of the practical use of ℵ–hypothesis. In the specified work, the physical dimensional parameters can be described by the formula $\boldsymbol{L}^{l} \cdot \boldsymbol{M}^{m} \cdot \boldsymbol{T}^{t} \cdot \boldsymbol{\Theta}^{\Theta}$. Based on (14), we find z'-β' = 850. While examining the heat transfer to a thin layer of material frozen on a moving cooled cylindrical wall, theoretical calculations and experimental data were introduced in the DS form. The final DS temperature of the outer surface of the material $\Theta_s^\circ=(\Theta_s-\Theta_e)/\Theta_{cr}-\Theta_e)$ is presented in the form of a correlation function of multiplication of six (z''-β'') independent one-parameter DS complexes where Θ_{cr}, Θ_s, Θ_e are the absolute temperatures respectively of freezing of a material, outer surface of a material layer and evaporating of the refrigerant; $\Delta\Theta_{cr}$, $\Delta\Theta_s$, $\Delta\Theta_e$ are the absolute errors of measurement of these temperatures. Then, considering Θ_{cr}=272ºK, Θ_s=259ºK, Θ_e= 243ºK, $\Delta\Theta_{cr}$=0.1ºK, $\Delta\Theta_s=\Delta\Theta_e$=0.5ºK, you can find an absolute DS error of the indirect measurement $(\Delta\Theta_s^\circ)_{exp}$, reached in the experiment [26]:

$$(\Delta\Theta_s^\circ)_{exp} = (\Delta\Theta_s+\Delta\Theta_e) / (|\Theta_{cr}-\Theta_e|) + |\Theta_s-\Theta_e|/ ((\Delta\Theta_{cr}+\Delta\Theta_e)\cdot|\Theta_{cr}-\Theta_e|^2) \approx 0.066. \qquad (22)$$

From (9), using calculated values $\aleph_{SI}$ и z'-β', you get a DS error value of $(\Delta\Theta_s^\circ)_{pmm}$ for the chosen PMM:

$$(\Delta\Theta_s^\circ)_{pmm} \le \Theta_{smax}^\circ \cdot ((z'-\beta')/\aleph_{SI} + (z''-\beta'')/(z'-\beta'))= \\ = 0.93\cdot [850/38\,265+6/850] = 0.027, \qquad (23)$$

where Θ_{smax}° is a given range of changes of the DS final temperature [25], allowed by the chosen mathematical model.

From (22) and (23), we get $(\Delta\Theta_s^\circ)_{exp}>(\Delta\Theta_s^\circ)_{pmm}$, i.e., an actual error in the experiment is 2.4 times (0.066/0.027) more than the minimum. It means, at the chosen number of DS criteria the existing accuracy of DL variable's measurement is not enough. The further experimental work is required to change devices to a higher grade of accuracy satisfactorily in order to confirm/refine the elaborated PMM.

Hence, the use of ℵ – hypothesis helps to a researcher find the minimum value of the *required* experimental error for the confirmation of the eligibility of the chosen PMM. This error will correspond to the error inherent in the model and caused only by its finiteness.

C. Quantum Mechanics Application

For a possible practice usage, $\aleph_{SI}$ formula (15) can be applied when selecting a DS form of the Heisenberg uncertainty relation - the theoretical limit of accuracy of any measurements for the standard deviation Δx of coordinates and standard deviation Δp of the momentum

$$\Delta x \cdot \Delta p \ge \hbar/2, \qquad (24)$$

where $\hbar = h/(2\pi)$, h – Plank constant.

Using (15) and (24), the strong DS Heisenberg uncertainty relation can be introduced in the following form

$$\Delta x'\cdot\Delta p' > (1+100\,\beta^2)\cdot(2\,(a_{cos}/\pi^2))^{-1} = 6.6771, \qquad (25)$$

$$\Delta x' = (\Delta x \cdot m_e \cdot c)/\hbar, \qquad (26)$$

$$\Delta p' = (\Delta p \cdot c)/eV, \qquad (27)$$

where $\Delta x'$– DS standard deviation of coordinates, $\Delta p'$– DS standard deviation of momentum.

Thus, the uncertainty in the macroscopic measurements gets practical meaning. The uncertainty of the DS variables, which are derived from DS Heisenberg uncertainty relation (25), is much closer to the limits of achievable accuracy of field experiments.

VII. Conclusion

Theory of measurements (TOM) and its concepts remain a correct science today, in the twenty-first century, and will continue to be right forever (a paraphrase of Prof. L.B. Okun [27]). The use of ℵ-hypothesis only limits the domain of applicability of TOM for errors that are larger than the error of PMM caused by its finiteness.

Within the proposed approach, an experimental error of the researched variable measured in conventional field studies will be always higher than the error Δ_{pmm} caused by the finiteness of the physical-mathematical model. This error is calculated according to the formulated $\varkappa$-hypothesis.

The physical meaning of Δ_{pmm} lies in the fact that at the schematization of any event or process, there is a discrepancy between PMM and MO, called the threshold mismatch [28]. The value Δ_{pmm}, due to the threshold mismatch, should always be no more than the permissible error of measurement. Otherwise, it is necessary to redefine the model before carrying out the experiment. Within the above approach, Δ_{pmm} represents a sort of "model noise" (similar to the "thermal noise").

Along with the already mentioned functions of inherent Δ_{pmm} (criterion validity of the proposed physical-mathematical model, the measure of evaluation of sufficient accuracy calculations), it is necessary to draw attention to the following fact. The error Δ_{pmm} can also be used in carrying out numerical experiments using the theory of planning experiment on computers. The feasibility of this approach is dictated by the need to calculate the reproducibility dispersion and the Fisher criterion. In turn, the Fisher criterion determines the times of cessation of screening influencing factors, which are important in the study.

The author hopes that, in case of usage of the proposed approach in practice, the avalanche of growth of scientific publications related to the numerical calculations of mathematical models of heat- and mass-transfer phenomena and processes will be stopped. This can be explained by the fact that the researchers will have to analyze more precisely the results obtained by comparing the experimental measurement error with errors caused by the finiteness of the physical-mathematical model and limited amount of recorded variables.

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A. Computer simulation by planning of experiment

Unfortunately, the sizable body

In order to reduce the volume of computations and to obtain the simple analytical dependence of dimensionless length ξ from essential construction and technological DF parameters, the active experiment planning theory, including the random balance method, was realized (Hartman et al., 1977).

As a result of a 3-stage "shifting" experiment and numerical results obtaining by the least squares method, the goal function ξ may be written in the following form: