Voltage Calculation in Periodically Grounded Multiconductor Transmission Lines

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Abstract— In this paper, the propagation of voltages and currents in lossless multiconductor transmission lines with one or more conductors periodically grounded is discussed. An general procedure for the computation of the characteristic impedance matrix is shown. The presented procedure is efficient, and can be applied to an arbitrary number of ungrounded and grounded conductors. Numerical results are shown for a typical line configuration, considering a direct lightning on the shield wire and discussing the effect that the termination has on the voltage propagation.

Keywords—Characteristic impedance, Direct lightning, Multiconductor transmission line, Nonsymmetric Algebraic Riccati Equation (NARE), Periodical grounding.

I. Introduction

SEVERAL numerical tools are nowadays available for the analysis of power lines, based on different solution methods and including several typologies of devices connected to the lines. They are able to consider networks of multiconductor transmission lines (MTL) with different kinds of conductors' geometries and complex configurations [1-5].

All the numerical tools have the common constrain to consider finite length lines. When the reference problem is an infinite length line, the problem is overcome by trimming the MTL to a chosen distance and by substituting each remaining semi-infinite part with its characteristic impedance matrix. If the MTL is not grounded on any wire, at the termination the characteristic impedance matrix will produce no reflections, just like the equivalent semi-infinite line.

However, in several practical applications the MTL have some periodically grounded wires. This is common in transmission lines, where periodically grounded shield wires are placed over the power wires in order to intercept direct lightning [6], and also in distribution lines, for the mitigation of induced overvoltages due to indirect lightning [7-9].

In all the applications where an MTL is periodically grounded on one or more wires, the characteristic impedance matrix is significantly different with respect to the non-grounded case and plays a different role [10]. In fact, when a signal propagates along such a line, reflections occur at the periodical grounding points. In addiction, part of the current flowing trough the grounded wires is deviated to the ground at each grounding point. So, in order to be equivalent to the semi-infinite line, the characteristic impedance matrix must have a proper frequency behavior, that have to reproduce all the expected reflections and attenuations [11].

The particular behavior of the characteristic impedance matrix of an MTL with some periodically grounded wires is often neglected in numerical simulations. The characteristic impedance matrix of the non-grounded MTL is generally adopted for the grounded line. Therefore numerical simulations may be affected by an error due to the improper modeling of the system. The error occurs since the MTL terminations do not reproduce the correct reflections.

In this paper we show a general procedure to compute the characteristic impedance matrix of a multiconductor transmission line periodically grounded on some wires. Then, for a practical configuration, we consider the direct lightning of the shield wire and we compute the voltages along the line, showing the different results obtained considering exact and improper line terminations.

II. Voltages and Currents Propagation

Let us consider a semi-infinite MTL with p non-grounded wires and s periodically grounded wires, as shown in Fig. 1. Then we name with $m=p+s$ the total number of wires.

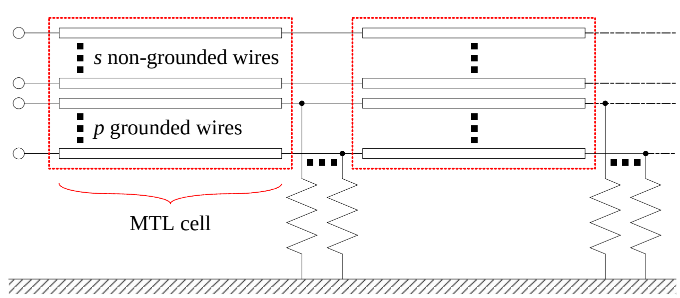

Fig. 1 Scheme of the MTL cells with the periodical grounding.

The MTL is assumed to be lossless, this is acceptable due to the small length of each MTL cell (no more than some hundreds of metres). It has been already shown that, for practical values of the p.u.l. resistance, its influence on the characteristic impedance matrix is almost neglectable [12].

Due to this assumption, the per-unit-length inductance and capacitance matrixes $\mathbf{L}$ and $\mathbf{C}$ satisfy the relationship $\mathbf{LC} = \mathbf{1}/c^2$, being $\mathbf{1}$ an identity matrix and c the speed of light in the free space. In addition, the characteristic impedance of

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the non-grounded MTL can be analytically expressed as $\dot{\mathbf{Z}}_0 = \mathbf{L}c = \mathbf{C}^{-1}/c$. The wires are numbered, independently from the effective spatial position, so that the wires from *1* to *p* are non-grounded and from *p+1* to *m* are periodically grounded.

Let us define with $\overline{\mathbf{V}}(z)$ and $\overline{\mathbf{I}}(z)$ the vectors of the voltages and currents of the different wires in the frequency domain, respectively. Among all the voltage and current vectors, the ones calculated in correspondence of the grounding points are the most relevant ones. Therefore we call them as $\overline{\mathbf{V}}_n = \overline{\mathbf{V}}(z = n\ell)$ and $\overline{\mathbf{I}}_n = \overline{\mathbf{I}}(z = n\ell)$ with n = 0, 1, …, being ℓ the distance between two grounding points.

Then, let us define as R_g the grounding resistance of the wires and **G** an *m*x*m* matrix where all the elements are zero but the last *s* ones on the main diagonal, namely $G_{i,i} = 1/R_g$ with $i = p+1, \ldots, m$.

With these assumptions it is possible to express the chain matrix of the n-th elementary cell, namely

$$\begin{pmatrix} \overline{\mathbf{V}}_n \\ \overline{\mathbf{I}}_n \end{pmatrix} = \mathbf{T} \begin{pmatrix} \overline{\mathbf{V}}_{n-1} \\ \overline{\mathbf{I}}_{n-1} \end{pmatrix} , \tag{1}$$

where

$$\mathbf{T} = \begin{pmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{pmatrix} = \begin{pmatrix} \cos(\varpi)\mathbf{1} & -\sin(\varpi)\dot{\mathbf{Z}}_0 \\ -\cos(\varpi)\mathbf{G} - j\sin(\varpi)\dot{\mathbf{Z}}_0^{-1} & j\sin(\varpi)\mathbf{G}\dot{\mathbf{Z}}_0 + \cos(\varpi)\mathbf{1} \end{pmatrix} \tag{2}$$

being $\varpi = \omega\ell/c$ the normalised frequency.

If a finite length MTL is considered, including *N* cells, it has to be connected to a termination network. In general we call $\dot{\mathbf{Z}}_N$ the impedance matrix representing the termination network, then at the end of the line it is of course verified the relation

$$\overline{\mathbf{V}}_N = \dot{\mathbf{Z}}_N \overline{\mathbf{I}}_N . \tag{3}$$

In order to evaluate the voltages and currents in this configuration, according to (1) and to the properties of the chain matrix, it is valid

$$\begin{pmatrix} \overline{\mathbf{V}}_N \\ \overline{\mathbf{I}}_N \end{pmatrix} = \mathbf{T}^N \begin{pmatrix} \overline{\mathbf{V}}_0 \\ \overline{\mathbf{I}}_0 \end{pmatrix} . \tag{4}$$

If we define

$$\mathbf{S} = \begin{pmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{pmatrix} = \mathbf{T}^N , \tag{5}$$

yielding the (3), then it is found that

$$\overline{\mathbf{I}}_0 = \left(\dot{\mathbf{Z}}_N \mathbf{S}_{22} - \mathbf{S}_{12}\right)^{-1} \left(\mathbf{S}_{11} - \dot{\mathbf{Z}}_N \mathbf{S}_{21}\right) . \tag{6}$$

So once the initial voltages $\overline{\mathbf{V}}_0$ or currents $\overline{\mathbf{I}}_0$ are known, relations (6) and (1) allow to compute the voltages and currents at every grounding point.

A. Computational remarks

Although the procedure in (4)-(6) is formally correct, since some of the eigenvalues of the matrix **T** are greater than 1, for high values of *N* some elements **S** diverge and so the computation of (6) may be numerically unstable.

To overcome this problem, we can consider that, at the generic grounding point at distance $z = i\,\ell$, it is verified the relation

$$\overline{\mathbf{V}}_i = \dot{\mathbf{Z}}_i \overline{\mathbf{I}}_i , \tag{7}$$

being $\dot{\mathbf{Z}}_i$ the generic equivalent impedance at distance $i\,\ell$. From (2), it is possible to obtain a recurrence relation that allows to compute the succession of the equivalent impedance matrixes, according to

$$\dot{\mathbf{Z}}_{i-1} = \left(\mathbf{T}_{11} - \dot{\mathbf{Z}}_i \mathbf{T}_{21}\right)^{-1} \left(\dot{\mathbf{Z}}_i \mathbf{T}_{22} - \mathbf{T}_{12}\right) . \tag{8}$$

So, starting from the termination matrix $\dot{\mathbf{Z}}_N$, it is possible to iteratively compute all the other equivalent matrixes. This procedure leads to the same results obtained using the (4)-(6), but avoiding numerical problems.

B. Considerations on the line termination

From the previous equations it is clear that the termination matrix $\dot{\mathbf{Z}}_N$ plays a key role in the computation of the voltages and currents.

If an ungrounded MTL is terminated on its characteristic impedance matrix $\dot{\mathbf{Z}}_0$, then the line will effectively have the behavior of a semi-infinite line. Then, if an MTL with some periodically grounded conductors is terminated on its proper characteristic impedance matrix, then proper reflections will occur at the terminations and the line will have again the behavior of a semi-infinite line.

Otherwise a different behavior of the voltages and currents is found, due to the incorrect reflections occurring at the terminations.

In this paper we investigate the influence of the termination matrix on the voltages and currents evaluations. However, in order to perform the comparisons, the characteristic impedance matrix of the considered network has to be computed at first. This task is not so trivial, since it requires the solution of second order matrix equation.

III. Correct Computation of the Characteristic Impedance Matrix

The calculation of the characteristic impedance matrix that includes the effect of the periodical grounding is not a trivial task [13]. In order to simplify the procedure, it is possible to introduce a similarity transformation in order to decouple the transmission line phasor equations. In this case we introduce the transformations $\mathbf{T}_i = \sqrt{c/\mathbf{L}}\,\mathbf{T}_\mathrm{i}$ and $\mathbf{T}_v = \mathbf{T}_i^{-1}$, and so the modal voltages $\tilde{\mathbf{V}} = \mathbf{T}_v\overline{\mathbf{V}}$ and currents $\tilde{\mathbf{I}} = \mathbf{T}_i\overline{\mathbf{I}}$. It is worth noting that the chosen transformations are frequency independent and are applicable for every line configuration.

In the transformed domain the characteristic impedance of the non-grounded MTL cell becomes an unitary matrix, i.e. $\mathbf{T}_v\dot{\mathbf{Z}}_0\mathbf{T}_i^{-1} = \mathbf{1}$, this simplify the chain matrix (2). By mean of this similarity transformation, the chain matrix (2) becomes

$$\tilde{\mathbf{T}} = \begin{pmatrix} \cos(\varpi)\mathbf{1} & -\sin(\varpi)\mathbf{1} \\ -\cos(\varpi)\tilde{\mathbf{G}} - j\sin(\varpi)\mathbf{1} & j\sin(\varpi)\tilde{\mathbf{G}} + \cos(\varpi)\mathbf{1} \end{pmatrix} \quad , \qquad (9)$$

being $\tilde{\mathbf{G}} = \mathbf{T}_i\mathbf{G}\mathbf{T}_v^{-1} = c\sqrt{\mathbf{L}}\mathbf{G}\sqrt{\mathbf{L}}$. Despite the matrix $\mathbf{G}$ is almost empty, the transformed matrix $\tilde{\mathbf{G}}$ is a singular real positive full matrix.

If we introduce a the periodically grounded MTL characteristic impedance matrix in the modal domain $\tilde{\mathbf{Z}}_c$, it verifies the relation $\tilde{\mathbf{V}}_k = \tilde{\mathbf{Z}}_c\tilde{\mathbf{I}}_k$ for every k = 0, 1, … . Then, with some simple manipulations, from (9) it is found

$$\tilde{\mathbf{Z}}_c\left[1 - j\cot(\varpi)\tilde{\mathbf{G}}\right]\tilde{\mathbf{Z}}_c - \tilde{\mathbf{Z}}_c\tilde{\mathbf{G}} - \mathbf{1} = \mathbf{0} \quad , \qquad (10)$$

for every $\varpi \neq k\pi$, with k = 0, 1, …

So the expression of the characteristic impedance matrix can be found by solving a Nonsymmetric Algebraic Riccati Equations (NARE) with complex coefficients [14-17]. Riccati equation are second order non-linear equations that appears in several problems of physics and engineering [18-26]: the non-symmetric ones with complex coefficients are probably the worst cases and the less studied in literature. It is not possible to find an analytical solution of such a problem, however a numerical procedure can be specifically applied for (10).

At first, it is possible to make some consideration on the solution. It is possible to observe that all the quantities have a periodical dependence by the frequency, so even $\tilde{\mathbf{Z}}_c$ will have a periodical behavior, being the period $\Delta\varpi = \pi$, that is to say $\Delta\omega = \pi c/\ell$. This means that, once the solution if found in one period, all the spectrum is computed. This result also implies that the characteristic impedance matrix can't be represented in terms of just rational functions and so it can be synthesized in time domain as a purely passive network.

It is possible to find a solution of (10) by studying the Hamiltonian matrix associated to the equation. It is a *m*x*m* matrix defined as:

$$\mathbf{H} = \begin{pmatrix} \tilde{\mathbf{G}} & 1 - j\cot(\varpi)\tilde{\mathbf{G}} \\ \mathbf{1} & \mathbf{0} \end{pmatrix} \quad . \qquad (11)$$

It can be found that (11) can be always diagonalized for $\varpi \neq k\pi$. Then, by analyzing the eigenvalues of (11), it comes out that, on varying of the frequency, there are:

- *p* distinct complex eigenvalues with phase between 0 and π;
- *p* distinct complex eigenvalues with phase always between 0 and –π;
- the real eigenvalue +1 with algebraic multiplicity *s*;
- the real eigenvalue –1, with algebraic multiplicity *s*.

Now, support to sort the eigenvectors' matrix placing in the first *s* columns the ones corresponding to the eigenvalue –1, that the *p* eigenvectors corresponding to

- the complex eigenvalues with positive phase if $k \le \varpi \le 0.5 + k$, for k = 0, 1, …;
- the complex eigenvalues with negative phase if $0.5 + k \le \varpi \le 1 + k$, for k = 0, 1, …;

and finally the remaining eigenvectors.

According to this choice, let us represent the corresponding eigenvector matrix as

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{U}_{21} & \mathbf{U}_{22} \end{pmatrix} \quad . \qquad (12)$$

It is also found that $\mathbf{U}_{11}$ is nonsingular, and the solution of (10) is

$$\dot{\mathbf{Z}}_c = -\mathbf{U}_{21}\mathbf{U}_{11}^{-1} \quad . \qquad (13)$$

Equation (10) has also other solutions, but the characteristic impedance matrix found with this specific procedure correspond to the only one physical solution.

In order to show the characteristic impedance behavior in a practical case, we can consider the real 77 kV power line geometry as shown in Fig. 2, consisting of two three phase lines and a periodically grounded shield wire on the top.

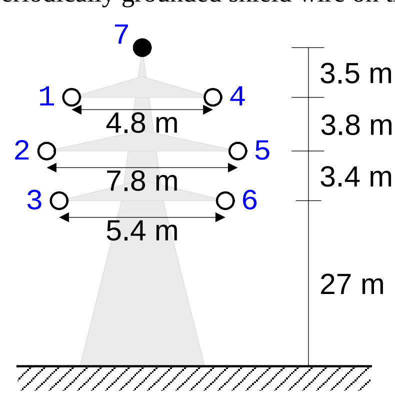

Fig. 2 Line geometry: conductors 1-6 are phase conductors, conductor 7 is the periodically grounded shield wire.

The conductors cross section is 150 mm^2. In our investigation we consider two borderline cases for the distance between grounding points: a very short distance (ℓ = 50 m) and a very long one (ℓ = 300 m).

In Figs. 3 and 4 we show, as example, the real and imaginary parts of the term $\dot{\mathbf{Z}}_{c,(11)}$ and $\dot{\mathbf{Z}}_{c,(7,7)}$ of the characteristic impedance matrix in one period, for a distance between grounding points of 50 m..

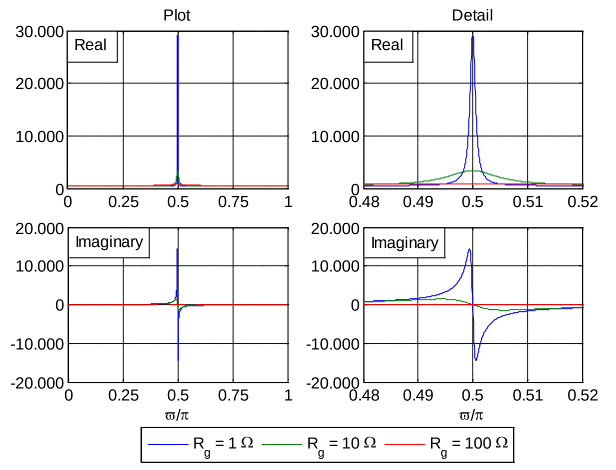

Fig. 3 Coefficient $\dot{\mathbf{Z}}_{c,(11)}$: plot over an entire period (left hand side) and zoom in (right hand side). Distance ℓ = 50 m.

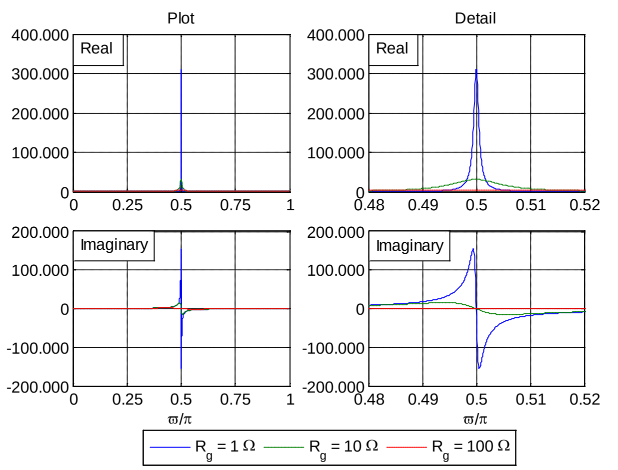

Fig. 4 Coefficient $\dot{\mathbf{Z}}_{c,(7,7)}$: plot over an entire period (left hand side) and zoom in (right hand side). Distance ℓ = 50 m.

The computed values have been verified with a numerical circuit solver. A MTL cell with the last conductor connected to ground and ended on a termination network is considered, then the equivalent impedance at the beginning of the MTL is computed. If, and only if, the termination network corresponds to the characteristic impedance matrix, it also corresponds to the computed equivalent entrance matrix. A very good agreement between

The real part has always an even behavior in the period, while the imaginary part is odd. As expected the terms of the characteristic impedance of the periodically grounded exhibit a strong dependence by the frequency, while all the terms of the characteristic impedance matrix in case of ungrounded MTL are not dependant by the frequency. Then, for higher values of the grounding resistance, the coefficients of the characteristic impedance matrix exhibit a smoother behavior.

In Figs. 5 we show the same term of Figs. 3, but for a distance between grounding points of 300 m..

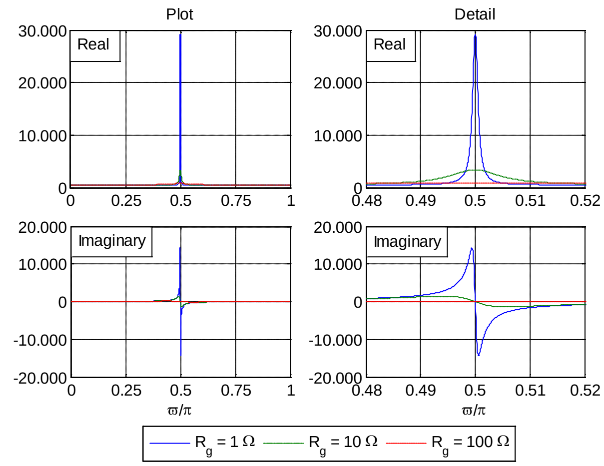

Fig. 5 Coefficient $\dot{\mathbf{Z}}_{c,(11)}$: plot over an entire period (left hand side) and zoom in (right hand side). Distance ℓ = 300 m.

By comparing Fig. 3 and Fig. 5, it is found that the characteristic impedance seems to be invariant with respect to the grounding points, the difference is neglectable. This result is found for all the terms of the matrix, so we avoid to show further impedance plot.

However, it must be remembered that the normalized periods used in the plots proportional to the distance between the groundings (i.e. $\varpi = \omega\ell/c$). So the effective periods in the plots in Fig. 3 and Fig. 5 are different: this will influence the time domain simulations.

IV. Numerical Results in Time-Domain

In this section we show, in a practical case, how the termination impedance matrix can affect the simulation of the voltage and current propagation in time-domain.

We consider the case of a direct lightning hit on the shield wire of an infinite-length line, which conductors configuration is the one shown in Fig. 2. For simplicity we assume that the shield wire is hit at the grounding point. To evaluate the voltages and currents along the line, we have considered the model depicted in Fig. 6.

The used lightning current is described by the sum of two Heidler functions [27], namely

$$i(t) = \frac{I_{01}}{\eta_1}\frac{(t/\tau_{11})^{n_1}}{1+(t/\tau_{11})^{n_1}}e^{-t/\tau_{12}} + \frac{I_{02}}{\eta_2}\frac{(t/\tau_{21})^{n_2}}{1+(t/\tau_{21})^{n_2}}e^{-t/\tau_{22}} \quad , \qquad (14)$$

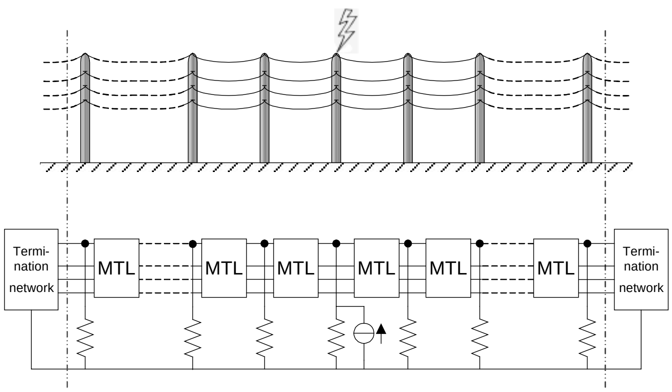

Fig. 6 Configuration considered (top) and equivalent circuit (down).

The adopted parameters are shown in Table I, which are representative of a subsequent stroke.

LIGHTNING CURRENT PARAMETERS

I_{01} (kA)	n_1	τ_{11} (μs)	τ_{21} (μs)	I_{02} (kA)	n_2	τ_{12} (μs)	τ_{22} (μs)
10.7	2	0.25	2.5	6.5	2	2.1	230

As termination network we consider two solutions commonly adopted in literature and the one proposed in the previous section, that is to say:

- the matrix $\dot{\mathbf{Z}}_0$, related to the ungrounded MTL;
- the matrix $\dot{\mathbf{Z}}_c$, related to the periodically grounded MTL and computed according to the procedure presented in the previous section;
- the matrix $\dot{\mathbf{Z}}_d$, related to an approximated solution where each phase conductor is terminated on its characteristic impedance computed in absence of the other conductors and the shield wires are directly terminated to earth [28].

We model each semi-infinite line (left and right of the hit point) by using just 2 MTL cell, including the grounding, and then we place the termination network. In such a configuration it is possible to observe that the different terminations influence the voltage and current computation. Of course only the termination correctly representing a semi-infinite line with its grounding will allow to compute the right voltages and currents.

In order to have a benchmark to verify the voltages and currents in the previous case, we perform a further simulation by considering 20 MTL cells instead of 2, obviously much more time consuming. In such a configuration the voltages at the beginning of the line are practically independent by the termination, and so the computed values can be considered as the "correct solution" and used as term of comparison.

In Fig. 7 we show the voltage produced on the conductor 1 (top-left conductor) at the hit point, assuming a small value of the grounding resistance (R_g = 1 Ω) and considering a distance between the groundings of ℓ = 50 m. It is observed that the voltages have the same peak value, but a difference in the descending part can be seen. So, a proper termination has a practical effect just on the tail part of the voltage.

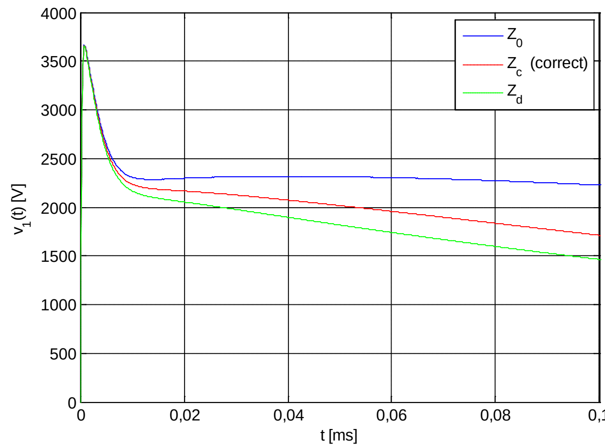

Fig. 7 Voltage v_1(t) at the hit point. R_g = 1 Ω, ℓ = 50 m.

In Fig. 8 we show the same, but this time for an higher value of the grounding resistance (R_g = 100 Ω). In this case the voltages differ in all their waveshape. If the line is terminated on the impedance matrix $\dot{\mathbf{Z}}_0$, the peak value is overestimated of about 30 %, while with the termination $\dot{\mathbf{Z}}_d$ the peak value is underestimated of about 15 %. In addiction, due to the higher value of the grounding resistance, the voltages are higher as well, of course.

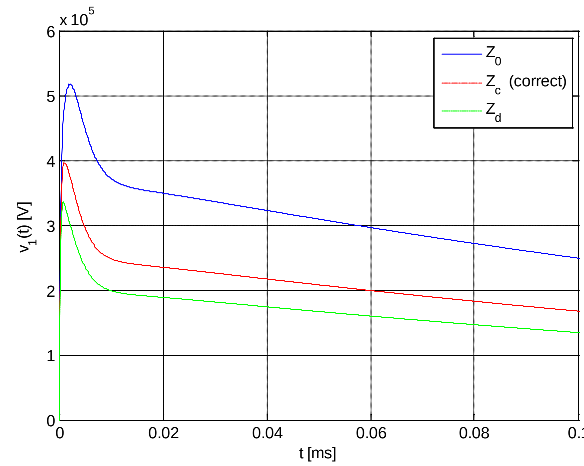

Fig. 8 Voltage v_1(t) at the hit point. R_g = 100 Ω, ℓ = 50 m.

If we consider a larger distance between the groundings (ℓ = 300 m), the previously observed behaviors are mitigated. For this distance in Fig. 9 we show as well the voltage produced on the conductor 1 at the hit point, assuming as grounding resistance R_g = 1 Ω. In this case the voltages have the same peak value and an almost similar behavior also in the tail part of the voltage.

Then, in Fig. 10 we show the same for R_g = 100 Ω. Also in this case the voltages differ in all their waveshape and are overestimated when the line is terminated on $\dot{\mathbf{Z}}_0$ and underestimated when the line is terminated on $\dot{\mathbf{Z}}_d$. In addition, due to the wider distance between the grounding points, a ringing effect produced by the reflections is much more evident, especially when the line is terminated on $\dot{\mathbf{Z}}_0$.

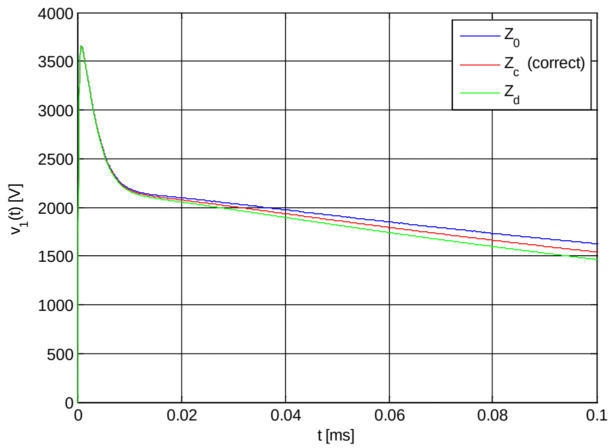

Fig. 9 Voltage v_1(t) at the hit point. R_g = 1 Ω, ℓ = 300 m.

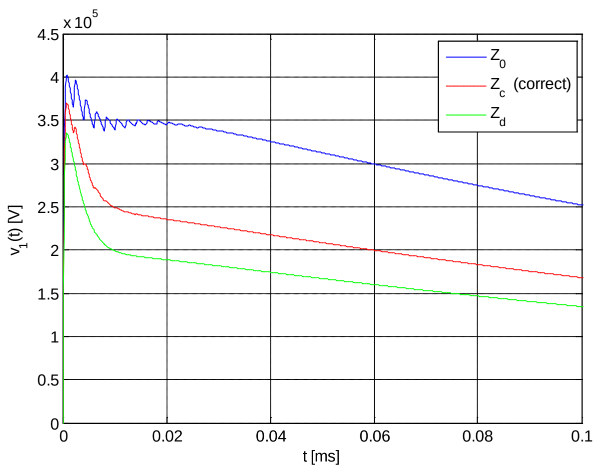

Fig. 10 Voltage v_1(t) at the hit point. R_g = 100 Ω, ℓ = 300 m.

Finally, just for the case of R_g = 100 Ω, where the differences are more evident, we also show the current flowing through the grounding at the hit point. In particular, in Fig. 11 we show the case of a distance between the groundings ℓ = 50 m and in Fig. 12 the case of ℓ = 300 m. The waveshapes are similar to the one observed for the voltages, so similar consideration can be done. It is also observed, as expected, that for higher value of the grounding resistance the current flowing through the grounding decrease, while the voltages notably increases.

The above calculations in time-domain are just a simple example. However they clearly show, in a practical case, the importance of properly terminating a periodically grounded line. The obtained results are quite general and can be assumed as valid for the most common configuration lines.

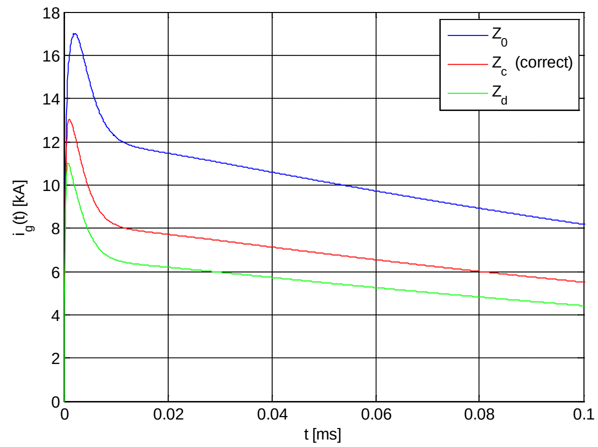

Fig. 11 Current i_g(t) flowing through the grounding at the hit point. R_g = 100 Ω, ℓ = 50 m.

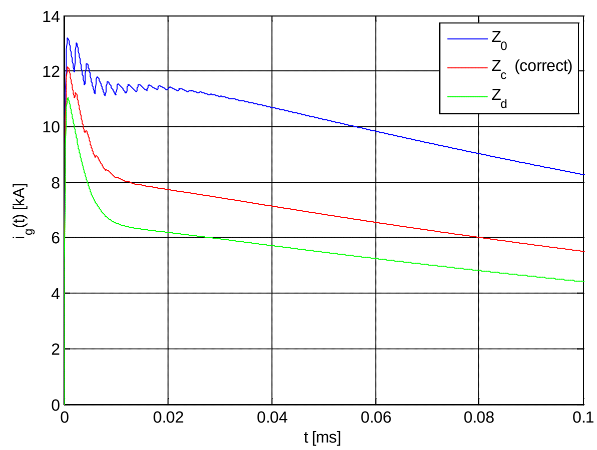

Fig. 12 Current i_g(t) flowing through the grounding at the hit point. R_g = 100 Ω, ℓ = 300 m.

V. Conclusions

The paper shows a concrete method for the computation of the characteristic impedance matrix of a multiconductor transmission line with an arbitrary number of conductors periodically grounded. The problem involves the solution of a non-symmetric Riccati algebraic equation. The characteristic impedance matrix exhibit a periodical frequency behavior, significantly different from the case of ungrounded MTLs. It has also been shown that the characteristic impedance matrix is almost independent by the distance between the groundings, if it is computed in period normalized to the distance itself.

Then, it has been shown, by means of simulations in time domain, how the proper termination of the transmission line affects the voltages at the hit point and the current flowing through the groundings. It has been observed that the effect is more relevant for higher values of the grounding resistance.

The proposed method can be simply used both in frequency and time domain [13].

It has also been shown how the distance between the groundings influences the effect of the terminations on the computation of the voltages and currents. It has been observed that the effect is more relevant when the distance between the grounding is short. So this aspect is more relevant for distribution than for transmission lines.

Finally, it is worth to make some considerations on the adopted model: a limitation used in the present paper is related to the assumption that the MTL is lossless. This hypothesis can be removed with minimal effort [12], in fact the introduction of conductive losses in the MTL just affect the chain matrix (9). By introducing more complex similarity transformation, it would be possible to reduce the problem to a NARE as in (10) with more complex coefficients. Then, the procedure proposed in Section III can be used as well, with different constraints on the eigenvalues to choose. However it has been observed that, due to the shortness of each MTL cell, the conductive losses just introduce a complication in the solution calculation, with no appreciable effect on the characteristic impedance matrix. So it is acceptable to neglect them.

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