Comparison of Single-phase and Two-phase Flow Dynamics in the HLTP for Microalgae Culture

Ujjwal Kumar Deb, Kittisak Chayantrakom, Yongwimon Lenbury

***Abstract*—** This paper aims to show dynamic behavior of microalgae suspension in a Horizontal Loop Tubular Photobioreactor (HLTP). Two models of a single-phase flow and a two-phase flow have been proposed taking into account the light irradiance. The governing equations describing a single-phase flow are the continuity equation and the Navier-Stokes equation. The viscosity of the microalgae suspension is a function of microalgae cell concentration which varies in time. Using the governing equations of a single-phase flow together with the Cahn-Hiliard mass conservation equation, we can describe the dynamic behavior of a two-phase flow. The results obtained from both models are compared. It is noted that both results are significantly different. In the two-phase flow model, the mass transfer rate and the shear rate are higher than those obtained from the single-phase model.

***Keywords*—** CFD model, microalgae, light irradiance, tubular reactor.

I. Introduction

MICROALGAE has been emerged as a natural source of biomass in bio-fuel production. They also provide high value compounds for pharmaceutical and food sectors [1], [2]. In waste water treatment microalgae are widely used to uptake heavy metal from aquatic media including carbon, phosphorus and nitrogen which are used as nutrients for their growth [3], [4]. For microalgae production, a closed Photo-bioreactor (PBR) system is considered to be better alternative than traditional open pond system for large scale production and to avoid the contamination risk although this method is more expensive [5], [6]. However, bio-fuel technology from microalgae has been progressing relatively slowly because of the lacks of proper design of photo-bioreactor and of appropriate growth model for microalgae cells.

Tubular Photo-bioreactors (PBR) are widely known as the most efficient option compared with other closed methods including annular, flat plate, helical, torus, stirred tank, spiral, vertical column, plastic bags etc. of outdoor microalgae cultivation because of its wide illumination area for light penetration inside the culture, fairly good biomass productivity and relatively cheaper maintenance cost [7]. The main optimal factors for microalgae cultivations are light, CO_2 and temperature. Light is needed for photosynthesis to obtain energy, whereas CO_2 is the main carbon source for photosynthetic culture of microalgae [8]. Studies suggested that, for the proper growth of microalgae, the CO_2 requirement varies with different species of algae and geometries of PBR. To improve the design of PBR, the amount of CO_2 gas injected in the culture should be considered. The light penetration and distribution inside the PBR influenced by gas injection methods [9]. The air enriched with 5% or 10% (v/v) CO_2 is found better for mass culture of microalgae [10]. However, 15% CO_2 concentration shows higher biomass productivity than that of 20% for a helical reactor [11]. In 2000, Miron *et al*. [12] found that liquid velocity and gas-hold up were two important parameters influenced by light availability in airlift driven tubular reactors. As light gradients occur frequently, the diameter of the tube is important for light penetration especially in dense culture. From the point of view of the biomass optimization, the cell fragility due to shear stress is the main problem in a closed photobioreactor system for microalgae cultivation [13].

The viscosity of the algal suspension can be considered as a function of relative viscosity related to concentration and growth rate of the microalgae [14]. The average irradiance depends on incident irradiance which varies with the geographical location on the Earth's surface. As growth rate of microalgae depends on light availability for outdoor mass culture, the geographical location of the photo-bioreactor setup is also significant [15].

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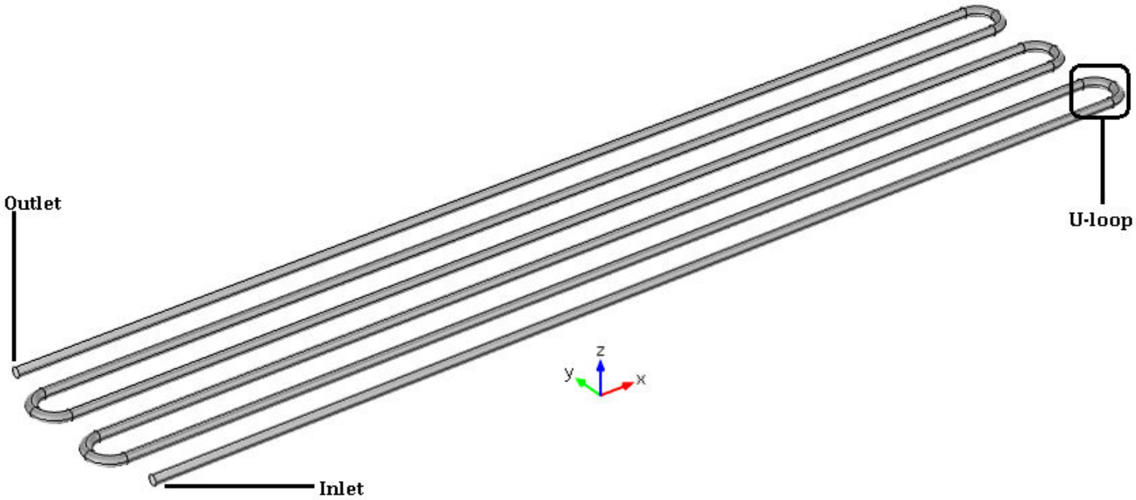


Fig. 1: Computational domain for a microalgae flow in a Horizontal Loop Tubular Photo-bioreactor (HLTP) with U-loop marked by a square, inlet and outlet surfaces.

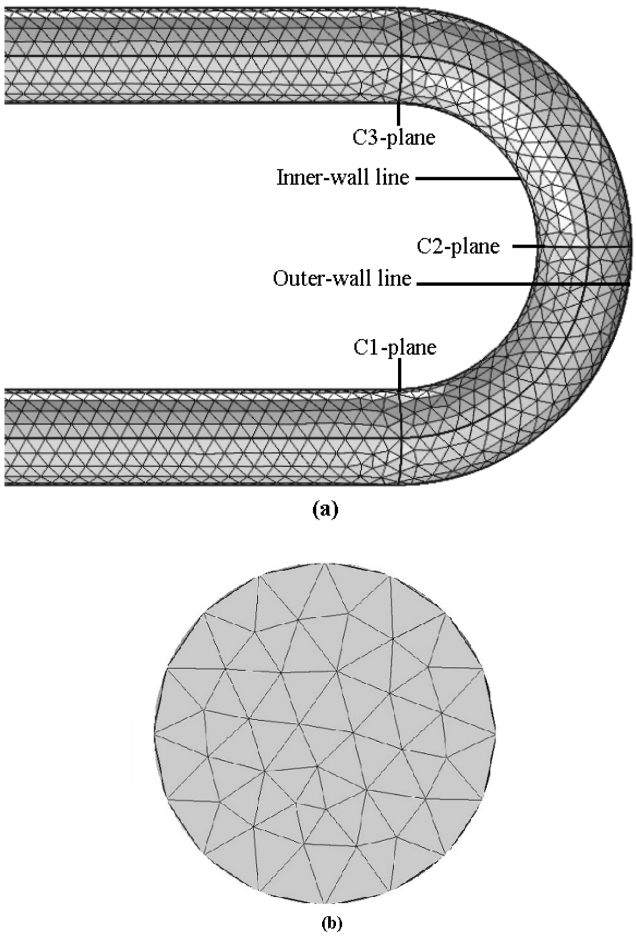


Fig. 2: Mesh design for finite element approximation at the U-loop (a) and the inlet/outlet surface (b) of the HLTP.

In this study, we analyze the local flow behavior of microalgae suspension in a HLTP. Firstly, a single-phase model (SPM) of microalgae suspension is considered. Afterwards CO_2 is taken into account in the model. The flow model becomes a two-phase flow model (TPM). Using the governing equations of a single-phase together with the Cahn-Hiliard mass conservation equation, the dynamic behavior of a two-phase flow can be described. The model with no-slip condition on the wall and zero outward normal stress at the outlet of the HLTP is used to describe the fluid phenomena inside the HLTP.

The rest of our paper is organized as follows. In Section II, the mathematical model of microalgae flow is presented. In Section III, numerical results for two-phase flow and single-phase flow are shown and discussed. Finally, some conclusions are presented in section IV.

II. Mathematical Models

The ultimate goal of this study is to improve our understanding of fluid dynamics phenomena after injecting CO_2 gas. Our model is developed to simulate the fluid dynamic behavior in an airlift-driven HLTP. Tredici's study [16] suggested that an airlift system provided uniform mixing by bubbling gas directly or indirectly in a tubular photobioreactor. Our assumption for using a typical airlift system is because of uniform mixing of liquid and gas phases. In this coupled nonlinear model, microalgae suspension and CO_2 including other nutrients occupied in the HLTP, the turbulent flow appears.

A. Computational Domain and Mesh Design

In this study, a Horizontal Loop Tubular Photo-bioreactor as depicted in Fig. 1 has the total length of 32 *m* and the radius of 0.025 *m*. The working volume and the surface area are

0.06119 m^3 and 4.946 m^2, respectively. It is notable that mesh size plays an important role in terms of accuracy of numerical results. To achieve a satisfactory computational accuracy we continually change our meshes until the results obtained from two trials lead to very close to each other. A fine mesh is used for our numerical simulation with 376,057 elements and 3,033,933 degrees of freedom for two-phase flow model, whereas same number of elements used in the two-phase model and 589,543 degrees of freedom for single-phase flow model. The mesh design for a U-loop and the inlet are shown in Fig. 2. It is also noted that three cross sections which are marked at the beginning, the middle and the end of the U-loop as shown in Fig. 2(a); are used to investigate the flow dynamics of microalgae-gas flow in this study.

B. Governing Equations

The uniform mixture of algal suspension and CO_2 is considered as an incompressible two-phase Newtonian fluid and the flow problem is assumed to be laminar. Therefore the flow phenomenon is governed by the following equations:

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

$$\rho\frac{\partial \vec{u}}{\partial t} + \rho(\vec{u}\cdot\nabla)\vec{u} = \nabla\cdot\left[-pI + \eta(t)\left(\nabla\vec{u} + (\nabla\vec{u})^T\right)\right] + \rho\vec{g} + \vec{F}_{st}, \tag{2}$$

where $\vec{u}$ denotes the velocity of the two-phase fluid; ρ, η and p are density, viscosity and pressure, respectively; $\vec{g}$ is the gravity, I is the identity matrix; $\vec{F}_{st}$ is the surface tension force. Together with (1) and (2), the separation of microalgae suspension and CO_2 is described by the Cahn- Hilliard equation [17] as follows:

$$\frac{\partial\phi}{\partial t} + \vec{u}\cdot\nabla\phi = \nabla\cdot\frac{\gamma\lambda}{\varepsilon_{pf}^2}\nabla\varphi, \tag{3}$$

where ϕ represents the dimensionless phase field variable. The function φ is given by:

$$\varphi = -\nabla\cdot\varepsilon_{pf}^2\nabla\phi + (\phi^2 - 1)\phi\frac{\varepsilon_{pf}^2}{\lambda}\frac{\partial f}{\partial\phi}, \tag{4}$$

where ε_{pf} is a parameter controlling interface thickness, γ is the mobility, and λ is the mixing energy density. The three parameters are related as follows:

$$\lambda = \frac{3\varepsilon_{pf}\sigma}{\sqrt{8}}, \tag{5}$$

$$\gamma = \chi\varepsilon_{pf}^2, \tag{6}$$

where χ is the mobility tuning parameter.

The term $\partial f/\partial\phi$ in (4) denotes the phi-derivative of external free energy and σ in (5) is the surface tension coefficient. The density and viscosity of single-phase flow are constant whereas the density and the viscosity of the mixture are the function of volume fraction of microalgae suspension V_m, which is given by $V_m = (1+\phi)/2$, i.e.

$$\rho = \rho_c + (\rho_m - \rho_c)V_m, \tag{7}$$

$$\eta = \eta_c + (\eta_m - \eta_c)V_m, \tag{8}$$

where the subscripts m and c are used for the microalgae and CO_2 gas, respectively.

As bubble distribution plays an important role in coalescence, we need to consider the surface tension force in the two-phase flow. The surface tension force in (2) depending on chemical potential G is given by

$$\vec{F}_{st} = G\nabla\phi. \tag{9}$$

Proliferation of microalgae cell in the culture induces the change of viscosity and subsequently the concentration with respect to time, hence we define the relative viscosity (η_r) to be a ratio between microalgae suspension viscosity (η_m) and water viscosity (η_w) as follows:

$$\eta_m = \eta_w\eta_r(t). \tag{10}$$

According to Wu and Merchuk's study [18], if we assume a microalgae cell to be a small sphere in our study, then the relative viscosity relating to concentration is determined by Einstein's relative viscosity equation as follows:

$$\eta_r(t) = 1 + \in C(t), \tag{11}$$

where $\in$ is the Einstein's coefficient [19], and $C(t)$ is the cell concentration depending on time. Based on the experimental data obtained by Hon-nami and Kunito [14], a logistic relation between concentration and growth rate (μ) is

$$C(t) = C_0 + \frac{a}{b\exp(-\mu t)}, \tag{12}$$

where C_0 is the initial concentration of the microalgae suspension, and a and b are constants. Fig. 3 represents the logistic curve of cell concentration obtained from (12).

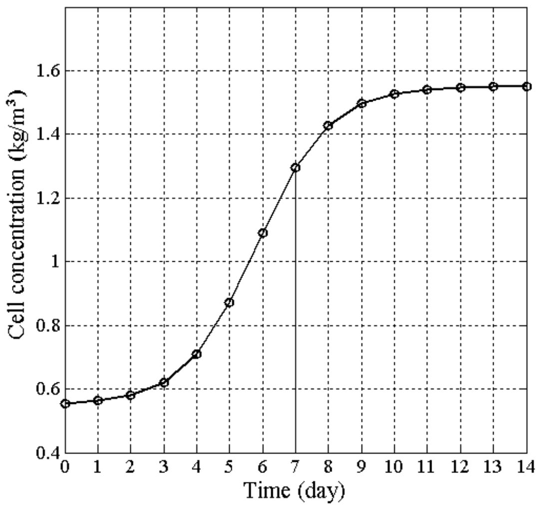


Fig. 3: The logistic curve for cell concentration of microlalgae with respect to time.

As microalgae are photosynthetic micro-organism, solar energy is required for their rapid growth. In this context, the growth equation relating to average irradiance (I_{av}) and maximum growth rate ($\mu_{\max}$) according to Molina's study [20] is used:

$$\mu = \frac{\mu_{\max} I_{av}}{I_k + I_{av}}, \tag{13}$$

where I_k is a constant depending upon micro-algal culture condition. If we tune out the physiological behavior of microalgae cell, primarily the available incident irradiance (I_0) on the reactor wall, the tube diameter, and PBR geographical position are the key factors to calculate the average irradiance, i.e.

$$I_{av} = \frac{I_0}{DK_a C(t)}[1 - \exp(-DK_a C(t))], \tag{14}$$

where K_a is the extinction coefficient of the biomass, $D = \frac{d_t}{\cos\theta}$, d_t is the diameter of the PBR tube and θ is the angle of incident of direct radiation depending on a function of five parameters including the declination (δ), solar hour (sh), geographical latitude (ψ), surface slope (β), surface azimuth angle (τ), and the hour angle (ω) as [21]:

$$\begin{aligned} \cos\theta &= \sin\delta\sin\psi\cos\beta - \sin\delta\cos\psi\sin\beta\cos\tau \\ &\quad + \cos\delta\cos\psi\cos\beta\cos\omega \\ &\quad + \cos\delta\cos\psi\sin\beta\cos\tau\cos\omega \\ &\quad + \cos\delta\sin\beta\sin\tau\sin\omega. \end{aligned} \tag{15}$$

Camacho et al. [22] pointed out that in order to utilize the maximum solar light with respect to change in solar hour and to reduce the reflection of energy, the solar collector of PBR's should be placed horizontally. In this context, the surface slope (β) is set to zero degree. Hence, we get the simplest form of (15) as:

$$\cos\theta = \sin\delta\sin\psi + \cos\delta\cos\psi\cos\omega, \tag{16}$$

where

$$\delta = 23.45\sin\left[\frac{360}{365}(284 + N)\right], \tag{17}$$

and N is the day of the year [23]. Now only one unknown ω in (16) is needed to estimate. Here, we adopted the concept of Duffie and Beckman [23]. An hour of Earth rotation from east to west is equivalent to 15 degree angular displacement, and the value of an hour angle is negative for morning hours and positive for afternoon hours. They suggested that an hour angle can be estimated by

$$\omega = 15(sh - 12), \tag{18}$$

where sh is the solar hour determined by Fig. 4.

C. Boundary and Initial Conditions:

The boundary conditions used for our simulation are considered as a uniform flow velocity $|\vec{u}| = U_{in}$ at the inlet, non-slip on the tube wall of the HLTP, i.e., $\vec{u} = 0$ and zero normal stress at the outlet of the domain which is given by

$$\left[-pI + \eta(t)\left(\nabla\vec{u} + (\nabla\vec{u})^T\right)\right]\vec{n} = 0. \tag{19}$$

III. Numerical results and discussions

The main goal of this study is to develop our in-depth understanding of fluid behavior after injecting CO_2 gas. There

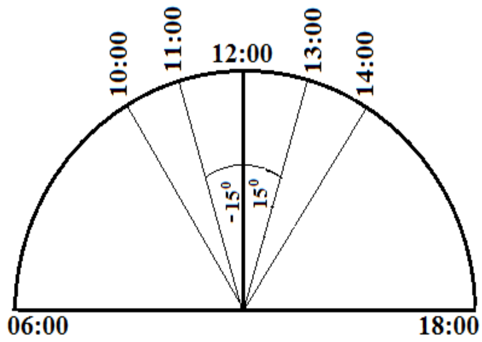


Fig.4: Calculation of an hour angle.

are considerable evidences that CO_2 supply plays an important role to increase the mass transfer rate. The COMSOL Multiphysics package version 4.2a has been used to solve the problem. The initial solution $\vec{u} = 0$ is assumed for the entire domain except at the inlet. The HLTP solar collector tube is assumed to be illuminated by sunlight with varying incident intensity and the culture temperature is set to 25^0C. The volume fractions of CO_2 and microalgae suspension are 0.05 and 0.95 respectively. The simulation was carried out on the seventh day the of the microalgae culture using the parameter values according to Table 1.

TABLE I
Parameters value using for simulation

Symbol	Quantity	Values
$\mu_{\max}$	Maximum growth rate	0.0000175 s^{-1}
I_k	Constant	114.67 $\mu molm^{-2}s^{-1}$
I_0	Incident Irradiance	1630 $\mu molm^{-2}s^{-1}$
E	Einstein co-efficient	2500 m^3kg^{-1}
C_0	Initial concentration	0.55 kgm^{-3}
a	Constant	1
b	Constant	200
K_a	Extinction coefficient	36.9 m^2kg^{-1}
η_c	CO_2 viscosity	0.000625 Pa·s
η_w	Water viscosity	0.001 Pa·s
ρ_c	CO_2 density	0.001799 kgm^{-3}
ρ_m	Microalgae density	1020 kgm^{-3}
N	Day of the year	172
$\frac{\partial f}{\partial \phi}$	Phi-derivative of external free energy	0.01
σ	Surface tension coefficient	0.07197 Nm^{-1}
χ	Mobility tuning Parameter	1 $m \cdot skg^{-1}$
ε_{pf}	Parameter controlling Interface thickness	0.01
U_{in}	Inlet initial velocity	0.5 ms^{-1}
d_t	Tube diameter	0.05 m
ψ	Geographic latitude (Phayathai, Bangkok)	13°45'32"
g	Gravity	9.8 ms^{-2}

The geographical location of our HLTP setup position is Phyathai, Bangkok, Thailand. Thailand locates in the northern hemisphere; therefore the 21st June is the longest day [24], which is chosen in our simulation. For the optimal accuracy, a parameter controlling interface thickness is assumed to be half of the maximum element size in our model. The mobility tuning parameter determining the time scale of Cahn-Hilliard diffusion is set to unity for a good starting point in our simulation. The surface tension coefficient of liquid-gas phase is assumed as the one of water-air phase. The element discretization is considered in the finite element formulation. The second order elements for the velocity components and linear elements are used for the pressure.

CFD simulations are carried out for both models in the same culture. The velocity profile, shear rate and pressure distributions are obtained for the entire HLTP. The velocity magnitudes for SPM of microalgae suspension at three different cross sections around the 1st U-loop are depicted in Fig. 5. The parabolic flow profile is observed in the straight parts of the tube and become slightly skewed to the inner wall when it reaches to the entrance of the U-loop. When the flow arrives in the middle part of the U-loop, a swirling flow is introduced. After passing the cross section C3 the flow becomes parabolic profile. This scenario implies that the local flow depends strongly on the geometry of the tube. Similar behavior is observed for the two-phase model (TPM) around the U-loop area. As described about the two-phase flow behavior elaborately in our previous study [25], it is observed that the velocity magnitude is generally high at the middle of the tube and higher speed occurs in the TPM. The maximum speed attains approximately 0.805 ms^{-1} in SPM, whereas it is about 0.933ms^{-1} in TPM as shown in Fig. 6.

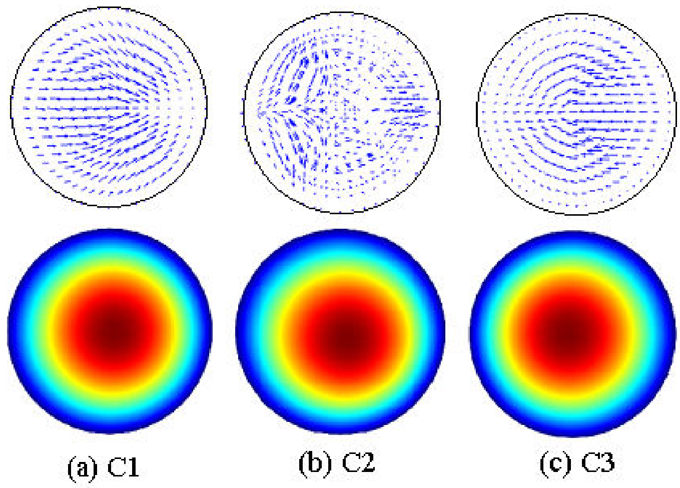


Fig. 5: Vector plot and surface plot of velocty field at three different cross sections: (a) C1 (b) C2 (c) C3.

Comparisons of flow speed obtained from SPM and TPM at three cross sectional views are presented by line graph in Fig. 7. These indicate that gas flow increases the mass transfer rate. It is also observed that the flow behavior is not similar at the three cross sections.

It is well known that motion of any kind of fluid incurs shear stress on the wall of the domain. In a tubular photo-bioreactor, wall shear stress might be the main reason for cell fragility and it prevents the optimization of biomass production. Therefore, we pay special attention to investigate the shear rate for the entire domain in this study.

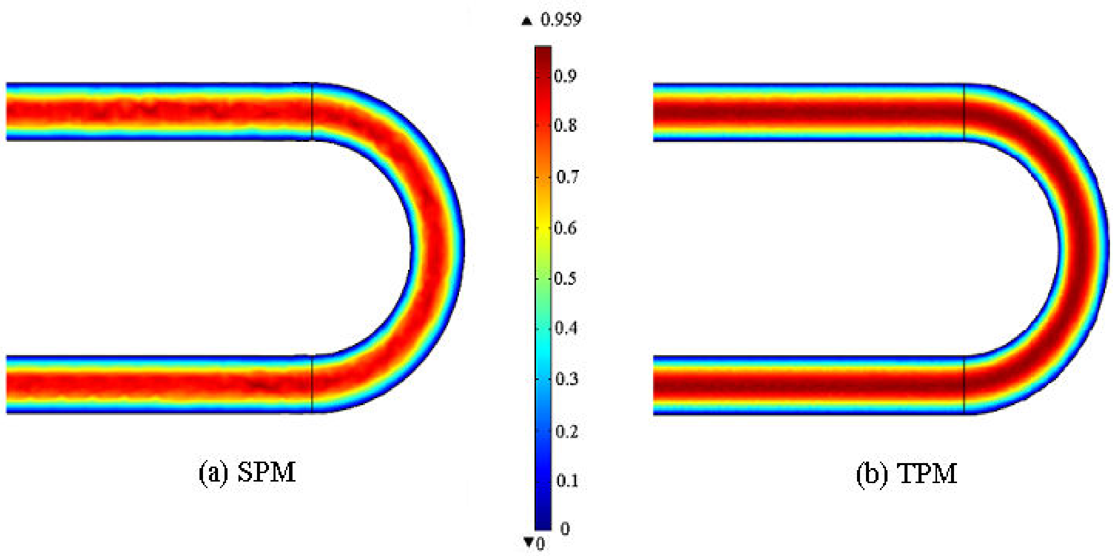


Fig 6: Mean speed of microalgae suspension obtained from two different models: (a) a single-phase flow model (SPM); (b) a two-phase flow model (TPM).

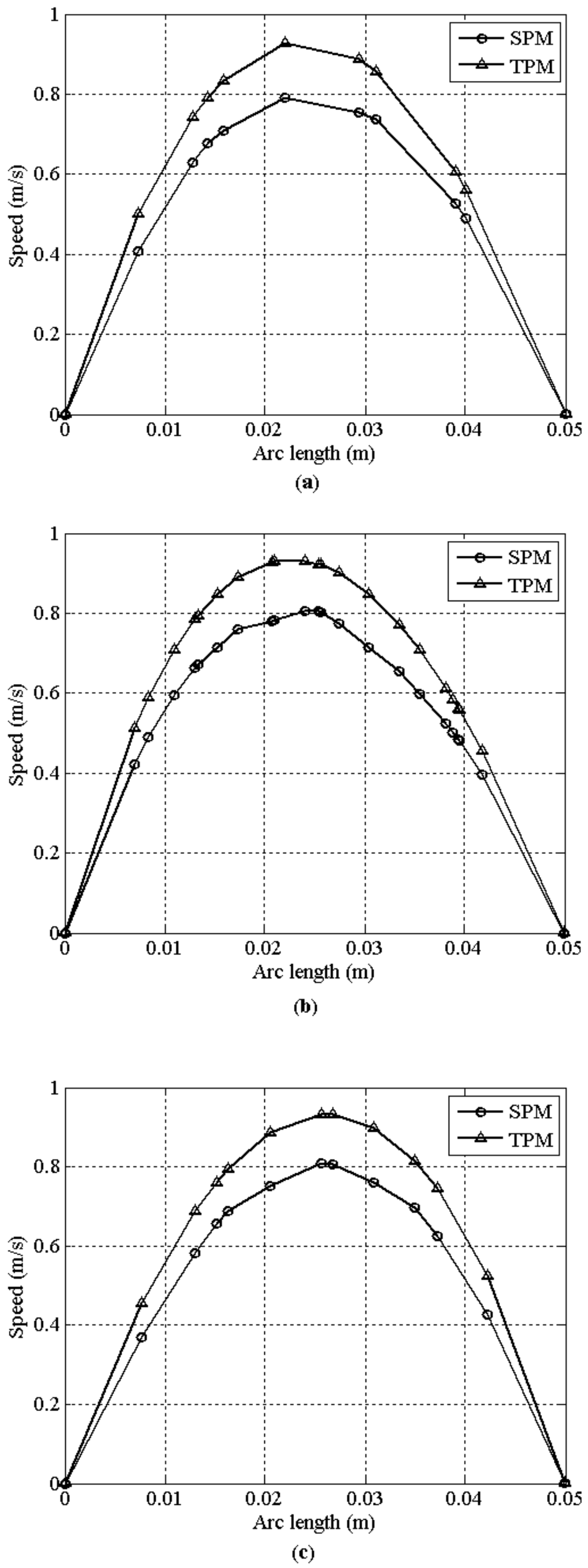


Fig. 7: Flow speed obtained from SPM and TPM at three different cross sections: a) C1, b) C2, and c) C3.

Fig. 8 demonstrates distribution of wall shear rate along the outer line at time 18:00 obtained from both models. It is clear that the shear rate obtained from TPM is higher than the one from SPM. It is also noticed that there is less shear rate in the straight tube and high shear rate occur in the U-loop areas. A maximum value of wall shear rate occurs closer to the entrance of the U-loop. Unlike TPM, the variation of wall shear rate at a straight tube obtained from the SPM is not smooth. As shear rate of the flow is related with the velocity of the fluid, this scenario facilitates us to accept the increment of speed in the case of TPM.

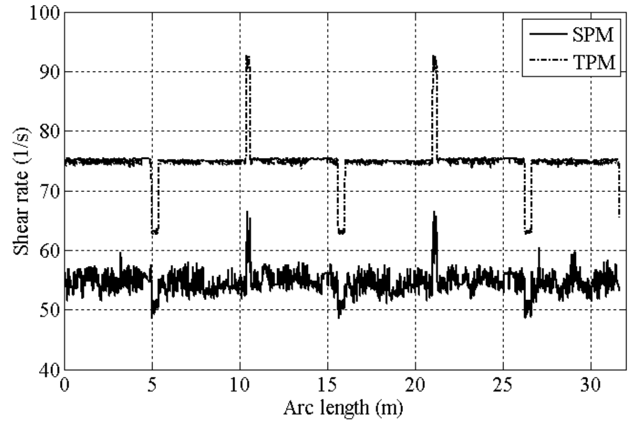
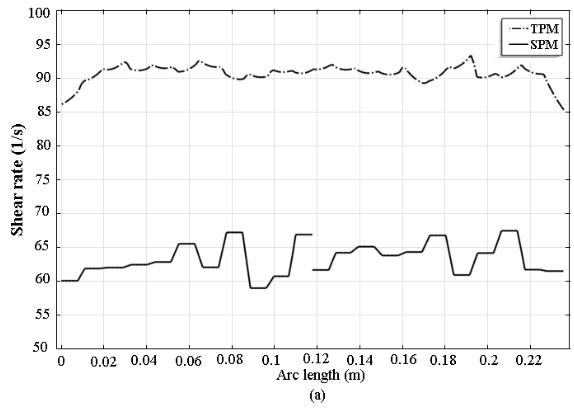


Fig. 8: Wall shear rate along the inlet to outlet along the outer-line (see Fig. 2) at time 18:00 obtained from SPM and TPM.

We also investigate the shear rate pattern along the inner and the outer lines of the 1st U-loop obtained from SPM and TPM as shown in Fig. 9. It is found that the shear rate along the inner wall is higher than the one along the outer wall. This shows that splitting flow arises around the U-loop after passing the cross-section C1. We observed that flow patterns at all U-loop are the same. These flow phenomena allow us to understand that the inner wall is more responsible for cell damage than the outer wall around U-loop area of the HLTP.



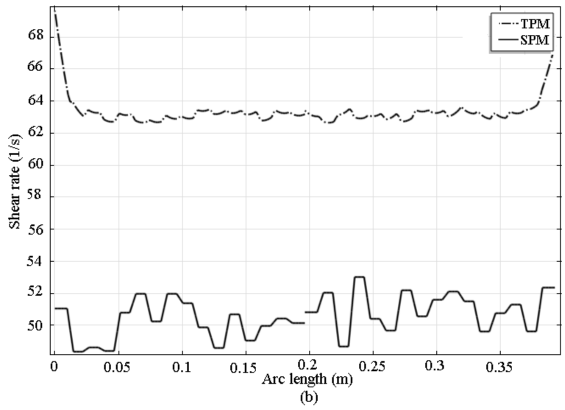


Fig. 9: Comparison of shear rate distribution around the U-loop: (a) along the inner-wall line (b) along the outer-wall line.

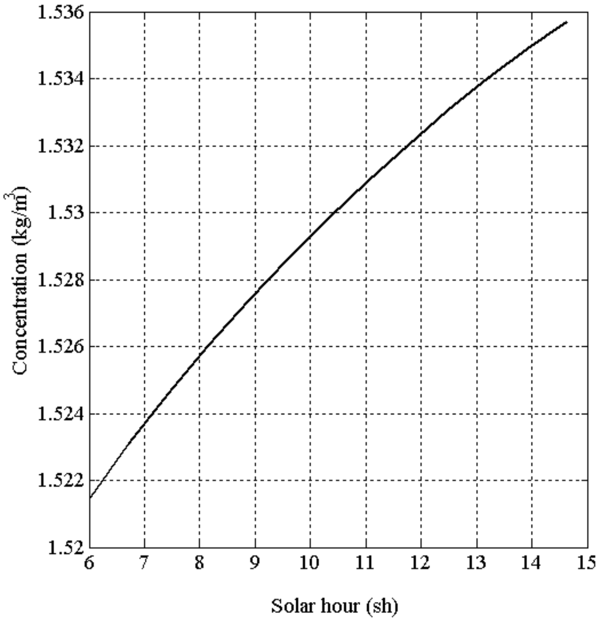


Fig. 10: Concentration of microalage cell from 06:00 to 18:00 on the seventh day of the culture.

Fig. 10 presents a graph of cell concentration versus time. The cell concentration of microalgae culture on the seventh day from the morning (06:00) to the evening (18:00) increases from 1.52 to 1.53 kg/m^3. A very slow increase of concentration is observed. From this result we can interpret that the growth related to concentration of microalgae is not constant but increased with the day length with respect to continuous light. The comparison of pressure profile obtained from both models at 18:00 is shown in Fig. 11. A uniform pressure drop from the inlet to outlet is found from both models. Like velocity and shear rate, pressure is also lower in SPM comparing with TPM. It is noted that for both models pressure is found to be lower at time 18:00 comparing with the pressure at time 06:00 on the day seventh of the culture.

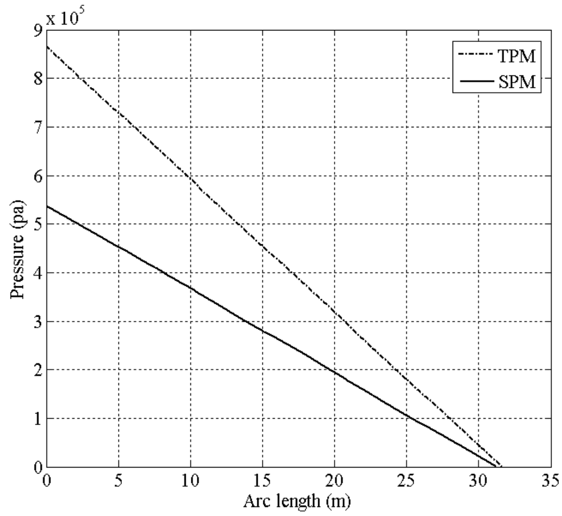


Fig. 11: The pressure profile along the tube axis at an instant time 18:00.

IV. Conclusion

In this study numerical result of velocity profile, shear rate distributions and pressure profile have been obtained from single-phase flow and two-phase flow models of microalgae flow in the HLTP on the longest day of the year in Bangkok, Thailand. It is noted that effect of CO_2 injection on fluid flow behavior such as velocity profile and shear rate distribution is significant. The flow speed and the shear rate of fluid in HLTP obtained from TPM are observed to be higher than those obtained from the single-phase model. Regarding the flow structures, the flows in the straight tube and in the curve tube are dissimilar. The shear rate distribution is higher around the U-loop region than the straight tube portion. This indicates that the geometry of the PBR also plays a vital role in flow dynamics. A uniform linear pressure drop is observed from the inlet to the outlet of the HLTP.

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