Digital Control of Unstable and Integrating Time-delay Processes

V. Bobál, P. Chalupa, P. Dostál, and M. Kubalčík

Abstract—Time-delay (dead time) is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. A part of time-delay systems can be unstable or have integrating properties. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns or some types of chemical reactors.

This paper deals with a design of algorithms for digital control of the unstable and integrating time-delay processes using one suitable modification of the Smith Predictor (SP). This digital modification of the Smith Predictor is based on Linear Quadratic (LQ) method. A minimization of the quadratic criterion is realized using spectral factorization. The designed algorithms have universal usage; they are suitable for control of stable, non-minimum phase, unstable and integrating time-delay processes. The main contribution of this paper is design and simulation verification of this Smith Predictor for control of the unstable and integrating processes, because classical continuous-time Smith Predictors are not suitable for control of such processes. The designed algorithms for control of individual processes influenced by external disturbance were verified. The program system MATLAB/SIMULINK was used for simulation verification of designed algorithms.

Keywords—Digital control, Integrating process, LQ control, Polynomial approach, Simulation of control loops, Smith predictor, Time-delay, Unstable process.

I. Introduction

TIME-delay appear in many processes in industry and other fields, including economical and biological systems They are caused by some of the following phenomena [1]:

- the time needed to transport mass, energy or information,
- the accumulation of time lags in a great numbers of low order systems connected in series,
- the required processing time for sensors, such as analyzers; controllers that need some time to implement a complicated control algorithms or process.

The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robust stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

A part of time-delay systems can be unstable or have integrating properties. Most authors are designing continuous-time algorithms for control of such processes. Integrating and unstable processes with a time-delay often cannot be controlled by usual controllers designed without consideration of the dead-time. There are various ways to control such systems. Several tuning rules for PI or PID controllers in the classical feedback closed-loop continuous-time structure have been presented in literature for these systems, see e.g. [2] – [7]. But when processes include long time-delay, the performances of these classical controllers become worsen [8]. In these cases, the use of a time-delay compensator in the structure of the closed-loop control system can be available [9].

The first time-delay compensation algorithm was proposed by Smith [10] in 1957. This time-delay compensator (TDC) known as the Smith predictor (SP) contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm. Control results of a good quality can be achieved by modified Smith predictor methods, see e.g. in [11] – [17]. The control scheme 2DOF (Two Degrees Of Freedom) is used in [18] - [20]. The design of controllers using polynomial approach [21], [22] can be found in [23] and the control system structure with two feedback controllers is proposed in [24]. The idea of the IMC (Internal Model Control) is employed in [25].

The problems of continuous-time control of integrating or unstable time-delay systems including the robustness, disturbance rejection and the extension of suitable compensators have been analyzed in other articles, see e.g. [26] - [34].

Historically first modifications of time-delay algorithms were proposed for continuous-time (analog) controllers. In industrial practice the implementation of the time-delay compensators on analog technique was difficult. Therefore the Smith Predictors and its modified versions can be implemented since 1980s together with the use of microprocessors in the industrial controllers. In spite of the fact that all these algorithms are implemented in digital platforms, most of the literature analyzes, as mentioned above,

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only the continuous-time version.

The first digital time-delay compensators are presented e.g. in [35] – [38]. Some Self-tuning Controller (STC) modifications of the digital Smith Predictors (STCSP) are designed in [39] – [41]. Two versions of the STCSP were implemented into MATLAB/SIMULINK Toolbox [42], [43]. The scope of paper [44] is a design and an analysis of 2DOF discrete time-delay compensators for stable and integrating processes, the simple robust discrete time-delay compensator for unstable processes is proposed in [45].

It is well known that classical analog Smith Predictor is not suitable for control of unstable and integrating time-delay processes. The designed digital LQ Smith Predictor eliminates this drawback.

The paper is organized in the following way. The general problem of a control of the time-delay systems is described in Section 1. Nine types of the continuous-time unstable and integrating processes with time-delay, that were analyzed and simulated in the control-loop systems are introduced in Section 2. The principle of the digital Smith Predictor is described in Section 3. The primary LQ controller of the digital Smith Predictor is proposed in Section 4. The simulation verification of individual control-loops with their results are presented in Section 5. Section 6 concludes this paper.

II. Process Models

Consider a continuous-time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay L.

$$G_L(s) = G(s)e^{-Ls} \tag{1}$$

where $G(s)$ is an unstable or an integrating time-delay free part of the process and the transfer function of a pure transportation lag is e^{-Ls} , where s is complex variable. A more complete description of the process must include external disturbances, which are normally represented in the linear model as an additive signal at process output.

This paper presents digital control of the unstable second order systems and the integrating systems with time-delay which can be described by the following continuous-time transfer function:

1) System with one unstable pole:

$$G_1(s) = \frac{K}{(T_1 s+1)(T_2 s-1)} e^{-Ls} \tag{2}$$

2) System with two unstable poles:

$$G_2(s) = \frac{K}{(T_1 s-1)(T_2 s-1)} e^{-Ls} \tag{3}$$

3) Oscillatory unstable system:

$$G_3(s) = \frac{K}{T^2 s^2 + 2\xi T s - 1} e^{-Ls}; \quad 0 \langle \ \xi \ \langle \ 1 \tag{4}$$

4) Non-minimum phase unstable system with one unstable pole:

$$G_4(s) = \frac{K(T_3 s-1)}{(T_1 s+1)(T_2 s-1)} e^{-Ls} \tag{5}$$

5) Non-minimum phase unstable system with two unstable poles:

$$G_5(s) = \frac{K(T_3 s-1)}{(T_1 s-1)(T_2 s-1)} e^{-Ls} \tag{6}$$

6) Oscillatory non-minimum phase unstable system:

$$G_6(s) = \frac{K(T_3 s-1)}{T^2 s^2 + 2\xi T s - 1} e^{-Ls} \tag{7}$$

where K is static gain, T, T_1, T_2 are time constants. Parameter T_3 is the derivative component and ξ is damping factor.

The following integrating systems were chosen for verification of the proposed digital SP algorithm:

7) Integrating system with one stable pole:

$$G_7(s) = \frac{K}{s(T_1 s+1)} e^{-Ls} \tag{8}$$

8) Integrating system with one unstable pole:

$$G_8(s) = \frac{K}{s(T_1 s-1)} e^{-Ls} \tag{9}$$

9) Double integrating system:

$$G_9(s) = \frac{K}{s^2} e^{-Ls} \tag{10}$$

III. Digital Smith Predictor

The discrete versions of the SP and its modifications are more suitable for time-delay compensation in industrial practice. The block diagram of a digital SP (see [39], [40]) is shown in Fig. 1. The function of the digital version is similar to the classical analog version.

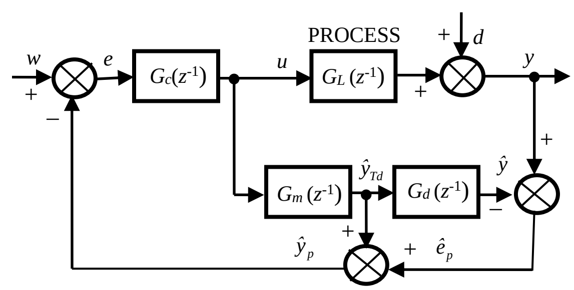

Fig. 1 Block diagram of a digital Smith Predictor

Number of higher order industrial processes can be approximated by a reduced order model with a pure time-delay. In this paper the following second-order linear model with a time-delay is considered

$$G_L(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \tag{11}$$

The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period.

The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The numerator in transfer function (11) is replaced by its static gain $B(1)$, i.e. for $z = 1$. This is to avoid problem of controlling a model with a $B(z^{-1})$, which has non-minimum phase zeros caused by a high sampling period or fractional delay. Since $B(z^{-1})$ is not controllable as in the case of a time-delay, it is moved out of the prediction model $G_m(z^{-1})$ and is treated together with the time-delay block, as shown in Fig. 1. The difference between the output of the process y and the model including time-delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown in Fig. 1, whereas e and v are the error and the measured disturbance, w is the reference signal. The primary (main) controller $G_c(z^{-1})$ can be designed by different approaches (for example digital PID control or methods based on polynomial approach). The outward feedback-loop through the block in Fig. 1 is used to compensate load disturbances and modelling errors.

For the second order model (11) first compensator has the form

$$G_m\left(z^{-1}\right)=\frac{\left(b_1+b_2\right)z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}=\frac{b_r z^{-1}}{1+a_1z^{-1}+a_2z^{-2}};\quad b_r=b_1+b_2 \tag{12}$$

and second compensator is given by the transfer function

$$G_d\left(z^{-1}\right)=\frac{b_1z^{-1}+b_2z^{-2}}{b_r z^{-1}}z^{-d} \tag{13}$$

A. Design of Primary Polynomial 2DOF Controller

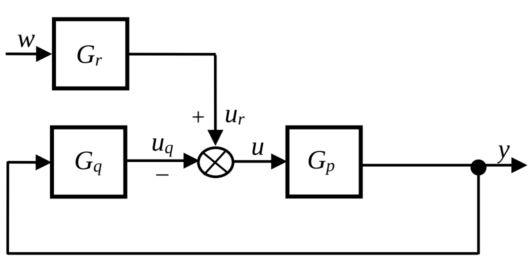

Fig. 2 Block diagram of a closed loop 2DOF control system

Polynomial control theory is based on the apparatus and methods of a linear algebra (see e.g. [21], [22], [46], [47]). The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 2.

The controlled process is given by the transfer function in the form

$$G_p(z^{-1})=\frac{Y(z)}{U(z)}=\frac{B(z^{-1})}{A(z^{-1})} \tag{14}$$

where A and B are the second order polynomials. The controller contains the feedback part G_q and the feedforward part G_r. Then the digital controllers can be expressed in the form of a discrete transfer functions

$$G_r\left(z^{-1}\right)=\frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)}=\frac{r_0}{\left(1+p_1z^{-1}\right)\left(1-z^{-1}\right)} \tag{15}$$

$$G_q\left(z^{-1}\right)=\frac{Q\left(z^{-1}\right)}{P\left(z^{-1}\right)}=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{\left(1+p_1z^{-1}\right)\left(1-z^{-1}\right)} \tag{16}$$

According to the scheme presented in Fig. 2 and equations (11) and (14) – (16), it is possible to derive the characteristic polynomial

$$A(z^{-1})P(z^{-1})+B_r(z^{-1})Q(z^{-1})=D_4(z^{-1}) \tag{17}$$

where $B_r(z^{-1}) = b_r z^{-1}$ and

$$D_4\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+d_4z^{-4} \tag{18}$$

is the fourth degree characteristic polynomial.

The procedure leading to determination of polynomials Q, R and P in (15) and (16) can be briefly described as follows (see [48]). A feedback part of the controller is given by a solution of the polynomial Diophantine equation (17). A feedback controller to control a second-order system with time-delay will be derived from equation (17). A system of linear equations can be obtained using the uncertain coefficients method

$$\begin{bmatrix} b_r & 0 & 0 & 1 \\ 0 & b_r & 0 & a_1-1 \\ 0 & 0 & b_r & a_2-a_1 \\ 0 & 0 & 0 & -a_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} d_1+1-a_1 \\ d_2+a_1-a_2 \\ d_3+a_2 \\ d_4 \end{bmatrix} \tag{19}$$

An asymptotic tracking is provided by a feedforward part of the controller given by a solution of the polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right)+B_r(z^{-1})R(z^{-1})=D_4(z^{-1}) \tag{20}$$

For a step-changing reference signal value, polynomial $D_w(z^{-1}) = 1 - z^{-1}$ and S is an auxiliary polynomial which does not enter into controller design.

For a step-changing reference signal value it is possible to solve equation (20) by substituting $z = 1$

$$R=r_0=\frac{D(1)}{B(1)}=\frac{1+d_1+d_2+d_3+d_4}{b_r} \tag{21}$$

The 2DOF controller output is given by

$$\begin{aligned} u(k)&=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)+\\ &\quad+\left(1-p_1\right)u(k-1)+p_1u(k-2) \end{aligned} \tag{22}$$

B. Minimization of LQ Criterion

The linear quadratic control methods try to minimize the quadratic criterion by penalization the controller output

$$J=\sum_{k=0}^{\infty}\left\{\left[w(k)-y(k)\right]^2+q_u\left[u(k)\right]^2\right\} \tag{23}$$

where q_u is the so-called penalization constant, which gives the rate of the controller output on the value of the criterion (where the constant at the first element of the criterion is considered equal to one). In this paper, criterion minimization will be realized through the spectral factorization for an input-output description of the system.

For the coefficients of the second order characteristic polynomial

$$D_2\left(z^{-1}\right)=1+d_1 z^{-1}+d_2 z^{-2} \tag{24}$$

of the closed loop the following expressions were derived [48]

$$d_1=\frac{m_1}{\delta+m_2};\ \ d_2=\frac{m_2}{\delta} \tag{25}$$

The parameters m_1, m_2 and δ are computed as follows:

$$\delta=\frac{\gamma+\sqrt{\lambda^2-4m_2{}^2}}{2};\ \ \gamma=\frac{m_0}{2}-m_2+\sqrt{\left(\frac{m_0}{2}+m_2\right)^2-m_1{}^2} \tag{26}$$

$$m_0=q_u\ (1+a_1{}^2+a_2{}^2)+b_r^2;\quad m_1=q_u(a_1+a_1 a_2);\quad m_2=q_u\ a_2$$

IV. Primary LQ Controller of Digital SP

From the previous paragraph, it is obvious that using analytical spectral factorization, only two parameters d_1 and d_2 of the second degree polynomial $D_2(z^{-1})$ can be computed. This approach is applicable only for control of processes without time-delay (out of Smith Predictor). The primary controller in the digital Smith Predictor structure requires usage of the fourth degree polynomial $D_4(z^{-1})$ (18) in equations (17) and (20). The polynomial $D_2(z^{-1})$ has two different real poles α, β or one complex conjugated pole $z_{1,2}=\alpha\pm j\beta$ (in the case of oscillatory systems). These poles must be included into polynomial $D_4(z^{-1})$ (18). A suitable pole assignment was designed for both types of the processes:

1st possibility:
Polynomial (18) has two different real poles α, β (computed from (24)) and user-defined real poles γ, δ. Then it is possible to write polynomial (18) as a product root of factor

$$D_4(z)=(z-\alpha)(z-\beta)(z-\gamma)(z-\delta) \tag{27}$$

and its individual parameters can be expressed as

$$\begin{aligned} d_1&=-(\alpha+\beta+\gamma+\delta)\\ d_2&=\alpha\beta+\gamma\delta+(\alpha+\beta)(\gamma+\delta)\\ d_3&=-\left[(\alpha+\beta)\gamma\delta+(\gamma+\delta)\alpha\beta\right]\\ d_4&=\alpha\beta\gamma\delta \end{aligned} \tag{28}$$

2nd possibility:
Polynomial (18) has the complex conjugate pole $z_{1,2}=\alpha\pm j\beta$ (computed from (24)) and user-defined real poles γ, δ. Then polynomial (18) has the form

$$D_4(z)=(z-\alpha-j\beta)(z-\alpha+j\beta)(z-\gamma)(z-\delta) \tag{29}$$

and its individual parameters can be expressed as

$$\begin{aligned} d_1&=-(2\alpha+\gamma+\delta)\\ d_2&=2\alpha\left(\gamma+\delta\right)+\alpha^2+\beta^2+\gamma\delta\\ d_3&=-\left[2\alpha\gamma\delta+\left(\alpha^2+\beta^2\right)\left(\gamma+\delta\right)\right]\\ d_4&=\left(\alpha^2+\beta^2\right)\gamma\delta \end{aligned} \tag{30}$$

The control algorithm based on the LQ control method contains the following steps:

The parameters of the polynomial $D_2(z^{-1})$ are computed according to equations (25) and (26).

If the polynomial (24) has the real poles α, β, its parameters are computed according to equations (28), otherwise, they are computed according to equations (30).

The controller parameters are computed using matrix equation (19) and equation (21).

The controller output is given by equation (22).

Penalization of the controller output is performed by setting $q_u\geq 0$.

With increased penalization constant, the amplitude of the controller output decreases and thereby, the flow of the process output is smoothened and any possible oscillations or instability are damped.

V. Simulation Verification and Results

Simulation is useful tool for the synthesis of control systems, allowing us not only to create mathematical models of a process but also to design virtual controllers in a computer. The mathematical models provided are sufficiently close to a real object that simulation can be used to verify the dynamic characteristics of control loops when the structure or parameters of the controller change. The models of the processes may also be excited by various random noise generators which can simulate the stochastic characteristics of the processes noise signals with similar properties to disturbance signals measured in the machinery. A simulation verification of the designed predictive algorithm was performed in MATLAB/SIMULINK environment. It is possible to influence the output of the process with the non-measurable disturbance d. The designed digital Smith Predictor has universal usage for control of a large group of processes with time-delay.

The following models with an exacting dynamic behavior were used for simulation experiments:

System with one unstable pole – (2)

$$G_1(s)=\frac{2}{(5s+1)(2s-1)}e^{-8s} \tag{31}$$

System with two unstable poles – (3)

$$G_2(s)=\frac{2}{(5s-1)(3s-1)}e^{-8s} \tag{32}$$

• Oscillatory unstable system – (4)

$$G_3(s)=\frac{2}{4s^2+2s-1}e^{-8s};\quad T=2;\quad \xi=0.5 \tag{33}$$

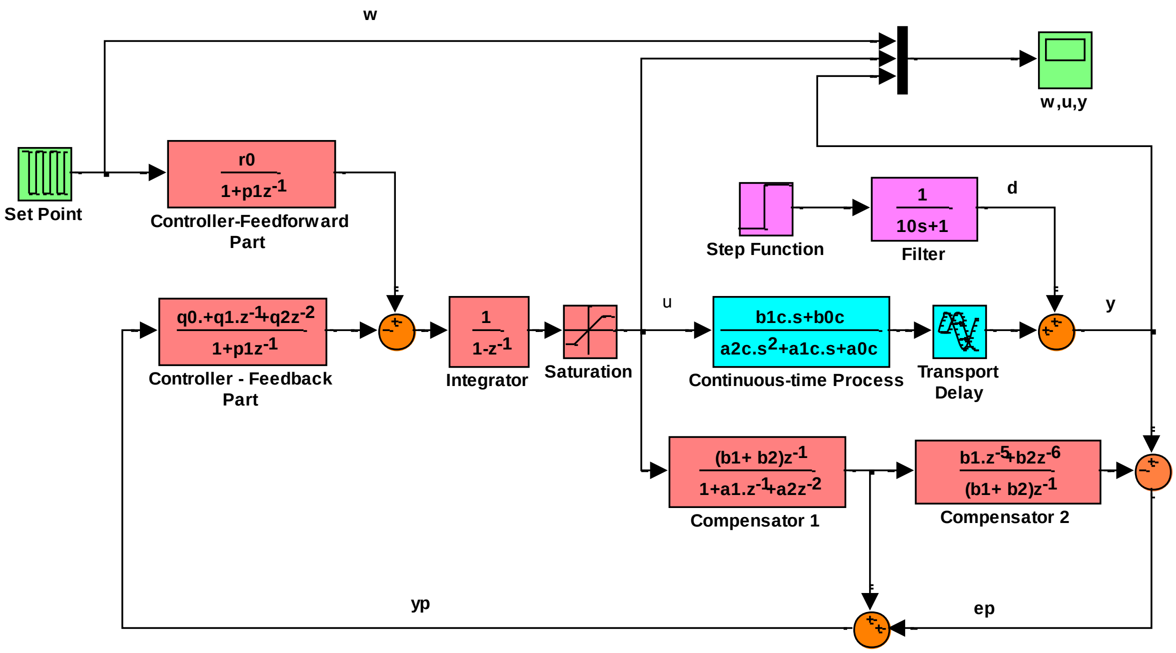

Fig. 3: Simulink control scheme

- Non-minimum phase unstable system with one unstable pole – (5)

$$G_4(s)=\frac{2(1-4s)}{(5s-1)(2s+1)}e^{-8s} \tag{34}$$

- Non-minimum phase unstable system with two unstable poles – (6)

$$G_5(s)=\frac{2(1-4s)}{(5s-1)(3s-1)}e^{-8s} \tag{35}$$

- Oscillatory unstable non-minimum phase system – (7)

$$G_6(s)=\frac{2(1-4s)}{4s^2+2s-1}e^{-8s} \tag{36}$$

Integrating system with one unstable pole – (8)

$$G_7(s)=\frac{2}{s(5s+1)}e^{-8s} \tag{37}$$

Integrating system with one unstable pole – (9)

$$G_8(s)=\frac{2}{s(5s-1)}e^{-8s} \tag{38}$$

(33)

- Double integrating system – (10)

$$G_9(s)=\frac{2}{s^2}e^{-8s} \tag{39}$$

Let us now discretize (31) - (39) using a sampling period $T_0=2\,\mathrm{s}$. The discrete forms of these transfer functions are (see (11))

$$G_1(z^{-1})=\frac{0.3414z^{-1}+0.2804z^{-2}}{1-1.8597z^{-1}+0.5488z^{-2}}z^{-4} \tag{40}$$

$$G_2(z^{-1})=\frac{0.3841z^{-1}+0.5482z^{-2}}{1-3.4396z^{-1}+2.9057z^{-2}}z^{-4} \tag{41}$$

$$G_3(z^{-1})=\frac{0.7946z^{-1}+0.5768z^{-2}}{1-2.0536z^{-1}+0.3679z^{-2}}z^{-4} \tag{42}$$

$$G_4(z^{-1})=\frac{-0.9431z^{-1}+1.5649z^{-2}}{1-1.8957z^{-1}+0.5488z^{-2}}z^{-4} \tag{43}$$

$$G_5(z^{-1})=\frac{-1.4369z^{-1}+2.3718z^{-2}}{1-3.4369z^{-1}+2.9057z^{-2}}z^{-4} \tag{44}$$

$$G_6(z^{-1})=\frac{-2.1695z^{-1}+3.5409z^{-2}}{1-2.0536z^{-1}+0.3679z^{-2}}z^{-4} \tag{45}$$

$$G_7(z^{-1})=\frac{0.7032z^{-1}+0.6155z^{-2}}{1-1.6703z^{-1}+0.6703z^{-2}}z^{-4} \tag{46}$$

$$G_8(z^{-1})=\frac{0.9182z^{-1}+1.0491z^{-2}}{1-2.4918z^{-1}+1.4918z^{-2}}z^{-4} \tag{47}$$

$$G_7(z^{-1})=\frac{0.7032z^{-1}+0.6155z^{-2}}{1-1.6703z^{-1}+0.6703z^{-2}}z^{-4} \tag{48}$$

The processes which are described by the above mentioned transfer functions were used in the Simulink control scheme for the verification of the dynamical behavior of the individual closed control loops. In time 500 – 800 s an exponential external disturbance

$$d(t) = 0.2\left(1 - e^{-0.1t}\right) \quad (37)$$

acted on the system output. The computed poles α, β and user-defined real poles γ, δ are introduced for individual simulation experiments including characteristic polynomial (18). For all experiments, the penalization factor was chosen $q_u = 1$.

A. *Simulation Control of Model* $G_1\left(z^{-1}\right)$

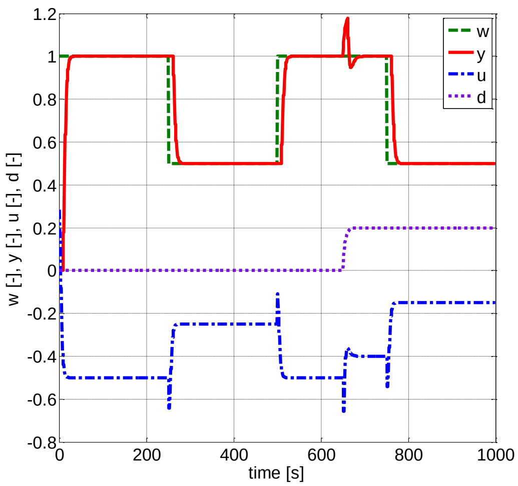

Fig. 4 Control of model $G_1\left(z^{-1}\right)$

The poles: $\alpha, \beta = 0.3912 \pm 0.1488i; \quad \gamma = 0.1; \quad \delta = 0.5$

The characteristic polynomial:

$$D_4(z) = z^4 - 1.3824z^3 + 0.6947z^2 - 0.1442z + 0.0088$$

The courses of the control variables are shown in Fig. 4.

B. *Simulation Control of Model* $G_2\left(z^{-1}\right)$

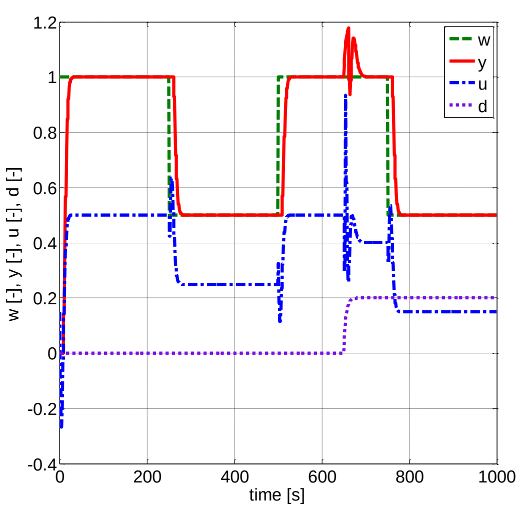

Fig. 5 Control of model $G_2\left(z^{-1}\right)$

The poles: $\alpha, \beta = 0.4737 \pm 0.1826i; \quad \gamma = 0.1; \quad \delta = 0.75$

The characteristic polynomial:

$$D_4(z) = z^4 - 1.5473z^3 + 0.8761z^2 - 0.2020z + 0.0129$$

The courses of the control variables are shown in Fig. 5.

C. *Simulation Control of Model* $G_3\left(z^{-1}\right)$

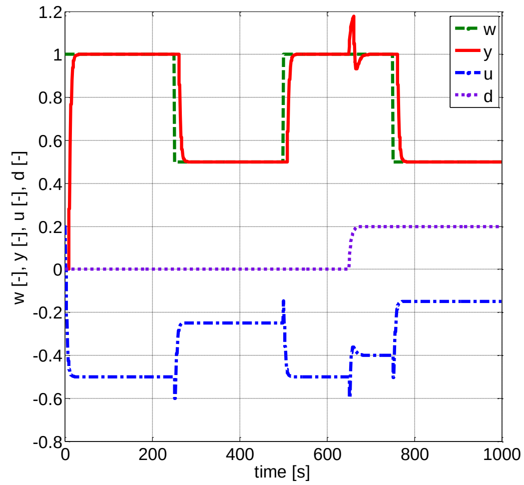

Fig. 6 Control of model $G_3\left(z^{-1}\right)$

The poles: $\alpha, \beta = 0.2188 \pm 0.1137i; \quad \gamma = 0.1; \quad \delta = 0.5$

The characteristic polynomial:

$$D_4(z) = z^4 - 1.0375z^3 + 0.3733z^2 - 0.0583z + 0.0030$$

The courses of the control variables are shown in Fig. 6.

D. *Simulation Control of Model* $G_4\left(z^{-1}\right)$

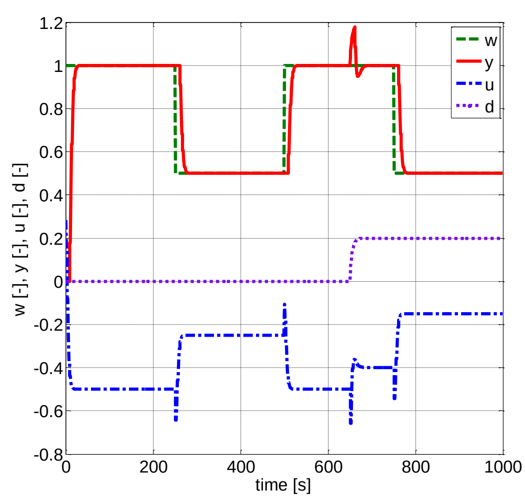

Fig. 7 Control of model $G_4\left(z^{-1}\right)$

The poles: $\alpha, \beta = 0.3912 \pm 0.1488i; \quad \gamma = 0.1; \quad \delta = 0.75$

The characteristic polynomial:

$$D_4(z) = z^4 - 1.6324z^3 + 0.9153z^2 - 0.2076z + 0.0131$$

The courses of the control variables are shown in Fig. 7.

E. Simulation Control of Model $G_5\left(z^{-1}\right)$

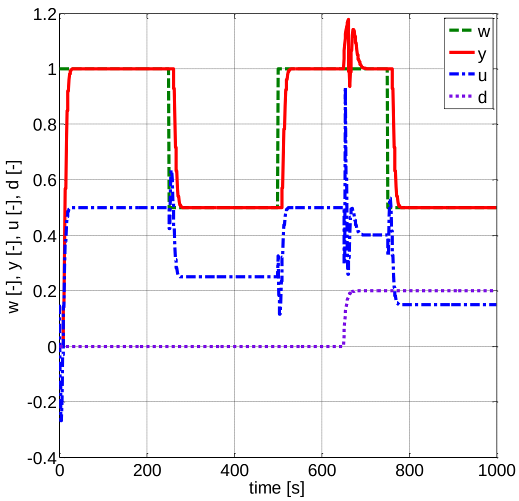

Fig. 8 Control of model $G_5\left(z^{-1}\right)$

The poles: $\alpha,\beta = 0.4737 \pm 0.1826i; \quad \gamma = 0.1; \quad \delta = 0.5$
The characteristic polynomial:

$$D_4\left(z\right) = z^4 - 1.5473z^3 + 0.8761z^2 - 0.2020z + 0.0129$$

The courses of the control variables are shown in Fig. 8.

F. Simulation Control of Model $G_6\left(z^{-1}\right)$

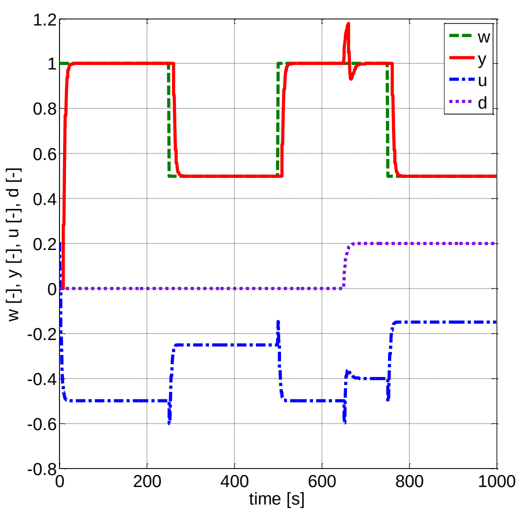

Fig. 9 Control of model $G_6\left(z^{-1}\right)$

The poles: $\alpha,\beta = 0.2188 \pm 0.1137i; \quad \gamma = 0.1; \quad \delta = 0.5$
The characteristic polynomial:

$$D_4\left(z\right) = z^4 - 1.0375z^3 + 0.3733z^2 - 0.0583z + 0.0030$$

The courses of the control variables are shown in Fig. 9.

G. Simulation Control of Model $G_7\left(z^{-1}\right)$

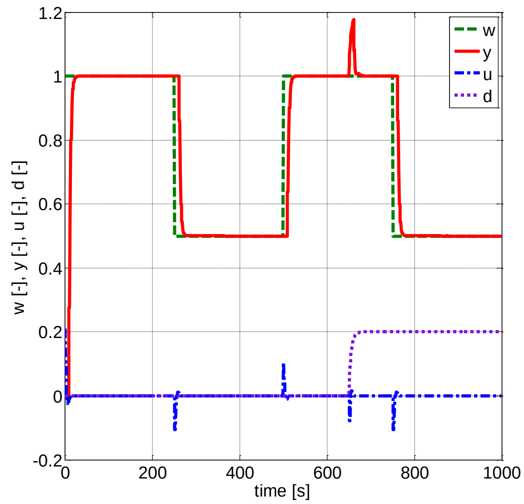

Fig. 10 Control of model $G_7\left(z^{-1}\right)$

The poles: $\alpha,\beta = 0.2652 \pm 0.2752i; \quad \gamma = 0.1; \quad \delta = 0.5$
The characteristic polynomial:

$$D_4\left(z\right) = z^4 - 1.1305z^3 + 0.5144z^2 - 0.1142z + 0.0073$$

The courses of the control variables are shown in Fig. 10.

H. Simulation Control of Model $G_8\left(z^{-1}\right)$

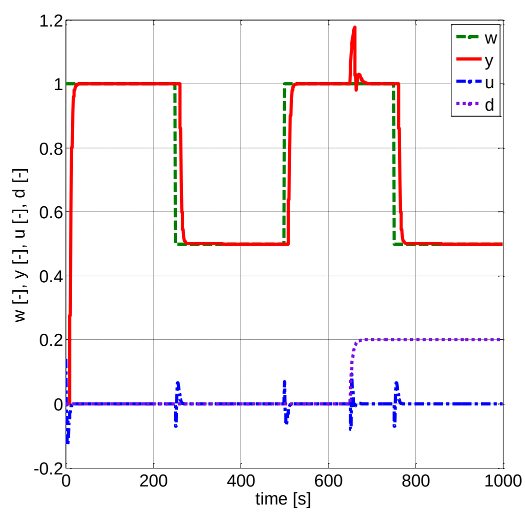

Fig. 11 Control of model $G_8\left(z^{-1}\right)$

The poles: $\alpha,\beta = 0.2652 \pm 0.2752i; \quad \gamma = 0.1; \quad \delta = 0.5$
The characteristic polynomial:

$$D_4\left(z\right) = z^4 - 1.1305z^3 + 0.5144z^2 - 0.1142z + 0.0073$$

The courses of the control variables are shown in Fig. 11.

I. Simulation Control of Model $G_9\left(z^{-1}\right)$

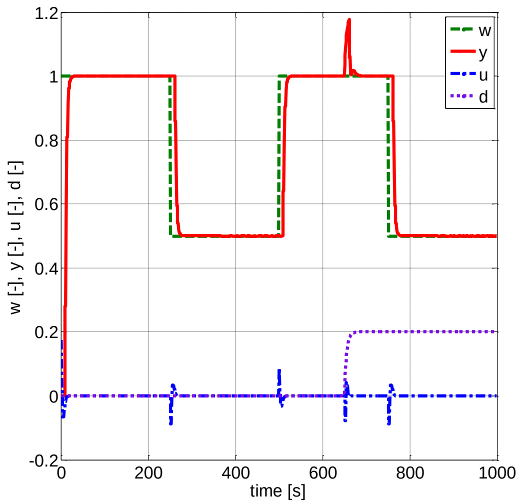

Fig. 12 Control of model $G_9\left(z^{-1}\right)$

The poles: $\alpha,\beta = 0.2640 \pm 0.2870i;\quad \gamma = 0.1;\quad \delta = 0.5$

The characteristic polynomial:

$$D_4\left(z\right) = z^4 - 1.1281z^3 + 0.5190z^2 - 0.1177z + 0.0076$$

The courses of the control variables are shown in Fig. 12.

It is obvious from Figs. 4 to 12 that the courses of the control variables have very good time behavior, the control quality is also very good in all cases. The designed controllers eliminate satisfactorily an influence of the non-measurable disturbance *d*.

VI. Conclusion

The paper presents a new unified approach for design of the digital LQ Smith Predictor for control of unstable and integrating systems with time-delay. The primary controller is based on minimization of the linear quadratic criterion. Minimization of the criterion is realized through spectral factorization. This controller was derived purposely by analytical way (without utilization of numerical methods) to obtain algorithms with easy implementability in industrial practice. Nine models (six unstable and three integrating) were used for simulation verification. Main contribution of the designed method is the universal applicability of this Smith Predictor for digital control of a large spectrum of processes (stable, unstable, non-minimum phase, integrating) with time-delay.

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