Relative gain array Interaction Analysis of UPFC Device for damping Oscillations

N. Magaji and M. W. Mustafa

Abstract: UPFC devices are used to improve real power, reactive power, improve bus voltage and eliminate line loses in ac systems. An additional task of UPFC is to enhance transmission capacity as result of power oscillation damping. This paper use relative gain array (RGA) for analyzing the interactions among the UPFC inputs/ output signals. RGA analyses for UPFC inputs with power oscillation damping controller and without POD controller are treated based on the multi-input multi-output (MIMO) transfer function matrix. With the help singular value and RGA, the dynamic input-output interactions are also analyzed. Then, the input signals for supplementary controller for oscillations damping is selected. A two area four machine power system with different operational conditions is simulated for the validations of the proposed approach.

Keywords—Flexible AC Transmission system, UPFC, MIMO, SISO, Power system oscillations, RGA.

I. Introduction

The Flexible AC Transmission Systems (FACTS) based on power electronics offer an opportunity to enhance controllability, stability, and power transfer capability of AC transmission systems[1, 2] . For this reason, control of FACTS devices has received a lot of attention in power system stability enhancement [1-5]. Eigenvalue sensitivities are one important outcome of the modal analysis and control of oscillatory behavior and dynamic stability in power systems. The pioneering work of [3]considers the local oscillation of a single machine by means of a transfer function model. The usually complex pattern of oscillations in a large power system can be studied through linear, time invariant, state-space models based on the perturbations of the system state variables from their nominal values at a specific operating point.

Flexible AC Transmission System (FACTS) Increase the trustworthiness of AC grids and reduce power delivery costs. They improve transmission class and efficiency of power transmission by providing inductive or reactive power to the grid. FACTS are electronic devices that offer dynamic control of the power system parameters such as: voltage, line impedance and phase angle [4, 22].

Power system oscillations occur due to the lack of damping torque at the generators rotors. Damping torque analysis is well recognized on the Phillips-Heffron model of single-machine infinite-bus power systems and based on the idea of damping torque contribution to the rotor motion of synchronous generators and classical control theory [3-20]. The oscillation of the generators rotors cause the oscillation of other power system variables (bus voltage, bus frequency, transmission lines active and reactive powers, etc.). Power system oscillations are usually in the range between 0.1 and 2 Hz depending on the number of generators involved in [5,24]. Local oscillations lie in the upper part of that range and consist of the oscillation of a single generator or a group of generators against the rest of the system. In contrast, inter-area oscillations are in the lower part of the frequency range and comprise the oscillations among groups of generators. In addition, power system oscillations exhibit low damping compared to oscillations found in other dynamic systems: an oscillation of 10% damping is commonly accepted as well damped. To improve the damping of oscillations in power systems, supplementary control laws can be applied to existing devices. These supplementary actions are referred to as power oscillation damping (POD) control.

Skogestad [6, 21] has explained detail on using RGA and condition number for controllability analysis. The following authors has done much in input-output controllability analysis of MIMO system [6, 7]

The interaction between FACTS normal control and the damping control function was reported by [8]. While selection of FACTS signals using RGA analysis has been reported by the following [9-11].
This paper detail RGA analysis for MIMO UPFC controller. The concepts of RGA peak, singular value decomposition, Right hand plane zeros and condition number related to RGA has been explained for UPFC connected in Kundur system or 11 bus system.

II. Power System and UPFC Model

A Two areas four machine with UPFC

In this study, a two area interconnected four machine power system shown in Fig.1 is considered. The system consists of four machines arranged in two areas inter-connected by a weak tie line [12].

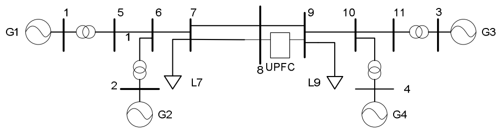

Fig. 1 Two area test system with TCSC

B UPFC Model

The Unified Power Flow Controller can provide simultaneous control of all basic power system parameters (transmission voltage, impedance and phase angle). The controller can fulfill functions of reactive shunt compensation, series compensation and phase shifting, meeting multiple control objectives. From a functional perspective, the objectives are met by applying a DC capacitor, shunt connected transformer and voltage source converter in parallel branch and dc capacitor, voltage source convertor and series injected transformer in the series branch The two voltage source converters are also called "back to back" AC to DC voltage source converters operated from a common DC link capacitor, Figure 2. The shunt converter is primarily used to provide active power demand of the series converter through the common DC link. Converter 1 can also generate or absorb reactive power, if it is desired, and thereby provides independent shunt reactive compensation for the line. Converter 2 provides the main function of the UPFC by injecting a voltage with controllable magnitude and phase angle in series with the line.

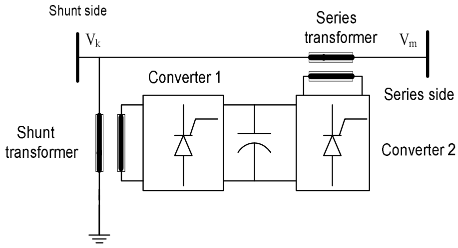

Fig. 2 UPFC back-to-back voltage source converters

The series voltage source and the shunt current source are defined as follows:

$$\bar{v}_S = (vp+jvq)e^{j\phi} = r\,\bar{V}_k e^{j\gamma} \quad (1)$$

The current of the shunt source is then given by as follow

$$\bar{i}_{SH} = (i_p+ji_q)e^{j\theta_k} \quad (2)$$

For lossless UPFC device, power in converter1 is equal to power in converter 2. Therefore, the active and reactive power supplied by the series voltage source is:

$$P_{km} = brV_kV_m sin(\gamma+\theta_k-\theta_m) - brV_k^2 \sin\gamma \quad (3)$$

$$Q_{km} = brV_kV_m cos(\gamma+\theta_k-\theta_m) + brV_k^2 cos\gamma + br^2V_k^2 \quad (4)$$

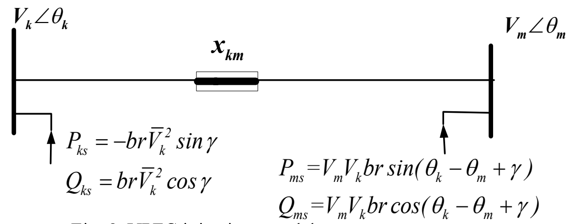

Fig. 3 UPFC injections model

The UPFC dc link capacitor dynamics can be expressed as follows with harmonics and UPFC losses neglected [13]:

$$CV_{DC}\frac{dV_{DC}}{dt} = \left(P_{conv1} - P_{conv2}\right) \quad (5)$$

Where Pconv2, the real power supplied by the series voltage source converter; C, the DC capacitor magnitude; r, compensating voltage ratio; and γ, phase difference between series injected voltage and UPFC bus voltage

1) UPFC main and supplementary control

For UPFC, normally there are three control functions to be performed; that are Power flow control, AC voltage control and DC voltage control. If a controller is assigned to each of these functions, then there are three possible controllers to be designed. This sort of arrangement can be seen in figure 4

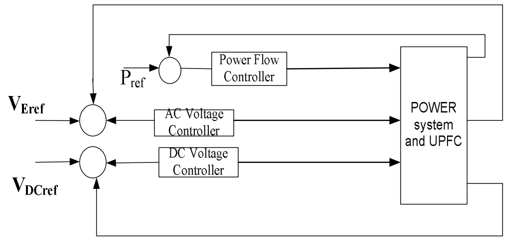

Fig. 4 UPFC MIMO Controller for

2) UPFC POD Controller Design

Supplementary control action applied to *UPFC* devices to increase the system damping is called Power Oscillation Damping (POD). In order to improve system damping, a supplementary control (POD) can be added with its output signal used to modulate the available signals either local or remote to provide damping effects. Figure 4 shows the considered closed-loop system where G(s) represents the power system including UPFC devices and H(s) *UPFC* POD controller. Figure 5 shows a power flow controller together with damping controller; in this paper the following are alternative for placing damping controller.

i) Superimposed with power flow controller
ii) Superimposed with AC voltage controller or
iii) Superimposed with DC voltage controller

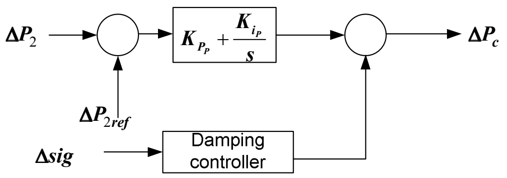

Fig. 5 PI- type power-flow controller with damping controller

Conventional controllers are used for normal UPFC control. In conventional method, P-I type controllers are considered for power-flow controller, voltage magnitude control and DC-voltage regulator. Figure 6 shows the block diagram P-I type power-flow controller and damping controller, similar structure is consider for voltage magnitude control and DC-voltage regulator control with damping controller.

III. Relative Gain Array (RGA) and NI

To measure the degree of coupling or interaction in a system, the concept of relative gain array can be used. The original technique is based upon the open loop steady state gains of the process and is relatively simple to interpret. The relative gain array was first introduced by Bristol[15]at steady state as the ratio of open loop and closed loop gains between input j and output i when all output $y_l \neq i$ are perfectly controlled using the inputs $u_h \neq j$.

$$\lambda_{ij} = \frac{\left(\partial y_i / \partial u_j\right) u_{k\neq j} constant}{\partial y_i / \partial u_j \big|_{yl\neq i} constant} = g_{ij}\left[G^{-1}(\mathbf{0})\right]_{ji} \quad (6)$$

$$\lambda_{ij} = \frac{open\text{-}loop\ gain}{closed\text{-}loop\ gain} \quad (7)$$

Consider a multivariable process transfer function matrix G (s) with inputs u and outputs y: Where G(s) is an m x n process transfer function

$$y(s)= G(s)u(s) \quad (8)$$

y(s) is an m x 1 output vector

u(s) is an n x 1 input vector.

Properties of RGA can be shortly described as follows:

- ✓ *RGA(i,i)=1* , there is no interaction with other control;
- ✓ *RGA(i,j)=* 0 , manipulated input *i* , does not affect the output *j* ;
- ✓ *RGA(i,j)=* 0.5, there is a high degree of interaction;
- ✓ 0.5 < *RGA(i,j)* < 1, there is an interaction between the control loops. However, this would be the preferable paring as it would minimize interactions;
- ✓ *RGA (i,j)* > 1, the interaction reduces the effect gain of the control loop. Higher controller gains are required;
- ✓ *RGA (i,j)* < 0, care must be taken with negative RGA elements. A negative off-diagonal element indicates that closing the loop will change the sign of effective gain.

For stability condition NI yields more information more than the RGA, because in the RGA the terms are combined into $\lambda_{ii} = \frac{g_{ii} detG^{ii}}{det\, G}$ so we may have cases where two negative determinants result in a positive RGA, element Nevertheless, the RGA is usually the preferred tool because it does not have to be recomputed for each pairing. Niederlinski index (NI) state that, if all n loops are closed, the multi-loop system will be unstable for all possible (any) values of controller parameters (i.e., it will be ‘‘structurally monotonic unstable’’), if the NI is negative [19], i.e.

$$NI = \frac{detG(j0)}{\prod_{i=1}^{n} g_{ii}(j0)} < 0 \qquad (9)$$

Where det[G(j0)] denotes the determinant of matrix G(j0). The sign of NI, i.e., NI > 0, provides a necessary stability condition and consequently, constitutes a complementary tool to the RGA in variable pairing selection.

RGA-Based loop pairing criteria

The pairing rules based on RGA and NI is that manipulated and controlled variables in a decentralized control system should be paired in such a way:

i) the paired RGA elements are closest to 1.0;
ii) the NI is positive,
iii) all paired RGA elements are positive; and
iv) Large RGA elements should be avoided.

A. RGA peak

The RGA peaks at a particular frequency indicate that the plant is difficult to control (or close to instability). (For example, for frequencies in the range from 0.2 to 2 Hz the RGA peaks identify electromechanical modes with small damping). Furthermore, for a given input and output signal, a high value of the RGA element at a particular frequency indicates the interaction between the corresponding input and output. From figure 6 the RGA peak of the entire diagonal element is closed to the bandwidth and RGA of Active power control loop (g11) is very small. Hence, this indicate an interaction for this loop with others therefore a damping controller with the input and the output linked with this RGA element could be used to get better the damping of the mode corresponding to this frequency [15].

B. Interaction between Control Loops

During the study of the interactions between voltage and power flow control loops, it was found that the RGA clearly identifies these interactions at the frequency of the electromechanical modes. Therefore, in accordance to [10, 15], the inverse-based controllers should not be used in voltage and power control loops of the system significantly involved in the electromechanical modes. This is especially true for the UPFC controller where the POD is to be installed.

C. Relation between RGA and Right Half Plane (RHP) Zeros

A theorem given in [5] represents a tool for identifying the RHP transmission zeros and transfer function zeros, through their relationship to the change of sign of the RGA gains between the steady state and high frequencies. This is an important issue as both RHP transmission and transfer function zeros are known to be problematic for the control [16]. From figure 6, the validity of this theorem was tested and it was found that the diagonal elements of the RGA of DC voltage control loop and AC voltage control loop change sign from steady state to high frequency indicating presence of RHP zeros in the system. This method however, does not guarantee the absence of the RHP zeros, as it provides only a sufficient condition for their existence.

D. Condition Number

The condition number has been used as an input and output controllability measure and in particular it has been assumed that ill-conditioned plants with a large condition number are often believed to be sensitive to uncertainty and will results in poor robust performance of the system [17].

$$\gamma(\mathrm{G}) = \frac{\bar{\sigma}(\mathrm{G})}{\underline{\sigma}(\mathrm{G})} \qquad (10)$$

A system with a large condition number (say,larger than 10) this may indicate control problems. For a nonsingular square matrix. $\underline{\sigma}(G) = 1/\bar{\sigma}(G^{-1})$ *so* $\gamma(G)=\bar{\sigma}(G)\bar{\sigma}(G^{-1})$. It then follows from equations 9 that the condition number is large if both G and G^{-1} have large elements [5].

$$\|A\|_{max} \le \bar{\sigma}(A) \le \sqrt{lm}\ \|A\|_{max} \qquad (10)$$

A large condition number may be caused by:

i) A small value of minimum singular $\underline{\sigma}$ which is generally undesirable
ii) A maximum singular value $\bar{\sigma}(G)$ need not necessarily be a problem
iii) large minimized condition number or large RGA elements which indicates fundamental control problems

From Figure 7, the minimum singular value is very small likewise from Figure 8, the condition number is relatively small at low frequency but it reach peak at a range of frequency which is a bandwidth frequency. This means that there will be a serious of problems in achieving control

E. *Diagonal dominance*

In RGA analysis the input and output variables should be paired so that the diagonal elements of the RGA are as close as possible to unity, thus shows less interaction. It is not desirable for a plant to have large RGA elements. The RGA can be used to measure diagonal dominance, by the simple quantity

$$\textit{RGA-number} = \|\Lambda(G) - \mathrm{I}\|_{sum} \qquad (11)$$

The lower the RGA number, the more preferred is the control structure. For decentralized control RGA-number close to 0 is prefer pairings at crossover frequencies. From Figure 9 it can be seen that RGA number is small at crossover frequency but not close to zero. Hence a damping controller is needed to achieve a required control.

F. *Selection of the feedback signals*

There is need for selecting feedback signal for supplementary controller of FACTS device for small signal stability. In general a high-quality feedback signal must have the following attractive properties:

i High sensitivity to the oscillatory modes,
ii Locally accessible, and

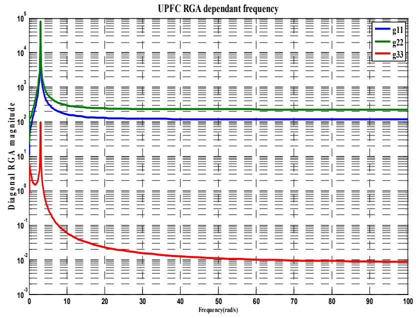

Fig.6 Frequency dependent RGA for UPFC

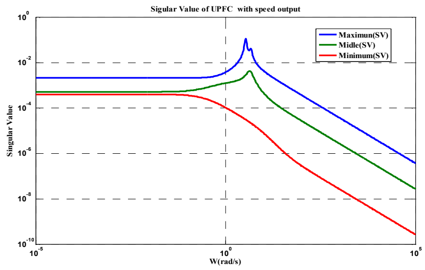

Fig. 7 Singular value of the UPFC

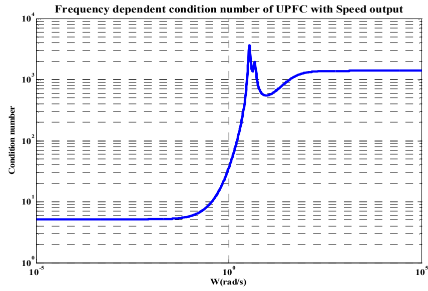

Fig.8 Frequency dependant condition number

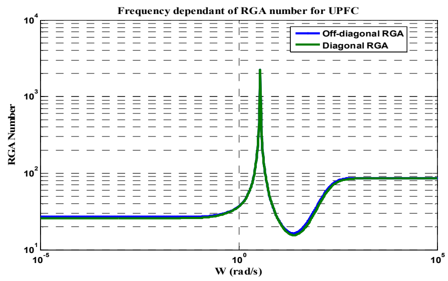

Fig. 9 RGA- Number for UPFC Controller

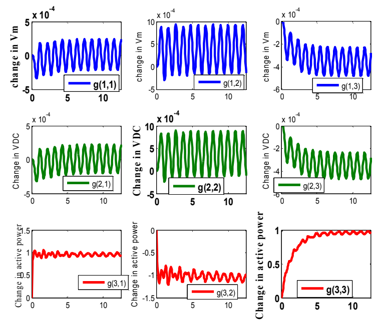

Fig. 10 step response of uncontrolled UPFC outputs for different input/output combination iii Small model uncertainties for a large range of working condition variations.

sensitivity means that the signal should be sensitive to the inter-area oscillations [23]; local accessibility implies the availability of the signal locally for the purpose of feedback control; model variation or model uncertainty is a vital consideration in damping controller design. The lesser the model uncertainties, the better closed-loop performance can be achieved. Since the dynamics of the power system changes with the change in the operating conditions, it is desirable to select a feedback signal with which the model variations under all possible operating conditions are minimal [23].

The best location of UPFC devices is line 10-11 which is not shown here. Four set of locally measurable local signals and one set of remote signal are considered. The possible choice of feedback damping signals for lines closer to location of UPFC device is as follows:

I5-6, I6-7, I7-8, I8-9, I10-9 and I10-11

P5-6, P6-7, P7-8, P8-9, P10-9 and P10-11

Q5-6, Q6-7, Q7-8, Q8-9, Q10-9 and Q10-11

V6, V7, V8, V9 and V10

ω_1- ω_3, ω_2- ω_3, and ω_4- ω_3

Where I= line current flow, P= real power, Q= reactive power ,V= Bus voltage and ω= rotor speed

RHP-zeros of free fault and post fault system are calculated for these selected signals and the results are shown in Table 1. The 'No' signifies that there is no encounter of RHP zero, while 'Yes' indicates the encounter of the RHP zero of the closed loop system with the selected signal. For free faults conditions, among the line currents in different lines, I7-8 is the only signal that encounter RHP zero hence is discarded and HSV analysis was carried out for the rest of the signals in this group. The HSV of these candidates are shown in Figures 11 to Figure 15. The RHP zeros results for the other categories of the signals from Table 1 gives P5-6, P6-7, P9-8, P9-10 and P10-11 were selected from the second group. Only Q9-10 of the Reactive power signals encountered RHP zero therefore the remaining signals in the third group candidates are selected; fourth group has V8 and V10 signals that meat RHP zeros therefore they are discarded and HSV analysis were carried out for the remaining signals. Speed deviation of all the signals does not encounter RHP zeros as a result all signals are chosen as candidates for group five.

TABLE I

Right hand pole zero encounter for 11 bus system with **UPFC Device**

S/N	Signal	Ff	Pf
1	I5-6	No	Yes
2	I6-7	No	Yes
3	I7-8	Yes	Yes
4	I9-8	No	Yes
5	I9-10	No	No
6	I10-11	No	yes
7	P5-6	No	No
8	P6-7	No	Yes
9	P7-8	No	No
10	P9-8	yes	yes
11	P9-10	No	No
12	P10-11	No	Yes
13	Q5-6	No	No
14	Q6-7	No	No
15	Q7-8	No	Yes
16	Q9-8	No	No
17	Q9-10	yes	yes
18	Q10-11	No	No
19	ω_1- ω_3	No	yes
20	ω_2- ω_3	No	No
21	ω_4- ω_3	No	No
22	V6	No	yes
23	V7	No	yes
24	V8	Yes	yes
25	V9	No	yes
26	V10	Yes	No

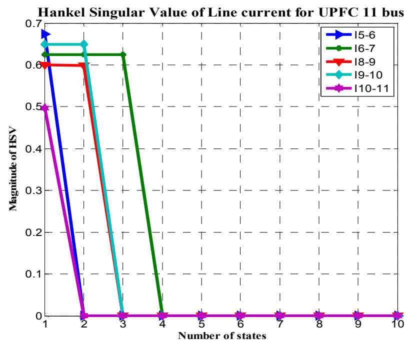

Fig. 11: HSV of Line current

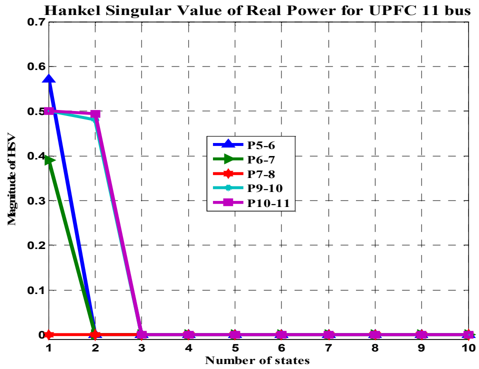

Fig. 12 HSV of Real Power

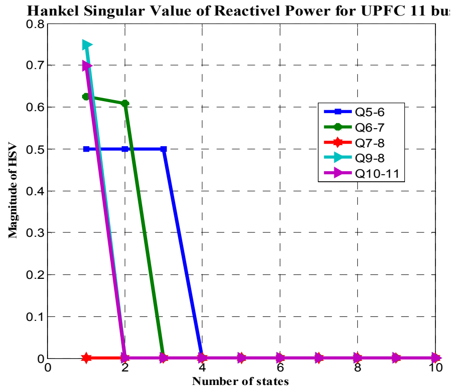

Fig. 13 HSV of Reactive

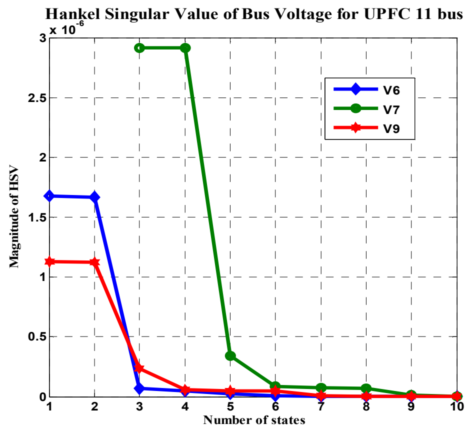

Fig. 14 HSV of Bus Voltage

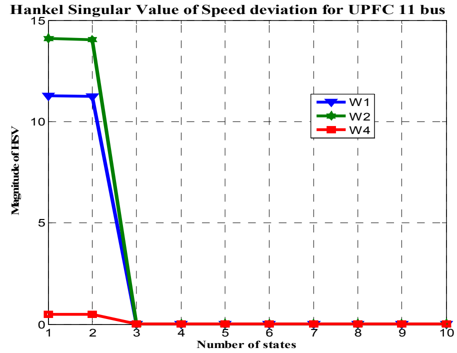

Fig. 16 HSV of Speed deviation

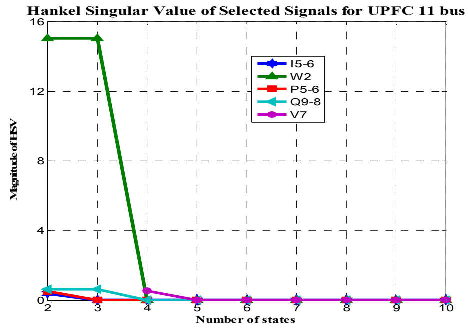

Fig. 15 HSV of selected Signals

IV Simulation results

The effectiveness of the proposed method of UPFC Controller designed was tested on two-area four -machine systems. Figure 10 shows the steps response of the system with all the possible combination of inputs and outputs.

A three phase fault is applied for second test model at the bus 8 and cleared after 74ms. The original system is restored upon the fault clearance. The transient stability performances of the system without UPFC controller and system with UPFC controller are shown in Figures 16-18. The oscillations of the system from Figure 16 to 18 are well damped with UPFC controller.

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Conclusion

This paper has reviewed methods for analysis and control of power system oscillations with

UPFC device based on RGA analysis of eigenstructure of MIMO linear model of the power system. Frequency dependent of condition number, RGA-number, and SVD related to RGA has been explained. Although eigenvalues based methods are very powerful, the complexity of the power system stability problem requires the complementary use of other methods such as non-linear time domain simulation. All the simulations were done with PST toolbox in Matlab environment.

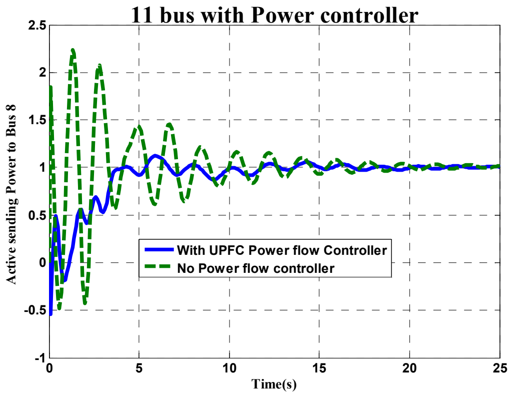

Fig.16 Active power flow with and without UPFC in line 8-9

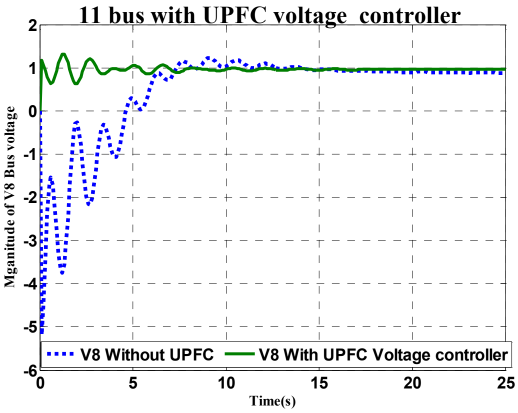

Fig.17 Bus 8 voltage magnitude with and without UPFC controller

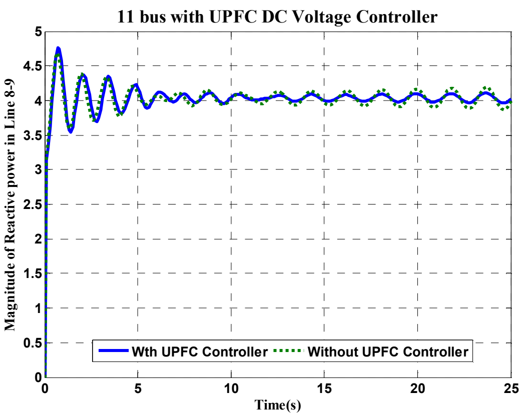

Fig.18 Reactive power response with and without UPFC controller for line 8-9

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Appendix A

Appendix A

Theorem A1: Consider a transfer function matrix with stable elements and no zeros or poles at s =0. Assume $\lim_{s\to\infty} \lambda_{ij}(s)$ is infinite and different from zero_ If $\lambda_{ij}(\infty)$ and $\lambda_{ij}(0)$ have different signs then at least one of the following must be true:

(a) The element gij (s) has a RHP-zero.

(b) The overall plant G(s) has a RHP-zero.

(c) The subsystem with input *j* and output *i* removed Gij (s) has a RHP zero.

Any such zero may be detrimental for decentralized control. In most cases the pairings are chosen such that $\lambda_{ij}(\infty)$ is positive (usually close to 1). Bristol!s claim that a negative $\lambda_{ij}(0)$ implies there is a RHP zero in some subsystem