Decentralized Controller Design, a Case Study

M. R. Hojjati, S. Akraminejad, H. quanbari

***Abstract*—** Large scale system are complex to be modeled and controlled by centralized controllers. In this paper, a multi variable non-linear system (two inverted pendulum coupled by a spring) is linearized about equilibrium point and formed to decentralized optimized control decentralized control law is designed basis on existing theorems and modern techniques and its stability is surveyed. This opens a possibility that based on this model to formulate some benchmark problems to simulate a wide research interest in large scale control.

***Keywords*—** Decentralized Control, Benchmark problems, Controller design, Stability

I. Introduction

It is generally recognized that, in an ever-increasingly inter-connected technological society, large-scale system control becomes more and more important [1]. Nonlinear large-scale systems are difficult to control due to various reasons, such as lack of centralized computing capability, system non-linearity, interconnection of subsystems, and system uncertainty [2]. Many interconnected systems found in real world such as industry manipulators and electric power systems are often composed of a set of subsystems. The centralized control for interconnected systems may be impractical due to a large amount of communications among the subsystems. A decentralized control system based only on local information is highly desirable. The fundamental uncertainties encountered in the decentralized controller design are the strength of the interactions among the subsystems [3].

Some of the difficulties associated with a centralized control scheme can be alleviated via a decentralized control structure in which information transfer between subsystems is avoided. An important problem in control is that of constructing decentralized control systems, where instead of a single controller connected to a physical system, one has multiple separate controllers each with access to different measured information and authority over different decision or actuation variables [4]. Traditionally, due to the practical limitations of available means of communications, research in large-scale system control is mainly under a theme of decentralized control [5]. Decentralized control is considered as an effective method to deal with large-scale interconnected systems [6]. Most of the early works in decentralized control of large scale systems are based on the assumptions that the interconnections are bounded by either constraints or first order polynomials in states. However, in practice, there do exist large scale systems where the interconnections among the subsystems are of high order. The high order interconnections can potentially destabilize an interconnected system if the decentralized controller does not explicitly account for these interconnections.

Also, decentralized control methods are appealing in coordination of multiple vehicles due to their demand for long-range communication and their robustness to single point failures [7]. Despite its importance and potentials, it seems that the impact of the research in this area is not as great as it could be. There are indeed many successful applications of large-scale system control, for example to electrical power systems [8]. However, these applications are mainly developed by domain experts. All applications in this area are "large-scale", i.e., the number of state variables is very big, and special knowledge is normally required for the formulation of the problem. In the general control communities, due to lack of simple yet meaningful examples, the interests in this area are not matched with its importance and potentials [6]. System of double pendulums coupled by a spring (Figure 1) was used in [9] to demonstrate some important theoretical results achieved in decentralized control. By simply adding more pendulums and springs to the existing system, this can be extended to a system of n-inverted-pendulums coupled by (n-1)-springs [9].

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II. Problem Formulation

A. Theoretical Background

Consider a system

$$S : \dot{x} = A_D x + B_D u + E_D k_D(t, C_D x, u) + f_C(t, x, u) \quad (1)$$

that is an interconnection of N subsystems

$$S_i : \dot{x}_i = A_i x_i + B_i u_i + E_i k_i(t, C_i x_i, u_i),$$
$$i = 1, 2, \dots, N \quad (2)$$

Where $x_i \in \mathcal{R}^{n_i}$ is the state and $u_i \in \mathcal{R}^{m_i}$ is the input of (2) at time $t \in R$, and A_i and B_i are constant matrixes of appropriate dimensions, which constitute stabilizable pairs A_i, B_i. In (1), $x = \left(x_1^T, x_2^T, \dots, x_N^T\right)^T$ is the state, $u = \left(u_1^T, u_2^T, \dots, u_N^T\right)^T$ is the input of the interconnected system and the system matrices are defined as

$$A_D = diag\{A_1, A_2 \dots, A_N\}, C_D = diag\{C_1, C_2, \dots, C_N\}$$
$$B_D = diag\{B_1, B_2, \dots, B_N\}, E_D = diag\{E_1, E_2, \dots, E_N\}$$

The $E_i k_i(t, C_i x_i, u_i)$ represents the structured uncertainty in (2), where E_i and C_i are $n_i \times q_i$ and $p_i \times m_i$ constant matrices, and $k_i: R \times R^{p_i} \times R^{m_i} \rightarrow R^{q_i}$, which is a vector component of $k_D = \left(k_1^T, k_2^T, \dots, k_N^T\right)^T$, satisfies the inequality

$$\|k_i(t, C_i x_i, u_i)\| \leq \|C_i x_i\| \qquad (3)$$

Finally, the function $k_D: R \times R^p \times R^m \rightarrow R^q$ and $f_C: R \times R^n \times R^m \rightarrow R^n$ are sufficiently smooth so that the solutions of (1) exist and are unique for all initial conditions and all fixed inputs $u(.)$.Furthermore, $k_D(t, 0,0) \equiv 0$, $f_C(t, 0,0) \equiv 0$ is assumed to be unique equilibrium of S when $u(t) \equiv 0$.

Assuming that a stabilizing control law for (2) exists, it can be computed as

$$u_i = -R_i^{-1} B_i^T P_i x_i \qquad (4)$$

Where P_i is the positive definite solution of the Ricatti equation

$$A_i^T P_i + P_i A_i - P_i B_i R_i^{-1} B_i^T P_i + \mu_i P_i E_i E_i^T P_i + \mu_i^{-1} C_i^T C_i + Q_i = 0 \qquad (5)$$

Q_i and R_i are positive definite matrices with proper dimensions, and μ_i is an appropriate positive number. Then, the decentralized control law is

$$u_D = -R_D^{-1} B_D^T P_D x \qquad (6)$$

Where

$$\begin{cases} P_D = diag\{P_1, P_2, \dots, P_N\} \\ R_D = diag\{R_1, R_2, \dots, R_N\} \end{cases}$$

For the above, the following Theorem is obtained in [3].

Theorem 1. If there exists positive numbers $d_i, i = 1,2, \dots, N$, such that the inequality

$$x^T \overline{Q}_D x - 2x^T \overline{P}_D f_c(t, x, u) + u^T \overline{R}_D u \geq \overline{\alpha}' x^T x + \overline{\beta}' u^T u \qquad (7)$$

holds for some positive numbers $\overline{\alpha}'$and $\overline{\beta}'$, where

$$\begin{cases} \overline{Q}_D = diag\{d_1 Q_1, d_2 Q_2, \dots, d_N Q_N\} \\ \overline{R}_D = diag\{d_1 R_1, d_2 R_2, \dots, d_N R_N\} \end{cases}$$

then the decentralized control u_D of (6) stabilizes the system of (1).

Furthermore, if the interconnection term in (1) has the following property

$$f_C(t, x, u) = B_D g_C(t, x, u) + h_C(t, x) \qquad (8)$$

Where

$$g_C = \left(g_1^T, g_2^T, \dots, g_N^T\right)^T, g_i: R \times R^n \times R^m \rightarrow R^{m_i}$$

,$h_C = \left(h_1^T, h_2^T, \dots, h_N^T\right)^T$, $h_i: R \times R^n \rightarrow R^{n_i}$ and g_i satisfies the inequality

$$\|g_i(t, x, u)\| \leq \acute{\zeta}_i \|x\| + \acute{\eta}_i \|u\| \text{ for all } (t, x, u) \in R \times R^n \times R^m \qquad (9)$$

For some positive numbers $\acute{\zeta}_i$ and $\acute{\eta}_i < \lambda_m^{1/2}(R_i)/\lambda_M^{1/2}(R_i)$, then the decentralized control law of (6) can be changed to

$$u_D^{\rho} = -\rho_D R_D^{-1} B_D^T P_D x \qquad (10)$$

Where $\rho_D = diag\{\rho_1 I_{m_1}, \rho_2 I_{m_2}, \dots, \rho_N I_{m_N}\}$, and $\rho_i > 1, i = 1, 2, \dots, N$.

Theorem 2. If there exists positive numbers $d_i, i = 1,2, \dots, N$, and a positive number v' such that

$$x^T \overline{Q}_D x - 2x^T \overline{P}_D h_C(t, x) \geq v' x^T x, \text{ for all } (t, x) \in R \times R^n \qquad (11)$$

Then the decentralized control u_D^{ρ} of (10) stabilizes the system of (1) [10].

B. Modeling

A system of two inverted pendulum coupled by a spring is shown in figure1. The variables of

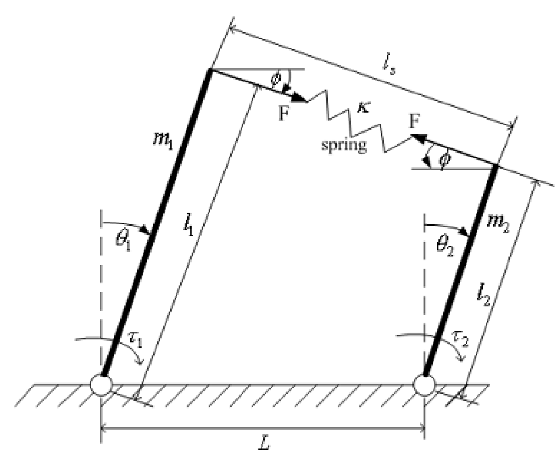

Fig 1. Two inverted pendulum coupled by spring

the system are:

θ_i: angular displace ment of pendulum I (i=1, 2)

τ_i: torque input generated by the actuator for pendulum I (i=1, 2)

F: spring force

ϕ: angular of the spring to the earth

and the constants are:

m_i: mass of pendulum

L: distance of two pendulums

κ: spring constant

The mass of each pendulum is uniformly distributed. The length of spring is chosen so that F=0 when $\theta_1 = \theta_2 = 0$,

which implies that $\left(\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2\right)^T = 0$ is an equilibrium of the system if $\tau_i = 0$. For simplicity, we assume that the mass of spring is zero.
The dynamic equations for the system of fig.1 are given as

$$[m_1(l_1)^2/3]\ddot{\theta}_1 = \tau_1 + m_1 g(l_1/2)\sin\theta_1 + l_1 F\cos(\theta_1 - \phi) \tag{12}$$

$$[m_1(l_2)^2/3]\ddot{\theta}_2 = \tau_2 + m_2 g(l_2/2)\sin\theta_2 + l_2 F\cos(\theta_2 - \phi) \tag{13}$$

where $g = 9.8\mathrm{m/s^2}$ is the constant of gravity and

$$F = \kappa\left(l_s - [L^2 + (l_2 - l_1)^2]^{1/2}\right) \tag{14}$$

$$l_s = [(L + l_2\sin\theta_2 - l_1\sin\theta_1)^2 + (l_2\cos\theta_2 - l_1\cos\theta_1)^2]^{1/2} \tag{15}$$

$$\phi = \tan^{-1}\left\{{l_1\cos\theta_1 - l_2\cos\theta_2}/{L + l_1\cos\theta_1 - l_2\cos\theta_2}\right\} \tag{16}$$

The following variables are used: $l_1 = 1\mathrm{m}$ ، $l_2 = 0.8\mathrm{m}$ ، $m_1 = 1\mathrm{kg}$ ، $m_2 = 0.8\mathrm{kg}$ ، $L = 1.2\mathrm{m}$ و $\kappa = 0.04\mathrm{N/m}$ [11].

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C. Controller design

The state variables are defined as

$$X = (x_1\quad x_2\quad x_3\quad x_4) = \left(\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2\right) \tag{17}$$

The obtained dynamic equations using the state variables are

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{3}{2l_1}g & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \frac{3}{2l_2}g & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ \frac{3}{m_1 l_1^2} & 0\\ 0 & 0\\ 0 & \frac{3}{m_2 l_2^2}\end{bmatrix}\begin{bmatrix}t_1\\ t_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{3}{2l_1}g(\sin q_1 - q_1)\\ 0\\ \frac{3}{2l_2}g(\sin q_2 - q_2)\end{bmatrix} + \begin{bmatrix}0\\ \frac{3}{m_1 l_1}F\cos(q_1 - f)\\ 0\\ -\frac{3}{m_2 l_2}F\cos(q_2 - f)\end{bmatrix} \tag{18}$$

Note that nonlinear function in each subsystem S_i satisfies the following condition:

$$|g(\sin\theta_i - \theta_i)| \le 0.45|\theta_i|, \qquad |\theta_i| \le \pi/6 \tag{19}$$

Considering the equations, $\frac{3}{2l_i}g(\sin\theta_i - \theta_i)$ can be written in form of $E_i k_i(C_i x_i)$ using the following relations:

$$E_i = \begin{bmatrix}0\\ \frac{3*0.45}{2l_i}\end{bmatrix},\ C_i = [1\quad 0],\ k_i(\theta_i) = g/0.45(\sin\theta_i - \theta_i) \tag{20}$$

The matricesA_i,B_i, Q_i and R_i are defined in the same way:

$$A_i = \begin{bmatrix}0 & 1\\ \frac{3}{2l_i}g & 0\end{bmatrix},\ B_i = \begin{bmatrix}0\\ \frac{3}{m_i l_i^2}\end{bmatrix},\ Q_i = \begin{bmatrix}m_i g & 0\\ 0 & m_i\end{bmatrix},\quad R_i = 1, \tag{21}$$

So the positive response of following Riccati equation

$$A_i^T P_i + P_i A_i - P_i B_i R_i^{-1} B_i^T P_i + \mu_i P_i E_i E_i^T P_i + \mu_i^{-1} C_i^T C_i + Q_i = 0, \tag{22}$$

By considering $\mu_i = 1$ is:

$$P_1 = \begin{bmatrix}17.5675 & 3.7756\\ 3.7756 & 1.0004\end{bmatrix}, P_2 = \begin{bmatrix}8.0031 & 1.2960\\ 1.2960 & 0.3176\end{bmatrix} \tag{23}$$

And finally, using the equation (10), decentralized control law will be

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}11.3269 & 3.0012 & 0 & 0\\ 0 & 0 & 7.5936 & 1.8611\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{24}$$

The nonlinear term which shows the interconnection relations is as follows:

$$f_C(x_1, x_3) = \begin{bmatrix}0\\ \frac{3}{m_1 l_1}F\cos(\theta_1 - \phi)\\ 0\\ -\frac{3}{m_2 l_2}F\cos(\theta_2 - \phi)\end{bmatrix} \tag{25}$$

In which

$$F = \kappa\left([3.08 - 2.4\sin\theta_1 + 1.92\sin\theta_2 - 1.6\cos(\theta_1 - \theta_2)]^{1/2} - 1.217\right) \tag{26}$$

and

$$\phi = \tan^{-1}\frac{\cos\theta_1 - 0.8\cos\theta_2}{1.2 + 0.8\sin\theta_2 - \sin\theta_1} \tag{27}$$

Using these equations, we can write

$$|F| \le \kappa(1.49|\theta_1| + 0.18|\theta_2|) \tag{28}$$

Consider that that the nonlinear term is independent from input and so the stated condition in (7) of theorem (1) is reduced to

$$x^T \overline{Q}_D x - 2x^T \overline{P}_D f_c(t, x) \ge \overline{\alpha}' x^T x \tag{29}$$

By defining $d_1 = d_2 = 1$, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}^T = \begin{bmatrix} m_1 g & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 g & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} - 2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{3}{m_1 l_1} F \cos(q_1 - f) \\ 0 \\ -\frac{3}{m_2 l_2} F \cos(q_2 - f) \end{bmatrix}$$
$$\geq \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}^T \begin{bmatrix} m_1 g & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 g & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} - 2 \begin{bmatrix} |x_1| \\ |x_2| \\ |x_3| \\ |x_4| \end{bmatrix}^T \begin{bmatrix} 11.327|F| \\ 3.001|F| \\ 6.075|F| \\ 1.489|F| \end{bmatrix} \qquad (30)$$

From inequality (30), we obtain the following results

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}^T W \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq \Pi_1 x_1^2 + \Pi_2 x_2^2 + \Pi_3 x_3^2 + \Pi_4 x_4^2 \qquad (31)$$

that

$$W = \begin{bmatrix} m_1 g - 33.754k & -4.471k & -11.091k & -2.219k \\ -4.471k & m_1 & -0.54k & 0 \\ -11.091k & -0.54k & m_2 g - 2.188k & -0.268k \\ -2.219k & 0 & -0.268k & m_2 \end{bmatrix} \qquad (32)$$

$$\Pi_1 = m_1 g - 33.754k - 4.471ke_1^{-1} - 11.091ke_2^{-1} - 2.219ke_5^{-1}$$
$$\Pi_2 = m_1 - 4.471ke_1^{-1} - 0.54ke_3$$
$$\Pi_3 = m_2 g - 2.188k - 11.091ke_2 - 0.54ke_3^{-1} - 0.268ke_3$$
$$\Pi_4 = m_2 - 0.268ke_4^{-1} - 2.219ke_5$$
$$e_i > 0 \quad \forall i = 1,2,\ldots,5$$

In terms of theorem (1), the following optimized problem can be stated to determine $\kappa_{s\,max}$

$$\begin{cases} \text{Min } \frac{1}{\kappa_{s\,max}} \\ \text{subject to: } \Pi_1 > 0, \Pi_2 > 0, \Pi_3 > 0, \Pi_4 > 0,\ \varepsilon_i > 0 \end{cases} \qquad (33)$$

For example, for $\varepsilon_1 = 0.9, \varepsilon_2 = 2.7, \varepsilon_3 = 0.3, \varepsilon_4 = 0.42, \varepsilon_5 = 1.3$ the value $\kappa_{s\,max} = 0.22$ is obtained. Also,

$$\begin{bmatrix} |x_1| \\ |x_2| \\ |x_3| \\ |x_4| \end{bmatrix}^T \begin{bmatrix} 0.002 & 0 & 0 & 0 \\ 0 & 0.0791 & 0 & 0 \\ 0 & 0 & 0.3498 & 0 \\ 0 & 0 & 0 & 0.025 \end{bmatrix} \begin{bmatrix} |x_1| \\ |x_2| \\ |x_3| \\ |x_4| \end{bmatrix} \geq 0.002\left(x_1^2 + x_2^2 + x_2^2 + x_2^2\right) \qquad (34)$$

That is true for $|x_1| \leq \pi/6$. So (32) is satisfied. Thus, the feedback of decentralized state (24) causes the system stabilization. While$\kappa > \kappa_{s\,max}$, the condition (32) is not established for each $d_1, d_2 > 0$. So, the decentralized control (6) can not be used for system (8) and high gains as feedback must be abstained fortunately, the nonlinear function $f_C(x_1, x_3)$ satisfies the following condition:

$$f_C(x_1, x_3) = \begin{bmatrix} 0 & 0 \\ \frac{3}{m_1 l_1} & 0 \\ 0 & 0 \\ 0 & \frac{3}{m_2 l_2} \end{bmatrix} \begin{bmatrix} F\cos(x_1 - \phi) \\ F\cos(x_3 - \phi) \end{bmatrix} \qquad (35)$$

Note that the nonlinear function in right-side has a limited gain consideringx_1, x_3. Then, condition (11) in theorem (2) is established for $h_C(t, x) = 0$ and decentralized control is selected as follows by considering high values for ρ_1, ρ_2:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = -\begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix} \Pi \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad (36)$$

In which

$$\Pi = \begin{bmatrix} 11.3269 & 3.0012 & 0 & 0 \\ 0 & 0 & 7.5936 & 1.8611 \end{bmatrix}.$$

Regarding theorem (2) and

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}^T \begin{bmatrix} m_1 g & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 g & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \geq 0.8\left(x_1^2 + x_2^2 + x_2^2 + x_2^2\right) \quad (37)$$

Decentralized control law (6) stabilizes system (1).

III. Problem Solution

The MATLAB/Simulink model and programs were provided for the simulation of system. By changing the values of parameters in the "data file", various simulation results can b obtained. The obtained results of system simulation in stead of regulation without disturbance, step disturbance of 0.1 Nm and step disturbance of 0.2 Nm is shown in figure 2-7.
Also, the results are compared with results by Yang [9]. The results are shown in figure 8 to 10. Figure 8 shows the response to non-zero condition. Figure 9 shows response to a step disturbance of 0.1N.m and finally, figure 10 response to a step disturbance of 2.0 N.m

figure 2 shows the responses of system in non-zero initial condition. these responses include angles of pendulums (θ_1 and θ_2). figure 3 shows the angular velocities of pendulums 1 and 2 with non-zero initial condition. figure 4 shows the system responses including angles θ_1 and θ_2 to step disturbance of 0.2 Nm. figure 5 shows system responses including angular velocities of two pendulums to a step disturbance of 0.2 Nm, figure 6 shows the system responses including Angles θ_1 and θ_2 of two pendulums to a step disturbance of 0.2 Nm. figure 7 shows the system responses including angular velocities of two pendulums in state of step disturbance of 0.2 Nm.
figure 8-10 shows the system response in method proposed by Yang. Figure 8 shows the system responses to non-zero condition. Figure 9 shows system response to a step disturbance of 0.1Nm. Figure 10shows system response to a step disturbance of 2.0 Nm.
The written programs in MATLAB are prepared in Appendix. These programs include these values for parameters:

$l_1 = 1\text{m}$ ،

$l_2 = 0.8\text{m}$ ،

$m_1 = 1\text{kg}$ ،

$m_2 = 0.8\text{kg}$،

$L = 1.2\text{m}$

$\kappa = 0.04\text{N/m}$

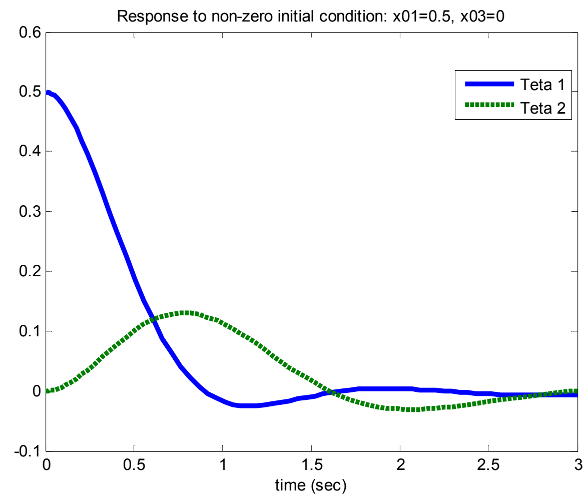

Fig 2. Angles θ_1 and θ_2 in non-zero condition

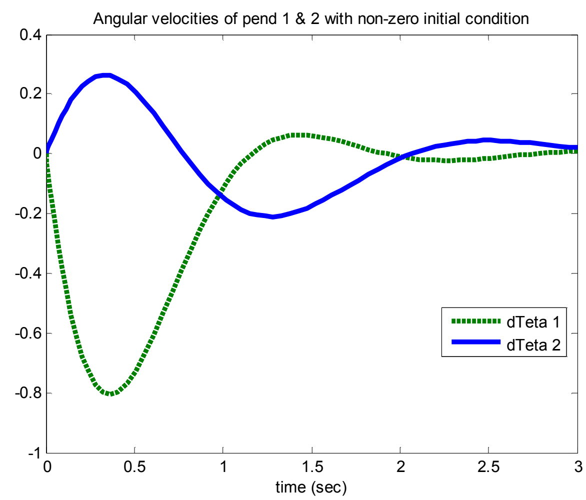

Fig 3. Angular velocities of two pendulums in non-zero condition

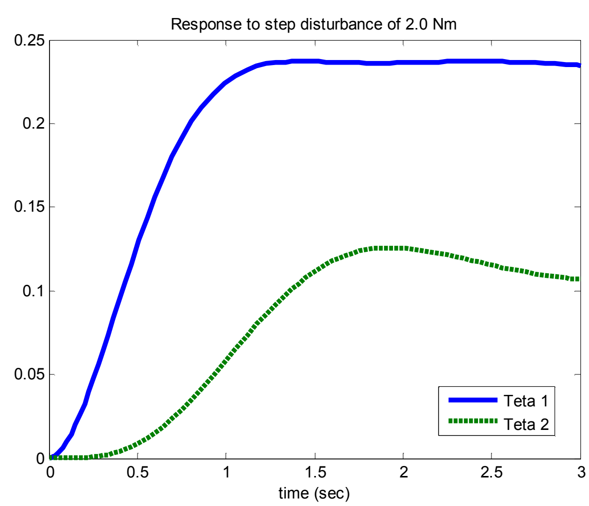

Fig 4. Angles $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$ in response to a step disturbance of 0.2 Nm

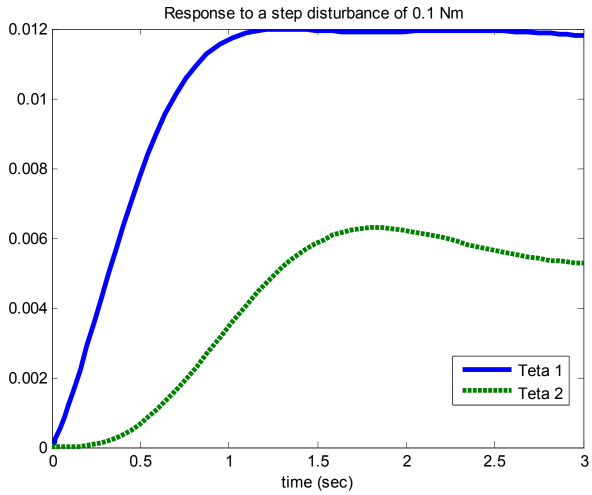

Fig 5. Angular velocities of two pendulums in response to a step disturbance of 0.2 Nm

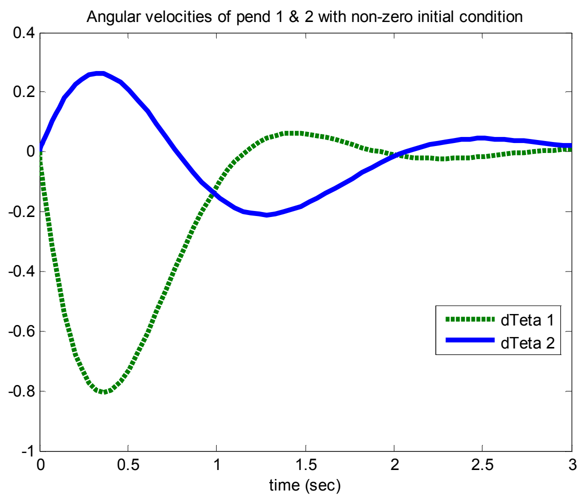

Fig 6. Angles θ_1 and θ_2 of two pendulums in response to a step disturbance of 0.2 Nm

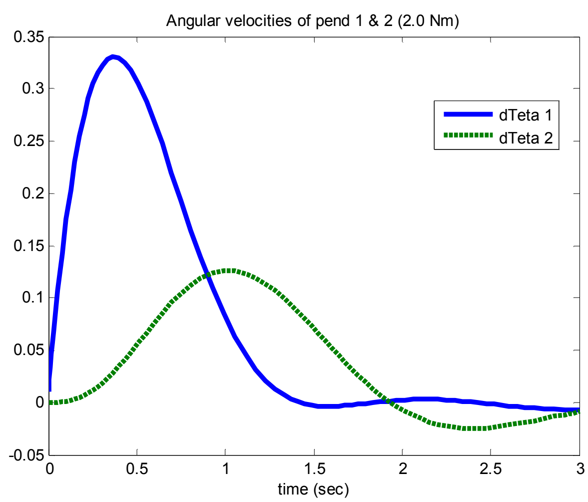

Fig 7. Angular velocities of two pendulums in state of step disturbance of 0.2 Nm

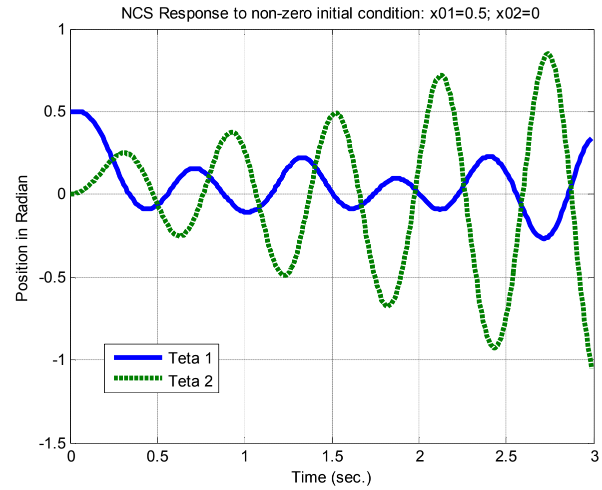

Fig 8. response to non-zero condition

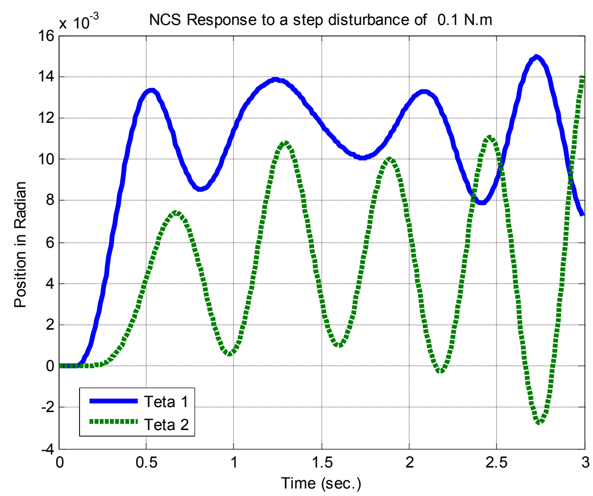

Fig 9. response to a step disturbance of 0.1N.m

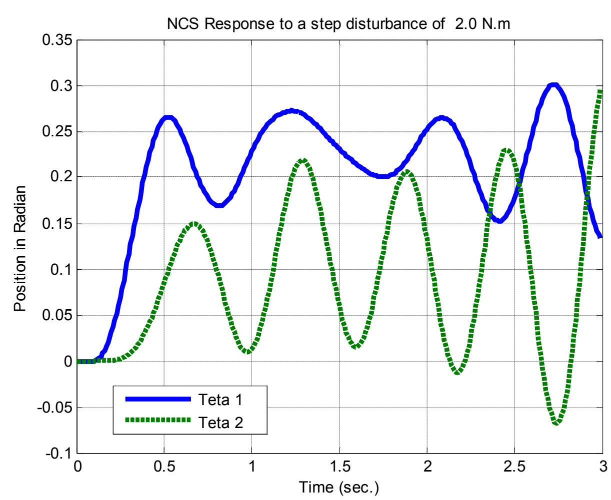

Fig 10. response to a step disturbance of 2.0 N.m

IV. CONCLUSION

In this paper, we have studied a model and redesigned a controller for a challenging task. Based on a model of multi-inverted pendulums coupled by multi-springs and other details, we plan to formulate a benchmark problem for the study of decentralized control and control beyond the limitation of decentralized control. By replacing point-to-point connections in a traditional control structure with networked communications, the example system can also be served as a benchmark problem for "networked scale systems" [11], which can be considered as a new structure for large scale system control. Using the Decentralized Control on this model led to stabilization of system.

In method proposed by Yang, the responses have less stability than results of model proposed in this paper due to using the sensors in simulation. it is related to the selected limitation with length of 0.6 in Yang's method with less secure margin while the obtained results in this paper have been achieved with higher stability.

Also, it seems that the results by Yang's method is becoming unstable by time increasing that is because of increasing the domains of response variations which are the angles of pendulums, here that means exiting from the desirable limitation with length of 0.6 that causes finally to instability. but these variations are not seen in responses obtained by the presented method in this paper.

other methods used to simulate the large scale systems are adaptive feedback linearizing decentralized controller architecture that is applicable to the control of nonlinear dynamical systems with a bound on the interconnections. to approximate on-line the inversion error, the single hidden layer neural network can be used. adaptive update laws can be derived from Lyapunov analysis. A robust adaptive signal is required in the analysis to shield the feedback linearizing control law from the interconnection effects [12].

another method for simulating the multivariable systems is applying decentralized adaptive control of nonlinear systems using radial basis neural networks. the results of using this method is that the dynamics for each subsystem are not required to be linear in a set of unknown coefficients due to the functional approximation capabilities of radial basis neural networks [13].

these two above methods are proposed adaptive algorithms that can be used in simulation to stabilize an interconnected double inverted pendulum which can be generalized to utilized in modeling the large scale systems.

the proposed theoretically guaranteed method for large scale interconnected uncertain non linear mechanical systems led to results that can verify simulation studies.

APPEMDIX

```
function dx = coupled_pendulums(t,x)
dx = zeros(4,1);    % a column vector
l1=1;    % lenth
m1=1;    % mass

% pendulum 2
l2=0.8;  % lenth
m2=0.8 ; % mass

%L is the distance of two penduli
L=1.2;
kappa = 4.0;

%g is the gravity constant
g=9.81;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ls=((L+l2*sin(x(3))-l1*sin(x(1)))^2+(l2*cos(x(3))-
l1*cos(x(1)))^2)^0.5;
F=kappa*(ls-[L^2+(l2-l1)^2]^0.5);
phi=atan((-l2*cos(x(3))+l1*cos(x(1)))/(L+l2*sin(x(3))-
l1*sin(x(1))));
K=[10.957  3.485  0   0
 0  0   7.500   2.375];
%K = [11.327  3.001  0   0 ;0  0   7.594   1.861];
u=-K*x+[.1;0];
dx(1) = x(2) ;
dx(2) = u(1)+(m1*g*(l1/2))*sin(x(1))+(l1)*F*cos(x(1)-
phi);
dx(3) = x(4);
dx(4)= u(2)+(m2*g*(l2/2))*sin(x(3))-(l2)*F*cos(x(3)-
phi);
```

```
clc
a=[0 1;1.5*9.8 0];
b=[0;3];Q=[9.8 0;0 1];
c=[1 0];
E=[0;1.5*.45];
B=b*b'-E*E';
QQ=Q+c'*c;
[P1] = care(a,b,QQ)

%-----------------------------
a=[0 1;1.5*9.8/.8 0];
b=[0;3/(.8)^3];Q=[0.8*9.8 0;0 .8];
c=[1 0];
E=[0;1.5*.45/0.8];
R=1;
B=b*b'-E*E';
QQ=Q+c'*c;
[P2] = care(a,b,QQ)
```

```
clc
a=[0 1;1.5*9.8 0];
b=[0;3];Q=[9.8 0;0 1];
c=[1 0];
E=[0;1.5*.45];
B=b*b'-E*E';
QQ=Q+c'*c;
[P1] = care(a,b,QQ)

%------------------------------
a=[0 1;1.5*9.8/.8 0];
b=[0;3/(.8)^3];Q=[0.8*9.8 0;0 .8];
c=[1 0];
E=[0;1.5*.45/0.8];
R=1;
B=b*b'-E*E';
QQ=Q+c'*c;
[P2] = care(a,b,QQ)
```

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