A Quasi Regression Model for Polytomous Data and Its Application for Measuring Service Quality

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Abstract—A quasi nonlinear regression model with polytomous response is considered. Unknown parameters are estimated using maximum likelihood method. Corresponding information matrix is presented. Gotten results are used for an evaluation of transport service quality in Riga Coach Terminal, different examples are considered.

Keywords—regression, categories, the maximum likelihood method, information matrix, quality of service.

I. Introduction

Considered problem was arisen as a result of collaboration with Riga Coach Terminal. The last one being a leader in the area of the passenger bus transportation services in Latvia provides the international, intercity and regional trips. Recent studies of the role of buses and coaches seem to confirm the already excellent safety, environmental and social record of bus and coach transport. In Latvia this mode of transport is in competition with railway (and also private cars) that's why the quality of services is very important from all points of view [4].

Many authors considered the different approaches for measuring service quality (Cronin, Parasuraman, Morgan, Penja) [3], [8]. One of the most used approaches considers quality as a function from several particular attributes - variables and the key step consists in definition of weight of each attribute.

The problems of the service quality provided by a coach terminal have been considered by the paper's authors several times [4], [5], [9], [10]. Suppose that random sample with size denoted n from population of users involves estimates of overall quality of service – y_i, ($i = 1,\ldots, n$) and estimates of attributes (particular quality indexes), which define quality of service – x_{ij}, for k concrete attributes ($i=1, \ldots, n; j=1, \ldots, k$). Assume that these estimates are made on the basis (0-5) scale. In previous researches [10] the theory of linear composite indicator constructing and statistical methods are being used, namely linear regression model with constraints on parameters' sign and value. The model constructed for a scalar quality indicator, allows estimating influence of particular quality indicators on the overall quality estimation and to simplify monitoring of quality indicators. But the models presented in paper [10] were based on a number of assumptions. Most critical one is that the overall estimations are continuous variables when in estimation and to simplify model development.

In the given work authors offer the new improved approach, which dismisses the above mentioned assumption and besides helps to estimate not only parameters (weights) of particular attributes of quality, but also intervals of categories to which values of the overall indicator of quality belong.

The described case is called in the literature as polytomous response or polytomous data [6]. McCullagh and Nelder wrote: "Often the categories are defined in a qualitative or non-numerical way". Our responses are numerical but considered approach allows using it for qualitative or non-numerical cases.

Our approach is based on regression theory [6], [7]. The next Section contains problem setting. Section 3 is devoted to the parameter estimation based on maximal likelihood method. An information matrix for unknown parameters is calculated in Section 4. The numerical example is considered in Section 5. Some concluding remarks are given in section 6. Appendix contains some technical calculations.

II. Problem Setting

We describe the considered problem following McCullagh and Nelder [6], chapter 5.

The response Y_i of a concrete individual or item i (i = 1, …, n) is one of the fixed set of possible values, let {1, 2, …, k}. These values are called *categories.* In our paper we suppose that the categories are ordered: the category j is "better" than i if $i < j$. The response probabilities $\pi = (\pi_1, \pi_2, \ldots, \pi_k)$ are function of vector of covariates or explanatory variables $x_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,d})$ associated with the i-th individual: $\pi = \pi(x_i)$. We have at our disposal the matrix of the covariates X and a vector of responses Y:

$$X = \begin{pmatrix} x_1 \\ \ldots \\ x_n \end{pmatrix}_{n\times d}, \quad Y = \begin{pmatrix} Y_1 \\ \ldots \\ Y_n \end{pmatrix}.$$

Our aim is to suggest the relationship between the response probability $\pi = \pi(x_i)$ and the explanatory variables $x_i = (x_{i,1}, x_{i,2}, ..., x_{i,d})$. For that aim we use an unobserved continuous random variable Z_i for the i-th individual:

$$Z_i = \sum_{j=1}^{d} \beta_j x_{i,j} + \zeta_i = x_i \beta + \zeta_i, \qquad (1)$$

where $\{\zeta_i\}$ are independent and identically normally distributed random variables with zero expectation and unknown variance σ^2, $\{\beta_j\}$ are unknown regression coefficients, $(\beta_1, ..., \beta_d)^T = \beta$.

Further, we introduce unknown parameters $\theta_1 < \theta_2 < ... < \theta_{k-1}$. If the unobserved variable Z_i lies in the interval $(\theta_{j-1}, \theta_j]$ then $y_i = j$ is recorded. Here $j = 1, …, k$, $\theta_0 = -\infty, \theta_k = \infty$.

We want to get maximal likelihood estimates of the next unknown parameters:

$$\beta = (\beta_1, ..., \beta_d)^T,\ \theta = (\theta_1, ..., \theta_{k-1})^T \text{ and } \sigma.$$

For that we have n observations with fixed values $\{Y_i\}$ *and* $x_i = (x_{i,1}, ..., x_{i,d})$.

III. The likelihood function and parameter estimation

Note that

$$\pi_j(x_i) = P\{Y_i = j\} = P\{\theta_{j-1} < Z_i \le \theta_j\} = = \Phi\left(\frac{\theta_j - x_i\beta}{\sigma}\right) - \Phi\left(\frac{\theta_{j-1} - x_i\beta}{\sigma}\right) \qquad (2)$$

We see that the unknown parameters $\beta = (\beta_1, ..., \beta_d)$, $\theta = (\theta_1, ..., \theta_{k-1})$ and σ can be estimated up to constant factor. Therefore we use united parameters $\tilde{\beta} = (\tilde{\beta}_1, ..., \tilde{\beta}_d) = \beta / \sigma$ and $\tilde{\theta} = (\tilde{\theta}_1, ..., \tilde{\theta}_{k-1}) = \theta / \sigma$.

Note that situation takes place often in the econometrics [1], [2].

To rewrite down the corresponding log-likelihood function, we rearrange our observation (individuals) as follows: at first observations of the first category are written, then the second category and so on. Let n_j be an observation size for the j-th category,

$$n_0 = 0,\ n_1 + ... + n_k = n,\ N_0 = 0,$$
$$N_j = N_{j-1} + n_j,\ j = 1, ..., k-1;\ N_k = n$$

Then the log-likelihood function can be written as

$$l(\tilde{\beta}, \tilde{\theta}; y) = \sum_{j=1}^{k} \sum_{i=N_{j-1}+1}^{N_j} \log\left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right) \qquad (3)$$

where *Φ* is the cumulative standard normal distribution function, $\tilde{\theta}_k = \infty$.

Following the usual technique, let us obtain the derivatives of the log-likelihood function with respect to $\tilde{\theta} = (\tilde{\theta}_1, ..., \tilde{\theta}_{k-1})$ and $\tilde{\beta} = (\tilde{\beta}_1, ..., \tilde{\beta}_d)$.

The derivatives of the log-likelihood function with respect to $\tilde{\theta}_j$ for $j = 1, …, k - 1$ are

$$\begin{aligned}\frac{\partial l}{\partial \tilde{\theta}_j} = &\sum_{i=N_{j-1}+1}^{N_j} \left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-1} \times \\ &\times \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) - \\ &- \sum_{i=N_j+1}^{N_{j+1}} .\left(\Phi(\tilde{\theta}_{j+1} - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_j - \tilde{\beta}x_i)\right)^{-1} \times \\ &\times \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right).\end{aligned} \qquad (4)$$

The derivatives of the log-likelihood function with respect to $\tilde{\beta} = (\tilde{\beta}_1, ..., \tilde{\beta}_d)^T$:

$$\begin{aligned}\frac{\partial l}{\partial \tilde{\beta}} = &-\sum_{j=1}^{k} \sum_{i=N_{j-1}+1}^{N_j} \left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-1} \times \\ &\times \frac{1}{\sqrt{2\pi}\sigma} \left\{\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) - \right. \\ &\left. - \exp\left(-\frac{1}{2}(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)^2\right)\right\} x_i.\end{aligned} \qquad (5)$$

Now we should solve the maximal likelihood equations:

$$\frac{\partial l}{\partial \tilde{\beta}} = 0;\ \frac{\partial l}{\partial \tilde{\theta}_j} = 0,\ j = 1, ..., k-1. \qquad (6)$$

Our experiences show that a solution is gotten easily by using standard computer programs. In other case it is necessary using Taylor expansion for a solution of the maximal likelihood equations (6). Corresponding expressions are presented in the Appendix.

If parameter estimates $\tilde{\theta}^* = (\tilde{\theta}_1^*, ..., \tilde{\theta}_{k-1}^*)^T$ and $\tilde{\beta}^* = (\tilde{\beta}_1^*, ..., \tilde{\beta}_d^*)^T$ are known, it is possible to estimate the probability of interest (2):

$$\pi*_j(x_i) = P*\{Y_i = j\} = \\ = P\{\tilde{\theta}*_{j-1} < Z_i \le \tilde{\theta}*_j\} = \\ \Phi(\tilde{\theta}*_j - x_i\tilde{\beta}*) - \Phi(\tilde{\theta}*_{j-1} - x_i\tilde{\beta}*) \quad (7)$$

IV. Information matrix and asymptotic distribution of the estimate

Now we consider information matrix of unknown parameters [6, 7]:

$$I = -\frac{1}{n}E\begin{pmatrix} \frac{\partial^2}{\partial\tilde{\beta}\partial\tilde{\beta}^T}l(\tilde{\beta},\tilde{\theta}) & \frac{\partial^2}{\partial\tilde{\beta}\partial\tilde{\theta}^T}l(\tilde{\beta},\tilde{\theta}) \\ \frac{\partial^2}{\partial\tilde{\theta}\partial\tilde{\beta}^T}l(\tilde{\beta},\tilde{\theta}) & \frac{\partial^2}{\partial\tilde{\theta}\partial\tilde{\theta}^T}l(\tilde{\beta},\tilde{\theta}) \end{pmatrix}, \quad (8)$$

where a dimension of matrix ***I*** is $(d+k-1)\times(d+k-1)$, and dimensions of its submatrices are $d\times d$, $d\times(k-1)$, $(k-1)\times d$ and $(k-1)\times(k-1)$.

From (4) we have for $j = 1, \ldots, k-1$:

$$\frac{\partial l^2}{\partial\tilde{\theta}_j^2} = -\sum_{i=N_{j-1}+1}^{N_j}\Big\{\left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-2} \times \\ \times\left(\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\right)^2 + \\ + \left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-1}(\tilde{\theta}_j - \tilde{\beta}x_i)\times \\ \times\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\Big\} + \\ + \sum_{i=N_j+1}^{N_{j+1}}\Big\{\left(\Phi(\tilde{\theta}_{j+1} - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_j - \tilde{\beta}x_i)\right)^{-2} \times \\ \times\left(\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\right)^2 + \\ + \left(\Phi(\tilde{\theta}_{j+1} - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_j - \tilde{\beta}x_i)\right)^{-1}(\tilde{\theta}_j - \tilde{\beta}x_i)\times \\ \times\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\Big\},$$

and for $j = 2, \ldots, k-1$:

$$\frac{\partial^2 l}{\partial\tilde{\theta}_j\partial\tilde{\theta}_{j+1}} = \sum_{i=N_j+1}^{N_{j+1}}\left(\Phi(\tilde{\theta}_{j+1} - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_j - \tilde{\beta}x_i)\right)^{-2} \times \\ \times\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left[(\tilde{\theta}_{j+1} - \tilde{\beta}x_i)^2 + (\tilde{\theta}_j - \tilde{\beta}x_i)^2\right]\right).$$

If $|i - j| > 1$ then the mixed derivative with respect to $\tilde{\theta}_i$ and $\tilde{\theta}_j$ equals zero.

Further from (5)

$$\frac{\partial^2 l}{\partial\tilde{\beta}\partial\tilde{\beta}^T} = \sum_{j=1}^{k}\sum_{i=N_{j-1}+1}^{N_j}\Big\{\left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-2} \times \\ \times\frac{1}{2\pi}\left(\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) - \exp\left(-\frac{1}{2}(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)^2\right)\right)^2 - \\ -\left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-1} \times \\ \times\frac{1}{\sqrt{2\pi}}\Big\{(\tilde{\theta}_j - \tilde{\beta}x_i)\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) - \\ -(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\exp\left(-\frac{1}{2}(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)^2\right)\Big\}x_i x_i^T.$$

$$\frac{\partial^2 l}{\partial\tilde{\beta}\partial\tilde{\theta}_j} = \sum_{i=N_{j-1}+1}^{N_j}\Big\{\left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-2} \times \\ \times\frac{1}{2\pi}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\times \\ \times\left[\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) - \exp\left(-\frac{1}{2}(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)^2\right)\right] + \\ + \left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)^{-1} \times \\ \times\frac{1}{\sqrt{2\pi}}(\tilde{\theta}_j - \tilde{\beta}x_i)\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\Big\}x_i - \\ - \sum_{i=N_j+1}^{N_{j+1}}\Big\{\left(\Phi(\tilde{\theta}_{j+1} - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_j - \tilde{\beta}x_i)\right)^{-2} \times \\ \times\frac{1}{2\pi}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\times \\ \times\left[\exp\left(-\frac{1}{2}(\tilde{\theta}_{j+1} - \tilde{\beta}x_i)^2\right) - \exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\right] - \\ -\left(\Phi(\tilde{\theta}_{j+1} - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_j - \tilde{\beta}x_i)\right)^{-1} \times \\ \times\frac{1}{\sqrt{2\pi}}(\tilde{\theta}_j - \tilde{\beta}x_i)\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right)\Big\}x_i.$$

Now to get information matrix ***I***, we must take expectation. The i-th observation (1) gives a nonzero contribution with probability (2). A value of this contribution is presented by above described formulas. Therefore, each this value must be multiplied by (2), it implies that a power of member $\left(\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) - \Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i)\right)$ will be increased by one.

It is known [6, 7] that asymptotic distribution of the maximal likelihood method's estimates is a normal one with zero expectation and covariance matrix $\frac{1}{n}I^{-1}$. Therefore now we are able to use various statistical procedures for the considered model.

V. Numerical Example

The model was constructed on the basis of results of questionnaire of 44 Latvian high-qualified transport experts, which was fulfilled in spring 2009. Initial data has been presented by an administration of Riga Coach Terminal. This fact, that respondents are the high-qualified transport specialists, allowed as to assume that the sample is homogeny and the assumption about equal variance of residual is fulfilled. The questionnaire included 7 groups of questions concerned the following groups of quality particular attributes: accessibility (availability); information; time characteristics of service; customer service; comfort; reliability and safety; infrastructure and environment (**see** Table 1).

Totally there were 22 particular attributes (X) of quality distributed among 7 groups, $x_i = \left(x_{i,1}, x_{i,2}, \ldots, x_{i,d}\right)$, d = 22 is number of factors.

Also the overall quality of service was evaluated. As well as particular attributes of quality the overall quality service was estimated on a scale (0-5). In total 44 questionnaires have been returned but some questions remained without the answer in two questionnaires (42 responses totally). According to questionnaire results we have 42 responses thus the response Y_i of a concrete expert *I* (i = 1, …, 42) is one of a fixed set {1, 2, 3, 4, 5} of possible values.

The analysis of coordination (consistency) of questionnaire questions was made by means of Cronbach alpha coefficient according the next formula:

$$\alpha = \frac{k}{k-1}\left(1 - \sum_{i=1}^{k} \frac{s_i^2}{s_{sum}^2}\right) \qquad (9)$$

where k – amount of questions (*in our case **22** – particular quality attributes),*

s_i^2 – variance of i question;

s_{sum}^2 – variance of sum of questions.

The results of questionnaire (estimates of particular attributes of quality) have demonstrated high indices of the internal coordination. A value of Cronbach alpha coefficient is equal to 0.933 and the standardized value is 0.93. It has allowed making an assumption about reliability of results.

Table I.

Particular Attributes of Quality

Title of chapter in questionnaire	Coding	Description of variable	Coding
1. Accessibility	W1	Accessibility for external participants of traffic	X1
		Accessibility for terminal passengers	X2
		Ticket booking	X3
2. Information	W2	General information in terminal	X4
		Information about trips in positive aspect	X5
		Information about trips in negative aspect	X6
3. Time	W3	Duration of trip	X7
		Punctuality	X8
		Reliability/trust	X9
		Bus time schedule	X10
4. Customer service	W4	Customer trust to terminal employees	X11
		Communication with customer	X12
		Requirements to employees	X13
		Physical services providing	X14
		Process of ticket booking	X15
		Services provided by bus crews during boarding/debarkation	X16
5. Comfort	W5	Cleanness and comfort in terminal premises and on terminal square	X17
		Additional opportunities/services providing in coach terminal	X18
6. Reliability /safety	W6	Protection from crimes	X19
		Protection from accidents	X20
7. Environment	W7	Dirtying, its prevention	X21
		Infrastructure	X22
	W8	**Overall estimation**	X23

In authors' previous researches which are considered in [10] one of the goals was to reduce dimension of the problem due to the insufficient amount of observation. With the purpose of comparing reduced and full factor set let's firstly describe our problem with not all 22 factors together, but the most significant factors according to previous investigations [10]. Least Squares Method (LSE) for a classical linear regression model was applied and because of partial quality attributes could correlate between each other, that's why the stepwise regression model definition procedure was used (Forward Stepwise Algorithm in SPSS package). Due to stepwise procedure five factors were selected as significant ones – X3 (Ticket booking), X8 (Punctuality), X11 (Customer trust to terminal employees), X13 (Requirements to employees), X22 (Infrastructure). Thus number of factors is *d* = **5.**

Build-in optimization block in Mathcad (functions *Given* and *Find*) was used to find optimal values of estimated parameters $\beta = (\beta_1, ..., \beta_5)^T$, $\theta = (\theta_1, ..., \theta_4)^T$. In program we set $\theta_0 = -3$, $\theta_5 = 8$. The initial values of the parameters are the next:

$$\beta = (0.01 \;\; 0.01 \;\; 0.01 \;\; 0.01 \;\; 0.01)^T$$

$$\theta = (1 \;\; 2 \;\; 3 \;\; 4)^T .$$

After optimization part we get the following estimations of unknown parameters:

$$\beta = (0.358 \;\; -0.203 \;\; 0.776 \;\; -0.24 \;\; 0.655)^T$$

$$\theta = (1.534 \;\; 2.268 \;\; 3.929 \;\; 6.841)^T .$$

The optimum value of likelihood function is $l(\tilde{\beta}^*, \tilde{\theta}^*) = -21.125$.

Checking gives correct results because all partial derivatives are equal to zero.

Table II contains values of expert estimates and predicted estimates according to formula (7) with estimated parameters.

Table II.

Real and predicted estimates (5 particular attributes)

Expert Number, *i*	1	2	3	4	5	6	7	8	9	10	11
Expert estimates	**1**	**2**	**3**	**3**	3	3	**3**	3	3	4	4
Predicted estimates	**2**	**3**	**4**	**4**	3	3	**4**	3	3	4	4
Expert Number, *i*	12	13	14	15	16	17	18	19	20	21	22
Expert estimates	4	4	4	4	4	4	4	4	4	4	4
Predicted estimates	4	4	4	4	4	4	4	4	4	4	4
Expert Number, *i*	23	24	25	26	27	28	29	30	31	32	33
Expert estimates	4	4	4	4	4	4	4	4	4	4	4
Predicted estimates	4	4	4	4	4	4	4	4	4	4	4
Expert Number, *i*	34	35	36	37	38	39	40	41	42		
Expert estimates	4	4	4	4	4	**5**	5	**5**	**5**		
Predicted estimates	4	4	4	4	4	**4**	5	**4**	**4**		

As we can see the predicted values are close to real response vector *Y*. We have eight observations, where the estimated value is not equal to the observed value (see Table II). In the table these values are in bold. The considerable quantity of deviations is observed in categories with small amount of observations. It can be connected with sample imperfection.

At the same time suggested approach allows using in model all 22 particular attributes, namely let $\beta = (\beta_1, ..., \beta_{22})^T$.

Applying previously described procedure we get the following estimations of unknown parameters:

$\beta_1 = 0.567$, $\beta_2 = 0.269$, $\beta_3 = 0.101$, $\beta_4 = -0.174$, $\beta_5 = -0.543$, $\beta_6 = 0.567$, $\beta_7 = -0.28$, $\beta_8 = 1.55$, $\beta_9 = -1.481$, $\beta_{10} = -1.112$, $\beta_{11} = 0.606$, $\beta_{12} = 0.113$, $\beta_{13} = 0.084$, $\beta_{14} = 0.177$, $\beta_{15} = 0.31$, $\beta_{16} = -0.641$, $\beta_{17} = 0.253$, $\beta_{18} = -0.227$, $\beta_{19} = 1.198$, $\beta_{20} = -0.826$, $\beta_{21} = 0.13$, $\beta_{22} = 0.642$;

$$\theta = (-2.091 \;\; -1.219 \;\; 1.675 \;\; 6.475)^T .$$

Now we have only two observations (in category $y_i = 5$) where the estimated value is not equal to the observed value (see Table III). It says about an advantage of the suggested approach.

Table III

Real and predicted estimates (22 particular attributes)

Expert Number, *i*	1	2	3	4	5	6	7	8	9	10	11
Expert estimates	1	2	3	3	3	3	3	3	3	4	4
Predicted estimates	1	2	3	3	3	3	3	3	3	4	4
Expert Number, *i*	12	13	14	15	16	17	18	19	20	21	22
Expert estimates	4	4	4	4	4	4	4	4	4	4	4
Predicted estimates	4	4	4	4	4	4	4	4	4	4	4
Expert Number, *i*	23	24	25	26	27	28	29	30	31	32	33
Expert estimates	4	4	4	4	4	4	4	4	4	4	4
Predicted estimates	4	4	4	4	4	4	4	4	4	4	4
Expert Number, *i*	34	35	36	37	38	39	40	41	42		
Expert estimates	4	4	4	4	4	5	5	**5**	**5**		
Predicted estimates	4	4	4	4	4	5	5	**4**	**4**		

Now let's consider the next method of forming the independent variables for regression model, in particular: we will form 7 new variables corresponding to 7 groups of attributes (categories of questions). Grouping of the initial attributes and calculation of new values on the basis of a arithmetic mean leads to a replacement of categorical variable x_i (i=1, .., 22) by interval w_l (l=1, .., 7).

Let's use method described above.

The initial values of the parameters are

$$\beta = (0.01 \;\; 0.01 \;\; 0.01 \;\; 0.01 \;\; 0.01 \;\; 0.01 \;\; 0.01)^T$$

$$\theta = (1 \;\; 2 \;\; 3 \;\; 4)^T .$$

After optimization part we get the following estimations of unknown parameters:

$$\beta = (1.18\ \ -0.639\ \ -0.719\ \ -0.48\ \ 0.522\ \ 1.002\ \ 0.51)^T$$
$$\theta = (-0.186\ \ 0.735\ \ 3.015\ \ 6.539)^T .$$

The optimum value of likelihood function is $l(\tilde{\beta}*, \tilde{\theta}*) = -20.384$.

Checking gives correct results because all partial derivatives are equal to zero.

Table IV contains values of expert estimates and predicted estimates according to formula (7).

As we can see the most significant difference in real and predicted estimates is followed up in those categories which are in minority, namely $y_2 = 2, y_3 = 3, y_5 = 5$.

Table IV

Real and predicted estimates (7 grouped particular attributes)

Expert Number, *i*	1	2	3	4	5	6	7	8	9	10	11
Expert estimates	1	**2**	**3**	3	3	3	3	3	3	4	4
Predicted estimates	1	**3**	**4**	3	3	3	3	3	3	4	4
Expert Number, *i*	12	13	14	15	16	17	18	19	20	21	22
Expert estimates	4	4	4	4	4	4	4	4	4	4	4
Predicted estimates	4	4	4	4	4	4	4	4	4	4	4
Expert Number, *i*	23	24	25	26	27	28	29	30	31	32	33
Expert estimates	4	4	4	4	4	4	4	4	4	4	4
Predicted estimates	4	4	4	4	4	4	4	4	4	4	4
Expert Number, *i*	34	35	36	37	38	39	40	41	42		
Expert estimates	**4**	4	4	4	4	**5**	**5**	**5**	**5**		
Predicted estimates	**5**	4	4	4	4	**4**	**4**	**4**	**4**		

A practical result of the given investigation is the following: it allows coach terminal management to estimate service quality on a quantitative basis not reducing dimension of the problem.

Our example shows that developed model for an integrated quality indicator allows not only to compare and reveal the significant categories of qualities influencing the general indicator, but also to predict customer choice.

VI. Conclusion

The problem of evaluation of service quality is a complicated task because it is the task with many variables and the data used to build models are subjective and need questioning a large number of respondents. One of the useful methods of solving this task is to build scalar indicator of quality as a function from a number of attributes on the basis of statistic analysis methods.

In this paper, we have considered a quasi regression model when the response of an individual in a study is restricted to one of a fixed set of possible values (categories). Unknown parameters of the model are estimated by the maximal likelihood methods. Expressions for the information matrix have been got. Numerical example concerns passenger evaluation of service quality in Riga Coach Terminal. This work considers three examples of evaluation of the service quality and includes different numbers of attributes. The first example considers five most significant attributes. After prediction there are eight observations, where the estimated value is not equal to the observed value The considerable quantity of deviations is observed in categories with small amount of observations. The second example considers all twenty two attributes with two deviations only. Due to this we may suggest that this method has a number of advantages. The third example considers seven grouped attributes where the possible application of the given method to the continuous variable (not qualitative one) is checked. The third example has seven deviations as well as in the first example they are observed in categories with a small number of observations, which must be connected with the imperfection of sample. The obtained results demonstrate good possibilities of the suggested approach.

Appendix

Below the Taylor expansions of derivatives (4) and (5) are presented for a solution of the maximal likelihood equations (6).

We begin with the derivative (4). If we use three members of Taylor expansion then for $j = 2, \ldots, k-2$

$$\Phi(\tilde{\theta}_{j-1} - \tilde{\beta}x_i) = \Phi(\tilde{\theta}_j - \tilde{\beta}x_i) + (\tilde{\theta}_{j-1} - \tilde{\theta}_j)\frac{\partial}{\partial\tilde{\theta}_j}\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) + \frac{1}{2}(\tilde{\theta}_{j-1} - \tilde{\theta}_j)^2\frac{\partial^2}{\partial\tilde{\theta}_j^2}\Phi(\tilde{\theta}_j - \tilde{\beta}x_i) + o\left((\tilde{\theta}_{j-1} - \tilde{\theta}_j)^2\right) =$$
$$= \Phi(\tilde{\theta}_j - \tilde{\beta}x_i) + (\tilde{\theta}_{j-1} - \tilde{\theta}_j)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) - \frac{1}{2}(\tilde{\theta}_{j-1} - \tilde{\theta}_j)^2\frac{1}{\sqrt{2\pi}}\left(-(\tilde{\theta}_j - \tilde{\beta}x_i)\right)\times$$
$$\times\exp\left(-\frac{1}{2}(\tilde{\theta}_j - \tilde{\beta}x_i)^2\right) + o\left((\tilde{\theta}_{j-1} - \tilde{\theta}_j)^2\right), \quad (10)$$

A substitution into (4) gives for $j = 2, \ldots, k-2$

$$\frac{\partial l}{\partial \tilde{\theta}_j} = \sum_{i=N_{j-1}+1}^{N_j} \left(\left(\tilde{\theta}_{j-1} - \tilde{\theta}_j\right) \left(\frac{1}{2}\left(\tilde{\theta}_{j-1} - \tilde{\theta}_j\right)\left(\tilde{\theta}_j - \beta x_i\right) - 1 \right) \right)^{-1} -$$
$$- \sum_{i=N_j+1}^{N_{j+1}} \left(\left(\tilde{\theta}_{j+1} - \tilde{\theta}_j\right) \left(1 - \frac{1}{2}\left(\tilde{\theta}_{j+1} - \tilde{\theta}_j\right)\left(\tilde{\theta}_j - \beta x_i\right) \right) \right)^{-1} = 0. \tag{11}$$

Furthermore

$$\frac{\partial l}{\partial \tilde{\theta}_1} = \sum_{i=1}^{N_1} \Phi\left(\left(\tilde{\theta}_1 - \tilde{\beta} x_i\right)\right)^{-1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\tilde{\theta}_1 - \tilde{\beta} x_i\right)^2 \right) -$$
$$- \sum_{i=N_1+1}^{N_2} \left(\left(\tilde{\theta}_2 - \tilde{\theta}_1\right) \left(1 - \frac{1}{2}\left(\tilde{\theta}_2 - \tilde{\theta}_1\right)\left(\tilde{\theta}_1 - \tilde{\beta} x_i\right) \right) \right)^{-1} = 0,$$

$$\frac{\partial l}{\partial \tilde{\theta}_{k-1}} = 0 =$$
$$= \sum_{i=N_{k-2}+1}^{N_{k-1}} \left(\left(\tilde{\theta}_{k-2} - \tilde{\theta}_{k-2}\right) \left(\frac{1}{2}\left(\tilde{\theta}_{k-2} - \tilde{\theta}_{k-1}\right)\left(\tilde{\theta}_{k-1} - \tilde{\beta} x_i\right) - 1 \right) \right)^{-1} -$$
$$- \sum_{i=N_{k-1}+1}^{n} \left(1 - \Phi\left(\tilde{\theta}_{k-1} - \tilde{\beta} x_i\right)\right)^{-1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\tilde{\theta}_{k-1} - \tilde{\beta} x_i\right)^2 \right) = 0.$$

Now we consider the derivative (5). The substitution (10) gives

$$\frac{\partial l}{\partial \beta} = \sum_{i=1}^{N_1} \left(\Phi\left(\tilde{\theta}_1 - \tilde{\beta} x_i\right)\right)^{-1} \frac{1}{\sqrt{2\pi}} \times$$
$$\times \left(-\exp\left(-\frac{1}{2}\left(\tilde{\theta}_1 - \tilde{\beta} x_i\right)^2 \right) \right) x_i +$$
$$+ \sum_{i=N_{k-1}+1}^{n} \left(1 - \Phi\left(\tilde{\theta}_{k-1} - \tilde{\beta} x_i\right)\right)^{-1} \times$$
$$\times \frac{1}{\sqrt{2\pi}} \left(\exp\left(-\frac{1}{2}\left(\tilde{\theta}_{k-1} - \tilde{\beta} x_i\right)^2 \right) \right) x_i +$$
$$+ \sum_{j=2}^{k-2} \sum_{i=N_{j-1}+1}^{N_j} \left(\tilde{\theta}_{j-1} - \tilde{\theta}_j\right)^{-1} \left(1 - \right.$$
$$\left. - \exp\left(-\frac{1}{2}\left(\tilde{\theta}_j^{\,2} - 2\tilde{\beta} x_i (\tilde{\theta}_j - \tilde{\theta}_{j-1}) + \tilde{\theta}_{j-1}^{\,2} \right) \right) \right) x_i = 0.$$

ACKNOWLEDGMENT

The article is written with the financial assistance of European Social Fund. Project Nr.2009/0159/1DP/1.1.2.1.2/09/IPIA/VIAA/006 (The Support in Realization of the Doctoral Programme “Telematics and Logistics” of the Transport and Telecommunication Institute).

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