

difference is about 1000 rad/m from $z = 0$ to 25 mm, and the difference becomes very small when z is more than 30 mm. On the other hand, d has little influence on the angular wavenumber of the BM mode. From Fig. 6(b), we can see that both modes have little dependence on d over the whole range of z when $f = 10000$ Hz.

IV. CONCLUSION

We investigated the cochlea, which has three regions separated by RM and BM, by using modal analysis. The dispersion diagrams of the RM mode and the BM mode showed that the wavenumber of the RM mode is constant from the base to the apex of the cochlea regardless of the acoustic frequency, while the wavenumber of the BM mode largely changes depending on the position along the cochlea. The analytical results of the displacement of the membranes demonstrated that when $f = 1000$ Hz, the RM mode vibrates only RM, while the BM mode vibrates both BM and RM, and that when $f = 10000$ Hz, each mode vibrates only its membrane. We also studied how the distance between BM and RM affects the dispersion relations of the RM mode and the BM mode. The results showed that when $f = 1000$ Hz, the angular wavenumber of the RM mode increases as the distance is reduced, whereas when $f = 10000$ Hz, both modes have little dependence on the distance over the whole range of the length along the cochlea duct. It was shown that the distance hardly affects the angular wavenumbers of both modes when it is more than 0.2 mm.

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