New non-linear adaptive command system for the aircrafts' attitude control

Mihai Lungu

***Abstract*—** The paper presents a new complex adaptive non-linear system with one input and one output (SISO) which is based on dynamic inversion. The stabilization command of the linearised system using as input the difference between closed loop system's output and the reference model's output is made by the linear dynamic compensator. The state vector of the linear dynamic compensator, the output and other state variables of the control system are used for the obtaining of the adaptive control law; this law is modeled by a neural network. The purpose of the adaptive command is to compensate the dynamic inversion error. Thus, the command law has two components: the first is the command given by the linear dynamic compensator and the second one is the adaptive command given by the neural network. As control system one chooses the non-linear model of the aircrafts' roll movements. One chooses a linear reference model. One obtains the structure of the adaptive control system of the roll angle and the Matlab/Simulink models of the adaptive command system's subsystems. Thus, characteristics that describe the adaptive command system's dynamics are obtained.

***Keywords*—** attitude, adaptive, neural network, dynamic inversion, aircrafts, roll angle.

I. INTRODUCTION

THE complexity and incertitude that appear in the non-linear and instable phenomena are the main reasons that require the projecting of non-linear adaptive structures for control and stabilization; in these cases the linear models are far from a good describe of the flying objects' dynamic [1], [2], [3]. Another reason is the non-linear character of the actuators. The observers must be easily adaptable and their project algorithms must allow the state's estimation of the flying objects even in the case of their failure or no use of the damaged sensors' signals. In these situations, it's good to use the real time adaptive control based on neural networks and dynamic inversion of the unknown or partial known non-linearities from the dynamic model of the flying object [4]. The neural network's training is based on the signals from state observers; these observers get information about the control system's error [5], [6], [7].

II. DYNAMIC SYSO SYSTEMS

Let's consider the dynamic system (A) with single input and single output described by equations

$$\begin{aligned} \dot{x} &= f(x,u), \\ y &= h(x), \end{aligned} \tag{1}$$

with $x(n \times 1), n -$ unknown f and $h -$ unknown nonlinear functions, u and $y -$ measurable.

One projects an adaptive control law v after (in rapport with) the output; the neural network (NN) models a function that depends on the values of input and output of the system (A) at different time moments so that $y(t)$ follows the finite $y(t)$. The feedback linearization may be made through transformation [8], [9]

$$v = \hat{h}_r(y,u), \tag{2}$$

where v is the pseudo-command signal and $\hat{h}_r(y,u) -$ the best approximation of $h_r(x,u) = h_r(x(y),\ u)$.

Equation (2) is equivalent with the following one

$$u = \hat{h}_r^{-1}(y,v) \ . \tag{3}$$

If $\hat{h}_r \equiv h_r$, one yields $y^{(r)} = v$; otherwise $\left(\hat{h}_r \neq h_r\right)$

$$y^{(r)} = v + \varepsilon, \tag{4}$$

where

$$\varepsilon = \varepsilon(x,u) = h_r(x,u) - \hat{h}_r(y,u) \tag{5}$$

is the approximation of function h_r (inversion error). Assessing y to follow $\bar{y}$, then v has form [8], [9], [10], [11]

The first part of this paper was published in the Proceedings of the International Conference on Circuits, Systems and Signals (IEEE.AM), Malta, September 15 – 17, 2010, pp. 39 – 44, ISSN: 1792-4324, ISBN: 978-960-474-226-4. After the conference, it received the recommendation to be extended and published by NAUN Journal.

Manuscript received October 1, 2010.

Mihai Lungu is Lecturer at the University of Craiova, Faculty of Electrical Engineering, Department of Avionics. 107, Decebal Blv., Craiova, Romania,

Email: Lma1312@yahoo.com, mlungu@elth.ucv.ro.

$$v = \bar{y}^{(r)} + v_{pd} - v_a + \bar{v}, \tag{6}$$

where v_{pd} is the output of the dynamic linear compensator for stabilization, used for liniarised dynamic (4), with $\varepsilon = 0, v_a$ – the adaptive command that must compensate ε and $\bar{v}$ has the form [8], [12]

$$\bar{v} = k_z\left(\left\|\hat{Z}\right\|_F + \bar{Z}\right)\frac{\bar{E}}{\left\|\bar{E}\right\|}\left\|\hat{E}\right\| + k_v\bar{E}, \tag{7}$$

with $k_z, k_v > 0$ gain constants, $\left\|\hat{Z}\right\|_F$ – the Frobenius norm of matrix $\hat{Z}, \bar{Z}$ – the ideal matrix of the neural network and $\bar{E} = \hat{E}P\bar{B}$, with $\hat{E}, P$ and $\bar{B}$ – matrices. The derivative $\bar{y}^{(r)}$ is introduced for the conditioning of the dynamic error $\tilde{y} = \bar{y} - y$. This derivative is given by a reference model (command filter) [9]. $\bar{y}^{(r)}$ may be cumulated with other signals and it results the component v_r of form (12).

Let's consider $H_d(\mathrm{s})$ – the transfer function of the linear subsystem of A (flying object) with the input u_n and the output y, having to the numerator a p order polynomial and at the denominator a r order polynomial; $p \le r - 1$. For this system the author proposes the command structure from fig. 1, with the linear part described by equations (9) ÷ (11).

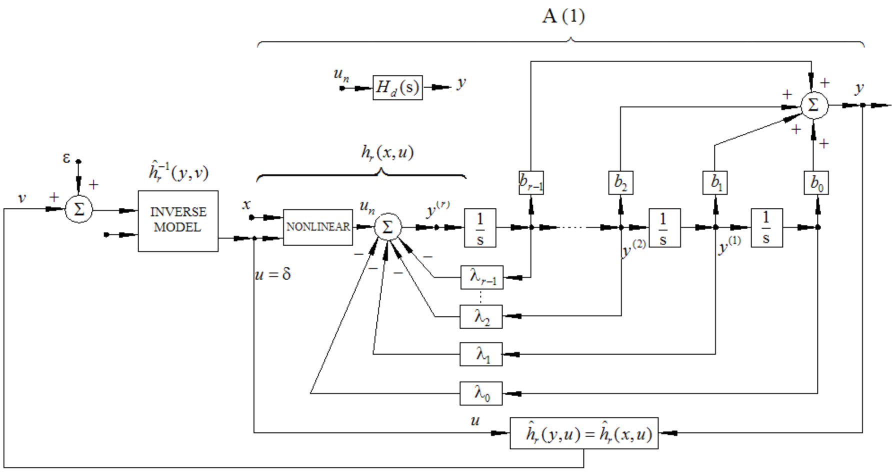

Fig. 1. The block diagram of the adaptive command system based on dynamic inversion

The transfer function of the linear system A with the input u_n and the output y is

$$H_d(\mathrm{s}) = \frac{b_p \mathrm{s}^p + b_{p-1}\mathrm{s}^{p-1} + \ldots + b_1\mathrm{s} + b_0}{s^r + \lambda_{r-1}\mathrm{s}^{r-1} + \ldots + \lambda_1\mathrm{s} + \lambda_0}, \; p \le r - 1. \tag{8}$$

Considering

$$\begin{aligned} &Y^T = \left[y \;\; \dot{y} \; \ldots \; y^{(r-1)}\right],\; Z^T = \left[v \;\; \dot{v} \; \ldots \; v^{(p)}\right], \\ &\lambda^T = \left[\lambda_0 \;\; \lambda_1 \; \cdots \; \lambda_{r-1}\right],\; b^T = \left[b_0 \;\; b_1 \; \cdots \; b_p\right], \end{aligned} \tag{9}$$

with $b_i, i = \overline{0, p}, \lambda_j, j = \overline{0, r-1}$ – the coefficients of the numerator and denominator of the transfer function for the system with input u_n and output y, the linear system with input v and output y is described by equation

$$y^{(r)} = -\lambda^T Y + b^T Z + \varepsilon. \tag{10}$$

If $p = 0$, then $Z = v, b = b_0$ and the previous equation becomes

$$y^{(r)} = -\lambda^T Y + b_0 v + \varepsilon. \tag{11}$$

In the particular case $y^{(r)} = \bar{y}^{(r)}$, one obtains

$$v_r = \frac{1}{b_0}\left(\bar{y}^{(r)} + \lambda^T \bar{Y}\right). \tag{12}$$

The compensator may be described by state equations

$$\begin{aligned} &\dot{\varsigma} = A_c\varsigma + b_c e, \\ &v_{pd} = c_c\varsigma + d_c e, \end{aligned} \tag{13}$$

where ς has at least dimension $(r-1)$,

$$\begin{aligned} &e = \tilde{y} = ce, e^{\mathrm{T}} = \left[e \;\; \dot{e} \; \cdots \; e^{(r-1)}\right], \\ &c = \left[1 \;\; 0 \;\; 0 \; \cdots \; 0\right]_{1\times r}. \end{aligned} \tag{14}$$

The state equation of the linear subsystem with input $(v + \varepsilon)$ and output y is

$$\begin{aligned} \dot{x} &= Ax + b(v + \varepsilon), \\ v &= v_{pd} - v_a + \overline{v}, \end{aligned} \tag{15}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{r\times r}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r\times 1}. \tag{16}$$

The stable state $\overline{x}\left(\dot{\overline{x}} = v = \varepsilon = 0\right)$ verifies equation $A\overline{x} = 0$ and, taking into account (15), leads to the equation of the error vector $e \equiv \widetilde{x} = \overline{x} - x$,

$$\dot{e} = Ae - bv_{pd} + b(v_a - \overline{v} - \varepsilon). \tag{17}$$

With notations

$$E = \begin{bmatrix} e \\ \varsigma \end{bmatrix}, \overline{A} = \begin{bmatrix} A - d_c bc & -bc_c \\ b_c c & A_c \end{bmatrix}, \overline{b} = \begin{bmatrix} b \\ 0 \end{bmatrix}, \overline{C} = \begin{bmatrix} c & 0 \\ 0 & I \end{bmatrix}, \tag{18}$$

where I is the identity matrix, one obtains

$$\begin{aligned} \dot{E} &= \overline{A}E + \overline{b}(v_a - \overline{v} - \varepsilon), \\ z &= \overline{C}E; \end{aligned} \tag{19}$$

A_c, b_c, c_c, d_c from (13) are calculated so that $\overline{A}$ is a Hurwitz matrix.

For the estimation of the vector E one uses a linear state observer of order $(2r - 1)$ described by equations

$$\begin{aligned} \dot{\hat{E}} &= \overline{A}\hat{E} + L(z - \hat{z}), \\ \hat{z} &= \overline{C}\hat{E}, \end{aligned} \tag{20}$$

with the gain matrix L calculated so that matrix $\widetilde{A} = \left(\overline{A} - L\overline{C}\right)$ is stable.

Considering $w -$ the sensor measure error, $y_m -$ the measured value of y, then $\widetilde{y}_m = \overline{y} - y_m = \widetilde{y} + w$ and the compensator's equations become

$$\begin{aligned} \dot{E} &= \overline{A}E + \overline{b}(v_a - \overline{v} - \varepsilon) + Gw, \\ z &= \overline{C}E + Hw. \end{aligned} \tag{21}$$

with $H^T = [1 \ 0], G^T = [-bd_c \ \ b_c]$.

If state ς of the compensator is known, one uses a reduced order observer for estimation of vector e

$$\begin{aligned} \dot{\hat{e}} &= \overline{A}\hat{e} + L_r(z_1 - \hat{z}_1), \\ z_1 &= c\hat{e}. \end{aligned} \tag{22}$$

The gain matrix L_r is obtained so that matrix $\widetilde{A} = \left(\overline{A} - L_r c\right)$ is stable. With vectors $\hat{e}$ and ς vector $\hat{E}^T = [\hat{e} \ \varsigma]$ is obtained. The signal $\overline{E} = \hat{E}^T P\overline{b}$ is used for neural network's adapting; the weights $\hat{W}$ and $\hat{V}$ are obtained with equations

$$\begin{aligned} \dot{\hat{W}} &= -\Gamma_W \left[2\left(\hat{\sigma} - \hat{\sigma}'\hat{V}^T\eta\right)\hat{E}^T P\overline{B} + k\left(\hat{W} - \hat{W}_0\right)\right], \\ \dot{\hat{V}} &= -\Gamma_V \left[2\eta\hat{E}^T P\overline{B}\hat{\sigma}' + k\left(\hat{V} - \hat{V}_0\right)\right], \end{aligned} \tag{23}$$

where the role of $\overline{B}$ is played by $\overline{b}$. In (23) σ is the sigmoid function

$$\sigma(z) = \frac{1}{1 + e^{-az}}, \tag{24}$$

$\hat{\sigma}' = \left.\frac{\mathrm{d}\hat{\sigma}(z)}{\mathrm{dz}}\right|_{z=z_0}$ is the Jacobian of vector $\hat{\sigma}$, $\hat{W}_0$ and $\hat{V}_0 -$ the initial values of weights $\hat{W}, \hat{V}, \Gamma_W, \Gamma_V > 0$, $k > 2\left(k_1^2 + \gamma_1^2\left\|P\overline{B}\right\|^2\right)$, $k_1 = k_2\alpha_1 + \left\|P\overline{B}\right\|\gamma_1$, $k_2 = \left\|P\overline{B}\right\| + \left\|\widetilde{P}\overline{B}\right\|$, P and $\widetilde{P} -$ the solutions of the Liapunov equations

$$\begin{aligned} \overline{A}^T P + P\overline{A} &= -Q, \\ \widetilde{A}^T \widetilde{P} + \widetilde{P}\widetilde{A} &= -\widetilde{Q} \end{aligned} \tag{25}$$

for $Q, \widetilde{Q} > 0, \lambda_{\min}(Q) > 1, \lambda_{\min}\left(\widetilde{Q}\right) > 1$. $\lambda_{\min}(Q)$ and $\lambda_{\min}\left(\widetilde{Q}\right)$ are the minimum eigenvalues of the matrices Q and $\widetilde{Q}$.

The structure of the neural network is the one from fig. 2 [5], [13]

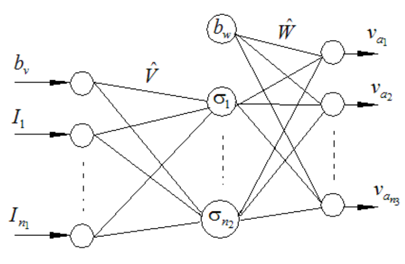

Fig. 2. The neural network

The input - output relationship for the neural network is

$$v_{a_k} = b_w \theta_{w,k} + \sum_{j=1}^{n_2} w_{j,k} \sigma_j, k = \overline{1, n_3}, \tag{26}$$

where

$$\sigma_j = \sigma\left(b_v \theta_{v,j} + \sum_{i=1}^{n_1} v_{i,j} I_i \right); \tag{27}$$

n_1, n_2, n_3 are respectively the input nodes' number, hidden layer nodes' number and output layer nodes' number. The sigmoid function is

$$\sigma(z) = \frac{1}{1 + e^{-az}}; \sigma_j(z) = \frac{1}{1 + e^{-a_j z_j}}; \tag{28}$$

σ is a vector with the elements $\sigma_j(z)$, a_j is the activation potential having a distinct value for every neuron. The matrices $\hat{V}$ and $\hat{W}$ are respectively

$$\hat{V} = \begin{bmatrix} \theta_{v,1} & \cdots & \theta_{v,n_2} \\ v_{1,1} & \cdots & v_{1,n_2} \\ \cdots & \cdots & \cdots \\ v_{n_1,1} & \cdots & v_{n_1,n_2} \end{bmatrix}, \hat{W} = \begin{bmatrix} \theta_{w,1} & \cdots & \theta_{w,n_3} \\ w_{1,1} & \cdots & w_{1,n_3} \\ \cdots & \cdots & \cdots \\ w_{n_2,1} & \cdots & w_{n_2,n_3} \end{bmatrix}. \tag{29}$$

One defines a new sigmoid vector

$$\hat{\sigma}(\mathrm{z}) = \left[b_w \;\; \sigma(\mathrm{z}_1) \; \sigma(\mathrm{z}_2) \cdots \sigma(\mathrm{z}_{\mathrm{n}_2}) \right]^T; \tag{30}$$

$b_w \geq 0$ allows to the threshold θ_w to be included in the matrix $\hat{W}$. Also, one defines the vector

$$\bar{x} = \left[b_v \;\; I_1 \;\; I_2 \cdots I_{n_1} \right]^T; \tag{31}$$

$b_v \geq 0$ is the bias which allows to the threshold θ_v to be included in matrix $\hat{V}$. Thus,

$$\boldsymbol{v}_a = \hat{W}^T \sigma\left(\hat{V}^T \bar{x} \right). \tag{32}$$

The derivative of the sigmoid vector $\hat{\sigma}(z)$ is

$$\hat{\sigma}' = \frac{\mathrm{d}\hat{\sigma}(z)}{\mathrm{dz}} = \begin{bmatrix} 0 & \cdots & 0 \\ \frac{\partial \sigma(\mathrm{z}_1)}{\partial z_1} & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \frac{\partial \sigma(\mathrm{z}_{\mathrm{n}_2})}{\partial z_{n_2}} \end{bmatrix}. \tag{33}$$

Conform to (36) there is a set of weights W^* and V^* that leads the output $\boldsymbol{v}_a$ of NN_c in a domain $D-$ the neighborhood of the inversion error $\boldsymbol{\varepsilon}$; the neighborhood has the maximum dimension μ^*. The matrices W^* and V^* are the matrices W and V that minimizes μ^*, value that can be made small by choosing of a sufficient number of neurons in the hidden layer (n_2) [14].

P from the signal used for the neural network's adapting is the solution of first equation (25) with $\overline{A} = (A - d_c bc)$. Second output of the compensator $(\tilde{y}_a)$ is used for obtaining of an error signal that is useful for adapting of the neural network's weights (fig. 3).

From (4) and (6) one yields

$$y^{(r)} = \bar{y}^{(r)} + v_{pd} - v_a + \bar{v} + \varepsilon, \tag{34}$$

equivalent with the dynamic error's equation

$$\tilde{y}^{(r)} = -v_{pd} + v_a - \bar{v} - \varepsilon. \tag{35}$$

Error ε may be approximated with the output of a linear neural network NN [8]

$$\varepsilon = W^T \Phi(\eta) + \mu(\eta), \|\mu\| < \mu^*, \tag{36}$$

where W is the weights' matrix for the connections between layer 2 and layer 3 (NN has 2 layers), $\mu(\eta)-$ the reconstruction error of the function and $\eta -$ the input vector of NN

$$\eta = \left[1 \;\; \bar{v}_d^T(t) \;\; \bar{y}_d^T(t) \right]^T, \tag{37}$$

where

$$\begin{aligned} \bar{v}_d^T(t) &= \left[v(t) \;\; v(t-d) \cdots v(t - (n_1 - r - 1)d) \right]^T, \\ \bar{y}_d^T(t) &= \left[y(t) \;\; y(t-d) \cdots y(t - (n_1 - 1)d) \right]^T, \end{aligned} \tag{38}$$

with $n_1 \geq n$ and $d > 0; v_a$ is projected so that

$$v_a = \hat{W}^T \Phi(\eta), \tag{39}$$

where $\hat{W}$ is the estimation of W.

The actuators' characteristics (time delays, nonlinearities with saturation zone) lead to neural network's adapting difficulties. This is why a block "PCH" is introduced; it limits the adaptive pseudo-control v_a and v by the mean of one component which represents an estimation of the actuator's dynamic (PCH – Pseudo control Hedging). PCH "moves back the reference model" introducing a correction of the reference

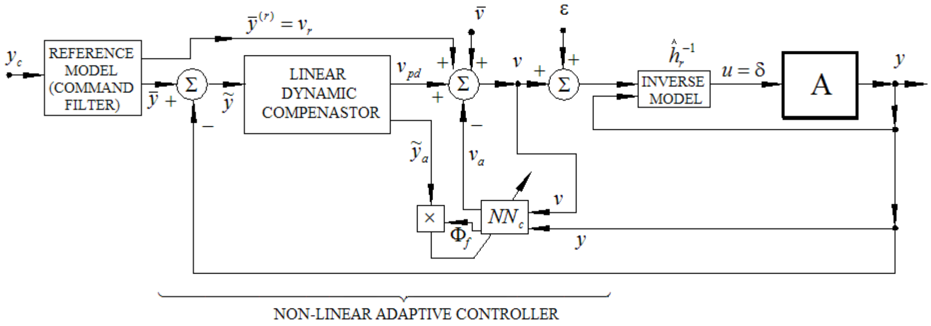

Fig. 3. Automat control system with non-linear adaptive controller

position [6], [12]. Because the dependence between δ and δ_c is expressed by a non-linear function h_a, one yields

$$\hat{h}_r(x,\delta_c) \neq \hat{h}_r(x,\hat{\delta}); \tag{40}$$

it results a difference between the two functions

$$v_h = \hat{h}_r(x,\delta_c) - \hat{h}_r(x,\hat{\delta}). \tag{41}$$

Taking into account that

$$\hat{h}_r(x,\delta_c) = \hat{h}_r\left(x, \hat{h}_r^{-1}(x,v)\right) = v, \tag{42}$$

function (41) becomes

$$v_h = v - \hat{h}_r(x,\hat{\delta}). \tag{43}$$

This signal is introduced in the reference model as an additional input [6]; one compares it with $\bar{y}^{(r)}$ inside of the reference model and, after integration, it leads to the modify of the signals $\bar{y}$ and $\tilde{y}$.

The existence and uniqueness of v_a is guaranteed by the following hypothesis [11], [15]: Conform to equations (2), (5), (6) and (8) one gets

$$\begin{aligned}\left|\frac{\partial\varepsilon}{\partial v_a}\right| &= \left|\frac{\partial\left(h_r - \hat{h}_r\right)}{\partial u}\frac{\partial u}{\partial v}\frac{\partial v}{\partial v_a}\right| = \left|\frac{\partial\left(h_r - \hat{h}_r\right)}{\partial u}\frac{\partial u}{\partial v}\right| = \\ &= \left|\frac{\partial\left(h_r - \hat{h}_r\right)}{\partial u}\frac{\partial u}{\partial \hat{h}_r}\right| = \left|\frac{\partial h_r / \partial u}{\partial \hat{h}_r / \partial u} - 1\right| < 1,\end{aligned} \tag{44}$$

condition that is equivalent with the following one

$$\operatorname{sgn}\left(\frac{\partial h_r}{\partial u}\right) = \operatorname{sgn}\left(\frac{\partial \hat{h}_r}{\partial u}\right), \quad \left|\frac{\partial \hat{h}_r}{\partial u}\right| > \frac{1}{2}\cdot\left|\frac{\partial h_r}{\partial u}\right|. \tag{45}$$

Conform to (35) and to the block diagram from fig. 3, one obtains the block diagram of the dynamic error's model (fig. 4); the equation of the compensator with the input $\tilde{y}$ and the outputs v_{pd} and $\tilde{y}_a$ is

$$\begin{bmatrix} v_{pd}(s) \\ \tilde{y}(s) \end{bmatrix} = \frac{1}{L_{pd}(s)}\begin{bmatrix} M_{pd}(s) \\ M_a(s) \end{bmatrix}\tilde{y} = \begin{bmatrix} H_{pd}(s) \\ H_a(s) \end{bmatrix}\tilde{y}(s). \tag{46}$$

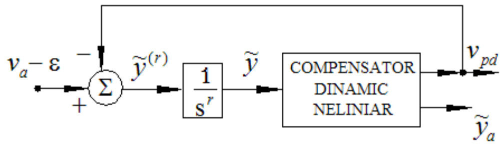

Fig. 4. Block diagram of the dynamic error's model

The equation (46) is equivalent with

$$\begin{aligned} H_{pd}(s) &= \frac{v_{pd}(s)}{\tilde{y}(s)} = \frac{M_{pd}(s)}{L_{pd}(s)}, \\ \tilde{H}_a(s) &= \frac{\tilde{y}_a(s)}{(v_a - \varepsilon)(s)} = \frac{H_a(s)}{s^r + H_{pd}(s)} = \frac{M_a(s)}{s^r L_{pd}(s) + M_{pd}(s)}. \end{aligned} \tag{47}$$

The polynomial $M_a(s)$ doesn't affect the stability of the system from fig. 4. For the stability the next condition must be fulfilled

$$q = \operatorname{grad} L_{pd}(s) \geq \operatorname{grad} M_{pd}(s) \geq r - 1. \tag{48}$$

The transfer function $\tilde{H}_a(s)$ is built so that it is strictly po-sitive real (SPR). From equations (47), (36) and (39) one gets

$$\tilde{y}_a(s) = \tilde{H}_a(s)(v_a - \bar{v} - \varepsilon) = \tilde{H}_a(s)\left(\tilde{W}^T\Phi(\eta) - \mu(\eta)\right), \tag{49}$$

where $\tilde{W} = \left(\hat{W} - W\right)$ is the error of the matrix's weights. If $r > 1$, $\tilde{H}_a(s)$ one can obtain a SPR using a filter with the operator $T(s)$ and the degree $(r-1)$; the resulted SPR function (conform to (47)) is

$$G(s) \overset{\Delta}{=} \widetilde{H}_a(s)T(s) = \frac{M_a(s)T(s)}{s^r L_{pd}(s) + M_{pd}(s)}. \tag{50}$$

Using the notations

$$\begin{aligned} \Phi_f &= T^{-1}(s)\Phi, \\ \mu_f(\eta) &= T^{-1}(s)\mu(\eta), \end{aligned} \tag{51}$$

the equation (49) becomes

$$\widetilde{y}_a(s) = G(s)\left[\widetilde{W}^T\Phi_f + \Delta(s) - \mu_f(\eta)\right], \tag{52}$$

where

$$\begin{aligned} &G(s) = \widetilde{H}_a(s)T(s), \\ &\Delta(s) = T^{-1}(s)\widetilde{W}^T\Phi - \widetilde{W}^T\Phi_f, \|\Delta\| \le c\left\|\widetilde{W}\right\|_F, c > 0. \end{aligned} \tag{53}$$

The polynomial $M_a(s)T(s)$ from the nominator of the transfer function $G(s)$ is chosen so that $G(s)$ is a SPR. For this, first one expresses $G(s)$ as

$$G(s) = \widetilde{H}_a(s)T(s) = \frac{b_{p-1}s^{p-1} + \cdots b_1 s + b_0}{s^p + a_{p-1}s^{p-1} + \cdots a_1 s + a_0}, \tag{54}$$

where $p = r + q; q = \text{grd}\left(L_{pd}(s)\right)$. The system with the transfer function $G(s)$, having the input $(v_a - \varepsilon)$ and the output $\widetilde{y}_a(s)$, may be described using the state equations in the canonical form

$$\begin{aligned} \dot{z} &= A_c z + B_c\left(\widetilde{W}^T\Phi_f + \Delta - \mu_f\right), \\ \widetilde{y}_a &= C_c z, \end{aligned} \tag{55}$$

where $z^T = \left[\widetilde{y}_a^{(p-1)}\ \widetilde{y}_a^{(p-2)}\ \cdots\ \widetilde{y}_a^{(1)}\ \widetilde{y}_a\right]$ is the state vector and

$$A_c = \begin{bmatrix} -a_{p-1} & -a_{p-2} & \cdots & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, B_c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, C_c^T = \begin{bmatrix} b_{p-1} \\ b_{p-2} \\ \vdots \\ b_1 \\ b_0 \end{bmatrix}. \tag{56}$$

$G(s)$ is SPR only if the conditions from the lemma Lefschetz – Kalman – Yakubovici (LKY) are fulfilled; that means $\exists Q_c > 0$ so that P_c is the solution of the Liapunov equation

$$A_c^T P_c + P_c A_c = -Q_c; C_c^T = P_c B_c. \tag{57}$$

Because $G(s)$ is a Hurwitz polynomial one results that $T(s)$ and $M_a(s)$ are Hurwitz polynomials too.

The filter with the transfer operator $T^{-1}(s)$ may be described by the state equations

$$\begin{aligned} \dot{z}_f &= A_c z_f + B_f\Phi, \\ \Phi_f &= C_f z_f, \end{aligned} \tag{58}$$

where z_f is the state vector of the filter. Also, the equation of the positive matrix P_f

$$A_f^T P_f + P_f A_f = -Q_f, Q_f > 0. \tag{59}$$

corresponds to the equation (58).

The signal Φ_f is used in the adapting law of NN_c [9]

$$\dot{\hat{W}} = -F\left(\widetilde{y}_a\Phi_f + \lambda_w\hat{W}\right), \tag{60}$$

where F and λ_w are adapting positive coefficients.

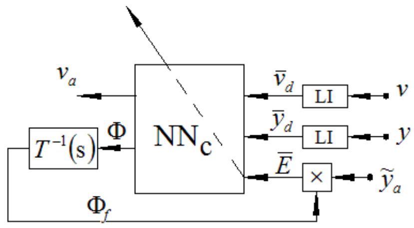

Fig. 5. Block diagram of the adaptive subsystem from din fig. 3

III. ADAPTIVE SYSTEM FOR THE COMMAND OF THE AIRCRAFTS' ROLL ANGLE

One considers the nonlinear model of the roll movement described by equations [16]

$$\begin{aligned} &\dot{x}_1 = x_2, \dot{x}_2 = f(x) + d_0 u, x_1 = \varphi, x_2 = \dot{\varphi}, u = \delta_e; \\ &f(x) = b_1 x_1 + b_2 x_2 + b_3|x_1|x_2 + b_4|x_2|x_2 + b_5 x_1^3; \end{aligned} \tag{61}$$

where

$b_1 = -0.0186, b_2 = 0.0152, b_3 = -0.0625, b_4 = 0.01, b_5 = 0.021,$ $d_0 = 1$.

The matriceal description of the movement is

$$\begin{aligned} &\dot{x} = Ax + h_r(x,u), x^T = \left[x_1\ \ x_2\right], A = \begin{bmatrix} 0 & 1 \\ b_1 & b_2 \end{bmatrix}, \\ &h_r(x,u) = b_3|x_1|x_2 + b_4|x_2|x_2 + b_5 x_1^3 + d_0 u. \end{aligned} \tag{62}$$

The system (61) is equivalent with equation

$$\ddot{\varphi} = b_2\dot{\varphi} + b_1\varphi + b_3|\varphi|\dot{\varphi} + b_4|\dot{\varphi}|\dot{\varphi} + b_5\varphi^3 + d_0 u, \tag{63}$$

where φ is the aircraft roll angle and $u = \delta_e -$ the ailerons deflection. From (63) it results the relative grade of system $(r = 2)$ and the transfer function $H_d(s)$

$$H_d(s) = \frac{1}{s^2 + \lambda_1 s + \lambda_0}. \quad (64)$$

One chooses the reference model described by equation

$$\bar{y} = \frac{\omega_{r0}^2}{s^2 + 2\xi_0\omega_{r0}s + \omega_{r0}^2} y_c, \quad (65)$$

with $\xi_0 = 0.7$ and $\omega_{r_0} = 1\text{rad/s}; y = \varphi, \bar{y} = \bar{\varphi}$. With (38), equation (10) becomes

$$\ddot{\varphi} = -\lambda_1\dot{\varphi} - \lambda_0\varphi + v + \varepsilon \quad (66)$$

and by elimination of $\ddot{\varphi}$ between this and equation (63) and identification, one yields

$$u = \delta_e = \frac{1}{d_0} v = \hat{h}_r^{-1}(x, v),$$
$$\varepsilon = (\lambda_0 + b_1)\varphi + (\lambda_1 + b_2)\dot{\varphi} + b_3|\varphi|\dot{\varphi} + b_4|\dot{\varphi}|\dot{\varphi} + b_5\varphi^3. \quad (67)$$

Equation (35) becomes

$$\ddot{\tilde{\varphi}} = -v_{pd} + v_a - \bar{v} - \varepsilon, \quad (68)$$

with v_{pd} of form [7]

$$v_{pd} = k_p\tilde{y} + k_d\dot{\tilde{y}}. \quad (69)$$

Implicit the dynamic equation of the error $\tilde{\varphi} = \bar{\varphi} - \varphi$ is

$$\begin{bmatrix} \dot{\tilde{y}} \\ \ddot{\tilde{y}} \end{bmatrix} = b_0 \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix} \begin{bmatrix} \tilde{y} \\ \dot{\tilde{y}} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (b_0 v_a - b_0\bar{v} - \varepsilon), \quad (70)$$

with $b_0 = 1$. One chooses $\lambda_0 = \lambda_1 = 1, k_p = \omega_0^2\ (\omega_0 = 1\text{rad/s})$ and $k_d = 2\xi\omega_0\ (\xi = 0.7)$.

One considers $\boldsymbol{E} = \boldsymbol{e} = [\tilde{\varphi}\ \dot{\tilde{\varphi}}]^T, \bar{C} = \bar{c} = [1\ 0], z = \tilde{y} = e$ and $\hat{z} = \hat{e}$; $\hat{\boldsymbol{E}}$ - the observer state (20) The gain matrix of the observer L is obtained so that matrix $\tilde{A} = (\bar{A} - LC)$ is stable; $\bar{A}$ is the matrix of system from equation (70) with $b_0 = 1$.

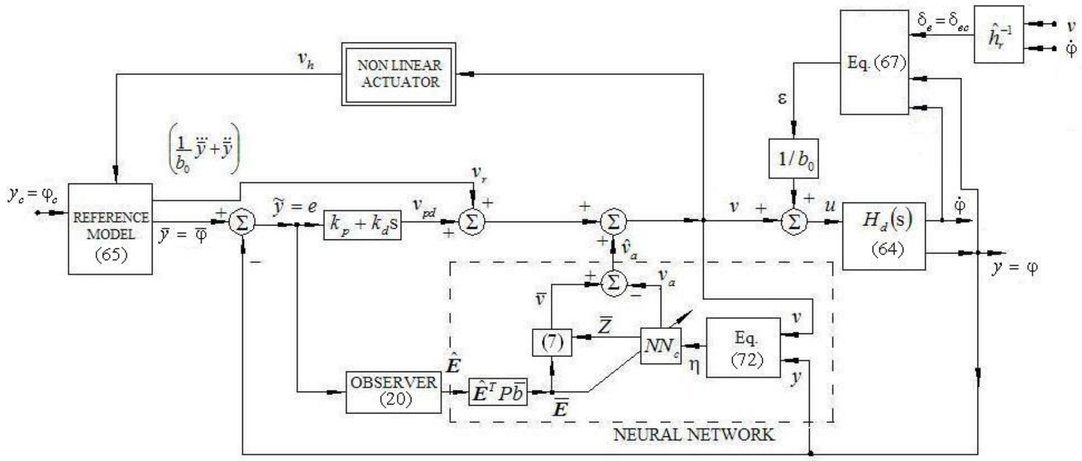

Fig. 6. Block diagram of the system for the control of aircrafts' roll angle

The component v_a is calculated using the equation

$$\boldsymbol{v}_a = \hat{W}^T\sigma(\hat{V}^T\boldsymbol{\eta}), \quad (71)$$

$P -$ the solution of the Liapunov equation (25) and $Q = I_2$; one chooses $\Gamma_V = 8, \Gamma_W = 10, k = 5$ and activation potentials between 0.1 and 1 [7]. The input vector η has the form (37) with components (38); $n_1 = n = 2, d = 0.05$;

$$\eta^T = [1\ v(t)\ v(t-d)\ v(t-2d)\ v(t-3d)\ y(t)\ y(t-d)]. \quad (72)$$

For the calculus of component $\bar{v}$ one uses (7), where $k_z = 0.6, k_v = 0.8, \bar{Z} = 30$.

The block diagram of the system for the control of aircrafts' roll angle is presented in fig. 6, while the block diagram of the reference model is the one from fig. 7 with $y = \varphi, y_c = \varphi_c$, $\bar{y} = \bar{\varphi}, \eta$ of form (45), $H_d(s)$ of form (64) and ε obtained with (67). One has chosen the initial values $\varphi(0) = 20\text{grd}$, $\dot{\varphi}(0) = 100\text{grd/s}$.

In fig. 8, the Matlab/Simulink model for the structure from fig. 6 is presented. The four subsystems ("NEURAL NET-

WORK r=2”, “Reference model r=2”, “Forming subsystem for vector deltaec (r=2)” and “Calculus subsystem for eps (r=2))” are presented in figures 9-12.

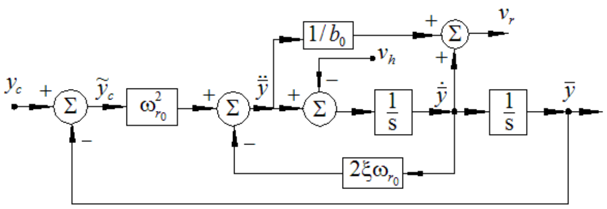

Fig. 7. The block diagram of the reference model

For $\varphi_c = 4^\circ; \bar{c} = [1\ \ 0]; \bar{b}^T = [0\ \ 1]$, one obtains

$$L = \begin{bmatrix} -0.13 \\ 0.11 \end{bmatrix}, P = \begin{bmatrix} 1.41 & 0.50 \\ 0.50 & 0.71 \end{bmatrix} \tag{73}$$

and matrices W and V after the neural network training are

$$W^T = [-20.7035\ -3.7136\ -1.7557\ -8.8042\ -6.8664\ -5.9634\ -6.1923\ -6.4364],$$

$$V = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.02 & 0.02 & 0.04 & 0.08 \\ 0 & 0 & 0.01 & 0.01 & 0.01 & 0.01 & 0.02 \\ 0.01 & 0.01 & 0.02 & 0.02 & 0.02 & 0.03 & 0.03 \\ -0.27 & -0.29 & -0.30 & -0.31 & -0.32 & -0.24 & -0.08 \\ 0.02 & 0.03 & 0.04 & 0.06 & 0.09 & 0.19 & 0.40 \\ 0.06 & 0.07 & 0.09 & 0.11 & 0.14 & 0.25 & 0.47 \\ 0.12 & 0.14 & 0.16 & 0.18 & 0.23 & 0.34 & 0.58 \end{bmatrix}.$$

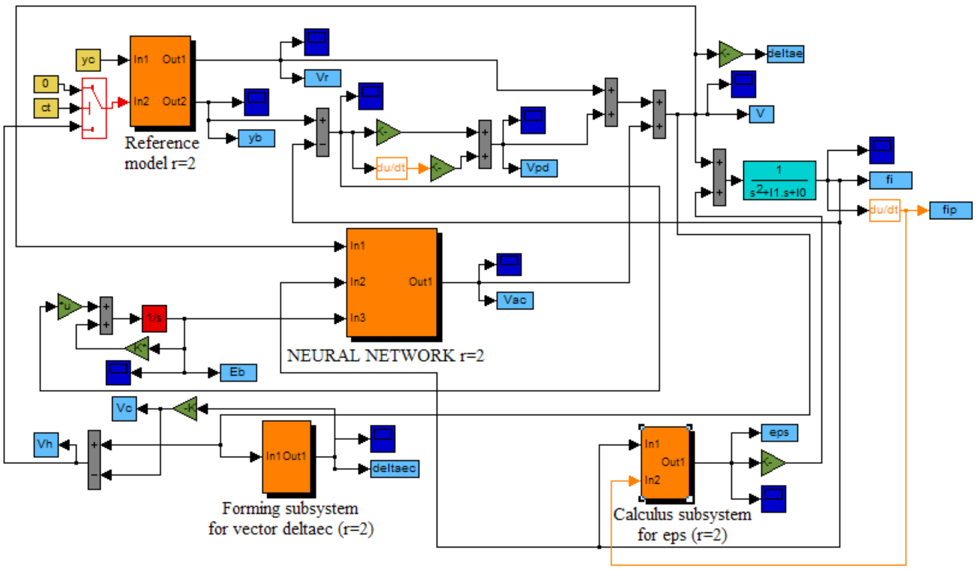

Fig. 8. Matlab/Simulink model for the structure from fig.6

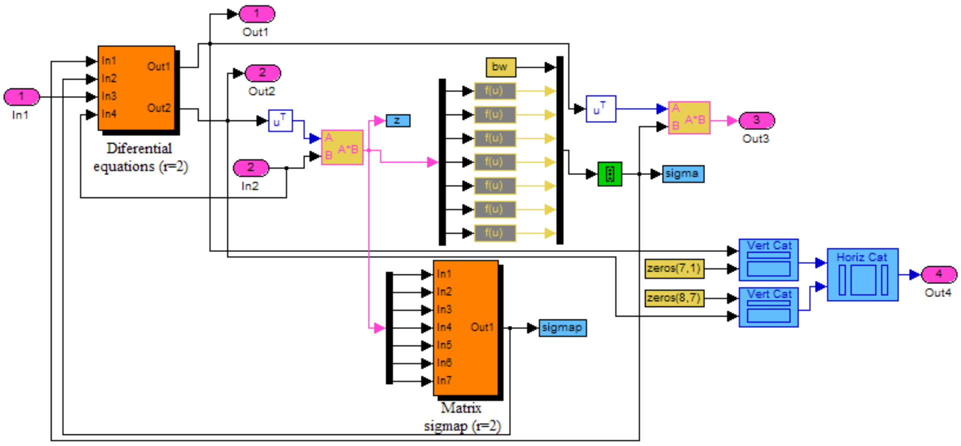

Fig. 9. Matlab/Simulink model for “NEURAL NETWORK r=2”

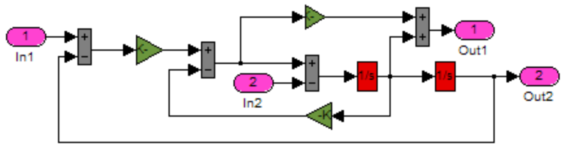

Fig. 10. Matlab/Simulink model for “Refecence model r=2”

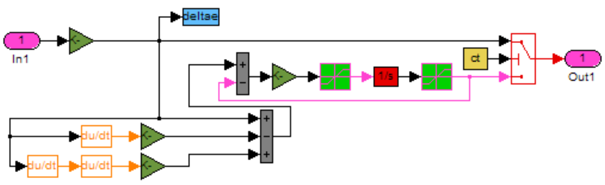

Fig. 11. Matlab/Simulink model of “Forming subsystem for vector deltaec (r=2)”

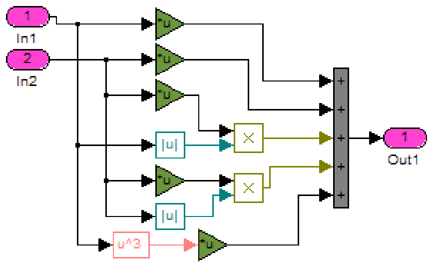

Fig. 12. Matlab/Simulink model of “Calculus subsystem for eps (r=2)”

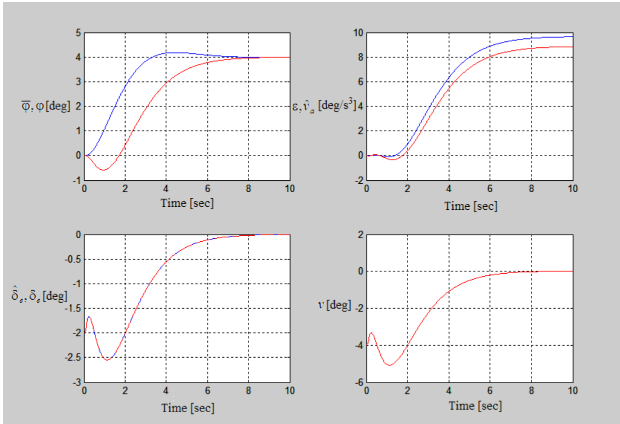

Fig. 13. Time characteristics in the case of linear actuator’s use

In fig. 13 the functions $\overline{\varphi}(t), \varphi(t), \varepsilon(t), \hat{v}_a(t), \hat{\delta}_e(t), \delta_e(t)$ and $v(t)$ ($\overline{\varphi}, \varepsilon, \hat{\delta}_e$ – with blue color, continuous line and $\varphi, \hat{v}_a, \delta_e$ with red color, dashed line) are presented.

If the actuator is non-linear one obtains the characteristics from fig. 14; additionally, characteristics $v_h(t)$ and $\dot{\varphi}(\varphi)$ appear. When $v_h = 0$ the actuator is in the saturation state and it works in the linear zone when $v_h \neq 0$. The characteristic $\dot{\varphi}(\varphi)$ (phase portrait of the system) shows that the non-linear system tends to a stable limit cycle.

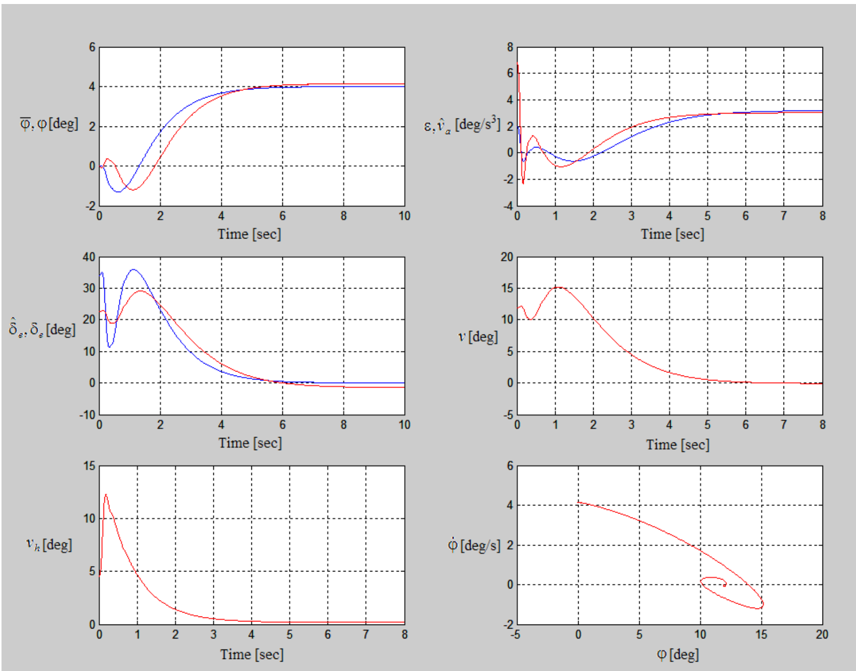

Fig. 14. Time characteristics in the case of non-linear actuator’s use

IV. CONCLUSIONS

The aim of the adaptive command is to compensate the dynamic inversion error. Thus, the command law has two components: the command given by the linear dynamic compensator and the adaptive command given by the neural network. As control system one chooses the non-linear model of aircrafts' dynamics in longitudinal plain. The reference model is linear. One obtains the structure of the adaptive control system of the roll angle and Matlab/ Simulink models of the adaptive command system's subsystems. Using these, some characteristics families are obtained; these describe the adaptive command system's dynamics with linear or non-linear actuator. The system is a stable one and has very good dynamic characteristics.

ACKNOWLEDGMENT

This work was supported by the strategic grant POSDRU/89/1.5/S/61968 (2009), co-financed by the European Social Fund within the Sectorial Operational Program Human Resources Development 2007 - 2013.

REFERENCES

[1] Chelaru T.V., Pana V. "Stability and Control of the UAV Formations Flight". *WSEAS Transactions on systems and Control,* Issue 1, vol. 5, January 2010.

[2] Yasemin I. "Pitch Rate Damping of an Aircraft by Fuzzy and Classical PD Controller". *WSEAS Transactions on Systems and Control,* Issue 7, vol. 5, July 2010.

[3] Chelaru T.V., Pana V., Chelaru A, " Dynamic and Flight Control of the UAV formations". *WSEAS Transactions on Systems and Control,* Issue 4, vol. 4, April 2010.

[4] A.J. Calise, "Flight Evaluation of an Adaptive Velocity Command System for Unmanned Helicopters". *AIAA Guidance, Navigation and Control Conference and Exhibit*, vol. 2, 11–14 August, 2003, Austin, Texas.

[5] S.M. Hosemi, M. Farrokhi, A.J. Kashkonei, "Robust Adaptive Control Systems Using Neural Networks". *The International Journal of Control*, vol. 3, 2006, 7 pp.

[6] E.N. Johnson, A.J. Calise, "Adaptive Guidance and Control for Autonomous Launch Vehicles". IEEE Aerospace Conference, Biy Ykg, MT, April, 2001, 13 pp.

[7] M. Lungu, " Flyght control systems". Sitech Publisher, Craiova, 2008, 329 pp.

[8] M. Lungu, R. Lungu, „Non-linear Adaptive Controllers with Linear Dynamic Compensator and Neural Network". Proceedings of the 11th WSEAS International Conference on Automatic Control, Modelling And Simulation (ACMOS '09), Istanbul, Turkey, May 30 - June 1, 2009, pp. 334 – 339.

[9] A.J. Calise, N. Hovakymyan, M. Idan, "Adaptive Output Control of Nonlinear Systems Using Neural Networks". *Automatica*, 37 (8), August, 2001, pp. 1201 – 1211.

[10] L.P. Gregory, "Adaptive Inverse Control of Plants with Disturbances". Stanford University, 1998.

[11] N. Hovakimyan, F. Nardi, N. Kim, A.J. Calise, "Adaptive Output Feedback Control of Uncertain Systems Using Single Hidden Layer Neural Networks". *IEEE Transactions on Neural Networks,* 13 (6), 2002, 22 pp.

[12] M. Sharma, A.J. Calise, "Adaptive Trajectory Control for Autonomous Helicopters". *AIAA Guidance, Navigation and Control Conference*, 14 – 17 August, 2001, Montreal, Canada, 9 pp.

[13] Johnson, E.N., Calise, A.J., Rysdyk, R., Shirbiny, El.A. „Feedback Linearization with Neural Network Augmentation Applied to X – 33 Attitude Control". *Proceedings of the AIAA Guidance, Navigation and Control Conference and Exhibit*, 14 – 17 August, 2000.

[14] Johnson, E.N., Calise, A.J. "Neural Network Adaptive Control of Systems with Input Saturation". *American Control Conference*, 2001, In Proceedings, 2001, 6 pp.

[15] N. Hovakimyan, N. Kim, A.J. Calise, J.V. Parasad, "Adaptive Output Feedback for High – Bandwidth Control of an Unmanned Helicopter". AIAA Guidance, Navigation and Control Conference, 6 – 9 August, Montreal, Canada, 11 pp.

[16] A.J. Calise, Y. Shin, M.D. Johnson, "Comparison Study of Conventional and Neural Network Based Adaptive Control of Wing Rock". AIAA Guidance, Navigation and Control Conference, Providence, RI, August, 2004. In Proceedings, 11 pp.