Fast Periodic Auto and Cross Correlation Algorithms for an Orthogonal Set of Real-Valued Perfect Sequences from the Huffman Sequence

Takahiro MATSUMOTO, Hideyuki TORII and Shinya MATSUFUJI

***Abstract*—** A perfect sequence has the optimal periodic autocorrelation function where all out-of-phase values are zero, and an orthogonal set has orthogonality that the periodic correlation function for any pair of distinct sequences in the set takes zero at the zero shift. The real-valued perfect sequences of period $N = 2^n$ are derived from a real-valued Huffman sequence of length $2^\nu + 1$ with $\nu \geq n$ whose out-of-phase aperiodic autocorrelation function takes zero except at the left and right shift-ends. This paper proposes fast periodic auto- and cross-correlation algorithms for an orthogonal set of real-valued perfect sequences of period 2^n. As a result, the number of multiplications and additions can be suppressed on the order $N \log_2 N$.

***Keywords*—** Finite-length sequence, Perfect sequence, Huffman sequence, Orthogonal set, Fast correlation algorithm.

I. Introduction

THE perfect sequence (also called the periodic orthogonal sequence) [1] has the optimal periodic autocorrelation function where all out-of-phase values are zero, and an orthogonal set has orthogonality that the periodic correlation function for any pair of distinct sequences in the set takes zero at the zero shift. The orthogonal set of perfect sequences is useful in various systems[1], such as synchronous code division multiple access (CDMA) systems, pulse compression radars and digital watermarkings[2].

A binary perfect sequence consisting of elements 1 and -1 is only $(1, 1, 1, -1)$ of period 4. On the other hand, any real-valued perfect sequence consisting of elements of real numbers is generated by its general solution [3]. In addition, the real-valued perfect sequence of period 2^n is derived from a real-valued Huffman sequence (also called shift-orthogonal finite-length sequence) [4], [5] of length $2^\nu + 1$ with positive integers n and ν, and $\nu \geq n$ whose out-of-phase aperiodic auto correlation function takes zero except at the left and right shift-ends [6], [7], [8]. Previously, a fast periodic auto correlation algorithm for a perfect sequence of period 2^n derived from a real-valued Huffman sequence of length $2^n + 1$ has been proposed [6].

Manuscript received October 30, 2011.

T. Matsumoto is assistant professor with Graduate School of Science and Engineering, Yamaguchi University, Ube, Yamaguchi, Japan e-mail: matugen@yamaguchi-u.ac.jp.

H. Torii is associate professor with Department of Information Network and Communication, Kanagawa Institute of Technology, Atsugi-shi, Kanagawa, Japan e-mail:torii@nw.kanagawa-it.ac.jp.

S. Matsufuji is associate professor with Graduate School of Science and Engineering, Yamaguchi University, Ube, Yamaguchi, Japan e-mail: s-matsu@yamaguchi-u.ac.jp.

In this paper, we propose fast periodic auto- and cross-correlation algorithms [9] for an orthogonal set of real-valued perfect sequences of period 2^n derived from the Huffman sequence of length $2^\nu + 1$, $\nu \geq n$ [7], [8]. By this principle, it is also easy to give fast periodic auto- and cross- correlation algorithm for a ZCZ set [10], [11] of perfect sequences of period 2^n[8].

II. The Huffman Sequence

A. Definition of the Huffman sequence

Let $\hat{a}_M^\ell$ be a real-valued finite-length sequence of length M consisting of real-elements , written as

$$\begin{aligned}\hat{a}_M^\ell &= \{\hat{a}_{M,i}^\ell\} = (\hat{a}_{M,0}^\ell, \cdots, \hat{a}_{M,i}^\ell, \cdots, \hat{a}_{M,M-1}^\ell), \\ &\quad \hat{a}_{M,i}^\ell \in R,\end{aligned}$$

where $\hat{a}_{M,i}^\ell = 0, i < 0, i > M-1$, i denotes the order variable, ℓ the sequence number and R the set of real numbers. Let $\hat{A}$ be a set of real-valued finite-length sequences $\hat{a}_M^\ell$, written as

$$\begin{aligned}\hat{A} &= \{\hat{a}_M^0, \cdots, \hat{a}_M^\ell, \cdots, \hat{a}_M^{\hat{L}-1}\} \\ &= \{\{\hat{a}_{M,i}^0\}, \cdots, \{\hat{a}_{M,i}^\ell\}, \cdots, \{\hat{a}_{M,i}^{\hat{L}-1}\}\},\end{aligned}$$

where $\hat{L}$ denotes the number of sequences in a sequence set, and is called the family size.

The aperiodic correlation function between sequences $\hat{a}_M^\ell$ and $\hat{a}_M^{\ell'}$ at shift i' is defined by

$$\begin{aligned}\hat{\rho}_{M,i'}^{\ell,\ell'} &= \frac{1}{M}\sum_{i=0}^{M-1} \hat{a}_{M,i}^\ell \hat{a}_{M,i-i'}^{\ell'} \\ &= \frac{1}{M}\left(\hat{a}_{M,i'}^\ell \otimes \hat{a}_{M,-i'}^{\ell'}\right), \qquad (1)\end{aligned}$$

where $\otimes$ denotes the convolution and $\hat{\rho}_{M,i'}^{\ell,\ell'} = 0$ for $|i'| > M-1$. If the aperiodic autocorrelation function satisfies

$$\hat{\rho}_{M,i'}^{\ell,\ell} = \begin{cases} 1 & ; i' = 0, \\ 0 & ; i' \neq 0, \pm(M-1), \\ \varepsilon_{M-1} & ; i' = \pm(M-1), \end{cases} \qquad (2)$$

where ε_{M-1} is called a shift-end value and $|\varepsilon_{M-1}| \leq 1/2$, the sequence $\hat{a}_M^\ell$ is called a real-valued Huffman sequence [4] or a real-valued shift-orthogonal finite-length sequence [5].

B. The Huffman sequence of short length

Let $\hat{a}^{\ell}_{M}$ be a real-valued Huffman sequence with a negative shift-end value $\varepsilon_{M-1} < 0$ and length M. Let $\hat{b}^{\ell}_{M}$ be a real-valued Huffman sequence with a positive shift-end value $\varepsilon'_{M-1} > 0$. It is easy to give the Huffman sequence of short length from definition of the sequence. From Eqs. (1) and (2), the Huffman sequence $\hat{a}^{0}_{2}$ of length 2 and a shift-end value $\varepsilon_1 < 0$ is solved as

$$\left.\begin{array}{rcl} \hat{a}^{0}_{2,0} &=& (\sqrt{1+2\varepsilon_1}+\sqrt{1-2\varepsilon_1})/\sqrt{2} \\ \hat{a}^{0}_{2,1} &=& (\sqrt{1+2\varepsilon_1}-\sqrt{1-2\varepsilon_1})/\sqrt{2} \end{array}\right\}. \quad (3)$$

From Eqs. (1) and (2), the Huffman sequence $\hat{b}^{0}_{2}$ of length 2 and a shift-end value $\varepsilon'_1 > 0$ is solved as

$$\left.\begin{array}{rcl} \hat{b}^{0}_{2,0} &=& (\sqrt{1+2\varepsilon'_1}+\sqrt{1-2\varepsilon'_1})/\sqrt{2} \\ \hat{b}^{0}_{2,1} &=& (\sqrt{1+2\varepsilon'_1}-\sqrt{1-2\varepsilon'_1})/\sqrt{2} \end{array}\right\}. \quad (4)$$

Similarly, from Eqs. (1) and (2), the Huffman sequence $\hat{b}^{0}_{5}$ of length 5 and a shift-end value $\varepsilon'_4 > 0$ is solved as

$$\left.\begin{array}{rcl} \hat{b}^{0}_{5,0} &=& \hat{b}^{0}_{5,4} = \sqrt{5\varepsilon'_4} \\ \hat{b}^{0}_{5,1} &=& -\hat{b}^{0}_{5,3} \\ &=& \sqrt{5\{2\sqrt{\varepsilon'_4(1+2\varepsilon'_4)}-4\varepsilon'_4\}} \\ \hat{b}^{0}_{5,2} &=& \sqrt{5}(\sqrt{1+2\varepsilon'_4}-2\sqrt{\varepsilon'_4}) \end{array}\right\}. \quad (5)$$

The sequence $\hat{b}^{0}_{m}$ of length $m \geq 3$ and a shift-end value $\varepsilon'_{m-1} > 0$ is obtained by insertion of zero values between neighboring values of the sequence $\hat{b}^{0}_{2}$ of length 2 and multiplication by the constant for normalization as follows.

$$\hat{b}^{0}_{m} = \sqrt{\frac{m}{2}}\Big(\hat{b}^{0}_{2,0}, \underbrace{0,0,\cdots,0}_{m-2}, \hat{b}^{0}_{2,1}\Big). \quad (6)$$

Note that a shift-end value ε'_{m-1} of the sequence $\hat{b}^{0}_{m}$ of length $m \geq 3$ is equal to one ε'_1 of the sequence $\hat{b}^{0}_{2}$ of length 2. In addition, the sequences $\hat{b}^{0}_{m}$ of length $m \geq 9$ and a shift-end value $\varepsilon'_{m-1} > 0$ are obtained by insertion of zero values between neighboring values of the sequence $\hat{b}^{0}_{5}$ of length 5 and multiplication by the constant for normalization as follows.

$$\begin{array}{rcl} \hat{b}^{0}_{m} &=& \sqrt{\frac{m}{5}}\Big(\hat{b}^{0}_{5,0}, \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}^{0}_{5,1}, \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \\ && \hat{b}^{0}_{5,2}, \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}^{0}_{5,3}, \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}^{0}_{5,4}\Big). \end{array} \quad (7)$$

Note that a shift-end value ε'_{m-1} of the sequence $\hat{b}^{0}_{m}$ of length $m \geq 9$ is equal to one ε'_4 of the sequence $\hat{b}^{0}_{5}$ of length 5.

Sequences $\hat{a}^{1}_{2}$ and $\hat{b}^{1}_{m}$, $m \geq 2$ with a sequence number 1 can be replaced by reversed sequences

$$\begin{array}{rcl} \hat{a}^{1}_{2} &=& (\hat{a}^{0}_{2,1}, \hat{a}^{0}_{2,0}), \\ \hat{b}^{1}_{m} &=& (\hat{b}^{0}_{m,m-1}, \cdots, \hat{b}^{0}_{m,i}, \cdots, \hat{b}^{0}_{m,0}). \end{array}$$

C. The Huffman sequence of length $2^{\nu}+1$ derived from the sequence of short length

It is difficult to give the Huffman sequence of long length from definition of the sequence. The real-valued Huffman sequence $\hat{a}^{\ell}_{M}$ of length $M = 2^{\nu}+1, \nu = 1, 2, \cdots$ and a shift-end value $\varepsilon_{M-1} < 0$ can be given by the ν-multiple convolution of a real-valued Huffman sequence $\hat{a}^{\ell_2}_{2}$ of length 2 and a shift-end value $\varepsilon_1 < 0$ and the sequences $\hat{b}^{\ell'_2}_{2}$, $\hat{b}^{\ell'_3}_{3}$, $\cdots$, $\hat{b}^{\ell'_{\frac{M+1}{2}}}_{\frac{M+1}{2}}$ of length 2, 3, $\cdots$, $\frac{M+1}{2}(= 2^{\nu-1}+1)$ and shift-end values $\varepsilon'_1, \varepsilon'_2, \cdots, \varepsilon'_{\frac{M-1}{2}} > 0$ as follows [5].

$$\begin{array}{rcl} \hat{a}^{\ell}_{M,i} &=& K\big(\hat{a}^{\ell_2}_{2,i} \otimes \hat{b}^{\ell'_2}_{2,i} \otimes \hat{b}^{\ell'_3}_{3,i} \otimes \cdots \\ && \otimes \hat{b}^{\ell'_m}_{m,i} \otimes \cdots \otimes \hat{b}^{\ell'_{\frac{M+1}{2}}}_{\frac{M+1}{2},i}\big), \end{array} \quad (8)$$

$$\begin{array}{rcl} K &=& K_3 K_5 \cdots K_{2m-1} \cdots K_M, \\ K_{2m-1} &=& \dfrac{\sqrt{2m-1}}{m\sqrt{1-2\varepsilon^2_{m-1}}}, \end{array} \quad (9)$$

where $m = 2^{\mu}+1$, $\mu = 0, 1, \cdots, \nu-1$, $\otimes$ denotes the convolution as

$$\hat{a}^{\ell}_{m,i} \otimes \hat{b}^{\ell'}_{n,i} = \sum_{k=0}^{m-1} \hat{a}^{\ell}_{m,k} \hat{b}^{\ell'}_{n,i-k}$$

and ℓ_2, ℓ'_2, ℓ'_3, $\cdots$, $\ell'_{\frac{M+1}{2}}$ denote the sequence numbers. The sequences $\hat{a}^{\ell_2}_{2}$ and $\hat{b}^{\ell'_m}_{m}$, $m = 2^{\mu}+1$, $\mu = 0, 1, \cdots, \frac{M+1}{2}$ are called elementary sequences. From a shift-end value ε_{M-1} of the Huffman sequence $\hat{a}^{\ell}_{M}$ of length M, shift-end values $\varepsilon'_{\frac{M-1}{2}}$, $\cdots$, ε'_2, ε'_1, ε_1 of elementary sequences $\hat{b}^{\ell'_{\frac{M+1}{2}}}_{\frac{M+1}{2}}$, $\cdots$, $\hat{b}^{\ell'_3}_{3}$, $\hat{b}^{\ell'_2}_{2}$ and $\hat{a}^{\ell_2}_{2}$ are solved as

$$\left\{\begin{array}{rcl} \varepsilon_{\frac{m-1}{2}} &=& -\sqrt{\frac{-\varepsilon_{m-1}}{1-2\varepsilon_{m-1}}} < 0, \\ \varepsilon'_{\frac{m-1}{2}} &=& -\varepsilon_{\frac{m-1}{2}} > 0, \\ m &=& M, \frac{M+1}{2}, \cdots, 5, 3. \end{array}\right. \quad (10)$$

These elementary sequences are derived from the Huffman sequence of length 2 shown in Eqs. (3), (4) and (6). or length 5 shown in Eqs. (5) and (7).

A set of the Huffman sequences $\hat{a}^{\ell}_{M}$ is constructed by combinations of original elementary sequences with a sequence number 0 and reversed elementary sequences with a sequence number 1. A sequence number ℓ of the Huffman sequence $\hat{a}^{\ell}_{M}$ is expressed in a binary notation as

$$\begin{array}{rcl} \ell &=& (\ell'_{\frac{M+1}{2}}, \cdots, \ell'_3, \ell'_2, \ell_2)_2 \\ &=& \ell_2 + 2\sum_{k=0}^{\nu-1} 2^k \ell'_{2^k+1}. \end{array}$$

As an example, we generate a set of real-valued Huffman sequences $\hat{a}^{\ell}_{9}$ of length $M = 9$ and a shift-end value $\varepsilon_8 = -1/(M-1) = -0.125$. From Eq. (8), the sequence $\hat{a}^{\ell}_{9}$ is synthesized by multiple convolutions of elementary sequences $\hat{a}^{\ell_2}_{2}$, $\hat{b}^{\ell'_2}_{2}$, $\hat{b}^{\ell'_3}_{3}$ and $\hat{b}^{\ell'_5}_{5}$. From Eq. (9), the constants for normalization are $K_3 = 1.18752$, $K_5 = 0.95232$ and $K_9 = 0.67082$, where these values are rounded off to five decimal places.

From Eq. (10), shift-end values of elementary sequences $\hat{b}_5^{\ell'_5}$, $\hat{b}_3^{\ell'_3}$, $\hat{b}_2^{\ell'_2}$ and $\hat{a}_2^{\ell_2}$ are $\varepsilon'_4 = 0.31623$, $\varepsilon'_2 = 0.44013$ and $\varepsilon'_1 = -\varepsilon_1 = 0.48382$, respectively, where these values are rounded off to five decimal places.

Elementary sequences are given by the sequence of only length 2, or length 2 and 5. First, let's give elementary sequences derived from the sequence of only length 2. Elementary sequences $\hat{a}_2^0$ and $\hat{b}_2^0$ are obtained from Eqs. (3) and (4), respectively. Similarly, elementary sequences $\hat{b}_3^0$ and $\hat{b}_5^0$ are obtained from Eqs. (4) and (6). Table I shows 4 elementary sequences $\hat{a}_2^0$, $\hat{b}_2^0$, $\hat{b}_3^0$ and $\hat{b}_5^0$ derived from the sequence of only length 2, and a set of 8 sequences with $M = 9$ and $\varepsilon_8 = -1/8$ combined by the original and reversed elementary sequences. Note that this set does not contain the sequence of $\ell'_2 = \ell_2$ whose element takes zero values. Next, let's give elementary sequences derived from the sequence of length 2 and 5. Elementary sequences $\hat{a}_2^0$, $\hat{b}_2^0$ and $\hat{b}_3^0$ are derived from the sequence of length 2. Elementary sequence $\hat{b}_5^0$ is obtained from Eqs. (5). Table II shows 4 elementary sequences $\hat{a}_2^0$, $\hat{b}_2^0$, $\hat{b}_3^0$ and $\hat{b}_5^0$ derived from the sequence of length 2 and 5, and a set of 16 sequences with $M = 9$ and $\varepsilon_8 = -1/8$ combined by the original and reversed elementary sequences.

III. A Perfect Sequence

A. Definition of a perfect sequence

Let $a_N^{j,s}$ be a real-valued periodic sequence of period N consisting of real elements, written as

$$a_N^{j,s} = \{a_{N,i}^{j,s}\} = (a_{N,0}^{j,s}, \cdots, a_{N,i}^{j,s}, \cdots, a_{N,N-1}^{j,s}), \quad a_{N,i}^{j,s} \in R,$$

where $a_{N,i+kN}^{j,s} = a_{N,i}^{j,s}$ with an integer k, i denotes the order variable, j the sequence number, s the set number and R the set of real numbers. Let A^s be a set of real-valued periodic sequences $a_N^{j,s}$, written as

$$A^s = \{a_N^{0,s}, \cdots, a_N^{j,s}, \cdots, a_N^{L-1,s}\},$$

where L is the number of sequences in a sequence set, and is called family size.

A periodic correlation function between sequences $a_N^{j,s}$ and $a_N^{j',s}$ at a shift i' is defined by

$$\rho_{N,i'}^{j,j',s} = \frac{1}{N}\sum_{i=0}^{N-1} a_{N,i}^{j,s} a_{N,(i-i') \bmod N}^{j',s} = \frac{1}{N}\Big(a_{N,i'}^{j,s} \otimes a_{N,-i'}^{j',s}\Big), \quad (11)$$

where $\otimes$ denotes the convolution and $\rho_{N,i'+kN}^{j,j',s} = \rho_{N,i'}^{j,j',s}$ for any integer k. Equation (11) means the autocorrelation function for $j = j'$ and the cross one for $j \neq j'$. If the periodic autocorrelation function satisfies

$$\rho_{N,i'}^{j,j,s} = \begin{cases} 1 & ; i' = 0, \\ 0 & ; i' \neq 0, \end{cases}$$

the sequence $a_N^{j,s}$ is called a real-valued perfect sequence [1] or a real-valued periodic orthogonal sequence [3]. In addition, if the periodic correlation function satisfies

$$\rho_{N,i'}^{j,j',s} = \begin{cases} 1 & ; i' = 0, j = j', \\ 0 & ; i' \neq 0, j = j', \\ 0 & ; i' = 0, j \neq j, \end{cases} \quad (12)$$

a set A^s of sequences $a_N^{j,s}$ is called an orthogonal set of real-valued perfect sequences.

B. Construction of an Orthogonal set of perfect sequences

An orthogonal set of real-valued perfect sequences $a_N^{j,s}$ of period $N = 2^n$ is derived from the Huffman sequences $\hat{a}_M^\ell$ of length $M = 2^\nu + 1$ with $\nu \geq n$. A perfect sequence $a_N^{j,s}$ of period N in an orthogonal set is expressed as

$$\begin{aligned} a_{N,i}^{j,s} &= \sqrt{\frac{N}{M(1+2\varepsilon_{M-1})}}\Big\{\hat{a}_{M,i}^\ell \otimes \Delta_{N,i-j}\Big\} \\ &= K\Big\{\Delta_{N,i-j} \otimes \hat{a}_{2,i}^{\ell_2} \otimes \hat{b}_{2,i}^{\ell'_2} \otimes \hat{b}_{3,i}^{\ell'_3} \otimes \cdots \\ &\quad \otimes \hat{b}_{m,i}^{\ell'_m} \otimes \cdots \otimes \hat{b}_{\frac{M+1}{2},i}^{\ell'_{\frac{M+1}{2}}}\Big\} \\ &= K\Big\{\Delta_{N,i-j} \otimes \alpha_{2,i}^{\ell_2} \otimes \beta_{2,i}^{\ell'_2} \otimes \beta_{3,i}^{\ell'_3} \otimes \cdots \\ &\quad \otimes \beta_{m,i}^{\ell'_m} \otimes \cdots \otimes \beta_{\frac{M+1}{2},i}^{\ell'_{\frac{M+1}{2}}}\Big\}, \\ &= K\Big\{\alpha_{2,i-j}^{\ell_2} \otimes \beta_{2,i-j}^{\ell'_2} \otimes \beta_{3,i-j}^{\ell'_3} \otimes \cdots \\ &\quad \otimes \beta_{m,i-j}^{\ell'_m} \otimes \cdots \otimes \beta_{\frac{M+1}{2},i-j}^{\ell'_{\frac{M+1}{2}}}\Big\}, \end{aligned} \quad (13)$$

$$\Delta_{N,i} = \sum_{k=-\infty}^{\infty} \delta_{i,kN} = \begin{cases} 1 & ; i = kN \\ 0 & ; i \neq kN \end{cases},$$

$$K = \sqrt{\frac{N}{2(1+2\varepsilon_{M-1})\prod_{\mu=0}^{\nu-1}(2^\mu+1)(1-2\varepsilon_{2^\mu}^2)}},$$

where $\Delta_{N,i}$ denotes the impulse sequence of period N, $\delta_{i,kN}$ the Kronecker delta, and $\alpha_2^{\ell_2}$ and $\beta_m^{\ell'_m}$ are periodic sequences of period N derived from the Huffman sequences $\hat{a}_2^{\ell_2}$ and $\hat{b}_m^{\ell'_m}$, respectively, and $\alpha_{2,i+kN}^{\ell_2} = \alpha_{2,i}^{\ell_2}$, $\beta_{m,i+kN}^{\ell'_m} = \beta_{m,i}^{\ell'_m}$ for any integer k. Note that the shift-end value ε_{M-1} of the Huffman sequence $\hat{a}_M^\ell$ should be chosen a value to reduce the absolute value of perfect sequences $a_N^{j,s}$ in the set. Periodic sequence $\alpha_2^{\ell_2}$ is expressed as

$$\{\alpha_{2,i}^{\ell_2}\} = \{\hat{a}_{2,i}^{\ell_2} \otimes \Delta_{N,i}\} = \Big(\hat{a}_{2,0}^{\ell_2}, \hat{a}_{2,1}^{\ell_2}, \underbrace{0, \cdots, 0}_{N-2}\Big).$$

On the other hand, periodic sequence $\beta_m^{\ell'_m}$ derived from the Huffman sequence of length 2 is expressed as

$$\{\beta_{m,i}^{\ell'_m}\} = \{\hat{b}_{m,i}^{\ell'_m} \otimes \Delta_{N,i}\} = \begin{cases} \Big(\hat{b}_{m,0}^{\ell'_m}, \underbrace{0, \cdots, 0}_{m-2}, \hat{b}_{m,m-1}^{\ell'_m}, \underbrace{0, \cdots, 0}_{N-m}\Big) & ; m \leq N, \\ \Big(\hat{b}_{m,0}^{\ell'_m} + \hat{b}_{m,m-1}^{\ell'_m}, \underbrace{0, \cdots, 0}_{N-1}\Big) & ; m > N. \end{cases}$$

TABLE I

ELEMENTARY SEQUENCES DERIVED FROM THE HUFFMAN SEQUENCE OF ONLY LENGTH 2, AND A SET OF 8 REAL-VALUED HUFFMAN SEQUENCES $\hat{a}_9^{\ell}$ OF LENGTH $M = 9$ AND A SHIFT-END VALUE $\varepsilon_{M-1} = -0.125$.

i	$\ell'_5\ell'_3\ell'_2\ell_2$	0	1	2	3	4	5	6	7	8
$\hat{a}_2^0$		1.11909	-0.86466							
$\hat{b}_2^0$		1.11909	0.86466							
$\hat{b}_3^0$		1.48719	0.00000	0.88784						
$\hat{b}_5^0$		2.10630	0.00000	0.00000	0.00000	0.75067				
$\hat{a}_9^1$	0001	-2.29947	1.19940	0.92672	0.71603	0.55324	0.42746	0.33028	0.25519	0.48924
$\hat{a}_9^2$	0010	2.29947	1.19940	-0.92672	0.71603	-0.55324	0.42746	-0.33028	0.25519	-0.48924
$\hat{a}_9^5$	0101	-1.37276	0.71603	-0.92672	1.19940	1.81023	0.25519	-0.33028	0.42746	0.81952
$\hat{a}_9^6$	0110	1.37276	0.71603	0.92672	1.19940	-1.81023	0.25519	0.33028	0.42746	-0.81952
$\hat{a}_9^9$	1001	-0.81952	0.42746	0.33028	0.25519	-1.81023	1.19940	0.92672	0.71603	1.37276
$\hat{a}_9^{10}$	1010	0.81952	0.42746	-0.33028	0.25519	1.81023	1.19940	-0.92672	0.71603	-1.37276
$\hat{a}_9^{13}$	1101	-0.48924	0.25519	-0.33028	0.42746	-0.55324	0.71603	-0.92672	1.19940	2.29947
$\hat{a}_9^{14}$	1110	0.48924	0.25519	0.33028	0.42746	0.55324	0.71603	0.92672	1.19940	-2.29947

Similarly, periodic sequence $\beta_m^{\ell'_m}$ derived from the Huffman sequence of length 5 is expressed as

$$\{\beta_{m,i}^{\ell'_m}\} = \begin{cases} \Big(\hat{b}_{m,0}^{\ell'_m}, \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,\frac{m-1}{4}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,\frac{m-1}{2}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,\frac{3(m-1)}{4}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,m-1}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{N-m}\Big) & ; m < N+1, \\ \Big(2\hat{b}_{m,0}^{\ell'_m}, \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,\frac{m-1}{4}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,\frac{m-1}{2}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{\frac{m-5}{4}}, \hat{b}_{m,\frac{3(m-1)}{4}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{\frac{m-5}{4}}\Big) & ; m = N+1, \\ \Big(2\hat{b}_{m,0}^{\ell'_m} + \hat{b}_{m,\frac{m-1}{2}}^{\ell'_m}, \\ \underbrace{0,\cdots,0}_{N-1}\Big) & ; m > N+1, \end{cases}$$

from Eq. (5).

A sequence number j and a family number s of the orthogonal set of perfect sequences derived from the Huffman sequence of only length 2 are expressed by

$$\begin{aligned} j &= (\ell'_{\frac{M+1}{2}}, \cdots, \ell'_m, \cdots, \ell'_3)_2 \\ &= \sum_{k=0}^{\nu-2} 2^k \ell'_{2^{k+1}+1}, \\ s &= \ell'_2, \end{aligned}$$

respectively, where $\ell'_2 \neq \ell_2$. Therefore, in the case of the orthogonal set derived from the Huffman sequence of only length 2, we can construct two orthogonal sets A^0 and A^1 of perfect sequences of period $N = M-1$ or $(M-1)/2$, family size $L = (M-1)/2$ and set number $s = \ell'_2 = 0$ and 1 with $\ell'_2 \neq \ell_2$. This orthogonal set with $N = (M-1)/2$ can reach the upper bound $L = N$ on family size [1].

Similarly, a sequence number j and a family number s of the orthogonal set of perfect sequences derived from the Huffman sequence of length 2 and 5 are expressed by

$$\begin{aligned} j &= (\ell'_{\frac{M+1}{2}}, \cdots, \ell'_m, \cdots, \ell'_9)_2 \\ &= \sum_{k=0}^{\nu-4} 2^k \ell'_{2^{k+3}+1}, \\ s &= (\ell'_5, \ell'_3, \ell'_2)_2 \\ &= \sum_{k=0}^{2} 2^k \ell'_{2^k+1}, \end{aligned}$$

respectively, where $\ell'_2 \neq \ell_2$. Note that a set with $\ell'_2 = \ell_2$ contains the sequence whose element takes zero values. Therefore, in the case of the orthogonal set derived from the Huffman sequence of length 2 and 5, we can construct 8 orthogonal sets A^0, A^1, $\cdots$, A^7 of perfect sequences of period $N = M-1$ or $(M-1)/2$ or $(M-1)/4$ or $(M-1)/8$, family size $L = (M-1)/8$ and set number $s = 0, 1, \cdots, 7$. This orthogonal set with $N = (M-1)/8$ can reach the upper bound $L = N$ on family size [1].

As an example of orthogonal sets, let us construct orthogonal sets of perfect sequences $a_8^{j,s}$ of period $N = 8$ derived from the Huffman sequences $\hat{a}_{17}^{\ell}$ of length $M = 17$. From Eq. (13), a perfect sequence $a_8^{j,s}$ is generated by convolution of the Huffman sequence $\hat{a}_{17}^{\ell}$ of length 17 and the impulse sequence $\Delta_{8,i-j}$ of period 8. In the case of the orthogonal set derived from the Huffman sequence of only length 2, we can construct two orthogonal sets A^0 and A^1 of perfect sequences of period 8 and family size $L = 8$ derived from the Huffman sequence of length 17 and the shift-end value $\varepsilon_{16} = -0.00073$. Table III shows two orthogonal sets A^0 and A^1 of 8 perfect sequences $a_8^{j,s}$ of period $N = 8$ derived from the Huffman sequence of only length 2. Similarly, in the case of the orthogonal set derived from the Huffman sequence of length 2 and 5, we can construct 8 orthogonal sets A^0, A^1, $\cdots$, A^7 of perfect

TABLE II

ELEMENTARY SEQUENCES DERIVED FROM THE HUFFMAN SEQUENCE OF LENGTH 2 AND 5, AND A SET OF 16 REAL-VALUED HUFFMAN SEQUENCES $\hat{a}_9^\ell$ OF LENGTH $M = 9$ AND A SHIFT-END VALUE $\varepsilon_{M-1} = -0.125$.

i	$\ell'_5\ell'_3\ell'_2\ell_2$	0	1	2	3	4	5	6	7	8
$\hat{a}_2^0$		1.11909	-0.86466							
$\hat{b}_2^0$		1.11909	0.86466							
$\hat{b}_3^0$		1.48719	0.00000	0.88784						
$\hat{b}_5^0$		1.25743	-0.92755	0.34210	0.92755	1.25743				
$\hat{a}_9^0$	0000	1.77669	-1.31058	0.48337	1.31058	1.14349	0.46708	-0.17227	-0.46708	-0.63320
$\hat{a}_9^1$	0001	-1.37276	1.72864	-0.34842	-0.79845	-0.18986	0.63590	1.09152	1.03198	0.81952
$\hat{a}_9^2$	0010	1.37276	-0.29659	-0.70794	2.04298	0.61558	1.02875	-0.46088	-0.17706	-0.81952
$\hat{a}_9^3$	0011	-1.06066	0.78240	0.85492	-1.62589	0.31110	0.06110	1.43206	0.78240	1.06066
$\hat{a}_9^4$	0100	1.06066	-0.78240	1.43206	-0.06110	0.31110	1.62589	0.85492	-0.78240	-1.06066
$\hat{a}_9^5$	0101	-0.81952	1.03198	-1.09152	0.63590	0.18986	-0.79845	0.34842	1.72864	1.37276
$\hat{a}_9^6$	0110	0.81952	-0.17706	0.46088	1.02875	-0.61558	2.04298	0.70794	-0.29659	-1.37276
$\hat{a}_9^7$	0111	-0.63320	0.46708	-0.17227	-0.46708	1.14349	-1.31058	0.48337	1.31058	1.77669
$\hat{a}_9^8$	1000	1.77669	-1.31058	0.48337	1.31058	1.14349	0.46708	-0.17227	-0.46708	-0.63320
$\hat{a}_9^9$	1001	-1.37276	1.72864	-0.34842	-0.79845	-0.18986	0.63590	1.09152	1.03198	0.81952
$\hat{a}_9^{10}$	1010	1.37276	-0.29659	-0.70794	2.04298	0.61558	1.02875	-0.46088	-0.17706	-0.81952
$\hat{a}_9^{11}$	1011	-1.06066	0.78240	0.85492	-1.62589	0.31110	0.06110	1.43206	0.78240	1.06066
$\hat{a}_9^{12}$	1100	1.06066	-0.78240	1.43206	-0.06110	0.31110	1.62589	0.85492	-0.78240	-1.06066
$\hat{a}_9^{13}$	1101	-0.81952	1.03198	-1.09152	0.63590	0.18986	-0.79845	0.34842	1.72864	1.37276
$\hat{a}_9^{14}$	1110	0.81952	-0.17706	0.46088	1.02875	-0.61558	2.04298	0.70794	-0.29659	-1.37276
$\hat{a}_9^{15}$	1111	-0.63320	0.46708	-0.17227	-0.46708	1.14349	-1.31058	0.48337	1.31058	1.77669

TABLE III

TWO ORTHOGONAL SETS A^0 AND A^1 OF 8 PERFECT SEQUENCES $a_8^{j,s}$ OF PERIOD $N = 8$ DERIVED FROM THE HUFFMAN SEQUENCES $\hat{a}_{17}^\ell$ OF LENGTH $M = 17$ AND THE SHIFT-END VALUE $\varepsilon_{16} = -0.00073$, WHERE THE HUFFMAN SEQUENCE $\hat{a}_{17}^\ell$ IS DERIVED FROM THE HUFFMAN SEQUENCE OF ONLY LENGTH 2.

	i	0	1	2	3	4	5	6	7
A^0	$a_8^{0,0}$	-1.72761	1.72840	1.10053	0.70074	0.44619	0.28410	0.18090	0.11518
	$a_8^{1,0}$	0.28410	-0.44619	0.70074	-1.10053	1.72840	1.72761	0.11518	-0.18090
	$a_8^{2,0}$	1.10053	0.70074	0.44619	0.28410	0.18090	0.11518	-1.72761	1.72840
	$a_8^{3,0}$	0.70074	-1.10053	1.72840	1.72761	0.11518	-0.18090	0.28410	-0.44619
	$a_8^{4,0}$	0.44619	0.28410	0.18090	0.11518	-1.72761	1.72840	1.10053	0.70074
	$a_8^{5,0}$	1.72840	1.72761	0.11518	-0.18090	0.28410	-0.44619	0.70074	-1.10053
	$a_8^{6,0}$	0.18090	0.11518	-1.72761	1.72840	1.10053	0.70074	0.44619	0.28410
	$a_8^{7,0}$	0.11518	-0.18090	0.28410	-0.44619	0.70074	-1.10053	1.72840	1.72761
A^1	$a_8^{0,1}$	1.72761	1.72840	-1.10053	0.70074	-0.44619	0.28410	-0.18090	0.11518
	$a_8^{1,1}$	0.28410	0.44619	0.70074	1.10053	1.72840	-1.72761	0.11518	0.18090
	$a_8^{2,1}$	-1.10053	0.70074	-0.44619	0.28410	-0.18090	0.11518	1.72761	1.72840
	$a_8^{3,1}$	0.70074	1.10053	1.72840	-1.72761	0.11518	0.18090	0.28410	0.44619
	$a_8^{4,1}$	-0.44619	0.28410	-0.18090	0.11518	1.72761	1.72840	-1.10053	0.70074
	$a_8^{5,1}$	1.72840	-1.72761	0.11518	0.18090	0.28410	0.44619	0.70074	1.10053
	$a_8^{6,1}$	-0.18090	0.11518	1.72761	1.72840	-1.10053	0.70074	-0.44619	0.28410
	$a_8^{7,1}$	0.11518	0.18090	0.28410	0.44619	0.70074	1.10053	1.72840	-1.72761

sequences of period 8 and family size $L = 2$ derived from the Huffman sequence of length 17. Table IV shows 8 orthogonal sets A^0, A^1, $\cdots$, A^7 of two perfect sequences $a_8^{j,s}$ of period $N = 8$ derived from the Huffman sequence of length 2 and 5.

IV. FAST CORRELATION ALGORITHM FOR AN ORTHOGONAL SET OF PERFECT SEQUENCES

A. Fast autocorrelation algorithm

Let d be an input sequence of length N consisting of real-elements, written as

$$d = \{d_i\} = (d_0, \cdots, d_i, \cdots, d_{N-1}), d_i \in R,$$

where $d_i = 0$, $i < 0$, $i > N - 1$. From Eq. (11), the periodic autocorrelation function between a perfect sequence $a_N^{j,s}$ and an input sequence d is given by

$$x_{i'} = \frac{1}{N} \sum_{i=0}^{N-1} d_i a^{j,s}_{N,(i-i') \bmod N}, \qquad (14)$$

where $x = \{x_{i'}\} = (x_0, \cdots, x_{i'}, \cdots, x_{N-1})$, $x_{i'} \in R$. In addition, Eq. (14) is replaced by a determinant of matrix as

$$\mathbf{X} = \frac{1}{N} \mathbf{A}_N^{j,s} \cdot \mathbf{D}, \qquad (15)$$

TABLE IV
8 ORTHOGONAL SETS $A^0, A^1, \cdots, A^7$ OF TWO PERFECT SEQUENCES $a_8^{j,s}$ OF PERIOD $N=8$ DERIVED FROM THE HUFFMAN SEQUENCES $\hat{a}_{17}^{\ell}$ OF LENGTH $M=17$, WHERE THE HUFFMAN SEQUENCE $\hat{a}_{17}^{\ell}$ IS DERIVED FROM THE HUFFMAN SEQUENCE OF LENGTH 2 AND 5.

	ε_{16}	i	0	1	2	3	4	5	6	7
A^0	$a_8^{0,0}$	-0.00045	-1.68595	1.78690	1.10389	0.68195	0.42129	0.26026	0.16078	0.09932
	$a_8^{1,0}$		-0.17357	-0.60564	0.43265	-0.52169	0.95484	-0.65903	1.61451	1.78636
A^1	$a_8^{0,1}$	-0.00045	1.68595	1.78690	-1.10389	0.68195	-0.42129	0.26026	-0.16078	0.09932
	$a_8^{1,1}$		-0.17357	0.60564	0.43265	0.52169	0.95484	0.65903	1.61451	-1.78636
A^2	$a_8^{0,2}$	-0.00073	-0.44619	0.70074	-1.10053	1.72840	1.72761	0.11518	-0.18090	0.28410
	$a_8^{1,2}$		0.99291	1.46081	-0.37116	-1.48938	1.01959	-0.17938	1.18709	0.20795
A^3	$a_8^{0,3}$	-0.00073	0.44619	0.70074	1.10053	1.72840	-1.72761	0.11518	0.18090	0.28410
	$a_8^{1,3}$		0.99291	-1.46081	-0.37116	1.48938	1.01959	0.17938	1.18709	-0.20795
A^4	$a_8^{0,4}$	-0.19999	-0.69044	-0.31542	0.75232	1.92378	-0.70145	1.47937	0.63958	-0.25931
	$a_8^{1,4}$		0.36233	-0.72040	-0.75068	0.74704	1.30214	-0.67149	1.91464	0.64486
A^5	$a_8^{0,5}$	-0.00010	0.34739	1.49141	1.46607	-0.36231	-0.89260	1.22770	-0.92086	0.47163
	$a_8^{1,5}$		-0.30226	-1.59827	1.61225	0.04595	0.41158	1.05306	1.10686	0.49926
A^6	$a_8^{0,6}$	-0.02941	0.62828	-0.20878	-0.53553	1.20070	0.67964	2.16089	-0.77240	-0.32439
	$a_8^{1,6}$		1.20073	0.53552	-0.20876	-0.62828	-0.32442	0.77240	2.16088	-0.67964
A^7	$a_8^{0,7}$	-0.00005	-0.31583	0.41060	0.85720	1.23056	1.00972	-0.22866	-1.55109	1.41593
	$a_8^{1,7}$		1.02309	-0.10988	-0.05277	-1.64331	1.62340	0.80377	0.23470	0.94942

where $\mathbf{X} = [x_0, \cdots, x_{i'}, \cdots, x_{N-1}]^T$, $\mathbf{D} = [d_0, \cdots, d_i, \cdots, d_{N-1}]^T$, $\mathbf{X}^T$ denotes a transposed matrix of $\mathbf{X}$, and

$$\begin{aligned}\mathbf{A}_N^{j,s} &= \Big[\{a_{N,i}^{j,s}\}, \{a_{N,i}^{j,s} \otimes \Delta_{N,i-1}\}, \cdots, \\ &\quad \{a_{N,i}^{j,s} \otimes \Delta_{N,i-(N-1)}\}\Big]^T \\ &= \begin{bmatrix} a_{N,0}^{j,s}, & a_{N,1}^{j,s}, & \cdots & a_{N,N-1}^{j,s} \\ a_{N,N-1}^{j,s}, & a_{N,0}^{j,s}, & \cdots & a_{N,N-2}^{j,s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N,1}^{j,s}, & a_{N,2}^{j,s}, & \cdots & a_{N,0}^{j,s} \end{bmatrix}.\end{aligned}$$

Therefore, the number of multiplications N_{mul} and additions N_{add} are given by N^2+N and $N(N-1)$, respectively.

From Eq. (13), Eq. (15) is factorized as

$$\begin{aligned}\mathbf{X} &= \frac{K}{N}\Big(\hat{\mathbf{B}}_{\frac{M+1}{2}}^{\ell'_{\frac{M+1}{2}}} \cdot \mathbf{R}^j\Big) \cdots \Big(\hat{\mathbf{B}}_m^{\ell'_m} \cdot \mathbf{R}^j\Big) \cdots \Big(\hat{\mathbf{B}}_3^{\ell'_3} \cdot \mathbf{R}^j\Big) \\ &\quad \cdot \Big(\hat{\mathbf{B}}_2^{\ell'_2} \cdot \mathbf{R}^j\Big) \cdot \Big(\hat{\mathbf{A}}_2^{\ell_2} \cdot \mathbf{R}^j\Big) \cdot \mathbf{D} \\ &= \frac{K}{N}\mathbf{R}^j \cdot \hat{\mathbf{B}}_{\frac{M+1}{2}}^{\ell'_{\frac{M+1}{2}}} \cdots \hat{\mathbf{B}}_m^{\ell'_m} \cdots \hat{\mathbf{B}}_3^{\ell'_3} \cdot \hat{\mathbf{B}}_2^{\ell'_2} \cdot \hat{\mathbf{A}}_2^{\ell_2} \cdot \mathbf{D},\end{aligned} \quad (16)$$

where

$$\begin{aligned}\hat{\mathbf{A}}_2^{\ell_2} &= \Big[\{\alpha_{2,i}^{\ell_2}\}, \{\alpha_{2,i}^{\ell_2} \otimes \Delta_{N,i-1}\}, \cdots, \\ &\quad \{\alpha_{2,i}^{\ell_2} \otimes \Delta_{N,i-(N-1)}\}\Big]^T \\ &= \begin{bmatrix} \hat{a}_{2,0}^{\ell_2}, & \hat{a}_{2,1}^{\ell_2}, & \cdots & 0 \\ 0, & \hat{a}_{2,0}^{\ell_2}, & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_{2,1}^{\ell_2}, & 0, & \cdots, & \hat{a}_{2,0}^{\ell_2} \end{bmatrix}, \\ \hat{\mathbf{B}}_m^{\ell'_m} &= \Big[\{\beta_{m,i}^{\ell'_m}\}, \{\beta_{m,i}^{\ell'_m} \otimes \Delta_{N,i-1}\}, \cdots, \\ &\quad \{\beta_{m,i}^{\ell'_m} \otimes \Delta_{N,i-(N-1)}\}\Big]^T, \\ \mathbf{R}^j &= \Big[\{\Delta_{N,i-j}\}, \{\Delta_{N,i-j-1}\} \cdots, \{\Delta_{N,i-j-(N-2)}\}, \\ &\quad \{\Delta_{N,i-j-(N-1)}\}\Big]^T.\end{aligned}$$

Note that a matrix $\mathbf{R}^0$ of size $N \times N$ is a unit matrix. Therefore, in the case of the orthogonal set derived from the Huffman sequence of only length 2, the number of multiplications N_{mul} and additions N_{add} are given by

$$\begin{aligned}N_{mul} &= 2N(\log_2 N + 1) + N \\ &= 2N\log_2 N + 3N, \end{aligned} \quad (17)$$

$$\begin{aligned}N_{add} &= N(\log_2 N + 1) \\ &= N\log_2 N + N. \end{aligned} \quad (18)$$

Similarly, in the case of the orthogonal set derived from the Huffman sequence of length 2 and 5, the number of multiplications N_{mul} and additions N_{add} are given by

$$N_{mul} = \begin{cases} 5N\log_2 N - 3N & ; N = M-1, \\ 5N\log_2 N + N & ; N < M-1, \end{cases} \quad (19)$$

$$N_{add} = \begin{cases} 4N\log_2 N - 5N & ; N = M-1, \\ 4N\log_2 N - 2N & ; N < M-1. \end{cases} \quad (20)$$

Figure 1 and 2 show the number of multiplications N_{mul} and additions N_{add} to calculate autocorrelation function of a perfect sequence $a_N^{j,s}$, respectively.

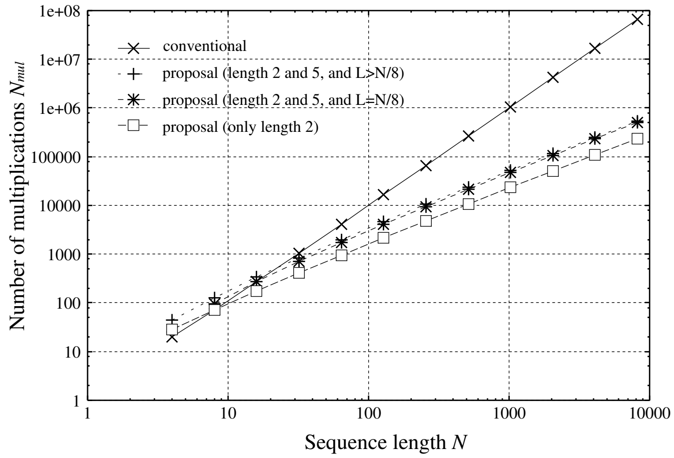

Fig. 1. The number of multiplications N_{mul} to calculate autocorrelation function of a perfect sequence $a_N^{j,s}$.

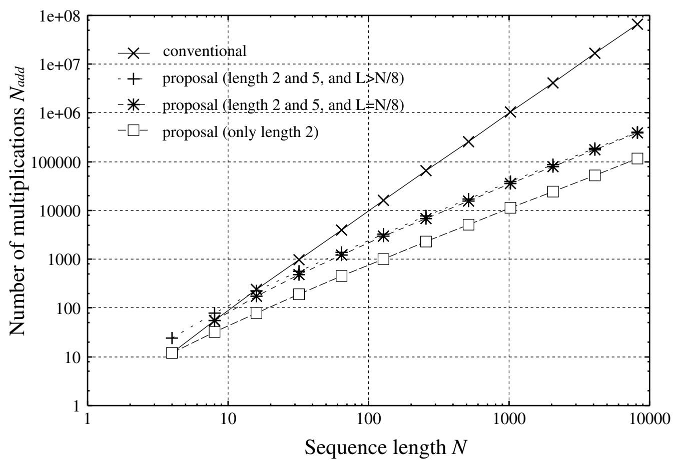

Fig. 2. The number of additions N_{add} to calculate autocorrelation function of a perfect sequence $a_N^{j,s}$.

As an example, Fig. 3 shows a signal flow of fast periodic autocorrelation algorithm for a perfect sequence $a_8^{2,1}$ of period $N = 8$, sequence number $j = 2$ and set number $s = 1$ derived from the Huffman sequences $\hat{a}_{17}^{10}$ of length $M = 17$ and sequence number $\ell = (\ell'_9, \ell'_5, \ell'_3, \ell'_2, \ell_2)_2 = (01010)_2 = 10$, where the Huffman sequence $\hat{a}_{17}^{10}$ is derived from the Huffman sequence of only length 2. Therefore, the number of multiplications N_{mul} and additions N_{add} are $72(= 2 \times 8(3 + 1) + 8)$ and $32(= 8(3 + 1))$, respectively. Similarly, Fig. 4 shows a signal flow of fast periodic autocorrelation algorithm for a perfect sequence $a_8^{0,5}$ of period $N = 8$, sequence number $j = 0$ and set number $s = 5$ derived from the Huffman sequences $\hat{a}_{17}^{10}$ of length $M = 17$ and sequence number $\ell = (\ell'_9, \ell'_5, \ell'_3, \ell'_2, \ell_2)_2 = (01010)_2 = 10$, where the Huffman sequence $\hat{a}_{17}^{10}$ is derived from the Huffman sequence of length 2 and 5. Therefore, the number of multiplications N_{mul} and additions N_{add} are $128(= 5 \times 8 + 88)$ and $80(= 4 \times 8 + 48)$, respectively.

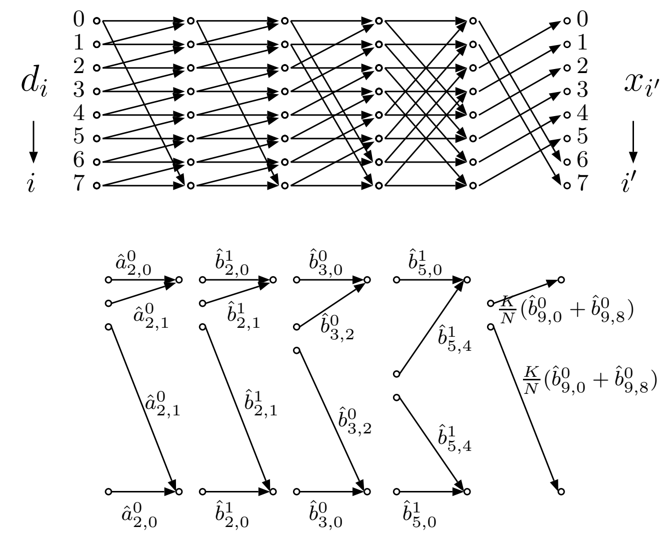

Fig. 3. A signal flow of fast periodic autocorrelation algorithm for a perfect sequence $a_8^{2,1}$ of period $N = 8$, sequence number $j = 2$ and set number $s = 1$ derived from the Huffman sequence $\hat{a}_{17}^{10}$ of length $M = 17$ and sequence number $\ell = 10$, where the Huffman sequence $\hat{a}_{17}^{10}$ is derived from the Huffman sequence of only length 2.

B. Fast crosscorrelation algorithm

From Eq. (11), the periodic crosscorrelation function between a perfect sequence $a_N^{j,s}$ and an input sequence d is given by

$$y_j = \frac{1}{N} \sum_{i=0}^{N-1} d_i a_{N,i}^{j,s}, \tag{21}$$

where $y = \{y_j\} = (y_0, \cdots, y_j, \cdots, y_{L-1})$, $y_j \in R$. In addition, Eq. (21) is replaced by a determinant of matrix as

$$\mathbf{Y} = \frac{1}{N} \mathbf{C}_N^s \cdot \mathbf{D}, \tag{22}$$

where $\mathbf{Y} = [y_0, \cdots, y_j, \cdots, y_{L-1}]^T$, $\mathbf{C}_N^s$ is a matrix of size $L \times N$ and

$$\begin{aligned} \mathbf{C}_N^s &= \left[a_N^{0,s}, \cdots, a_N^{j,s}, \cdots, a_N^{L-1,s}\right]^T \\ &= \begin{bmatrix} a_{N,0}^{0,s}, & a_{N,1}^{0,s}, & \cdots & a_{N,N-1}^{0,s} \\ a_{N,0}^{1,s}, & a_{N,1}^{1,s}, & \cdots & a_{N,N-1}^{1,s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N,0}^{L-1,s}, & a_{N,1}^{L-1,s}, & \cdots & a_{N,N-1}^{L-1,s} \end{bmatrix}. \end{aligned}$$

Therefore, the number of multiplications N_{mul} and additions N_{add} are given by $LN + L$ and $L(N - 1)$, respectively.

In the case of the orthogonal set derived from the Huffman sequence of only length 2, from Eq. (13), Eq. (22) is factorized as

$$\begin{aligned} \mathbf{Y} = \frac{K}{N} \hat{\mathbf{F}}_{\frac{M+1}{2}} \cdot \hat{\mathbf{C}}_{2^{\nu-2}+1} \cdots \hat{\mathbf{C}}_{2^k+1} \cdots \hat{\mathbf{C}}_3 \\ \cdot \hat{\mathbf{B}}_2^{\ell'_2} \cdot \hat{\mathbf{A}}_2^{\ell_2} \cdot \mathbf{D}, \end{aligned} \tag{23}$$

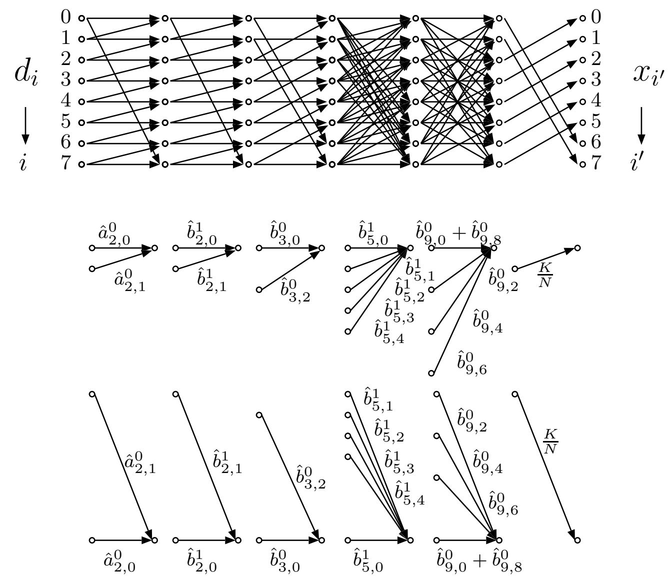

Fig. 4. A signal flow of fast periodic autocorrelation algorithm for a perfect sequence $a^{0,5}_8$ of period $N = 8$, sequence number $j = 0$ and set number $s = 5$ derived from the Huffman sequence $\hat{a}^{10}_{17}$ of length $M = 17$ and sequence number $\ell = 10$, where the Huffman sequence $\hat{a}^{10}_{17}$ is derived from the Huffman sequence of length 2 and 5.

where set number $s = \ell'_2$ and $\ell_2 \neq \ell'_2$, and $\hat{\mathbf{F}}_m$ is a matrix of size $L \times N$ and $\mathbf{C}_m$ is a matrix of size $N \times N$, and these matrices are given by

$$\begin{aligned}\hat{\mathbf{F}}_{\frac{M+1}{2}} &= \Big[\{\beta^0_{\frac{M+1}{2},i} \otimes \Delta_{N,i}\}, \{\beta^0_{\frac{M+1}{2},i} \otimes \Delta_{N,i-1}\}, \cdots, \\ &\quad \{\beta^0_{\frac{M+1}{2},i} \otimes \Delta_{N,i-(\frac{L}{2}-1)}\}, \{\beta^1_{\frac{M+1}{2},i} \otimes \Delta_{N,i-\frac{L}{2}}\}, \\ &\quad \cdots \{\beta^1_{\frac{M+1}{2},i} \otimes \Delta_{N,i-(L-1)}\}, \Big]^T,\end{aligned}$$

$$\begin{aligned}\hat{\mathbf{C}}_{2^k+1} &= \Big[\{\beta^0_{2^k+1,i} \otimes \Delta_{N,i}\}, \cdots, \\ &\quad \{\beta^0_{2^k+1,i} \otimes \Delta_{N,i-(2^{k-1}-1)}\}, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-2^{k-1}}\}, \cdots, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-(2^k-1)}\}, \\ &\quad \{\beta^0_{2^k+1,i} \otimes \Delta_{N,i-2^k}\}, \\ &\quad \cdots, \{\beta^0_{2^k+1,i} \otimes \Delta_{N,i-(3\cdot 2^{k-1}-1)}\}, \cdots, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-(N-2^{k-1})}\}, \cdots, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-(N-1)}\}\Big]^T.\end{aligned}$$

Therefore, the number of multiplications N_{mul} and additions N_{add} are given by

$$N_{mul} = \begin{cases} 2N\log_2 N + 3L & ; N = M-1, \\ 2N\log_2 N + 3N & ; N < M-1, \end{cases} \tag{24}$$

$$N_{add} = \begin{cases} N\log_2 N + L & ; N = M-1, \\ N\log_2 N + N & ; N < M-1. \end{cases} \tag{25}$$

Similarly, in the case of the orthogonal set derived from the Huffman sequence of length 2 and 5, from Eq. (13), Eq. (22) is factorized as

$$\begin{aligned}\mathbf{Y} &= \frac{K}{N}\hat{\mathbf{F}}_{\frac{M+1}{2}} \cdot \hat{\mathbf{C}}_{2^{\nu-2}+1} \cdots \hat{\mathbf{C}}_{2^k+1} \cdots \hat{\mathbf{C}}_9 \\ &\quad \cdot \hat{\mathbf{B}}^{\ell'_5}_5 \cdot \hat{\mathbf{B}}^{\ell'_3}_3 \cdot \hat{\mathbf{B}}^{\ell'_2}_2 \cdot \hat{\mathbf{A}}^{\ell_2}_2 \cdot \mathbf{D},\end{aligned} \tag{26}$$

where set number $s = (\ell'_5, \ell'_3, \ell'_2)_2$ and $\ell_2 \neq \ell'_2$, and a matrix $\mathbf{C}_m$ is given by

$$\begin{aligned}\hat{\mathbf{C}}_{2^k+1} &= \Big[\{\beta^0_{2^k+1,i} \otimes \Delta_{N,i}\}, \cdots, \\ &\quad \{\beta^0_{2^k+1,i} \otimes \Delta_{N,i-(2^{k-3}-1)}\}, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-2^{k-3}}\}, \cdots, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-(2^{k-2}-1)}\}, \\ &\quad \{\beta^0_{2^k+1,i} \otimes \Delta_{N,i-2^{k-2}}\}, \cdots, \\ &\quad \{\beta^0_{2^k+1,i} \otimes \Delta_{N,i-(3\cdot 2^{k-3}-1)}\}, \cdots, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-(N-2^{k-3})}\}, \cdots, \\ &\quad \{\beta^1_{2^k+1,i} \otimes \Delta_{N,i-(N-1)}\}\Big]^T.\end{aligned}$$

Therefore, the number of multiplications N_{mul} and additions N_{add} are given by

$$N_{mul} = \begin{cases} 2N + 30L & ; N = M-1, N = 8, \\ 4N + 45L & ; N = M-1, N = 16, \\ 5N\log_2 N & \\ -19N + 66L & ; N = M-1, N \geq 32, \\ 5N\log_2 N & \\ -4N + 5L & ; N < M-1, \end{cases} \tag{27}$$

$$N_{add} = \begin{cases} N + 16L & ; N = M-1, N = 8, \\ 2N + 31L & ; N = M-1, N = 16, \\ 4N\log_2 N & \\ -17N + 52L & ; N = M-1, N \geq 32, \\ 4N\log_2 N & \\ -5N + 3L & ; N < M-1. \end{cases} \tag{28}$$

Figures 5 and 6 show the number of multiplications N_{mul} and additions N_{add} to calculate crosscorrelation function of an orthogonal set of perfect sequences $a^{j,s}_N$ of family size $L = N/2$, respectively. Note that the number of multiplications N_{mul} and additions N_{add} to calculate crosscorrelation function of an orthogonal set of perfect sequences $a^{j,s}_N$ of family size $L = N$ is equal to Fig. 1 and 2, respectively.

As an example, Fig. 7 shows a signal flow of fast periodic crosscorrelation algorithm for an orthogonal set of perfect sequences $a^{j,s}_8$ of period $N = 8$, family size $L = 8$ and set number $s = \ell'_2$ derived from the real-valued Huffman sequences $\hat{a}^{\ell}_{17}$ of $M = 17$, where the Huffman sequence $\hat{a}^{\ell}_{17}$ is derived from the Huffman sequence of only length 2. Note that $\hat{b}^0_{9,0} + \hat{b}^0_{9,8}$ is equal to $\hat{b}^1_{9,0} + \hat{b}^1_{9,8}$ because $\hat{b}^0_9$ is a reversed sequence of $\hat{b}^1_9$. Therefore, the number of multiplications N_{mul} and additions N_{add} are $72(= 2 \times 8(3+1) + 8)$ and $32(= 8(3+1))$, respectively. Similarly, Fig. 8 shows a signal flow of fast periodic crosscorrelation algorithm for an orthogonal set of perfect sequences $a^{j,s}_8$ of period $N = 8$, family size $L = 2$ and set number $s = (\ell'_5, \ell'_3, \ell'_2)_2$ derived from the real-valued Huffman sequences $\hat{a}^{\ell}_{17}$ of $M = 17$, where the Huffman sequence $\hat{a}^{\ell}_{17}$ is derived from the Huffman sequence of length 2 and 5. Note that $\hat{b}^0_{9,0} + \hat{b}^0_{9,8}$ is equal to

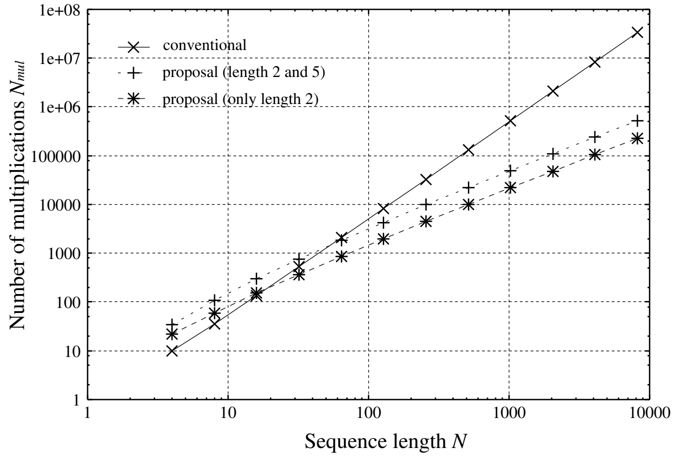

Fig. 5. The number of multiplications N_{mul} to calculate crosscorrelation function of an orthogonal set of perfect sequences $a_N^{j,s}$ of family size $L = N/2$.

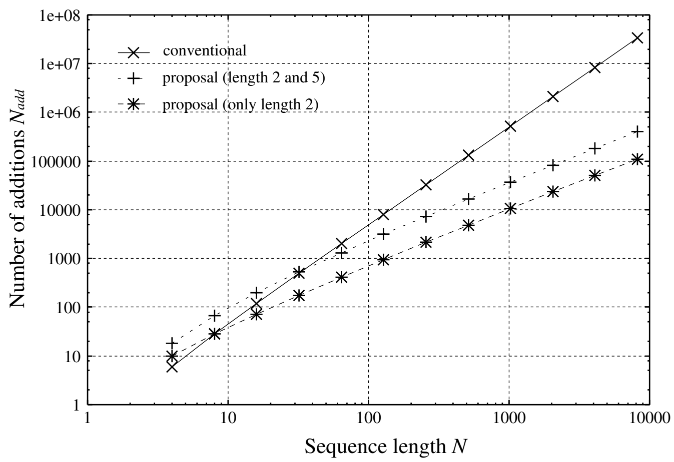

Fig. 6. The number of additions N_{add} to calculate crosscorrelation function of an orthogonal set of perfect sequences $a_N^{j,s}$ of family size $L = N/2$.

$\hat{b}^1_{9,0} + \hat{b}^1_{9,8}$ because $\hat{b}^0_9$ is a reversed sequence of $\hat{b}^1_9$. Therefore, the number of multiplications N_{mul} and additions N_{add} are $98(= 5 \times 8 + 48 + 10)$ and $62(= 4 \times 8 + 24 + 6)$, respectively.

V. Conclusion

In this paper, we have proposed fast periodic auto- and cross-correlation algorithms for an orthogonal set of real-valued perfect sequences of period $N = 2^n$ which are derived from the real-valued Huffman sequences of length $M = 2^\nu + 1$ with $n \leq \nu$. The sequence is synthesized by multiple convolutions of $\nu+1$ elementary sequences which are generated by combinations of original and reversed elementary sequences.

As a result, the number of multiplications and additions can be suppressed from the order of N^2 to the order of $N \log_2 N$ by proposed algorithms.

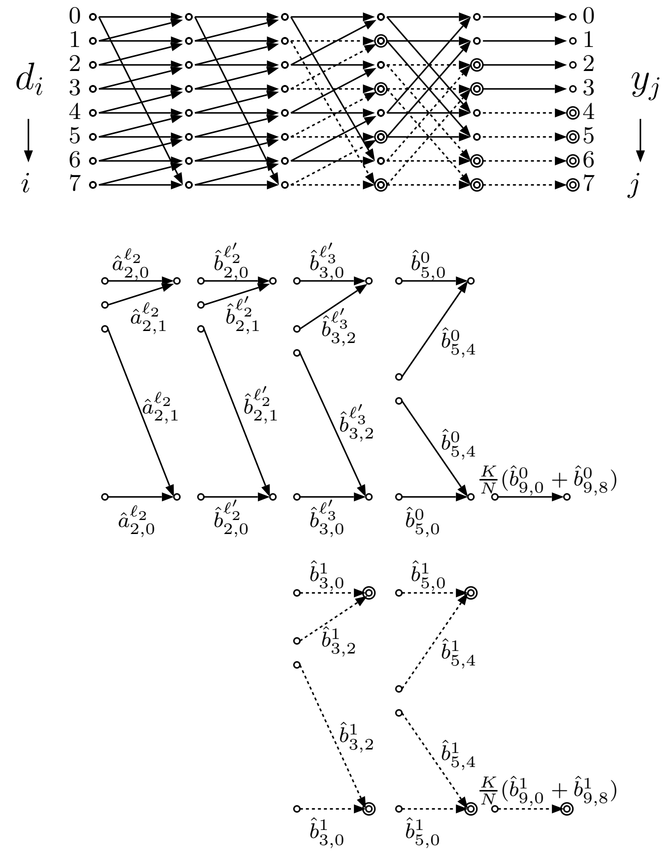

Fig. 7. A signal flow of fast periodic crosscorrelation algorithm for an orthogonal set of perfect sequences of period $N = 8$, family size $L = 8$ and set number $s = \ell'_2$ derived from the Huffman sequence $\hat{a}^\ell_{17}$ of length $M = 17$, where the Huffman sequence $\hat{a}^\ell_{17}$ is derived from only length 2.

Acknowledgment

This research was supported by the Ministry of Education,Culture,Sports,Science and Technology(MEXT), Grant-in-Aid for Young Scientists (B)(21760294).

References

[1] P. Z. Fan and M. Darnell, Sequence design for communications applications, *Research Studies Press*, 1996.

[2] M. Nakamura, H. Torii and C. Aoshima, Digital watermarking scheme using direct-sequence spread spectrum with adaptive voltage control for spread watermark signals, *Proc. 11th WSEAS International Conference on Multimedia Systems and Signal Processing*, 2011, pp. 93–97.

[3] Y. Tanada, General solution for orthogonal periodic real-number sequences, *IEICE Trans. on Fundamentals*, vol. 68-A, no. 5, 1985, pp. 451–457 (in Japanese).

[4] D. A. Huffman, The generation of impulse equivalent pulse trains, *IRE Trans. Information Theory*, vol. IT-8, 1962, pp. s10–s16.

[5] Y. Tanada, Synthesis of a set of real-valued shift-orthogonal finite-length PN sequences, *Proc. of the IEEE 4th International Symposium on Spread Spectrum Techniques and Applications*, 1996, pp. 58–62.

[6] T. Matsumoto and Y. Tanada, A real-valued periodic orthogonal sequence derived from a real-valued shift-orthogonal finite-length sequence and its fast periodic correlation algorithm, *IEICE Trans. on Fundamentals*, vol. J89-A, no. 5, 2006, pp. 323–332 (in Japanese).

[7] T. Matsumoto, S. Matsufuji, T. Kojima and U. Parampalli, Generation method of an orthogonal set of real-valued periodic orthogonal sequences from Huffman sequences, *Proc. of the 2011 Australian Communication Theory Workshop*, 2011, pp. 66–70.

[8] T. Matsumoto, S. Matsufuji, T. Kojima and U. Parampalli, Orthogonal and ZCZ sets of real-valued periodic orthogonal sequences from Huffman sequences, *IEICE Trans. Fundamentals*, vol. E94-A, no. 12, 2011 (to appear).

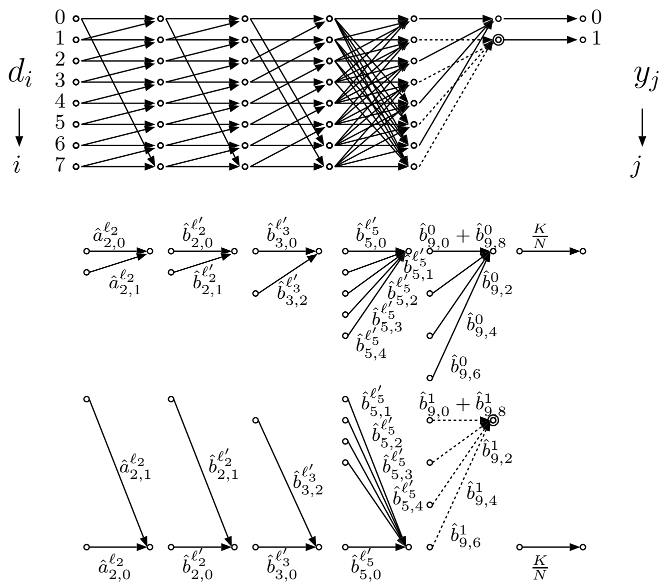

Fig. 8. A signal flow of fast periodic crosscorrelation algorithm for an orthogonal set of perfect sequences of period $N = 8$, family size $L = 2$ and set number $s = (\ell'_5, \ell'_3, \ell'_2$ derived from the Huffman sequence $\hat{a}^{\ell}_{17}$ of length $M = 17$, where the Huffman sequence $\hat{a}^{\ell}_{17}$ is derived from length 2 and 5.

[9] T. Matsumoto, H. Torii and S. Matsufuji, Fast correlation algorithm for an orthogonal set of real-valued perfect sequences from the Huffman sequences, *Proc. of the 11th WSEAS International Conference on Applied Computer Science*, 2011, pp. 146–151.

[10] H. Torii, M. Nakamura and N. Suehiro, New method for constructing polyphase ZCZ sequence sets, *Proc. 2nd WSEAS multiconference on applied and theoretical mathematics*, 2001, pp. 100–104.

[11] H. Torii and M. Nakamura, A study of asymmetry ZCZ sequence sets, *Proc. 11th WSEAS International Conference on Multimedia Systems and Signal Processing*, 2011, pp. 79–86.

Takahiro Matsumoto received his B. Eng. and M. Eng. degrees in Information and Computer Science from Kagoshima University, Japan, in 1996 and 1998, respectively, and his Ph. D. degree in Engineering from Yamaguchi University, Japan, in 2007. From 1998 to 2007, he was a Research Associate of the Department of Computer Science and Systems Engineering at Yamaguchi University, Japan. Since 2007, he has been an Assistant Professor of the Graduate School of Science and Engineering at Yamaguchi University. From 2010 to 2011, he was a visiting researcher at the University of Melbourne, Australia. His current research interests include spread spectrum systems and their applications. He received the Best Paper Award in the 10th WSEAS Intenational Conference on Applied Informatics and Communications in 2010. He is a member of IEICE of Japan.

Hideyuki Torii received the B.Eng., M.Eng., and Ph.D. degrees from the University of Tsukuba, Tsukuba, Japan in 1995, 1997, and 2000 respectively. In 2000, he joined the Department of Network Engineering, Kanagawa Institute of Technology as a Research Associate. He is currently an Associate Professor in the Department of Information Network and Communication at the same university. His research interests include spreading sequences, CDMA systems, and mobile communication systems. He is a member of IEEE and IEICE.

Shinya Matsufuji graduated from the Department of Electronic Engineering at Fukuoka University in 1977. He received the Dr. Eng. in Computer Science and Communication Engineering from Kyushu University, Fukuoka, Japan in 1993. From 1977 to 1984, he was a technical official at Saga University, Saga, Japan. From 1984 to 2002, he was a research associate in the Department of Information Science at Saga University. From 2002 to 2006, he was an associate professor of the Department of Computer Science and Systems Engineering at Yamaguchi University. Since 2006, he has been an associate professor of the Graduate School of Science and Engineering at Yamaguchi University. His current research interests include sequence design and spread spectrum systems. He received the Excellent Paper Award in IET International Communication Conference on Wireless Mobile and Computing, and the Best Paper Award in the 10th WSEAS Intenational Conference on Applied Informatics and Communications in 2009 and 2010, respectively. He is a member of IEEE, IEICE and WSEAS.