A New Approach to Non-fragile $\mathcal{H}_\infty$ Fuzzy Filter of Uncertain Markovian Jump Nonlinear Systems

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Abstract — This paper considers the problem of designing a non-fragile $\mathcal{H}_\infty$ fuzzy filter for uncertain Markovian jump nonlinear systems that the guarantees the $\mathcal{L}_2$-gain from an exogenous input to an estimate error output being less than or equal to a prescribed value. Sufficient conditions for the existence of the $\mathcal{H}_\infty$ fuzzy filter are given in terms of a set of LMIs. In this paper, the premise variables of the $\mathcal{H}_\infty$ fuzzy filter are allowed to be different from the premise variables of the TS fuzzy model of the plant such that the results are shown into two cases which are the premise variable of the fuzzy model be measurable and the premise variable assumed to be unmeasurable.

Keywords — Non-fragile $\mathcal{H}_\infty$ Filtering, Fuzzy systems, Lyapunov function, Fuzzy filtering, LMIs.

I. Introduction

In the last few years, many researchers have studied the $\mathcal{H}_\infty$ filter design for a general class of linear systems due to a great practical importance [1]-[9]. Solutions to the nonlinear $\mathcal{H}_\infty$ filtering are characterized in terms of the so-called Hamilton-Jacobi equation (HJE) in [7]. Until now, however, it is still very difficult to find a global solution to the HJE either analytically or numerically. The filtering problem can be stated as follows: given a dynamic system with exogenous input and measured output, design a filter to estimate an unmeasured output such that the mapping from the exogenous input to the filter error is minimized or no larger than some prescribed level in terms of the $\mathcal{H}_\infty$ norm. In [4] and [5], it has been shown that the existence of solution to $\mathcal{H}_\infty$ filtering problem is in fact related to the solvability of an appropriate algebraic Riccati equation. This result is then extended in [6] to a class of linear systems which are subject to parametric uncertainty. A sufficient condition for the existence of a solution is derived also via algebraic Riccati equations.

Recently, a great amount of effort has been made on the design of fuzzy $\mathcal{H}_\infty$ control and filter for a class of nonlinear systems which can be represented by a Takagi-Sugeno (TS) fuzzy model; see [10]-[25]. Fuzzy system theory enables us to utilise qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. Recent studies show that a fuzzy linear model can be used to approximate global behaviours of a highly complex nonlinear system; see for example, [10]-[25]. In this fuzzy linear model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling where a single model is used to describe the global behaviour of a system, the fuzzy modelling is essentially a multi-model approach in which simple sub-models (linear models) are combined to describe the global behaviour of the system. Based on this fuzzy model, a systematic model-based fuzzy control design was developed.

Markovian jump systems, sometimes called hybrid systems with a state vector, consists of two components; i.e., the state (differential equation) and the mode (Markov process). The Markovian jump system changes abruptly from one mode to another mode caused by some phenomenon such as environmental disturbances, changing subsystem interconnections and fast variations in the operating point of the system plant. The switching between modes is governed by a Markov process with the discrete and finite state space. Over the past few decades, the Markovian jump systems have been extensively studied by many researchers; see [26]-[36]. This is due to the fact that jumping systems have been a subject of the great practical importance.

The aim in this paper is to study the problem of designing a robust or non-fragile fuzzy filter for uncertain Markovian jump nonlinear signal processing systems that guarantees the $\mathcal{L}_2$-gain from an exogenous input to a filter error is less or equal to a prescribed value. Based on an LMI approach, solutions to the problem of the $\mathcal{H}_\infty$ filtering are derived in terms of a family of linear matrix inequalities. In this paper, the premise variables of the $\mathcal{H}_\infty$ fuzzy filter are allowed to be different from the premise variables of the TS fuzzy model of the plant such that the results are shown into two cases which are the premise variable of the fuzzy model be measurable and the premise variable assumed to be unmeasurable.

This paper is organized as follows. In Section II, system descriptions and definition are presented. Based on an LMI approach, we develop a technique in Section III for designing a non-fragile fuzzy $\mathcal{H}_\infty$ filter that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the filter error is less than a prescribed value. The validity of this approach is demonstrated by an example from the literature in Section IV. Finally, in Section IV, the conclusion is given.

II. System Descriptions and Definitions

In this section, we generalize the TS fuzzy system to represent a TS fuzzy system with Markovian jumps. In this paper, we examine a TS fuzzy system with Markovian jumps

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as follows:

$$\begin{aligned}\dot{x}(t) &= \textstyle\sum_{i=1}^{r} \mu_i(\nu(t))\Big[[A_i(\eta(t)) + \Delta A_i(\eta(t))]x(t) \\ &\quad +[B_{1_i}(\eta(t)) + \Delta B_{1_i}(\eta(t))]w(t) \\ &\quad +[B_{2_i}(\eta(t)) + \Delta B_{2_i}(\eta(t))]u(t)\Big], \quad x(0) = 0, \\ z(t) &= \textstyle\sum_{i=1}^{r} \mu_i(\nu(t))\Big[[C_{1_i}(\eta(t)) + \Delta C_{1_i}(\eta(t))]x(t) \\ &\quad +[D_{12_i}(\eta(t)) + \Delta D_{12_i}(\eta(t))]u(t)\Big] \\ y(t) &= \textstyle\sum_{i=1}^{r} \mu_i(\nu(t))\Big[[C_{2_i}(\eta(t)) + \Delta C_{2_i}(\eta(t))]x(t) \\ &\quad +[D_{21_i}(\eta(t)) + \Delta D_{21_i}(\eta(t))]w(t)\Big]\end{aligned} \tag{1}$$

where $\nu(t) = [\nu_1(t) \ \cdots \ \nu_\vartheta(t)]$ is the premise variable vector that may depend on states in many cases, $\mu_i(\nu(t))$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\mu_i(\nu(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(\nu(t)) = 1$), ϑ is the number of fuzzy sets, $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, $\mathrm{y}(t) \in \Re^\ell$ is the measurement, $z(t) \in \Re^s$ is the controlled output, and the matrix functions $A_i(\eta(t))$, $B_{1_i}(\eta(t))$, $B_{2_i}(\eta(t))$, $C_{1_i}(\eta(t))$, $C_{2_i}(\eta(t))$, $D_{12_i}(\eta(t))$, $D_{21_i}(\eta(t))$, $\Delta A_i(\eta(t))$, $\Delta B_{1_i}(\eta(t))$, $\Delta B_{2_i}(\eta(t))$, $\Delta C_{1_i}(\eta(t))$, $\Delta C_{2_i}(\eta(t))$, $\Delta D_{12_i}(\eta(t))$ and $\Delta D_{21_i}(\eta(t))$ are of appropriate dimensions. $\{\eta(t))\}$ is a continuous-time discrete-state Markov process taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, s\}$ with transition probability matrix $Pr \stackrel{\Delta}{=} \{P_{\imath k}(t)\}$ given by

$$\begin{aligned}P_{\imath k}(t) &= Pr(\eta(t+\Delta) = k|\eta(t) = \imath) \\ &= \begin{cases} \lambda_{\imath k}\Delta + O(\Delta) & \text{if } \imath \neq k \\ 1 + \lambda_{\imath\imath}\Delta + O(\Delta) & \text{if } \imath = k \end{cases}\end{aligned} \tag{2}$$

where $\Delta > 0$, and $\lim_{\Delta \longrightarrow 0} \frac{O(\Delta)}{\Delta} = 0$. Here $\lambda_{\imath k} \geq 0$ is the transition rate from mode $\imath$ (system operating mode) to mode k ($\imath \neq k$), and

$$\lambda_{\imath\imath} = -\sum_{k=1, k\neq \imath}^{s} \lambda_{\imath k}. \tag{3}$$

For the convenience of notations, we let $\mu_i \stackrel{\Delta}{=} \mu_i(\nu(t))$, $\eta = \eta(t)$, and any matrix $M(\mu, \imath) \stackrel{\Delta}{=} M(\mu, \eta = \imath)$. The matrix functions $\Delta A_i(\eta)$, $\Delta B_{1_i}(\eta)$, $\Delta B_{2_i}(\eta)$, $\Delta C_{1_i}(\eta)$, $\Delta C_{2_i}(\eta)$, $\Delta D_{12_i}(\eta)$ and $\Delta D_{21_i}(\eta)$ represent the time-varying uncertainties in the system and satisfy the following assumption.

Assumption 1:

$$\Delta A_i(\eta) = F(x(t), \eta, t) H_{1_i}(\eta),$$

$$\Delta B_{1_i}(\eta) = F(x(t), \eta, t) H_{2_i}(\eta),$$

$$\Delta B_{2_i}(\eta) = F(x(t), \eta, t) H_{3_i}(\eta),$$

$$\Delta C_{1_i}(\eta) = F(x(t), \eta, t) H_{4_i}(\eta),$$

$$\Delta C_{2_i}(\eta) = F(x(t), \eta, t) H_{5_i}(\eta),$$

$$\Delta D_{12_i}(\eta) = F(x(t), \eta, t) H_{6_i}(\eta),$$

$$\textit{and} \ \ \Delta D_{21_i}(\eta) = F(x(t), \eta, t) H_{7_i}(\eta)$$

where $H_{j_i}(\eta)$, $j = 1, 2, \cdots, 7$ are known matrices which characterize the structure of the uncertainties. Furthermore, there exists a positive function $\rho(\eta)$ such that the following inequality holds:

$$\|F(x(t), \eta, t)\| \leq \rho(\eta). \tag{4}$$

III. Non-fragile Filter Design

This section presents a technique of designing a non-fragile fuzzy filter for a TS fuzzy system with Markovian jumps and parametric uncertainties. We develop a technique for designing a non-fragile fuzzy filter such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the estimated error output is less than the prescribed value. The proposed design is given in terms of LMIs.

Without loss of generality, we assume $u(t) = 0$. Let us recall the system (1) with $u(t) = 0$ as follows:

$$\begin{aligned}\dot{x}(t) &= \textstyle\sum_{i=1}^{r} \mu_i\Big[[A_i(\eta) + \Delta A_i(\eta)]x(t) \\ &\quad +[B_{1_i}(\eta) + \Delta B_{1_i}(\eta)]w(t)\Big], \quad x(0) = 0 \\ z(t) &= \textstyle\sum_{i=1}^{r} \mu_i\Big[[C_{1_i}(\eta) + \Delta C_{1_i}(\eta)\Big]x(t) \\ y(t) &= \textstyle\sum_{i=1}^{r} \mu_i\Big[[C_{2_i}(\eta) + \Delta C_{2_i}(\eta)]x(t) \\ &\quad +[D_{21_i}(\eta) + \Delta D_{21_i}(\eta)]w(t)\Big].\end{aligned} \tag{5}$$

The aim is to design a full order dynamic $\mathcal{H}_\infty$ fuzzy filter of the form

$$\begin{aligned}\dot{\hat{x}}(t) &= \textstyle\sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\mu}_i\hat{\mu}_j\Big[\hat{A}_{ij}(\imath)\hat{x}(t) + \hat{B}_i(\imath)y(t)\Big] \\ \hat{z}(t) &= \textstyle\sum_{i=1}^{r} \hat{\mu}_i\hat{C}_i(\imath)\hat{x}(t)\end{aligned} \tag{6}$$

where $\hat{x}(t) \in \Re^n$ is the filter's state vector, $\hat{z} \in \Re^s$ is the estimate of $z(t)$, $\hat{A}_{ij}(\imath)$, $\hat{B}_i(\imath)$ and $\hat{C}_i(\imath)$ are parameters of the filter which are to be determined, and $\hat{\mu}_i$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\hat{\mu}_i \geq 0$ and $\sum_{i=1}^{r} \hat{\mu}_i = 1$), such that the following inequality holds

$$\begin{aligned}&\mathbf{E}\Big[\textstyle\int_0^{T_f} \Big\{\Big(z(t) - \hat{z}(t)\Big)^T\Big(z(t) - \hat{z}(t)\Big) \\ &\quad -\gamma^2 w^T(t)w(t)\Big\}\, dt\Big] \leq 0, \ x(0) = 0\end{aligned} \tag{7}$$

where $\mathbf{E}[\cdot]$ stands for the mathematical expectation and $(z(t) - \hat{z}(t))$ is the estimated error output, for all $T_f \geq 0$ and $w(t) \in [0, T_f]$.

Figure 1 shows the block diagram of a non-fragile fuzzy filtering problem associated with an uncertain fuzzy system. The major implication of this approach is that the structure of the filter has to take into a account the effect of uncertainty. The problem addressed is the design of a filter such that the induced operator norm of the mapping from the noise $w(t)$ to the filter error $e(t) = z(t) - \hat{z}(t)$ is kept within a prescribed bound for all admissible parameter uncertainties.

Clearly, in real control problems, all of the premise variables are not necessarily measurable, thus two cases will be considered in this section. Subsection A considers the case where the premise variable of the fuzzy model μ_i is measurable, while in Subsection B, the premise variable is assumed to be unmeasurable.

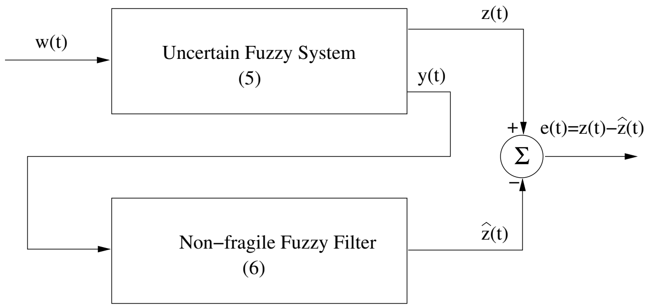


Fig. 1. Block diagram of an uncertain fuzzy system with a non-fragile $\mathcal{H}_\infty$ fuzzy filter.

A. Case I–$\nu(t)$ is available for feedback

The premise variable of the fuzzy model $\nu(t)$ is available for feedback which implies that μ_i is available for feedback. Thus, we can select our filter that depends on μ_i as follows:

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \Big[\hat{A}_{ij}(\imath)\hat{x}(t) + \hat{B}_i(\imath)y(t)\Big] \\ \hat{z}(t) & = & \sum_{i=1}^r \mu_i \hat{C}_i(\imath)\hat{x}(t). \end{array} \quad (8)$$

Figure 2 shows the block diagram of the non-fragile $\mathcal{H}_\infty$ filtering problem associated with uncertain fuzzy system in case that μ_i is available for feedback. Before presenting our

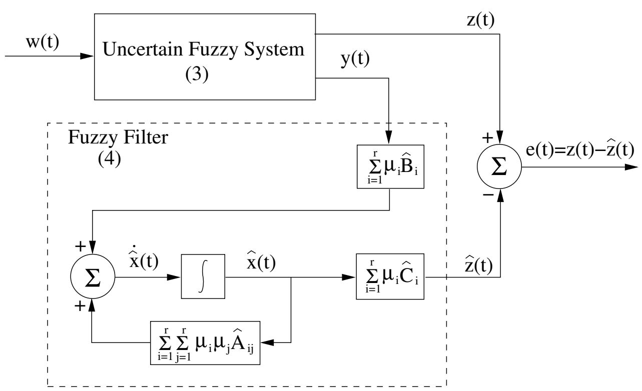


Fig. 2. Block diagram of an uncertain fuzzy system with a non-fragile $\mathcal{H}_\infty$ fuzzy filter in Case A.

next results, the following lemma is recalled.

Lemma 1: Consider the system (5). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and any positive constants $\delta(\imath)$, for $\imath = 1, 2, \cdots, s$, if there exist matrices $P(\imath) = P^T(\imath)$ such that the following linear inequalities hold:

$$P(\imath) \;>\; 0 \quad (9)$$

$$\begin{pmatrix} \begin{pmatrix} P(\imath)A_{cl}^{ij}(\imath) \\ +(A_{cl}^{ij}(\imath))^T P(\imath) \\ +\sum_{k=1}^s \lambda_{\imath k} P(k) \end{pmatrix} & (*)^T & (*)^T \\ (P(\imath)B_{cl}^{ij}(\imath))^T & -\gamma^2 I & (*)^T \\ C_{cl}^{ij}(\imath) & 0 & -I \end{pmatrix} < 0 \quad (10)$$

where $i, j = 1, 2, \cdots, r$,

$$A_{cl}^{ij}(\imath) = \begin{bmatrix} A_i(\imath) & 0 \\ \hat{B}_i(\imath)C_{2_j}(\imath) & \hat{A}_{ij}(\imath) \end{bmatrix},$$

$$B_{cl}^{ij}(\imath) = \begin{bmatrix} \tilde{B}_{1_i}(\imath) \\ \hat{B}_i(\imath)\tilde{D}_{21_j}(\imath) \end{bmatrix},$$

$$C_{cl}^{ij}(\imath) = [\tilde{C}_{1_i}(\imath) \quad \tilde{D}_{12}(\imath)\hat{C}_j(\imath)]$$

with

$$\tilde{B}_{1_i}(\imath) = \Big[\delta(\imath)I \;\; I \;\; 0 \;\; B_{1_i}(\imath) \;\; 0\Big]$$

$$\tilde{C}_{1_i}(\imath) = \Big[\frac{\gamma\rho(\imath)}{\delta(\imath)}H_{1_i}^T(\imath) \;\; \frac{\gamma\rho(\imath)}{\delta(\imath)}H_{5_i}^T(\imath)$$

$$\sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath) \;\; \sqrt{2}\aleph(\imath)C_{1_i}^T(\imath)\Big]^T$$

$$\tilde{D}_{12}(\imath) = \Big[0 \;\; 0 \;\; 0 \;\; -\sqrt{2}\aleph(\imath)I\Big]^T$$

$$\tilde{D}_{21_i}(\imath) = \Big[0 \;\; 0 \;\; \delta(\imath)I \;\; D_{21_i}(\imath) \;\; I\Big]$$

$$\begin{array}{rcl} \aleph(\imath) & = & \Bigg(1 + \rho^2(\imath)\sum_{i=1}^r\sum_{j=1}^r \Big[\|H_{2_i}^T(\imath)H_{2_j}(\imath)\| \\ & & +\|H_{7_i}^T(\imath)H_{7_j}(\imath)\|\Big]\Bigg)^{\frac{1}{2}}, \end{array}$$

then the inequality (7) is guaranteed.

Proof: The closed-loop state space form of the fuzzy system model (5) with the filter (8) is given by

$$\begin{array}{rcl} \dot{\breve{x}}(t) & = & \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \Big(A_{cl}^{ij}(\imath)\breve{x}(t) + B_{cl}^{ij}(\imath)\tilde{w}(t)\Big) \\ \breve{z}(t) & = & \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j C_{cl}^{ij}(\imath)\breve{x}(t) \end{array} \quad (11)$$

where $\breve{x}(t) = \big[x^T(t) \;\; \hat{x}^T(t)\big]^T$ and the matrix functions $A_{cl}^{ij}(\imath)$, $B_{cl}^{ij}(\imath)$ and $C_{cl}^{ij}(\imath)$ are defined in Lemma 1 and the disturbance is

$$\tilde{w}(t) = \begin{bmatrix} \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{1_i}(\imath)x(t) \\ F(x(t),\imath,t)H_{2_i}(\imath)w(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{5_i}(\imath)x(t) \\ F(x(t),\imath,t)H_{7_i}(\imath)w(t) \end{bmatrix}.$$

Let choose a stochastic Lyapunov function

$$V(\breve{x}(t),\imath) = \breve{x}^T(t)P(\imath)\breve{x}(t) \;\; \forall\, \imath \in \mathcal{S} \quad (12)$$

where $P(\imath)$ is a constant positive definite matrix for each $\imath$. For this choice, we have $V(0,\imath_0) = 0$ and $V(\breve{x}(t),\imath) \to \infty$ only when $\|\breve{x}(t)\| \to \infty$.

Consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{(\breve{x}(t),\imath), t \geq 0\}$, which is the stochastic analog of the deterministic derivative. $\{(\breve{x}(t),\imath), t \geq 0\}$ is a Markov process with infinitesimal operator given by [32],

$$\begin{aligned}\tilde{\Delta}V(\check{x}(t),\imath) &= \dot{\check{x}}^T(t)P(\imath)\check{x}(t)+\check{x}^T(t)P(\imath)\dot{\check{x}}(t)\\ &\quad +\check{x}^T(t)\sum_{k=1}^{s}\lambda_{\imath k}P(k)\check{x}^T(t)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(\check{x}^T(t)(A_{cl}^{ij}(\imath))^TP(\imath)\check{x}(t)\\ &\quad +\check{x}^T(t)P(\imath)A_{cl}^{ij}(\imath)\check{x}(t)\\ &\quad +\tilde{w}^T(t)(B_{cl}^{ij}(\imath))^TP(\imath)\check{x}(t)\\ &\quad +\check{x}^T(t)P(\imath)B_{cl}^{ij}(\imath)\tilde{w}(t)\\ &\quad +\check{x}^T(t)\sum_{k=1}^{s}\lambda_{\imath k}P(k)\check{x}^T(t)\Big). \qquad (13)\end{aligned}$$

Adding and subtracting

$$-\aleph^2(\imath)z^T(t)z(t)+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]$$

to and from (13), we get

$$\begin{aligned}&\tilde{\Delta}V(x(t),\imath)=-\aleph^2(\imath)z^T(t)z(t)\\ &\quad+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]\\ &\quad+\aleph^2(\imath)z^T(t)z(t)\\ &\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\begin{bmatrix}\check{x}(t)\\ \tilde{w}(t)\end{bmatrix}^T\times\\ &\left(\begin{array}{cc}\left(\begin{array}{c}(A_{cl}^{ij}(\imath))^TP(\imath)\\ +P(\imath)A_{cl}^{ij}(\imath)\\ +\sum_{k=1}^{s}\lambda_{\imath k}P(k)\end{array}\right) & (*)^T\\ (B_{cl}^{ij}(\imath))^TP(\imath) & -\gamma^2I\end{array}\right)\begin{bmatrix}\check{x}(t)\\ \tilde{w}(t)\end{bmatrix}. \qquad (14)\end{aligned}$$

Now let us consider the following terms:

$$\begin{aligned}&\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]\\ &=\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\times\\ &\qquad\begin{bmatrix}\frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{1_i}(\imath)x(t)\\ F(x(t),\imath,t)H_{2_i}(\imath)w(t)\\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{5_i}(\imath)x(t)\\ F(x(t),\imath,t)H_{7_i}(\imath)w(t)\end{bmatrix}^T\times\\ &\begin{bmatrix}\frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{1_m}(\imath)x(t)\\ F(x(t),\imath,t)H_{2_m}(\imath)w(t)\\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{5_m}(\imath)x(t)\\ F(x(t),\imath,t)H_{7_m}(\imath)w(t)\end{bmatrix}\\ &\le\frac{\gamma^2\rho^2(\imath)}{\delta^2(\imath)}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\check{x}^T(t)\times\\ &\left[\;H_{1_i}^T(\imath)H_{1_m}(\imath)+H_{5_i}^T(\imath)H_{5_m}(\imath)\;\right]\check{x}(t)\\ &+\aleph^2(\imath)\gamma^2w^T(t)w(t) \qquad (15)\end{aligned}$$

and

$$\begin{aligned}&\aleph^2(\imath)z^T(t)z(t)\\ &=\aleph^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\check{x}^T(t)\times\\ &\Big[C_{1_i}(\imath)+F(x(t),\imath,t)H_{4_i}(\imath)\quad-\hat{C}_j(\imath)\Big]^T\times\\ &\Big[C_{1_m}(\imath)+F(x(t),\imath,t)H_{4_m}(\imath)\quad-\hat{C}_n(\imath)\Big]\check{x}(t)\\ &\le\;\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\times\\ &\quad\Big(2\aleph^2(\imath)\check{x}^T(t)\Big[C_{1_i}(\imath)\quad-\hat{C}_j(\imath)\Big]^T\times\\ &\quad\Big[C_{1_m}(\imath)\quad-\hat{C}_n(\imath)\Big]\check{x}(t)+2\aleph^2(\imath)\rho^2(\imath)\check{x}^T(t)\times\\ &\quad H_{4_i}^T(\imath)H_{4_m}(\imath)\check{x}(t)\Big) \qquad (16)\end{aligned}$$

where $\aleph(\imath)\ \ge\ \Big(1+\rho^2(\imath)\Big[\|H_{2_i}^T(\imath)H_{2_j}(\imath)\|+\|H_{7_i}^T(\imath)H_{7_j}(\imath)\|\Big]\Big)^{\frac{1}{2}}$. Hence,

$$\begin{aligned}&\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]\\ &+\aleph^2(\imath)z^T(t)z(t)\\ &\le\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\times\\ &\Big(\check{x}^T(t)\Big[\tilde{C}_{1_i}(\imath)\quad\tilde{D}_{12_i}(\imath)\hat{C}_j(\imath)\Big]^T\times\\ &\Big[\tilde{C}_{1_m}(\imath)\quad\tilde{D}_{12_m}(\imath)\hat{C}_n(\imath)\Big]\check{x}(t)\Big)+\aleph^2(\imath)\gamma^2w^T(t)w(t) \qquad (17)\end{aligned}$$

where

$$\tilde{C}_{1_i}(\imath)=\Big[\frac{\gamma\rho(\imath)}{\delta(\imath)}H_{1_i}^T(\imath)\quad\frac{\gamma\rho(\imath)}{\delta(\imath)}H_{5_i}^T(\imath)\quad\sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath)\quad\sqrt{2}\aleph(\imath)C_{1_i}^T(\imath)\Big]^T$$

$$\tilde{D}_{12}(\imath)=\Big[0\ \ 0\ \ 0\ \ -\sqrt{2}\aleph(\imath)I\Big]^T$$

Substituting (17) into (14), we have

$$\begin{aligned}\tilde{\Delta}V(x(t),\imath)\ &\le\ -\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t)\\ &\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\times\\ &\quad\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\Omega_{ijmn}(\imath)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix} \qquad (18)\end{aligned}$$

where

$$\Omega_{ijmn}(\imath)=\left(\begin{array}{cc}\left(\begin{array}{c}(A_{cl}^{ij}(\imath))^TP(\imath)\\ +P(\imath)A_{cl}^{ij}(\imath)\\ +(C_{cl}^{ij}(\imath))^TC_{cl}^{mn}(\imath)\\ +\sum_{k=1}^{s}\lambda_{\imath k}P(k)\end{array}\right) & (*)^T\\ (B_{cl}^{ij}(\imath))^TP(\imath) & -\gamma^2I\end{array}\right). \qquad (19)$$

Using the fact

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n M_{ij}^T(\imath)N_{mn}(\imath)\\&\leq \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j[M_{ij}^T(\imath)M_{ij}(\imath)+N_{ij}(\imath)N_{ij}^T(\imath)],\end{aligned}$$

we can rewrite (19) as follows:

$$\begin{aligned}&\tilde{\Delta}V(x(t),\imath)\leq -\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t)\\&+\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\Omega_{ij}(\imath)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}\end{aligned}\tag{20}$$

where

$$\Omega_{ij}(\imath)=\begin{pmatrix}\begin{pmatrix}(A_{cl}^{ij}(\imath))^TP(\imath)\\+P(\imath)A_{cl}^{ij}(\imath)\\+(C_{cl}^{ij}(\imath))^TC_{cl}^{ij}(\imath)\\+\sum_{k=1}^{s}\lambda_{\imath k}P(k)\end{pmatrix} & (*)^T\\ (B_{cl}^{ij}(\imath))^TP(\imath) & -\gamma^2 I\end{pmatrix}.\tag{21}$$

Note that (21) is the Schur complement of (10). Using the inequality (10), we have

$$\tilde{\Delta}V(x(t),\imath)< -\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t).\tag{22}$$

Applying the operator $\mathbf{E}[\int_0^{T_f}(\cdot)dt]$ on both sides of (22), we obtain

$$\begin{aligned}&\mathbf{E}\left[\int_0^{T_f}\tilde{\Delta}V(x(t),\imath)dt\right]\\&<\mathbf{E}\left[\int_0^{T_f}(-\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right]\end{aligned}\tag{23}$$

From the Dynkin's formula [27], it follows that

$$\begin{aligned}&\mathbf{E}\left[\int_0^{T_f}\tilde{\Delta}V(x(t),\imath)dt\right]\\&=\mathbf{E}[V(x(T_f),\imath(T_f))]-\mathbf{E}[V(x(0),\imath(0))].\end{aligned}\tag{24}$$

Substitute (24) into (23) yields

$$\begin{aligned}0\ <\ &\mathbf{E}\left[\int_0^{T_f}(-\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right]\\&-\mathbf{E}[V(x(T_f),\imath(T_f))]+\mathbf{E}[V(x(0),\imath(0))].\end{aligned}$$

Using (22) and the fact that $V(x(0)=0,\imath(0))=0$ and $V(x(T_f),\imath(T_f))>0$, we have

$$\mathbf{E}\left[\int_0^{T_f}\left\{z^T(t)z(t)-\gamma^2w^T(t)w(t)\right\}dt\right]<0.\tag{25}$$

Hence the inequality (7) holds. This completes the proof of Lemma 1. ∎

Knowing that the filter's premise variable is the same as the plant's premise variable, the left hand side of (10) can be re-expressed as follows:

$$\begin{aligned}&P(\imath)A_{cl}^{ij}(\imath)+(A_{cl}^{ij}(\imath))^TP(\imath)\\&+\gamma^{-2}P(\imath)B_{cl}^{ij}(\imath)(B_{cl}^{ij}(\imath))^TP(\imath)\\&+\sum_{k=1}^{s}\lambda_{\imath k}P(k)+(C_{cl}^{ij}(\imath))^TC_{cl}^{ij}(\imath).\end{aligned}\tag{26}$$

Before providing LMI-based sufficient conditions for the system (1) with $u(t)=0$ to have an $\mathcal{H}_\infty$ performance, let us partition the matrix $P(\imath)$ given by Lemma 1 as follows:

$$P(\imath)=\begin{bmatrix}X(\imath) & Y^{-1}(\imath)-X(\imath)\\ Y^{-1}(\imath)-X(\imath) & X(\imath)-Y^{-1}(\imath)\end{bmatrix}\tag{27}$$

where $X(\imath)=X^T(\imath)\in\Re^{n\times n}$ and $Y(\imath)=Y^T(\imath)\in\Re^{n\times n}$. Utilizing the partition above, we define the new filter's input and output matrices as

$$\begin{aligned}\mathcal{B}_i(\imath)&\triangleq\left[Y^{-1}(\imath)-X(\imath)\right]\hat{B}_i(\imath)\\ \mathcal{C}_i(\imath)&\triangleq\hat{C}_i(\imath)Y(\imath).\end{aligned}\tag{28}$$

Using these changes of variable, we have the following theorem.

Theorem 1: Consider the system (5). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma>0$ and any positive constants $\delta(\imath)$, for $\imath=1,2,\cdots,s$, if there exist matrices $X(\imath)=X^T(\imath)$, $Y(\imath)=Y^T(\imath)$, $\mathcal{B}_i(\imath)$ and $\mathcal{C}_i(\imath)$, $i=1,2,\cdots,r$, satisfying the following linear matrix inequalities:

$$\begin{bmatrix}X(\imath) & I\\ I & Y(\imath)\end{bmatrix}>0\tag{29}$$

$$X(\imath)>0\tag{30}$$

$$Y(\imath)>0\tag{31}$$

$$\Psi_{11_{ii}}(\imath)<0,\quad i=1,2,\cdots,r\tag{32}$$

$$\Psi_{22_{ii}}(\imath)<0,\quad i=1,2,\cdots,r\tag{33}$$

$$\Psi_{11_{ij}}(\imath)+\Psi_{11_{ji}}(\imath)<0,\quad i<j\leq r\tag{34}$$

$$\Psi_{22_{ij}}(\imath)+\Psi_{22_{ji}}(\imath)<0,\quad i<j\leq r\tag{35}$$

where

$$\Psi_{11_{ij}}(\imath)=\begin{pmatrix}\begin{pmatrix}A_i(\imath)Y(\imath)\\+Y(\imath)A_i^T(\imath)\\+\lambda_{\imath\imath}Y(\imath)\\+\gamma^{-2}\tilde{B}_{1_i}(\imath)\tilde{B}_{1_j}^T(\imath)\end{pmatrix} & (*)^T & (*)^T\\ \begin{pmatrix}\tilde{C}_{1_i}(\imath)Y(\imath)\\+\tilde{D}_{12}(\imath)\mathcal{C}_j(\imath)\end{pmatrix} & -I & (*)^T\\ \mathcal{J}^T(\imath) & 0 & -\mathcal{Y}(\imath)\end{pmatrix}\tag{36}$$

$$\Psi_{22_{ij}}(\imath)=\begin{pmatrix}\begin{pmatrix}A_i^T(\imath)X(\imath)\\+X(\imath)A_i(\imath)\\+\mathcal{B}_i(\imath)C_{2_j}(\imath)\\+C_{2_i}^T(\imath)\mathcal{B}_j^T(\imath)\\+\tilde{C}_{1_i}^T(\imath)\tilde{C}_{1_j}(\imath)\\+\sum_{k=1}^{s}\lambda_{\imath k}X(k)\end{pmatrix} & (*)^T\\ \begin{pmatrix}\tilde{B}_{1_i}^T(\imath)X(\imath)\\+\tilde{D}_{21_i}^T(\imath)\mathcal{B}_j^T(\imath)\end{pmatrix} & -\gamma^2 I\end{pmatrix}\tag{37}$$

with

$$\begin{aligned}\mathcal{J}(\imath)=\Big[&\sqrt{\lambda_{1\imath}}Y(\imath)\ \cdots\ \sqrt{\lambda_{(i-1)\imath}}Y(\imath)\\&\sqrt{\lambda_{(i+1)\imath}}Y(\imath)\ \cdots\ \sqrt{\lambda_{s\imath}}Y(\imath)\Big]\end{aligned}$$

$$\mathcal{Y}(\imath)=\mathit{diag}\left\{Y(1),\ \cdots,\ Y(\imath-1),\ Y(\imath+1),\ \cdots,Y(s)\right\}$$

$$\tilde{B}_{1_i}(\imath) = [\delta(\imath)I \;\; I \;\; 0 \;\; B_{1_i}(\imath) \;\; 0]$$
$$\tilde{C}_{1_i}(\imath) = \Big[\frac{\gamma\rho(\imath)}{\delta(\imath)}H^T_{1_i}(\imath) \;\; \frac{\gamma\rho(\imath)}{\delta(\imath)}H^T_{5_i}(\imath)$$
$$\sqrt{2}\aleph(\imath)\rho(\imath)H^T_{4_i}(\imath) \;\; \sqrt{2}\aleph(\imath)C^T_{1_i}(\imath)\Big]^T$$
$$\tilde{D}_{12}(\imath) = \big[\; 0 \;\; 0 \;\; 0 \;\; -\sqrt{2}\aleph(\imath)I \;\big]^T$$
$$\tilde{D}_{21_i}(\imath) = [0 \;\; 0 \;\; \delta(\imath)I \;\; D_{21_i}(\imath) \;\; I]$$
$$\aleph(\imath) = \Bigg(1 + \rho^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\Big[\|H^T_{2_i}(\imath)H_{2_j}(\imath)\| + \|H^T_{7_i}(\imath)H_{7_j}(\imath)\|\Big]\Bigg)^{\frac{1}{2}},$$

then the prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable filter is of the form (8) with

$$\begin{aligned}\hat{A}_{ij}(\imath) &= \big[Y^{-1}(\imath) - X(\imath)\big]^{-1}\mathcal{M}_{ij}(\imath)Y^{-1}(\imath)\\ \hat{B}_i(\imath) &= \big[Y^{-1}(\imath) - X(\imath)\big]^{-1}\mathcal{B}_i(\imath)\\ \hat{C}_i(\imath) &= \mathcal{C}_i(\imath)Y^{-1}(\imath)\end{aligned} \tag{38}$$

where

$$\begin{aligned}&\mathcal{M}_{ij}(\imath) = -A^T_i(\imath) - X(\imath)A_i(\imath)Y(\imath)\\ &-\big[Y^{-1}(\imath) - X(\imath)\big]\hat{B}_i(\imath)C_{2_j}(\imath)Y(\imath)\\ &-\textstyle\sum_{k=1}^{s}\lambda_{\imath k}Y^{-1}(k)Y(\imath)\\ &-\tilde{C}^T_{1_i}(\imath)\Big[\tilde{C}_{1_j}(\imath)Y(\imath) + \tilde{D}_{12}(\imath)\hat{C}_j(\imath)Y(\imath)\Big]\\ &-\gamma^{-2}\Big\{X(\imath)\tilde{B}_{1_i}(\imath) + \big[Y^{-1}(\imath) - X(\imath)\big]\hat{B}_i(\imath)\tilde{D}_{21_i}(\imath)\Big\}\tilde{B}^T_{1_j}(\imath).\end{aligned} \tag{39}$$

Proof: Suppose there exist $X(\imath)$ and $Y(\imath)$ such that the inequalities (29) and (30)-(31) hold. The inequality (29) implies that the matrix $P(\imath)$ defined in (26) is a positive definite matrix. Using the partition (27), the filter (28) and multiplying (26) to the left by $\begin{bmatrix} Y(\imath) & I \\ Y(\imath) & 0 \end{bmatrix}$ and to the right by $\begin{bmatrix} Y(\imath) & Y(\imath) \\ I & 0 \end{bmatrix}$, we have

$$\begin{bmatrix} \Phi_{11_{ij}}(\imath) & 0 \\ 0 & \Phi_{22_{ij}}(\imath) \end{bmatrix} \tag{40}$$

where

$$\begin{aligned}\Phi_{11_{ij}}(\imath) &= A_i(\imath)Y(\imath) + Y(\imath)A^T_i(\imath) + \lambda_{\imath\imath}Y(\imath)\\ &\quad+[Y(\imath)\tilde{C}^T_{1_i}(\imath) + \mathcal{C}^T_i(\imath)\tilde{D}^T_{12}(\imath)] \times\\ &\quad[Y(\imath)\tilde{C}^T_{1_i}(\imath) + \mathcal{C}^T_i(\imath)\tilde{D}^T_{12}(\imath)]^T\\ &\quad+\gamma^{-2}\tilde{B}_{1_i}(\imath)\tilde{B}^T_{1_j}(\imath) + \mathcal{J}(\imath)\mathcal{Y}^{-1}(\imath)\mathcal{J}^T(\imath)\end{aligned} \tag{41}$$

$$\begin{aligned}\Phi_{22_{ij}}(\imath) &= A^T_i(\imath)X(\imath) + X(\imath)A_i(\imath)\\ &\quad+\mathcal{B}_i(\imath)C_{2_j}(\imath) + C^T_{2_i}(\imath)\mathcal{B}^T_j(\imath)\\ &\quad+\gamma^{-2}\Big[X(\imath)\tilde{B}_{1_i}(\imath) + \mathcal{B}_i(\imath)\tilde{D}_{21_j}(\imath)\Big] \times\\ &\quad\Big[\tilde{B}^T_{1_i}(\imath)X(\imath) + \tilde{D}^T_{21_i}(\imath)\mathcal{B}^T_i(\imath)\Big]\\ &\quad+\tilde{C}^T_{1_i}(\imath)\tilde{C}_{1_j}(\imath) + \sum_{k=1}^{s}\lambda_{\imath k}X(k).\end{aligned} \tag{42}$$

Note that $\Phi_{11_{ij}}$ and $\Phi_{22_{ij}}$ are the Schur complements of $\Psi_{11_{ij}}$ and $\Psi_{22_{ij}}$. Using (32)-(35), we have (40) less than zero. Hence, by Theorem 1, we learn that the inequality (7) holds. ■

B. Case II–$\nu(t)$ is unavailable for feedback

Now, the premise variable of the fuzzy model $\nu(t)$ is unavailable for feedback which implies μ_i is unavailable for feedback. Hence, we cannot select our filter which depends on μ_i. Thus, we select our filter as follows.

$$\begin{aligned}\dot{\hat{x}}(t) &= \textstyle\sum_{i=1}^{r}\sum_{j=1}^{r}\hat{\mu}_i\hat{\mu}_j\Big[\hat{A}_{ij}(\imath)\hat{x}(t) + \hat{B}_i(\imath)y(t)\Big]\\ \hat{z}(t) &= \textstyle\sum_{i=1}^{r}\hat{\mu}_i\hat{C}_i(\imath)\hat{x}(t)\end{aligned} \tag{43}$$

where $\hat{\mu}_i$ depends on the premise variable of the filter which is different from μ_i.

By applying the same technique used in Case A, we have the following theorem.

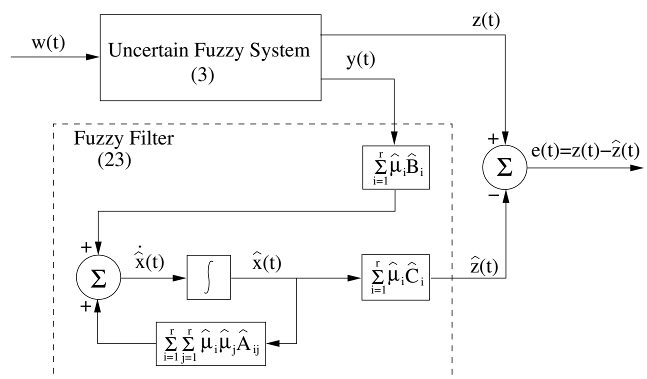


Fig. 3. Block diagram of an uncertain fuzzy system with a non-fragile $\mathcal{H}_\infty$ fuzzy filter in Case B.

Theorem 2: Consider the system (5). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ and any positive constants $\delta(\imath)$, for $\imath = 1, 2, \cdots, s$, if there exist matrices $X(\imath) = X^T(\imath)$, $Y(\imath) = Y^T(\imath)$, $\mathcal{B}_i(\imath)$ and $\mathcal{C}_i(\imath)$, $i = 1, 2, \cdots, r$, satisfying the following linear matrix inequalities:

$$\begin{bmatrix} X(\imath) & I \\ I & Y(\imath) \end{bmatrix} > 0 \tag{44}$$
$$X(\imath) > 0 \tag{45}$$
$$Y(\imath) > 0 \tag{46}$$
$$\Psi_{11_{ii}}(\imath) < 0, \quad i = 1, 2, \cdots, r \tag{47}$$
$$\Psi_{22_{ii}}(\imath) < 0, \quad i = 1, 2, \cdots, r \tag{48}$$
$$\Psi_{11_{ij}}(\imath) + \Psi_{11_{ji}}(\imath) < 0, \quad i < j \le r \tag{49}$$
$$\Psi_{22_{ij}}(\imath) + \Psi_{22_{ji}}(\imath) < 0, \quad i < j \le r \tag{50}$$

where

$$\Psi_{11_{ij}}(\imath)=\begin{pmatrix} \begin{pmatrix} A_i(\imath)Y(\imath) \\ +Y(\imath)A_i^T(\imath) \\ +\lambda_{\imath\imath}Y(\imath) \\ +\gamma^{-2}\tilde{\bar{B}}_{1_i}(\imath)\tilde{\bar{B}}_{1_j}^T(\imath) \end{pmatrix} & (*)^T & (*)^T \\ \begin{pmatrix} \tilde{\bar{C}}_{1_i}(\imath)Y(\imath) \\ +\tilde{\bar{D}}_{12}(\imath)\mathcal{C}_j(\imath) \end{pmatrix} & -I & (*)^T \\ \mathcal{J}^T(\imath) & 0 & -\mathcal{Y}(\imath) \end{pmatrix} \quad (51)$$

$$\Psi_{22_{ij}}(\imath) = \begin{pmatrix} \begin{pmatrix} A_i^T(\imath)X(\imath) \\ +X(\imath)A_i(\imath) \\ +\mathcal{B}_i(\imath)C_{2_j}(\imath) \\ +C_{2_i}^T(\imath)\mathcal{B}_j^T(\imath) \\ +\tilde{\bar{C}}_{1_i}^T(\imath)\tilde{\bar{C}}_{1_j}(\imath) \\ +\sum_{k=1}^{s}\lambda_{\imath k}X(k) \end{pmatrix} & (*)^T \\ \begin{pmatrix} \tilde{\bar{B}}_{1_i}^T(\imath)X(\imath) \\ +\tilde{\bar{D}}_{21_i}^T(\imath)\mathcal{B}_j^T(\imath) \end{pmatrix} & -\gamma^2 I \end{pmatrix} \quad (52)$$

with

$$\mathcal{J}(\imath) = \Big[\sqrt{\lambda_{1\imath}}Y(\imath) \ \cdots \ \sqrt{\lambda_{(i-1)\imath}}Y(\imath) \ \sqrt{\lambda_{(i+1)\imath}}Y(\imath) \ \cdots \ \sqrt{\lambda_{s\imath}}Y(\imath)\Big]$$

$$\mathcal{Y}(\imath) = \mathit{diag}\Big\{Y(1), \ \cdots, \ Y(\imath-1), \ Y(\imath+1), \ \cdots, Y(s)\Big\}$$

$$\tilde{\bar{B}}_{1_i}(\imath) = [\delta(\imath)I \ I \ 0 \ B_{1_i}(\imath) \ 0]$$

$$\tilde{\bar{C}}_{1_i}(\imath) = \Big[\frac{\gamma\bar{\rho}(\imath)}{\delta(\imath)}\bar{H}_{1_i}^T(\imath) \ \ \frac{\gamma\bar{\rho}(\imath)}{\delta(\imath)}\bar{H}_{5_i}^T(\imath) \ \ \sqrt{2}\bar{\aleph}(\imath)\bar{\rho}(\imath)\bar{H}_{4_i}^T(\imath) \ \ \sqrt{2}\bar{\aleph}(\imath)C_{1_i}^T(\imath)\Big]^T$$

$$\tilde{\bar{D}}_{12}(\imath) = \big[\ 0 \ \ 0 \ \ 0 \ \ -\sqrt{2}\bar{\aleph}(\imath)I \ \big]^T$$

$$\tilde{\bar{D}}_{21_i}(\imath) = [0 \ \ 0 \ \ \delta(\imath)I \ \ D_{21_i}(\imath) \ \ I]$$

$$\bar{\aleph}(\imath) = \Bigg(1+\bar{\rho}^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\Big[\|\bar{H}_{2_i}^T(\imath)\bar{H}_{2_j}(\imath)\| + \|\bar{H}_{7_i}^T(\imath)\bar{H}_{7_j}(\imath)\|\Big]\Bigg)^{\frac{1}{2}},$$

then the prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable filter is of the form (43) with

$$\begin{aligned} \hat{A}_{ij}(\imath) &= \big[Y^{-1}(\imath)-X(\imath)\big]^{-1}\mathcal{M}_{ij}(\imath)Y^{-1}(\imath) \\ \hat{B}_i(\imath) &= \big[Y^{-1}(\imath)-X(\imath)\big]^{-1}\mathcal{B}_i(\imath) \\ \hat{C}_i(\imath) &= \mathcal{C}_i(\imath)Y^{-1}(\imath) \end{aligned} \quad (53)$$

where

$$\begin{aligned} &\mathcal{M}_{ij}(\imath) = -A_i^T(\imath) - X(\imath)A_i(\imath)Y(\imath) \\ &-\big[Y^{-1}(\imath)-X(\imath)\big]\hat{B}_i(\imath)C_{2_j}(\imath)Y(\imath) \\ &-\sum_{k=1}^{s}\lambda_{\imath k}Y^{-1}(k)Y(\imath) \\ &-\tilde{\bar{C}}_{1_i}^T(\imath)\Big[\tilde{\bar{C}}_{1_j}(\imath)Y(\imath)+\tilde{\bar{D}}_{12}(\imath)\hat{C}_j(\imath)Y(\imath)\Big] \\ &-\gamma^{-2}\Big\{X(\imath)\tilde{\bar{B}}_{1_i}(\imath)+\big[Y^{-1}(\imath)-X(\imath)\big]\hat{B}_i(\imath)\tilde{\bar{D}}_{21_i}(\imath)\Big\}\tilde{\bar{B}}_{1_j}^T(\imath). \end{aligned} \quad (54)$$

Proof: It can be shown by employing the same technique used in the proof for Theorem 1. ■

IV. Illustrative Examples

Example 1: Consider the tunnel diode circuit shown in Figure 4 where the tunnel diode is characterized by

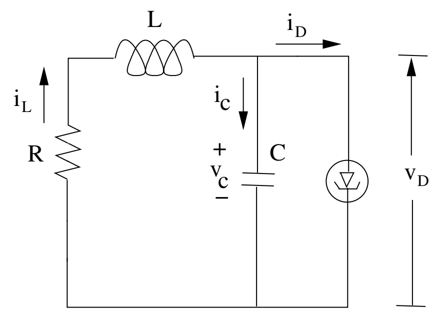


Fig. 4. Tunnel diode circuit.

$$i_D(t) = 0.002v_D(t) + \alpha v_D^3(t)$$

where α is the characteristic parameter. The circuit is governed by the following state equations:

$$\begin{aligned} C\dot{x}_1(t) &= -0.002x_1(t) - \alpha x_1^3(t) + x_2(t) \\ L\dot{x}_2(t) &= -x_1(t) - Rx_2(t) + 0.1w_2(t) \\ y(t) &= Jx(t) + 0.1w_1(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \quad (55)$$

where $w(t)$ is the disturbance noise input, $y(t)$ is the measurement output, $z(t)$ is the state to be estimated and J is the sensor matrix. Note that the variables $x_1(t)$ and $x_2(t)$ are the deviation variables (variables deviate from the desired trajectories). The parameters in the circuit are given as follows: $C = 20 \ mF$, $L = 1000 \ mH$ and $R = 10 \ \Omega$. Suppose that this system is aggregated into 3 modes as shown in Table 6.1:

TABLE I
SYSTEM TERMINOLOGY.

Mode $\imath$	$\alpha(\imath) \pm \Delta\alpha(\imath)$
1	0.01 $\pm 10\%$
2	0.02 $\pm 10\%$
3	0.03 $\pm 10\%$

with the nominal transition probability matrix that relates the three operation modes

$$P_{\imath k} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$

With these parameters, (55) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= -0.1x_1(t) - \Big(\tfrac{[\alpha(\imath)+\Delta\alpha(\imath)]}{C}x_1^2(t)\Big)\cdot x_1(t) \\ &\quad +50x_2(t) \\ \dot{x}_2(t) &= -x_1(t) - 10x_2(t) + 0.1w_2(t) \\ y(t) &= Jx(t) + 0.1w_1(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \end{aligned} \quad (56)$$

For the sake of simplicity, we will use as few rules as possible. Assuming that $|x_1(t)| \leq 3$, the nonlinear network system (56) can be approximated by the following TS fuzzy model:

Plant Rule 1: IF $x_1(t)$ is $M_1(x_1(t))$ THEN

$$\begin{aligned}\dot{x}(t) &= [A_1(\imath) + \Delta A_1(\imath)]x(t) + B_{1_1}(\imath)w(t), \quad x(0) = 0,\\ z(t) &= C_{1_1}(\imath)x(t),\\ y(t) &= C_{2_1}(\imath)x(t) + D_{21_1}(\imath)w(t).\end{aligned}$$

Plant Rule 2: IF $x_1(t)$ is $M_2(x_1(t))$ THEN

$$\begin{aligned}\dot{x}(t) &= [A_2(\imath) + \Delta A_2(\imath)]x(t) + B_{1_2}(\imath)w(t), \quad x(0) = 0,\\ z(t) &= C_{1_2}(\imath)x(t),\\ y(t) &= C_{2_2}(\imath)x(t) + D_{21_2}(\imath)w(t)\end{aligned}$$

where

$$A_1(1) = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, \quad A_2(1) = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix},$$

$$A_1(2) = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, \quad A_2(2) = \begin{bmatrix} -9.1 & 50 \\ -1 & -10 \end{bmatrix},$$

$$A_1(3) = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, \quad A_2(3) = \begin{bmatrix} -13.6 & 50 \\ -1 & -10 \end{bmatrix},$$

$$B_{1_1}(\imath) = B_{1_2}(\imath) = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_{1_1}(\imath) = C_{1_2}(\imath) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_{2_1}(\imath) = C_{2_2}(\imath) = J, \quad D_{21_1}(\imath) = D_{21_2}(\imath) = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$

$$\Delta A_1(\imath) = F(x(t), \imath, t)H_{1_1}(\imath)$$

$$\text{and} \quad A_2(\imath) = F(x(t), \imath, t)H_{1_2}(\imath).$$

Now, by assuming that $\|F(x(t), \imath, t)\| \leq \rho(\imath) = 1$, we have

$$H_{1_1}(1) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad H_{1_2}(1) = \begin{bmatrix} -0.45 & 0 \\ 0 & 0 \end{bmatrix},$$

$$H_{1_1}(2) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad H_{1_2}(2) = \begin{bmatrix} -0.9 & 0 \\ 0 & 0 \end{bmatrix},$$

$$H_{1_1}(3) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } H_{1_2}(3) = \begin{bmatrix} -1.35 & 0 \\ 0 & 0 \end{bmatrix}.$$

Note that the plot of the membership function Rules 1 and 2 is the same as in Figure 5.

Case I-$\nu(t)$ is available for feedback

In this case, $x_1(t) = \nu(t)$ is assumed to be available for feedback; for instance, $J = [1 \quad 0]$. This implies that μ_i is available for feedback. Using the LMI optimization algorithm and Theorem 1 with $\gamma = 1$ and $\delta(1) = \delta(2) = \delta(3) = 1$, we obtain

$$X(1) = \begin{bmatrix} 1.3527 & 4.1536 \\ 4.1536 & 23.7154 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} 15.9976 & -0.2409 \\ -0.2409 & 0.5000 \end{bmatrix},$$

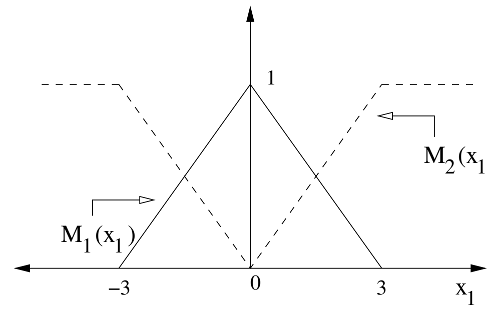


Fig. 5. Membership functions for the two fuzzy set.

$$\hat{A}_{11}(1) = \begin{bmatrix} -50.5324 & -1.7600 \\ -9.7924 & -0.5462 \end{bmatrix},$$

$$\hat{A}_{12}(1) = \begin{bmatrix} -50.5324 & -1.7600 \\ -9.7924 & -0.5462 \end{bmatrix},$$

$$\hat{A}_{21}(1) = \begin{bmatrix} -53.3639 & -1.8542 \\ -19.4469 & -0.3911 \end{bmatrix},$$

$$\hat{A}_{22}(1) = \begin{bmatrix} -53.3639 & -1.8542 \\ -19.4469 & -0.3911 \end{bmatrix},$$

$$\hat{B}_1(1) = \begin{bmatrix} 0.2743 \\ -0.9846 \end{bmatrix}, \quad \hat{B}_2(1) = \begin{bmatrix} 0.3067 \\ -1.2423 \end{bmatrix},$$

$$\hat{C}_1(1) = \begin{bmatrix} -35.3553 & -1.1213 \end{bmatrix},$$

$$\hat{C}_2(1) = \begin{bmatrix} -35.3553 & 0.1110 \end{bmatrix},$$

$$X(2) = \begin{bmatrix} 1.1422 & 3.3069 \\ 3.3069 & 19.7273 \end{bmatrix},$$

$$Y(2) = \begin{bmatrix} 8.8351 & -0.1880 \\ -0.1880 & 0.3363 \end{bmatrix},$$

$$\hat{A}_{11}(2) = \begin{bmatrix} -52.3064 & -2.3475 \\ -3.8388 & -0.5670 \end{bmatrix},$$

$$\hat{A}_{12}(2) = \begin{bmatrix} -52.3064 & -2.3475 \\ -3.8388 & -0.5670 \end{bmatrix},$$

$$\hat{A}_{21}(2) = \begin{bmatrix} -58.4742 & -2.4526 \\ -25.9706 & -0.1006 \end{bmatrix},$$

$$\hat{A}_{22}(2) = \begin{bmatrix} -58.4742 & -2.4526 \\ -25.9706 & -0.1006 \end{bmatrix},$$

$$\hat{B}_1(2) = \begin{bmatrix} 0.4488 \\ -1.6417 \end{bmatrix}, \quad \hat{B}_2(2) = \begin{bmatrix} 0.0851 \\ -0.5918 \end{bmatrix},$$

$$\hat{C}_1(2) = \begin{bmatrix} -35.3553 & -0.1998 \end{bmatrix},$$

$$\hat{C}_2(2) = \begin{bmatrix} -35.3553 & -0.2554 \end{bmatrix},$$

$$X(3) = \begin{bmatrix} 0.9146 & 2.5472 \\ 2.5472 & 16.0807 \end{bmatrix},$$

$$Y(3) = \begin{bmatrix} 5.8540 & -0.1805 \\ -0.1805 & 0.2579 \end{bmatrix},$$

$$\hat{A}_{11}(3) = \begin{bmatrix} -53.3336 & -2.8124 \\ -0.7319 & -0.7547 \end{bmatrix},$$

$$\hat{A}_{12}(3) = \begin{bmatrix} -53.3336 & -2.8124 \\ -0.7319 & -0.7547 \end{bmatrix},$$

$$\hat{A}_{21}(3) = \begin{bmatrix} -63.4126 & -3.1736 \\ -22.7881 & -0.0209 \end{bmatrix},$$

$$\hat{A}_{22}(3) = \begin{bmatrix} -63.4126 & -3.1736 \\ -22.7881 & -0.0209 \end{bmatrix},$$

$$\hat{B}_1(3) = \begin{bmatrix} 0.7630 \\ -2.9262 \end{bmatrix}, \quad \hat{B}_2(3) = \begin{bmatrix} 0.0795 \\ -0.7686 \end{bmatrix},$$

$$\hat{C}_1(3) = \begin{bmatrix} -35.3553 & -1.6653 \end{bmatrix},$$

$$\hat{C}_2(3) = \begin{bmatrix} -35.3553 & 0.2665 \end{bmatrix}.$$

The resulting fuzzy filter is

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & \sum_{i=1}^{2}\sum_{j=1}^{2} \mu_i\mu_j \hat{A}_{ij}(\imath)\hat{x}(t) \\ & & +\sum_{i=1}^{2} \mu_i \hat{B}_i(\imath) y(t) \\ \hat{z}(t) & = & \sum_{i=1}^{2} \mu_i \hat{C}_i(\imath)\hat{x}(t) \end{array} \tag{57}$$

where

$$\mu_1 = M_1(x_1(t)) \text{ and } \mu_2 = M_2(x_1(t)).$$

Case II-$\nu(t)$ is unavailable for feedback

In this case, $x_1(t) = \nu(t)$ is assumed to be unavailable for feedback; for instance, $J = [0 \quad 1]$. This implies that μ_i is unavailable for feedback. Using the LMI optimization algorithm and Theorem 2 with $\gamma = 1$ and $\delta(1) = \delta(2) = \delta(3) = 1$, we obtain

$$X(1) = \begin{bmatrix} 1.3721 & 4.2243 \\ 4.2243 & 24.1080 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} 14.7533 & -0.2063 \\ -0.2063 & 0.4399 \end{bmatrix},$$

$$\hat{A}_{11}(1) = \begin{bmatrix} -50.7139 & -1.7308 \\ -22.5449 & -0.0146 \end{bmatrix},$$

$$\hat{A}_{12}(1) = \begin{bmatrix} -50.7139 & -1.7308 \\ -22.5449 & -0.0146 \end{bmatrix},$$

$$\hat{A}_{21}(1) = \begin{bmatrix} -53.6150 & -1.7741 \\ -24.4667 & -0.8441 \end{bmatrix},$$

$$\hat{A}_{22}(1) = \begin{bmatrix} -53.6150 & -1.7741 \\ -24.4667 & -0.8441 \end{bmatrix},$$

$$\hat{B}_1(1) = \begin{bmatrix} 0.1802 \\ -0.7387 \end{bmatrix}, \quad \hat{B}_2(1) = \begin{bmatrix} 0.5358 \\ -1.8729 \end{bmatrix},$$

$$\hat{C}_1(1) = \begin{bmatrix} -35.3553 & 1.0222 \end{bmatrix},$$

$$\hat{C}_2(1) = \begin{bmatrix} -35.3553 & 0.1221 \end{bmatrix},$$

$$X(2) = \begin{bmatrix} 1.1564 & 3.3632 \\ 3.3632 & 20.0687 \end{bmatrix},$$

$$Y(2) = \begin{bmatrix} 8.1386 & -0.1553 \\ -0.1553 & 0.2925 \end{bmatrix},$$

$$\hat{A}_{11}(2) = \begin{bmatrix} -52.9363 & -2.1627 \\ -15.3598 & 0.3097 \end{bmatrix},$$

$$\hat{A}_{12}(2) = \begin{bmatrix} -52.9363 & -2.1627 \\ -15.3598 & 0.3097 \end{bmatrix},$$

$$\hat{A}_{21}(2) = \begin{bmatrix} -59.2867 & -2.2823 \\ -28.1564 & -0.9187 \end{bmatrix},$$

$$\hat{A}_{22}(2) = \begin{bmatrix} -59.2867 & -2.2823 \\ -28.1564 & -0.9187 \end{bmatrix},$$

$$\hat{B}_1(2) = \begin{bmatrix} 0.5723 \\ -1.6360 \end{bmatrix}, \quad \hat{B}_2(2) = \begin{bmatrix} 1.0338 \\ -1.7367 \end{bmatrix},$$

$$\hat{C}_1(2) = \begin{bmatrix} -35.3553 & -1.4211 \end{bmatrix},$$

$$\hat{C}_2(2) = \begin{bmatrix} -35.3553 & 0 \end{bmatrix},$$

$$X(3) = \begin{bmatrix} 0.9254 & 2.5969 \\ 2.5969 & 16.4096 \end{bmatrix},$$

$$Y(3) = \begin{bmatrix} 5.4341 & -0.1491 \\ -0.1491 & 0.2228 \end{bmatrix},$$

$$\hat{A}_{11}(3) = \begin{bmatrix} -54.8946 & -2.9091 \\ -12.0349 & 0.4766 \end{bmatrix},$$

$$\hat{A}_{12}(3) = \begin{bmatrix} -54.8946 & -2.9091 \\ -12.0349 & 0.4766 \end{bmatrix},$$

$$\hat{A}_{21}(3) = \begin{bmatrix} -64.5265 & -2.7580 \\ -24.0698 & -1.2716 \end{bmatrix},$$

$$\hat{A}_{22}(3) = \begin{bmatrix} -64.5265 & -2.7580 \\ -24.0698 & -1.2716 \end{bmatrix},$$

$$\hat{B}_1(3) = \begin{bmatrix} 1.0373 \\ -1.1620 \end{bmatrix}, \quad \hat{B}_2(3) = \begin{bmatrix} 1.1281 \\ -1.5550 \end{bmatrix},$$

$$\hat{C}_1(3) = \begin{bmatrix} -35.3553 & 1.4877 \end{bmatrix},$$

$$\hat{C}_2(3) = \begin{bmatrix} -35.3553 & -0.3775 \end{bmatrix}.$$

The resulting fuzzy filter is

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & \sum_{i=1}^{2}\sum_{j=1}^{2}\hat{\mu}_i\hat{\mu}_j\hat{A}_{ij}(\imath)\hat{x}(t) \\ & & +\sum_{i=1}^{2}\hat{\mu}_i\hat{B}_i(\imath)y(t) \\ \hat{z}(t) & = & \sum_{i=1}^{2}\hat{\mu}_i\hat{C}_i(\imath)\hat{x}(t) \end{array} \tag{58}$$

where

$$\hat{\mu}_1 = M_1(\hat{x}_1(t)) \text{ and } \hat{\mu}_2 = M_2(\hat{x}_1(t)).$$

Remark 1: Figures 6(a)-6(b), respectively, show the responses of $x_1(t)$ and $x_2(t)$ in Cases I and II. Figure 7 shows the result of the changing between modes during the simulation with the initial mode 2. The disturbance input signal, $w(t)$, which was used during the simulation is given in Figure 8. The simulation results for the ratio of the filter error energy to the disturbance input noise energy obtained by using the non-fragile $\mathcal{H}_\infty$ fuzzy filter are depicted in Figure 9. After 15 seconds, the ratio of the filter error energy to the disturbance input noise energy tends to a constant value which is about 0.33 in Case I and 0.38 in Case II. Thus, in Case I where $\gamma = \sqrt{0.33} = 0.574$ and in Case II where $\gamma = \sqrt{0.38} = 0.616$, both are less than the prescribed value 1. □

Example 2: Consider the following nonlinear system.

$$\begin{array}{rcl} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} & = & \begin{bmatrix} -\frac{0.05}{J(\imath)} & \frac{0.005}{J(\imath)}x_2(t) \\ -0.005x_2(t) & -10 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ & & +\begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} w(t) \\ & & +\begin{bmatrix} -\frac{0.05}{\Delta J(\imath)} & \frac{0.005}{\Delta J(\imath)}x_2(t) \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ z(t) & = & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ y(t) & = & Sx(t) + \begin{bmatrix} 0 & 0.1 \end{bmatrix} w(t) \end{array} \tag{59}$$

where $x(t) = [x_1^T(t) \;\; x_2^T(t)]^T$ is the state variables, $w(t) = [w_1^T(t) \;\; w_2^T(t)]^T$ is the disturbance input, $z(t)$ is the controlled output, $y(t)$ is the measured output and S is the sensor matrix.

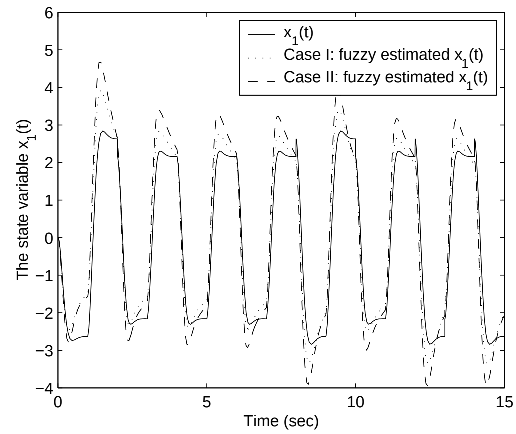
Assume that, the system is aggregated into 3 modes as shown in Table II:

TABLE II
SYSTEM TERMINOLOGY.

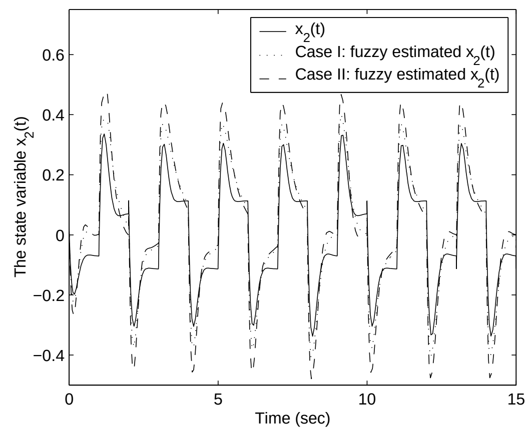
Mode $\imath$	$J(\imath) \pm \Delta J(\imath)$
1	0.0005 $\pm 10\%$
2	0.005 $\pm 10\%$
3	0.05 $\pm 10\%$

The transition probability matrix that relates the three operation modes is given as follows:

$$P_{\imath k} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$



(a) The histories of $x_1(t)$



(b) The histories of $x_2(t)$

Fig. 6. The histories of $x_1(t)$ and $x_2(t)$ in Cases I and II.

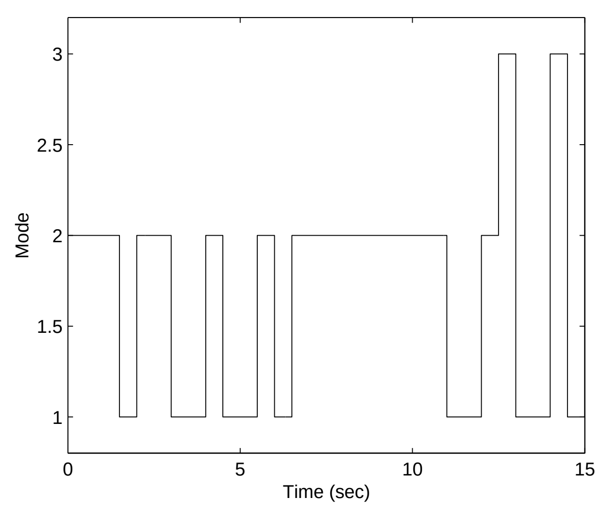


Fig. 7. The result of the changing between modes during the simulation with the initial mode 2.

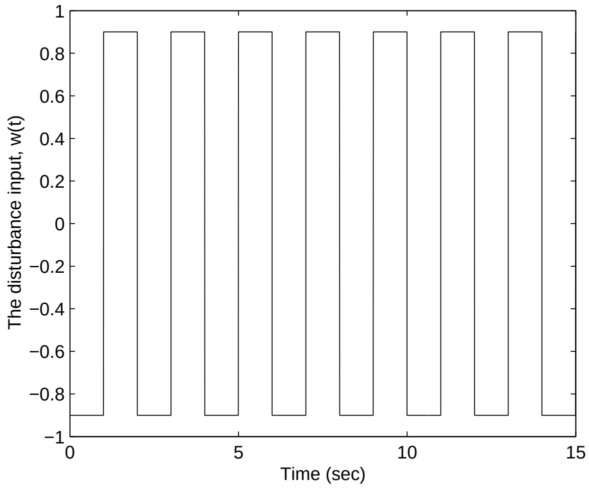


Fig. 8. The disturbance input noise, $w(t)$.

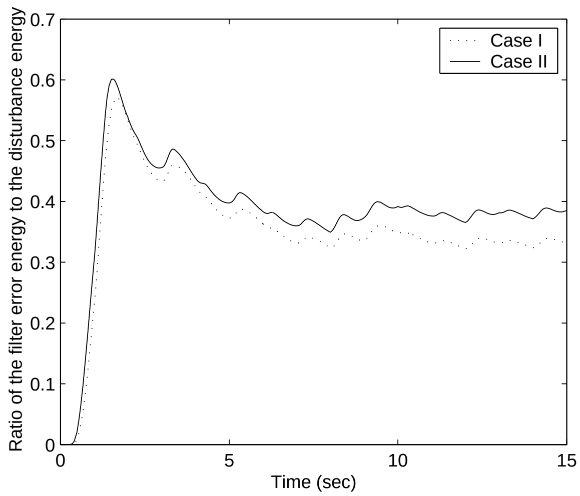


Fig. 9. The ratio of the filter error energy to the disturbance noise energy: $\left(\frac{\int_0^{T_f}(z(t)-\hat{z}(t))^T(z(t)-\hat{z}(t))dt}{\int_0^{T_f} w^T(t)w(t)dt}\right)$.

Note that Figure 10 shows the plot of the membership function represented by

$$M_1(x_2(t)) = \frac{-x_2(t)+N_2}{N_2-N_1} \text{ and } M_2(x_2(t)) = \frac{x_2(t)-N_1}{N_2-N_1}.$$

Knowing that $x_2(t) \in [N_1 \ \ N_2]$, the nonlinear system (59) can be approximated by the following two rules TS model:
Plant Rule 1:

IF $x_2(t)$ is $M_1(x_2(t))$ THEN

$$\begin{aligned}\dot{x}(t) &= [A_1(\imath)+\Delta A_1(\imath)]x(t)+B_{1_1}(\imath)w(t), \quad x(0)=0,\\ z(t) &= C_{1_1}(\imath)x(t),\\ y(t) &= C_{2_1}(\imath)x(t)+D_{21_1}(\imath)w(t),\end{aligned}$$

Plant Rule 2:

IF $x_2(t)$ is $M_2(x_2(t))$ THEN

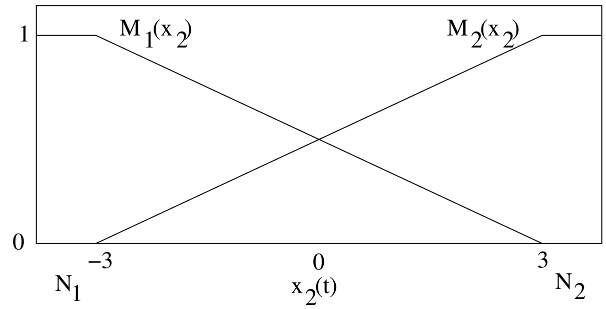


Fig. 10. Membership functions for the two fuzzy set.

$$\begin{aligned}\dot{x}(t) &= [A_2(\imath)+\Delta A_2(\imath)]x(t)+B_{1_2}(\imath)w(t), \quad x(0)=0,\\ z(t) &= C_{1_2}(\imath)x(t),\\ y(t) &= C_{2_2}(\imath)x(t)+D_{21_2}(\imath)w(t)\end{aligned}$$

where $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$,

$$A_1(1) = \begin{bmatrix} -100 & 10N_1 \\ -0.005N_1 & -10 \end{bmatrix},$$

$$A_2(1) = \begin{bmatrix} -100 & 10N_2 \\ -0.005N_2 & -10 \end{bmatrix},$$

$$A_1(2) = \begin{bmatrix} -10 & N_1 \\ -0.005N_1 & -10 \end{bmatrix},$$

$$A_2(2) = \begin{bmatrix} -10 & N_2 \\ -0.005N_2 & -10 \end{bmatrix},$$

$$A_1(3) = \begin{bmatrix} -1 & 0.1N_1 \\ -0.005N_1 & -10 \end{bmatrix},$$

$$A_2(3) = \begin{bmatrix} -1 & 0.1N_2 \\ -0.005N_2 & -10 \end{bmatrix},$$

$$B_{1_1}(1) = B_{1_1}(2) = B_{1_1}(3) = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix},$$

$$C_{1_1}(1) = C_{1_1}(2) = C_{1_1}(3) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_{2_1}(1) = C_{2_1}(2) = C_{2_1}(3) = S,$$

$$D_{21_1}(1) = D_{21_1}(2) = D_{21_1}(3) = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

$$B_{1_2}(1) = B_{1_2}(2) = B_{1_2}(3) = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix},$$

$$C_{1_2}(1) = C_{1_2}(2) = C_{1_2}(3) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_{2_2}(1) = C_{2_2}(2) = C_{2_2}(3) = S,$$

$$D_{21_2}(1) = D_{21_2}(2) = D_{21_2}(3) = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

$$\Delta A_1(\imath) = F(x(t),\imath,t)H_{1_1}(\imath)$$

$$\text{and } \Delta A_2(\imath) = F(x(t),\imath,t)H_{1_2}(\imath)$$

with $\|F(x(t),\imath,t)\| \leq 1$. Then we define

$$H_{1_1}(\imath) = \begin{bmatrix} -\frac{0.05}{J(\imath)} & \frac{0.05}{J(\imath)}N_1 \\ 0 & 0 \end{bmatrix}$$

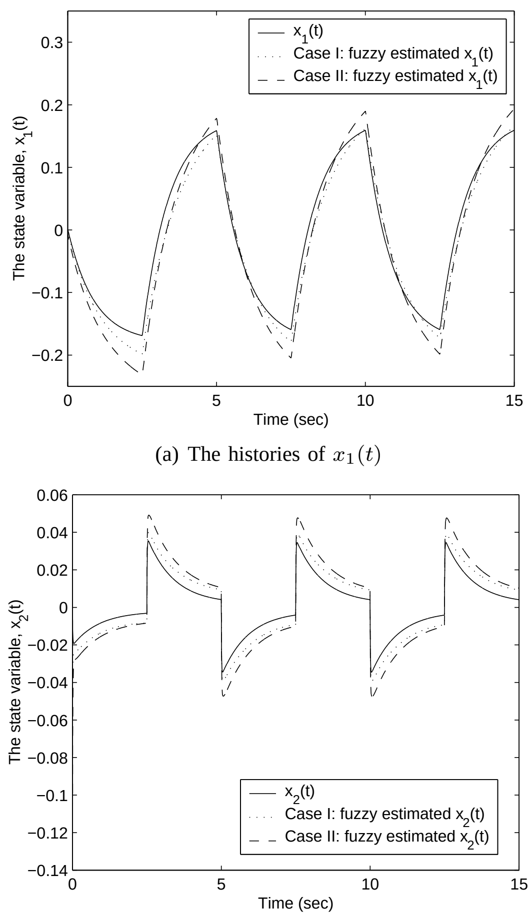
$$\text{and} \quad H_{1_2}(\imath) = \begin{bmatrix} -\frac{0.05}{J(\imath)} & \frac{0.05}{J(\imath)}N_2 \\ 0 & 0 \end{bmatrix}.$$

Applying Theorem 1 and 2, we obtain the results as shown in Figure 11-13. The resulting fuzzy filter is

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & \sum_{i=1}^{2}\sum_{j=1}^{2}\hat{\mu}_i\hat{\mu}_j\hat{A}_{ij}(\imath)\hat{x}(t) \\ & & +\sum_{i=1}^{2}\hat{\mu}_i\hat{B}_i(\imath)y(t) \\ \hat{z}(t) & = & \sum_{i=1}^{2}\hat{\mu}_i\hat{C}_i(\imath)\hat{x}(t) \end{array} \tag{60}$$

where

$$\hat{\mu}_1 = M_1(\hat{x}_1(t)) \ \text{ and } \ \hat{\mu}_2 = M_2(\hat{x}_1(t)).$$



(a) The histories of $x_1(t)$

(b) The histories of $x_2(t)$

Fig. 11. The histories of $x_1(t)$ and $x_2(t)$ in Cases I and II.

Remark 2: Figures 11(a)-11(b), respectively, show the responses of $x_1(t)$ and $x_2(t)$ in Cases I and II. Figure 7 shows the result of the changing between modes during the simulation with the initial mode 2. The disturbance input signal, $w(t)$, which was used during the simulation is given in Figure 12. The simulation results for the ratio of the filter error energy to the disturbance input noise energy obtained by using the non-fragile $\mathcal{H}_\infty$ fuzzy filter are depicted in Figure

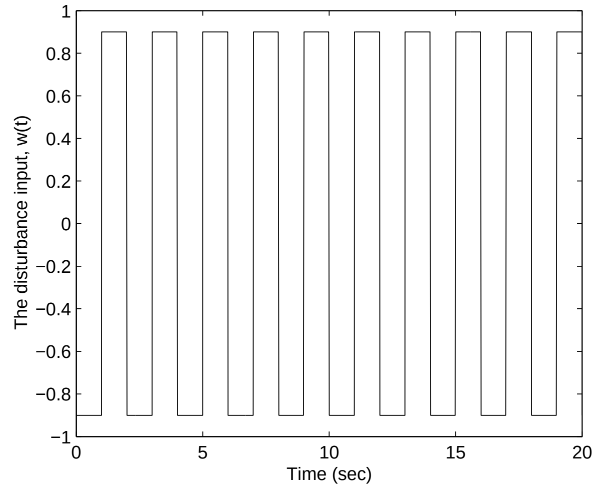


Fig. 12. The disturbance input noise, $w(t)$.

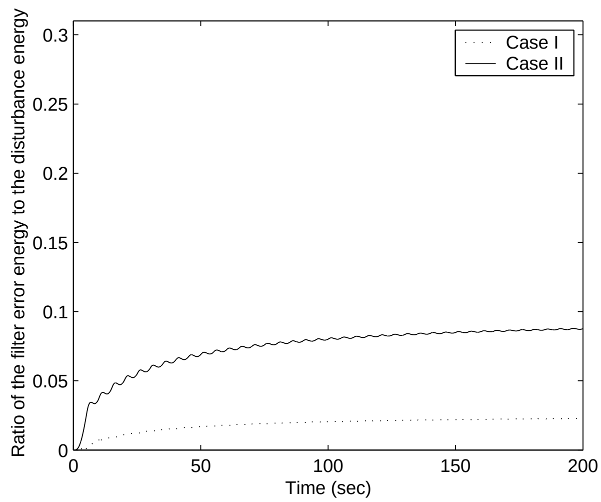


Fig. 13. The ratio of the filter error energy to the disturbance noise energy: $\frac{\int_0^t (z(t)-\hat{z}(t))^T(z(t)-\hat{z}(t))dt}{\int_0^t w^T(t)w(t)dt}$.

13. After 50 seconds, the ratio of the filter error energy to the disturbance input noise energy tends to a constant value which is about 0.02 in Case I and 0.08 in Case II. Thus, in Case I where $\gamma = \sqrt{0.02} = 0.141$ and in Case II where $\gamma = \sqrt{0.08} = 0.283$, both are less than the prescribed value 1. □

V. CONCLUSION

The aim of a filter is to estimate the values of internal system variables that are not measured from the available output. Estimation problems arise in diverse fields such as communication, control and signal processing. This paper addresses the problem of designing a non-fragile $\mathcal{H}_\infty$ filter for a class of robust uncertain Markovian jump nonlinear systems that guarantees the $\mathcal{L}_2$-gain from an exogenous input to a filter error is less or equal to a prescribed value. Based on an LMI approach, solutions to the problem of the non-fragile $\mathcal{H}_\infty$ fuzzy filtering are derived in terms of a family of linear matrix inequalities. In this article, the premise variables of the non-fragile $\mathcal{H}_\infty$ fuzzy filter are allowed to be different from the premise variables of the TS fuzzy model of the plant such that the results are shown into two cases which are the premise variable of the fuzzy model be measurable and the premise variable assumed to be unmeasurable.

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