Control of general time delay systems using Matlab toolbox

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***Abstract*—**The aim of this paper is to show an application of Matlab toolbox "Robust Control Toolbox for Time Delay Systems with Time Delay in Numerator and Denominator". The solved problem is robust control of time delay system with time delay in numerator and denominator of the controlled plant. This type of problem is usually solved in the ring of retarded quasipolynomial (RQ) meromorphic functions. This approach can solve the task for nominal plants but it is not easy to apply this technique if the plant has uncertain time delays. In this paper, the plant is defined as a system with uncertain time delays which can vary in predefined intervals. A method handling this problem in the robust sense is derived and implemented using both the D-K iteration and algebraic approach. The D-K iteration is a standard method in the structured singular value framework. However, some remaining issues are present, such as nonzero steady-state error and the necessity of approximation of the resulting controller with low order system due to its high complexity. A solution the algebraic approach combining the structured singular value, algebraic theory and global optimization method can give. Here, Differential Migration is used providing high efficiency in finding the global extreme and reliable results.

***Keywords*—**Algebraic approach, robust control, RQ-meromorphic functions, structured singular value, Uncertain time delay systems.

I. Introduction

THE paper is focused on control of uncertain time delay systems with time delay in numerator and denominator of the controlled plant. This type of plants is currently solved in the ring of retarded quasipolynomial (RQ) meromorphic functions (see [15] and [16]). However, the robustness is not easy to derive using this approach.

The toolbox presented in this paper implements a method handling the robustness and uncertainty in an easy way giving simple and easy to implement controllers. Typically, for a 1st order system the controller can be described as 4th order transfer function compared to awkward and hard to implement controllers obtained from the design in the ring of RQ-meromorphic functions, which can treat the uncertainty with difficulties.

The presented method takes into account the uncertainty using the procedure described in [3], which fully covers the varying time delays and guarantees the robust stability and performance. In order to obtain controllers that satisfy bounded-input bounded-output (BIBO) stability algebraic theory is used for pole placement. The task is accomplished via solving the Diophantine equation in the ring of Hurwitz-stable and proper rational functions ($\mathbf{R}_{PS}$). As a measure of robust stability and performance, structured singular value denoted μ is employed (see [11]).

Due to the multimodality of the cost function in the algebraic approach an algorithm of global optimization is used. For this task evolutionary algorithm (see [9], [10], [13] and [14]) proved reliable results. Differential Migration (see [1]) appears to be one of the most effective. Therefore, its application was chosen together with Nelder-Mead simplex method as a tool for the final tune-up of the pole placement.

As a reference method, the D-K iteration (see [5]) is implemented in the toolbox with entropy, LMI or DGKF formulae as the options in the D-K iteration part (see [6], [7] and [8]). The D-K iteration controller is compared with the proposed method in the simulations of the response to the step of the reference for different values of uncertain time delays. The controllers are connected in simple and two-degree-of-freedom feedback loop (1DOF and 2DOF, see [12]).

The following notation is used: $\|\cdot\|_\infty$ denotes $\mathbf{H}_\infty$ norm, $\bar{\sigma}(\cdot)$ is maximum singular value, $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\mathbf{I}_n$ is the unit matrix of dimension n and $\mathbf{R}_{PS}$ denotes the ring of Hurwitz-stable and proper rational functions.

II. Preliminaries

Define $\mathbf{\Delta}$ as a set of block diagonal matrices

$$\mathbf{\Delta} \equiv \{\mathrm{diag}[\delta_1 I_{r_1},\ldots,\delta_S I_{r_S},\Delta_1,\ldots,\Delta_F]: \delta_i \in \mathbf{C}, \Delta_j \in \mathbf{C}^{m_j\times m_j}\} \quad (1)$$

where S is the number of repeated scalar blocks,
F is the number of full blocks,
$r_1,\ldots, r_S$ and $m_1,\ldots, m_F$ are positive integers defining dimensions of scalar and full blocks.

For consistency among all the dimensions, the following condition must be held

$$\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n \quad (2)$$

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Definition 1: For $\mathbf{M} \in \mathbf{C}^{n \times n}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_{\mathbf{\Delta}}(\mathbf{M}) \equiv \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I} - \mathbf{M}\Delta) = 0\}} \tag{3}$$

If no such $\Delta \in \mathbf{\Delta}$ exists making $\mathbf{I} - \mathbf{M}\Delta$ singular, then $\mu_{\mathbf{\Delta}}(\mathbf{M}) = 0$.

Consider a complex matrix $\mathbf{M}$ partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \tag{4}$$

and suppose there is a defined block structure Δ_2 which is compatible in size with $\mathbf{M}_{22}$ (for any $\Delta_2 \in \mathbf{\Delta}_2$, $\mathbf{M}_{22}\Delta_2$ is square). For $\Delta_2 \in \mathbf{\Delta}_2$, consider the following loop equations

$$\begin{aligned} e &= \mathbf{M}_{11} d + \mathbf{M}_{12} w \\ z &= \mathbf{M}_{21} d + \mathbf{M}_{22} w \\ w &= \Delta_2 z \end{aligned} \tag{5}$$

If the inverse to $\mathbf{I} - \mathbf{M}_{22}\Delta_2$ exists, then e and d must satisfy $e = \mathbf{F}_L(\mathbf{M}, \Delta_2)d$, where

$$\mathbf{F}_L(\mathbf{M}, \Delta_2) = \mathbf{M}_{11} + \mathbf{M}_{12}\Delta_2(I - \mathbf{M}_{22}\Delta_2)^{-1}\mathbf{M}_{21} \tag{6}$$

is a linear fractional transformation on $\mathbf{M}$ by Δ_2, and in a feedback diagram appears as the loop in Fig. 2.

The subscript L on $\mathbf{F}_L$ pertains to the *lower* loop of $\mathbf{M}$ and is closed by Δ_2. An analogous formula describes $\mathbf{F}_U(\mathbf{M}, \Delta_1)$, which is the resulting matrix obtained by closing the *upper* loop of $\mathbf{M}$ with a matrix $\Delta_1 \in \mathbf{\Delta}_1$.

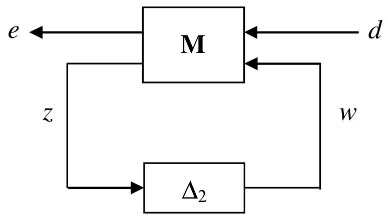

Fig. 2. LFT interconnection

Theorem 1: Let $\beta > 0$. For all $\Delta_2 \in \mathbf{\Delta}_2$ with $\left|\overline{\sigma}(\Delta_2)\right| < \frac{1}{\beta}$, the loop shown in Fig. 2 is well-posed, internally stable, and $\left\|\mathbf{F}_L(\mathbf{M}, \Delta_2)\right\|_\infty \le \beta$ if and only if

$$\sup_{\omega \in \Re} \mu_\Delta[\mathbf{M}(j\omega)] \le \beta \tag{7}$$

Proof: Proof is the same as in [4] and [11] except for the fact that perturbations are complex matrices, which simplifies the proof and complies with the definition of μ.

III. Algebraic μ-Synthesis

The algebraic μ-synthesis can be applied to any control problem that can be transformed to the loop in Fig. 1, where $\mathbf{G}$ denotes the generalized plant, $\mathbf{K}$ is the controller, Δ_{del} is the perturbation matrix, r is the reference and e is the output.

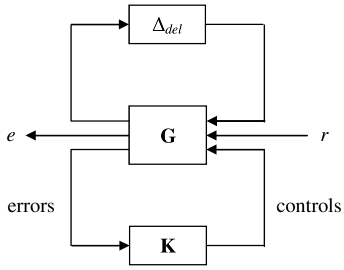

Fig. 1. Closed loop interconnection.

For the purposes of the algebraic μ-synthesis, the MIMO system with l inputs and l outputs has to be decoupled into l identical SISO plants. The nominal model is defined in terms of transfer functions:

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$$\mathbf{P}_{nom}(s) \equiv \begin{bmatrix} P_{11}(s) & \cdots & P_{1l}(s) \\ \vdots & \ddots & \vdots \\ P_{l1}(s) & \cdots & P_{ll}(s) \end{bmatrix} \tag{8}$$

For decoupling the nominal plant $\mathbf{P}_{nom}$ ($\mathbf{P}_{nom}$ invertible) it is satisfactory to have the controller in the form

$$\mathbf{K}(s) = K(s)\mathbf{I}_l \det[\mathbf{P}_{nom}(s)] \frac{1}{P_{xy}(s)} [\mathbf{P}_{nom}(s)]^{-1} \tag{9}$$

where P_{xy} is an element of $adj[\mathbf{P}_{nom}(s)] = \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}$ with the highest degree of numerator {$adj[\mathbf{P}_{nom}(s)]$ denotes adjugate matrix of $\mathbf{P}_{nom}$}. The choice of the decoupling matrix prevents the controller from cancelling any poles or zeros from the right half-plane so that internal stability of the nominal feedback loop is held. The MIMO problem is reduced to finding a controller $K(s)$, which is tuned via setting the poles of the nominal feedback loop with the plant

$$\begin{aligned} \mathbf{P}_{dec}(s) &= \frac{1}{P_{xy}(s)} \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}\mathbf{P}_{nom}(s) \\ &= \frac{1}{P_{xy}(s)} \det[\mathbf{P}_{nom}(s)]\mathbf{I}_l \end{aligned} \tag{10}$$

Define

$$P_{dec} \equiv \frac{1}{P_{xy}(s)} \det[\mathbf{P}_{nom}(s)] \tag{11}$$

Transfer function P_{dec} can be approximated by a system P^*_{dec} with lower order than P_{dec}

$$P^*_{dec}(s) = \frac{b(s)}{a(s)} \tag{12}$$

which can be rewritten in terms of its coefficients and transformed to the elements of $\mathbf{R}_{\text{PS}}$

$$P^*_{dec}(s) = \frac{\dfrac{b_0 + b_1 s + \ldots + b_n s^n}{(\alpha_1+1)(\alpha_2+1)\cdot\ldots\cdot(\alpha_n+1)}}{\dfrac{s^n + a_0 + a_1 s + \ldots + a_{n-1}s^{n-1}}{(\alpha_1+1)(\alpha_2+1)\cdot\ldots\cdot(\alpha_n+1)}} = \frac{B}{A},\ A,\ B \in \mathbf{R}_{\text{PS}} \tag{13}$$

The controller $K = N_K/D_K$ is obtained by solving the Diophantine equation

$$AD_K + BN_K = 1 \tag{14}$$

with A, B, D_K, $N_K \in \mathbf{R}_{\text{PS}}$. Equation (14) is often called the Bezout identity. All feedback controllers N_K/D_K are given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT}, \qquad N_{K_0}, D_{K_0} \in \mathbf{R}_{\text{PS}} \tag{15}$$

where N_{K_0}, $D_{K_0} \in \mathbf{R}_{\text{PS}}$ are particular solutions of (14) and T is an arbitrary element of $\mathbf{R}_{\text{PS}}$.

The controller K satisfying equation (14) guarantees the BIBO (bounded input bounded output) stability of the feedback loop in Fig. 3. This is a crucial point for the theorems regarding the structured singular value. If the BIBO stability is held, then the nominal model is internally stable and theorems related to robust stability and performance can be used. The BIBO stability also guarantees stability of $\mathbf{F}_L(\mathbf{G},\mathbf{K})$ making possible usage of performance weights with integration property implying non-existence of state space solutions using DGKF formulae (see [6]) due to zero eigenvalues of appropriate Hamiltonian matrices. Such methodology results in zero steady-state error in the feedback loop with the controller obtained as a solution to equation (14). This technique is neither possible in the scope of the standard μ-synthesis using DGKF formulae, nor using LMI approach (see [7]) leading to numerical problems in most of real-world applications.

The aim of synthesis is to design a controller which satisfies the condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G},\mathbf{K})(\omega,\alpha_1,\ldots,\alpha_{n+n_1+n_2},t_1,\ldots,t_{n_2})] \le 1,\ \omega \in (-\infty,+\infty) \tag{16}$$

where $n + n_1 + n_2$ is the order of the nominal feedback system, n_1 is the order of particular solution K_0, t_i are arbitrary parameters in $T = \dfrac{t_0 + t_1 s + \ldots + t_{n_2} s^{n_2}}{(\alpha_{n_1+1}+s)\cdot\ldots\cdot(\alpha_{n_1+n_2}+s)}$ and $\mu_{\mathbf{\Delta}}$ denotes the structured singular value of LFT on generalized plant $\mathbf{G}$ and controller $\mathbf{K}$ with

$$\Delta \equiv \begin{bmatrix} \Delta_{del} & 0 \\ 0 & \Delta_F \end{bmatrix} \tag{17}$$

where Δ_{del} denotes the perturbation matrix and Δ_F is a full-block matrix corresponding with the robust performance condition.

Tuning parameters are positive and constrained to the real axis since parameters of the transfer function have to be real and due to the fact that non-real poles cause oscillations of the nominal feedback loop.

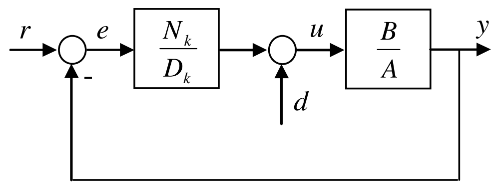

Fig. 3. Nominal feedback loop

A crucial problem of the cost function in (16) is the fact that many local extremes are present. Hence, local optimization does not yield a suitable or even stabilizing solution. This can be overcome via evolutionary optimization, which solves the task very efficiently.

IV. Problem Formulation

The problem to solve is general 1st order system with uncertain time delays:

$$P(s) \equiv \frac{b_0 e^{-\tau_{01}s}}{a_1 s + e^{-\tau_{02}s}},\ \tau_{01} \in [0, T_{01}],\ \tau_{02} \in [0, T_{02}] \tag{18}$$

This family of plants has uncertain retarded quasi-polynomial in the denominator. The delays vary in the intervals of zero to a predefined value representing the upper bound for each time delay.

This set of plants is treated via LFT using the scheme in Fig. 4. The weights W_{del1} and W_{del2} are obtained from the inequalities:

$$|W_{deli}| > \left|1 - e^{j\omega T_{di}}\right|, i = 1, 2 \tag{19}$$

The perturbation matrix has the form:

$$\mathbf{\Delta}_{del} \equiv \begin{bmatrix} \delta_{del1} & 0 \\ 0 & \delta_{del2} \end{bmatrix},\ |\delta_{del1}| < 1,\ |\delta_{del2}| < 1,\ \delta_{del1}, \delta_{del2} \in \mathbf{C} \tag{20}$$

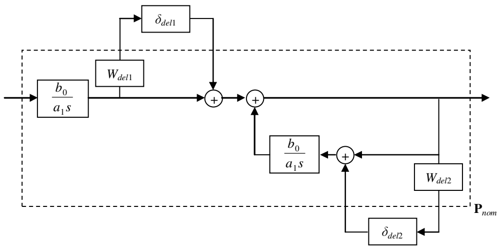

Fig. 4 LFT model of plant

and performance weight is a 3rd order transfer function:

$$W_1 = \frac{b_2 s^2 + b_1 s^1 + b_0}{a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0} \quad (21)$$

The weights W_{del1} and W_{del2} should satisfy (19) with very low conservatism.

The performance condition is of the form:

$$\|W_1 S\|_\infty < 1 \quad (22)$$

where S is the sensitivity function and weight W_1 is designed so that the asymptotic tracking is achieved.

V. Problem Solution

A. Structured Singular Value Framework

The problem defined in previous section can be solved using interconnection in Fig. 6 and 5. Here, **G** denotes the generalized plant partitioned to

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & G_{22} \end{bmatrix} \quad (23)$$

where the block structure of **G** corresponds with the input and output variables in Fig. 1:

$$\begin{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} \\ v \end{bmatrix} = \mathbf{G} \cdot \begin{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix} \\ u \end{bmatrix} \quad (24)$$

Then the transfer function from d to e is the upper linear fractional transformation on **M** and Δ

$$e = \mathbf{F}_u(\mathbf{M}, \Delta_{del}) r = \mathbf{M}_{22} r + \mathbf{M}_{21} \Delta_{del} (1 - M_{11} \Delta_{del})^{-1} \mathbf{M}_{12} r \quad (25)$$

and perturbation matrix corresponding with Fig. 6 is of the form

$$\tilde{\mathbf{\Delta}}_{del} \equiv \begin{bmatrix} \delta_{del1} & 0 \\ 0 & \delta_{del2} \end{bmatrix}, \quad \delta_{del1}, \delta_{del2} \in \mathbf{C} \quad (26)$$

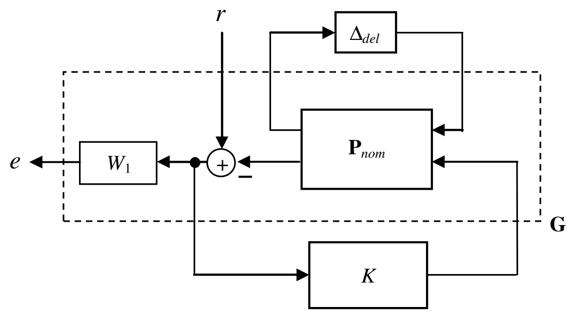

Fig. 6 Closed-loop interconnection for μ-synthesis

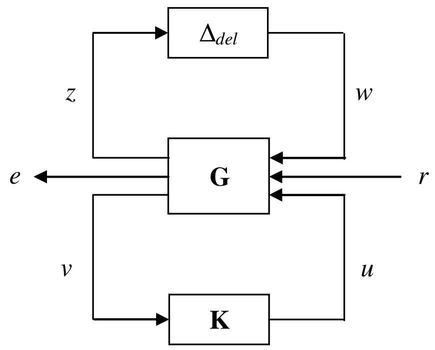

Fig. 5 Closed-loop interconnection

For performance and stability the following corollary of Theorem 1 holds.

Corollary 1: Closed loop in Fig. 6 is stable for all $\Delta_{del} \in \tilde{\mathbf{\Delta}}_{del}$ $|\overline{\sigma}(\Delta_{del})| < 1$, the performance condition (22) holds and $\|F_u(\mathbf{M}, \Delta_{del})\|_\infty \le 1$ if and only if conditions (19) hold and

$$\mu_{\tilde{\Delta}}(\mathbf{M}) \le 1 \quad (27)$$

for all frequencies.

The design objective is to find a stabilizing controller K such that

$$\sup_\omega \mu_{\tilde{\Delta}}[\mathbf{F}_l(\mathbf{G}, K)] \le 1 \quad (28)$$

where

$$\mathbf{M} = \mathbf{F}_l(\mathbf{G}, K) = \mathbf{G}_{11} + \mathbf{G}_{12} K (1 - G_{22} K)^{-1} \mathbf{G}_{21} \quad (29)$$

is the lower linear fractional transformation on generalized plant **G** and controller K (see Fig. 6) and $\mu_{\tilde{\Delta}}$ corresponds with the perturbation matrix from the set

$$\tilde{\mathbf{\Delta}} \equiv \begin{bmatrix} \tilde{\Delta}_{del} & 0 \\ 0 & \delta_P \end{bmatrix}, \; \tilde{\Delta}_{del} \in \tilde{\mathbf{\Delta}}_{del}, \; \delta_P \in \mathbf{C} \tag{30}$$

taking into account performance condition (22).

B. Algebraic Approach

The plant for which the controller is derived is the nominal system:

$$P_0(s) \equiv \frac{b_0}{a_0 s - 1} \tag{31}$$

Nominal plant P_0 can be transformed to:

$$P_0(s) = \frac{\dfrac{b_0}{\alpha_1 + 1}}{\dfrac{a_1 s - 1}{\alpha_1 + 1}} = \frac{B}{A}, \qquad A, B \in \mathbf{R}_{PS} \tag{32}$$

The controller is obtained as a solution to the Diophantine equation

$$AM + BN = 1 \tag{33}$$

with BIBO stable feedback controller N_K/D_K given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT} = \frac{\dfrac{n_{K_01}s + n_{K_00}}{(\alpha_2 + s)} - A\dfrac{t_2 s^2 + t_1 s}{(\alpha_3 + s)(\alpha_4 + s)}}{\dfrac{d_{K_01}s}{(\alpha_2 + s)} + B\dfrac{t_2 s^2 + t_1 s}{(\alpha_3 + s)(\alpha_4 + s)}} \tag{34}$$

The denominator of (34) is divisible by s so that asymptotic tracking for the stepwise reference signal can be achieved.

The aim of synthesis is to design a controller which satisfies condition

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_{\mathbf{\Delta}}[\mathbf{F}_L(\mathbf{G},K)(\omega,\alpha_1,\ldots,\alpha_4,t_1,t_2)] \le 1, \; \omega \in (-\infty,+\infty) \tag{35}$$

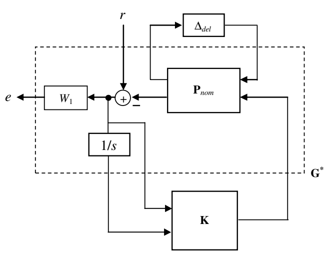

Fig. 8 Closed loop interconnection with integrator cascade

The controller has the form:

$$K_A(s) = \frac{n_K}{d_K} = \frac{n_{K,4}s^4 + \ldots + n_{K,0}}{s^4 + d_{K,3}s^3 + \ldots + d_{K,1}s} \tag{36}$$

In order to overcome the problem of non-integration structure of the D-K iteration controller a scheme with integrator that incorporates the integration property into the controller was used (see Fig. 8). The controller has the transfer function:

$$K_{D-K}(s) = \frac{n_{K,5}s^5 + \ldots + n_{K,0}}{s^5 + d_{K,4}s^4 + \ldots + d_{K,1}s} \tag{37}$$

VI. User Interface

The main window of the toolbox consists of three parts (see Fig. 7):

- System Definition
- Controller Design
- Simulation and Verification

A. System Definition

System definition has the button for displaying the dialog for entering parameters of the control plant. Here, the parameters of transfer function and the maximum value of time delay can be entered (Fig. 9).

Next two buttons display the dialogs for entering the parameters of the weight W_{del1} and W_{del2} treating uncertain time delay τ_{01} and τ_{02} (Fig. 10). In the dialogs, there is a button for showing the Bode plot of the weights W_{del1} and W_{del2} compared to the left side of (19) (see Fig. 11).

In the last part of system definition, button showing dialog for entering parameters of the performance weight W_1 is placed. The weight is the same for the D-K iteration and algebraic approach. The dialog has a button for showing the Bode plot of the weight.

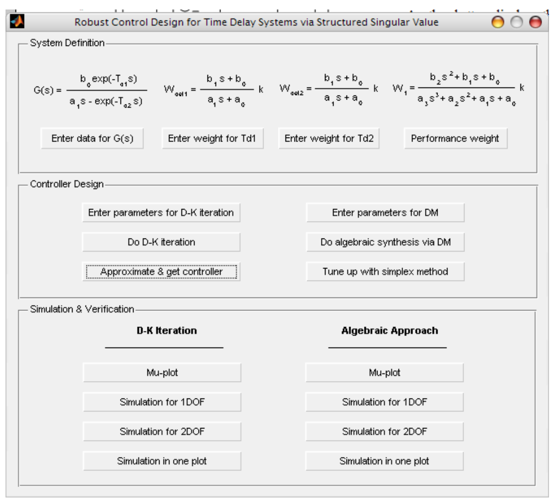

Fig. 7 The main window

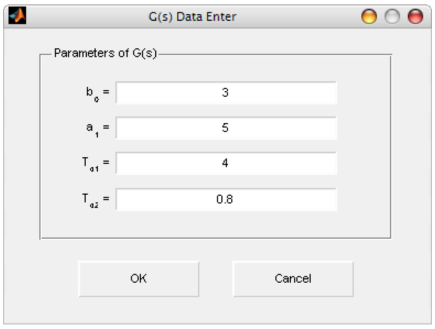

Fig. 9 Dialog for entering parameters of the control plant

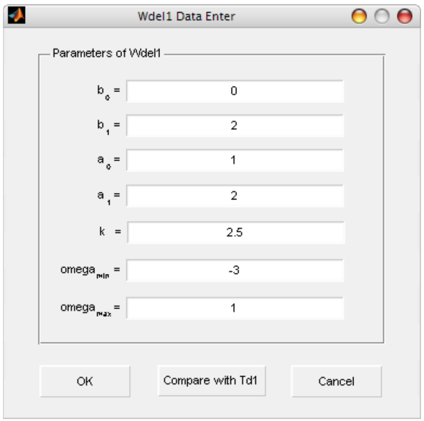

Fig. 10 Dialog for entering the parameters of the weight W_{del1}

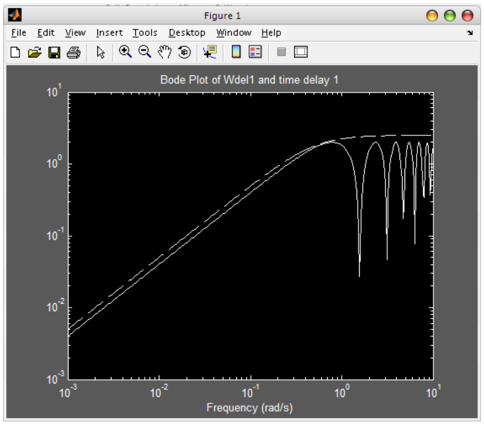

Fig. 11 Bode plot of the weight W_{del1} compared to the left side of (19)

B. Controller Design

The controller design part is divided into two sections – D-K iteration and algebraic approach. In the first row, there are the buttons for entering parameters for both the D-K iteration and algebraic approach.

In the second row, there are buttons for performing the design of controllers. The design is interactive and uses the command line window of the Matlab system for communication with user.

The D-K iteration asks the user for the starting mu-iteration. Then, the first gamma for the suboptimal controller is searched using the bisection method. Then, the user is prompted for the change of frequency range and bounds or tolerances. Then, the current step of the D-K iteration is finished and the μ-plot is displayed. In the next step, the μ-plot is approximated using scaling matrices D and D^{-1}. To this effect, the user is asked for his choice. Command apf can be used for auto-prefit, which automatically finds the parameters for this step. After exiting this part, using e command parameters for gamma search can be set. Then, the user is again prompted for change of the frequency range and bounds or tolerances. Finally, the μ-plot is calculated and displayed. These steps are repeated until the user terminates the whole process. Then, the resulting controller is obtained and displayed in the Matlab window.

The algebraic approach launches the evolutionary search, which performs the predefined number of migration loops defined in the parameters dialog for the algebraic approach. The search can be interrupted by pressing Ctrl+C. The controller can be obtained by pressing Approximate and get controller button.

Besides the evolutionary search, Nelder-Mead simplex method can be used for the tune up of the controller by pressing the button Tune up with simplex method.

C. Simulation and Verification

Simulation and verification part has two columns of buttons each for the particular design method, i.e. D-K iteration and algebraic approach.

In the first row, buttons for displaying the μ-plots are present. If the Mu-plot button in the algebraic approach is pressed then a comparison of both approaches can be viewed in terms of the μ-plots for both the D-K iteration and algebraic approach in one figure.

In the second row, buttons for simulation in Matlab Simulink are placed. The simulation can be performed for both simple feedback loop and two-degree-of-freedom (2DOF) feedback loop (see Fig. 14 and 13).

Finally, buttons for showing the simulation in one plot are at the bottom of the main window. If the button for algebraic approach is pressed then the simulation for D-K iteration is displayed in the same plot for comparison.

VII. Time Delay System Control for Uncertain Time Delay in Numerator and Denominator

Consider the set of anisochronic systems with time delay in the numerator and denominator:

$$P(s) \equiv \frac{3e^{-\tau_1 s}}{5s - e^{-\tau_2 s}}, \ \tau_1 \in [0, 4], \ \tau_2 \in [0, 0.8] \quad (38)$$

This set of plants is treated via LFT using the scheme in Fig. 4. Weights W_{del1} and W_{del2} can be obtained from the inequalities:

$$\left|W_{deli}\right| > \left|1 - e^{j\omega T_{di}}\right|, i = 1, 2; T_{d1} = 4, T_{d2} = 0.8 \tag{39}$$

It follows from Fig. 15 and 12 that

$$W_{del1} = \frac{2s}{2s+1} 2.5\,,\ W_{del2} = \frac{0.4s}{0.4s+1} 2.5 \tag{40}$$

satisfy (19) with very low conservatism.

Now, it is easy to create an open-loop interconnection with weighted sensitivity function as a performance indicator. Recall the closed-loop interconnection depicted in Fig. 6 with the open loop in dashed rectangle denoted **G**. The perturbation matrix has the form:

$$\mathbf{\Delta}_{del} \equiv \begin{bmatrix} \delta_{del1} & 0 \\ 0 & \delta_{del2} \end{bmatrix},\ \left|\delta_{del1}\right| < 1,\ \left|\delta_{del2}\right| < 1,\ \delta_{del1}, \delta_{del2} \in \mathbf{C} \tag{41}$$

and performance weight is a 3rd order transfer function:

$$W_1 = \frac{0.004}{10s^3 + 100s^2 + s + 1\cdot 10^{-5}} \tag{42}$$

The weight W_1 has a small factor for s^0 in the denominator so that the DGKF formulae can be used.

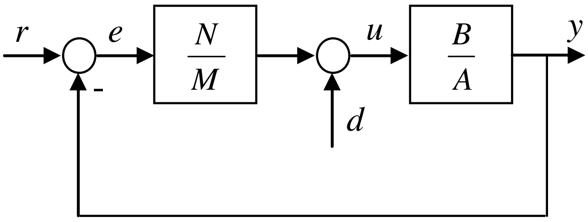

Fig. 14 Simple feedback loop

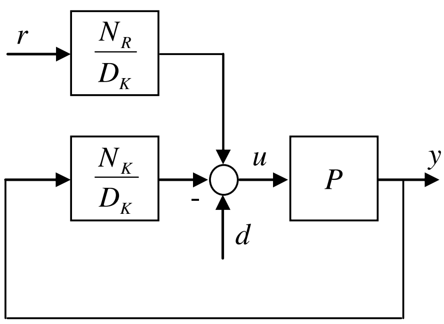

Fig. 13 2DOF feedback loop

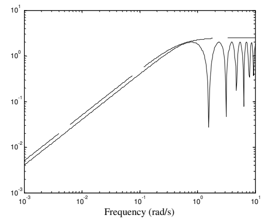

Fig. 15. Bode plot W_{del1} (dashed) and the right side of (19) (solid)

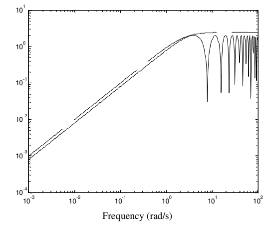

Fig. 12. Bode plot W_{del2} (dashed) and the right side of (19) (solid)

The plant for which the controller is derived is the nominal system:

$$P_0(s) \equiv \frac{3}{5s-1} \tag{43}$$

The instability of P_0 does not contradict stability of the nominal *feedback* loop. This is guaranteed by controller K satisfying (14).

Nominal plant P_0 can be transformed to:

$$P_0(s) = \frac{\dfrac{3}{\alpha_1+1}}{\dfrac{5s-1}{\alpha_1+1}} = \frac{B}{A}, \qquad A, B \in \mathbf{R}_{\mathrm{PS}} \tag{44}$$

The controller is obtained as a solution to the Diophantine equation (14) with all BIBO stable feedback controllers N_K/D_K given by (15).

For plant (31), the controller is a 4th order transfer function derived from (14) given as

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT} = \frac{\dfrac{n_{K_0 1}s + n_{K_0 0}}{(\alpha_2+s)} - A\dfrac{t_2 s^2 + t_1 s}{(\alpha_3+s)(\alpha_4+s)}}{\dfrac{d_{K_0 1}s}{(\alpha_2+s)} + B\dfrac{t_2 s^2 + t_1 s}{(\alpha_3+s)(\alpha_4+s)}} \tag{45}$$

The denominator of (34) is divisible by s so that asymptotic tracking for the stepwise reference signal can be achieved.

The aim of synthesis is to design a controller which satisfies condition (16). Evolutionary optimization by Differential

Migration gave the poles and arbitrary parameters as follows:

$$\alpha_1 = 0.023,\ \alpha_2 = 31.973,\ \alpha_3 = 23.264,\ \alpha_4 = 1.771 \quad (46)$$
$$t_1 = 24.50,\ t_2 = 44.89 \quad (47)$$

and controller

$$K_A(s) = \frac{n_K}{d_K} = \frac{29.16s^4 + 522.7s^3 + 1003s^2 + 389s + 1.159}{s^4 + 39.76s^3 + 538.6s^2 + 862.1s} \quad (48)$$

The *D-K* iteration for the interconnection in Fig. 6 yields the controller

$$K_{D-K}(s) = \frac{21.94s^4 + 210.3s^3 + 105.1s^2 + 1.203s + 0.003}{s^4 + 35.26s^3 + 248.3s^2 + 2.19s + 2\cdot 10^{-5}} \quad (49)$$

Both controllers satisfy condition (16) (see Fig. 16).

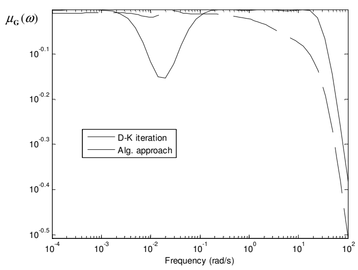

Fig. 16. Mu-plot for the *D-K* iteration (dashed) and algebraic approach (solid)

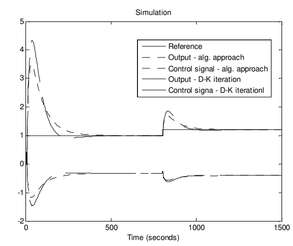

Fig. 18. Simulation for 1DOF structure ($\tau_1 = 4,\ \tau_2 = 0.8$)

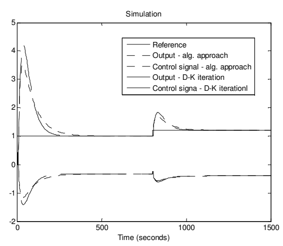

Fig. 17. Simulation for *D-K* iteration with $\mathbf{G}^*$ and 1DOF structure ($\tau_1 = 4,\ \tau_2 = 0.8$)

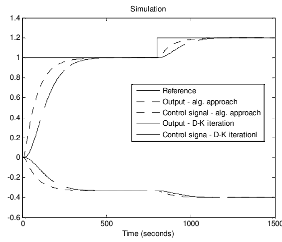

Fig. 22. Simulation for with 2DOF structure ($\tau_1 = 4$, $\tau_2 = 0.8$).

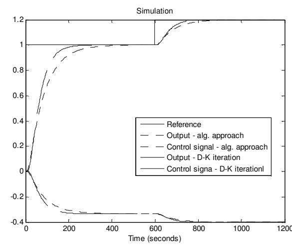

Fig. 21. Simulation for *D-K* iteration with $\mathbf{G}^*$ and 2DOF structure ($\tau_1 = 4,\ \tau_2 = 0.8$)

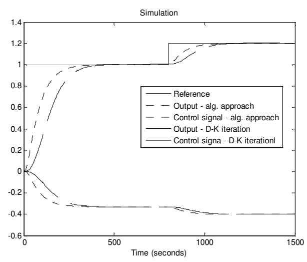

Fig. 19. Simulation for 2DOF structure ($\tau_1 = 2,\ \tau_2 = 0.4$)

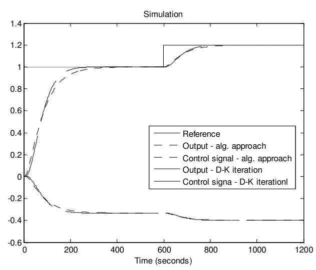

Fig. 20. Simulation for *D-K* iteration with $\mathbf{G}^*$ and 2DOF structure ($\tau_1 = 2,\ \tau_2 = 0.4$).

In order to overcome the problem of non-integration structure of the D-K iteration controller a scheme with integrator incorporating the integration property into the controller was used (see Fig. 8). The controller has the transfer function:

$$K^*_{D-K}(s) = \frac{23.38s^5 + 95.84s^4 + 138.3^3 + 68.59s^2 + 10.37s + 0.0326}{s^5 + 32.14s^4 + 108.7s^3 + 118.0s^2 + 24.41s} \quad (50)$$

Simulations have been performed for 1DOF and 2DOF feedback loop with real-plant *P*, i.e. with transport delays present in the simulation model. Two-degree-of-freedom controller for the D-K iteration has been obtained by putting n_R equal to the parameter with zero exponent of *s*, i.e., $n_R = 0.003$. The interconnection of 2DOF system is in Fig. 13. For details on 2DOF controllers in $\mathbf{R}_{PS}$ see [12].

Simulation for both controllers with 1DOF structure and stepwise reference signal is in Fig. 18. Simulation for 2DOF structure and the same reference signal is in Fig. 22. It is apparent that the *D-K* iteration has a non-zero steady-state error for both 1DOF and 2DOF interconnection, which is not the case of the algebraic approach. Set point tracking is faster for the algebraic approach with lower overshoot for 1DOF controller structure. The steady-state error is not present for the *D-K* iteration and generalized plant $\mathbf{G}^*$ with integrator cascade included (Fig. 17 and 21). The standard procedure yields faster tracking, however, the complexity of the controller is higher than for the algebraic approach and D-K iteration with no internal model.

The same simulations but with lower time delays are depicted in Fig. 19 and 20. It can be observed that the properties of feedback loop do not degrade if the time delays vary in the intervals of 0 to 4 s and 0 to 0.8 s for τ_1 and τ_2, respectively. For the 2DOF structure no overshoot is present, which is not true for 1DOF feedback loop.

VIII. Download

The Robust Control Toolbox for Time Delay Systems with Time Delay in Numerator and Denominator toolbox can be downloaded from:
http://web.fai.utb.cz/?id=0_5_2_8_2&lang=cs&type=0

IX. Conclusion

An application of the Robust Control Toolbox for Time Delay Systems with Time Delay in Numerator and Denominator to unstable time delay system with uncertain time delays in both numerator and denominator of the controlled plant has been presented. The simulation proved functionality of the algebraic approach and the method of treating uncertain time delays using linear fractional transformation and structured singular value even in the case of uncertain time delay in the denominator of the control plant.

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