Non-Euclidean geometrical transformation groups in the electric circuit theory with stabilization and regulation of load voltages

Alexandr Penin

Abstract— Restricted load powers, two–valued regulation characteristics, and interference of several loads is observed in power supply systems with limited power of voltage source.

In this paper a geometrical approach is presented for interpretation of changes or "kinematics" of load regimes; the definition of load regime parameters and regulators (in the relative and invariant form through different parameters) is grounded; suitable geometrical transformation groups, which describe the movement of an operating point along regulation trajectories, are proposed. To simplify the task and to reveal a matter of principle, the static regulation characteristics and idealized models of voltage regulators are considered.

Non-Euclidean geometry is a new mathematical apparatus in the electric circuit theory, adequately interprets "kinematics" of circuit, and proves the introduction and definition of the proposed concepts.

The obtained results are useful for the electric circuit theory education and a coordinated control of load voltage.

Keywords—Load influence, projective transformation, regulated characteristic, voltage source.

I. Introduction

THE limitation of load powers, two-valued regulation and load characteristics are appeared in power supply systems with limited capacity voltage sources. If such a power supply contains some quantity of loads with individual voltage regulators, the interference of loads takes place [1]. Distributed, autonomous or hybrid power supply systems with solar cells, fuel elements, and storage energy modules (battery, ultra capacitor) can be examples of such systems [2]–[4]. At the same time, the storage module represents a voltage source (with some internal resistance), which influences on load regimes also.

Therefore, it is necessary to take into account the internal resistance of voltage sources, to carry out analysis of the load interference, and obtain relationships for the definition of the regime and regulation parameters, for example, at a possible coordinated predictive control for preset load regimes [5]–[8].

But for that, it is desirable correctly or reasonably to determine the load regime parameters and regulators using admissible area of changes of the load voltage and load resistance; that is, to present the regime parameters in the normalized or relative form, using the parameters of characteristic regimes. In this case, the two-valued characteristics are eliminated.

To simplify the solution of this task and to reveal a matter of principle of this influence, it is expediently to consider the static regulation characteristics and idealized models of voltage regulators. Therefore, this problem relates to the electric circuit theory with changeable load regimes.

In the present paper, the results of interpretation of changes or "kinematics" of load regimes are used on the base of the conformal plane, hyperbolic, and projective geometry [9]. In this work, the definition of the load regime parameters and regulators (in the relative and invariant form through different parameters) is grounded; some geometrical transformations, which describe the movement of an operating point along regulation trajectories, are used.

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II. Analysis of voltage stabilization regimes of loads

Let us present the necessary results [9]. To do this, we consider a power supply system in Fig.1.

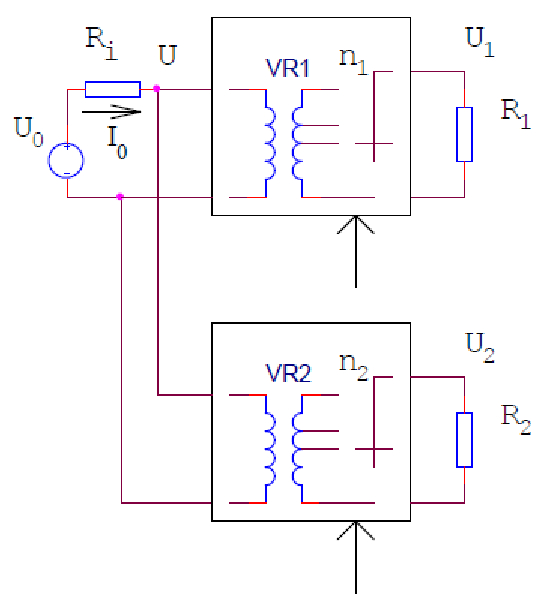


Fig.1 Power supply system with two voltage regulators VR_1, VR_2 and loads R_1, R_2

The power supply system includes two idealized regulated voltage converters (voltage regulators) VR_1, VR_2, and load resistances R_1, R_2. Generally, the voltage converters with a switched tapped secondary of transformers, multicell or multilevel converters, PWM converters, and so on can be examples of these regulators. The regulators define the transformation ratios n_1, n_2. At the same time, the internal resistance R_i determines the interference of the regulators on load regimes.

A. Case of one load

In this case, the transformation ratio $n_2 = 0$. Let us obtain an equation describing behavior or "kinematics" of this circuit at change of the parameter n_1.

By definition,

$$n_1 = \frac{U_1}{U}. \tag{1}$$

Load power

$$P_1 = \frac{U_1^2}{R_1}.$$

On the other hand, this power

$$P_1 = UI_0 = U\frac{U_0 - U}{R_i}.$$

Then, we get

$$\frac{R_i}{R_1}U_1^2 + U(U_0 - U) = 0.$$

It now follows that

$$\frac{R_i}{R_1}U_1{}^2 + \left(U - \frac{U_0}{2}\right)^2 = \frac{U_0{}^2}{4}. \tag{2}$$

For different values R_1 (R_1^1, R_1^2 and so on), this expression represents a bunch of circles (ellipses) by coordinates U_1, U in Fig.2.

Let a stabilized load voltage $U_{1=}$ be given. Then, the vertical line with coordinate $U_{1=}$ intersects the bunch of circles in two points generally. At the same time, the transformation ratio or variable n_1 is resulted by the stereographic projection of circle's points from the bottom pole $0,0$ on the tangent line at the upper pole [10]. For example, the load resistance R_1^1 corresponds to the variable n_1^1.

For minimum value of load resistance $R_{1\min}$, the circle is tangent to the vertical line. In this case, the voltage $U = 0.5U_0$.

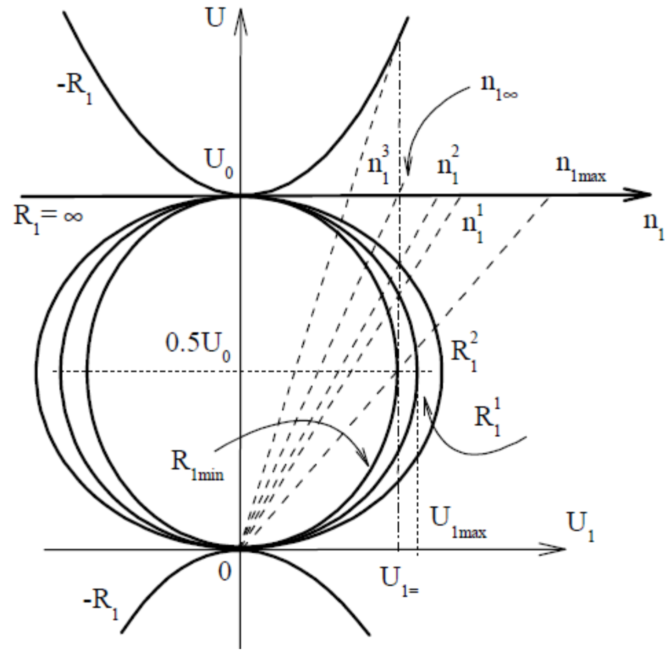


Fig.2 Stereographic projection of bunch of the ellipses $U(U_1, R_1)$ on the line n_1

Using (2), we get the corresponding condition

$$\frac{R_i}{R_{1\min}}(U_{1=})^2 = \frac{U_0{}^2}{4}.$$

Then, the minimum value of the load resistance

$$R_{1\min} = 4R_i\frac{(U_{1=})^2}{U_0{}^2}. \tag{3}$$

Also, the respective maximum allowable transformation ratio

$$n_{1\max} = 2\frac{U_{1=}}{U_0}. \tag{4}$$

The operating area of all the circles must be above the diameters of these circles; that is, $U \geq U_0/2$. Therefore, we use the upper point of the intersection. In other words, on some step of switching period at increase of the parameter n_1, a running point can pass over the diameter that is inadmissible. Therefore, it is better to use such groups of transformations or movements of points along the line n_1 when it is impossible to move out the running point over the diameter. So, we must decrease the next values n_1 by some rule. In this sense, we come to hyperbolic geometry.

A.1 Use of hyperbolic geometry

Let us consider the functional dependence $R_1\,(n_1, U_{1=})$, where the voltage $U_{1=}$ is a parameter. Similarly to [9], we must validate the definition of regime and its changes; find the invariants of regime parameters. To do this, we consider such a characteristic regime as $R_1 = \infty$. In this case, the ellipse degenerates into the two straight lines, $U = 0$, $U = U_0$. Then, for the voltage $U = U_0$, the transformation ratio

$$n_{1\infty} = \frac{U_{1=}}{U_0}. \tag{5}$$

However, the question arises about the range of transformation ratio as $0 < n_1 < n_{1\infty}$. According to Fig.2, this range corresponds to the negative load value $R_1 < 0$ and expression (2) determines a hyperbola. In this case, the load gives back energy and the voltage source U_0 consumes this energy as shown in Fig.3.

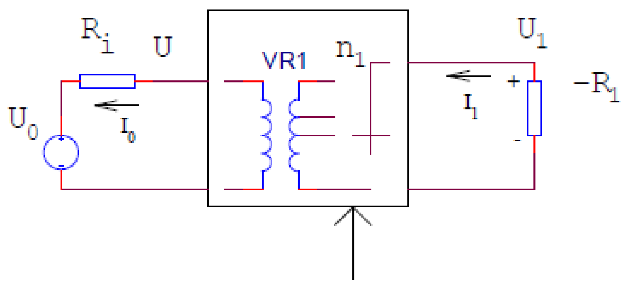
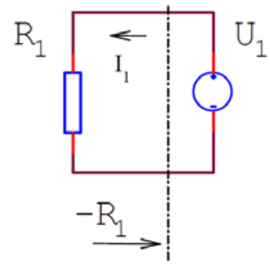
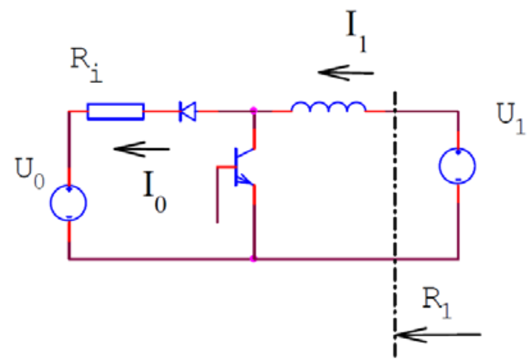


Fig.3 Negative load $R_1 < 0$ gives back energy

In this regard, we consider a physical realization of such a power source, as a negative resistance $-R_1$. For this purpose, we remind something of the electric circuit theory by the example of a circuit in Fig. 4(a). Let a voltage source be connected to a resistance R_1, then the negative resistance $-R_1$ and the drain current I_1 will be corresponding to this voltage source. We consider Fig.3 again. Then, it is possible to connect up the voltage source U_1 instead of the resistance $-R_1$. At the same time, the input resistance of this circuit must be equal to the constant value R_1 at change of the voltage value U_1. Such condition is satisfied due to the variable value n_1; there is a loss-free resistance [11]. The example of another circuit, as a high-power-factor boost rectifier, is given in Fig.4(b) [12].



(a)



(b)

Fig.4 Realization of a negative resistance: the negative resistance $-R_1$ corresponds to a source U_1 - (a); a boost PWM converter with the constant input resistance R_1 - (b)

Taking into account (1), (2), we obtain

$$U_1 = \frac{n_1 U_0}{1 + \frac{R_i}{R_1} n_1^{\,2}}. \tag{6}$$

Thus, we get the required relationships $n_1\,(R_1)$, $R_1\,(n_1)$

$$n_1^{\,2} - n_1 \frac{U_0}{U_1}\frac{R_1}{R_i} + \frac{R_1}{R_i} = 0, \quad \frac{R_1}{R_i} = \frac{n_1^{\,2}}{n_1 \frac{U_0}{U_1} - 1}, \tag{7}$$

where the voltage value $U_1 = U_{1=}$ is a parameter. The dependence $R_1\,(n_1)$ determines a hyperbola in Fig.5.

We have a single-valued mapping of hyperbola points on the axis n_1. This projective transformation preserves a cross ratio of four points [13]. Similarly to [9], let us constitute the cross ratio m_n^1 for the points $0,\ n_1^1,\ n_{1\infty},\ n_{1\max}$

$$m_n^1 = (\,0\ \ n_1^1\ \ n_{1\infty}\ \ n_{1\max}\,) = \\ = \frac{n_1^1 - 0}{n_1^1 - n_{1\max}} \div \frac{n_{1\infty} - 0}{n_{1\infty} - n_{1\max}}. \tag{8}$$

The points $0, n_{1\max}$ are base ones and point $n_{1\infty}$ is a unit one. The point n_1^1 is a point of an initial or running regime.

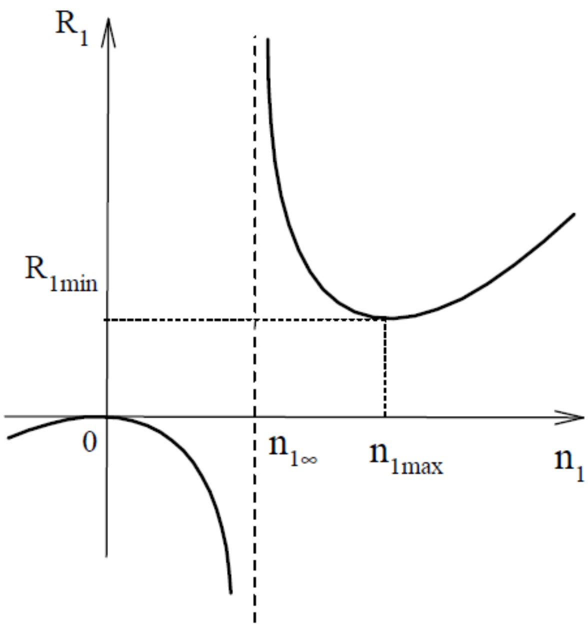


Fig.5 Hyperbola $R_1(n_1)$

Using (4), (5), we get

$$m_n^1 = \left(0 \;\; n_1^1 \;\; \frac{U_{1=}}{U_0} \;\; 2\frac{U_{1=}}{U_0}\right) = \frac{n_1^1}{2\frac{U_{1=}}{U_0} - n_1^1} = \frac{n_1^1}{n_{1\max} - n_1^1}. \tag{9}$$

The conformity of points n_1^1, m_n^1 is shown in Fig.6. In this case, the value m_n^1 is a nonhomogeneous (or non-uniform) coordinate of the value n_1^1. Further, the cross ratio m_n^{21}, which corresponds to a regime change $n_1^1 \rightarrow n_1^2$, has the form

$$m_n^{21} = (\,0 \;\; n_1^2 \;\; n_1^1 \;\; n_{1\max}\,) = \frac{n_1^2 - 0}{n_1^2 - n_{1\max}} \div \frac{n_1^1 - 0}{n_1^1 - n_{1\max}} = \frac{n_1^2(n_1^1 - n_{1\max})}{n_1^1(n_1^2 - n_{1\max})}. \tag{10}$$

Using normalized values

$$\overline{n}_1^2 = \frac{n_1^2}{n_{1\max}}, \;\; \overline{n}_1^1 \frac{n_1^1}{n_{1\max}},$$

we get the regime change (segment $\overline{n}_1^2 \; \overline{n}_1^1$) or cross ratio (10) in the view

$$m_n^{21} = (\,0 \;\; \overline{n}_1^2 \;\; \overline{n}_1^1 \;\; 1\,) = \frac{\overline{n}_1^2(\overline{n}_1^1 - 1)}{\overline{n}_1^1(\overline{n}_1^2 - 1)}. \tag{11}$$

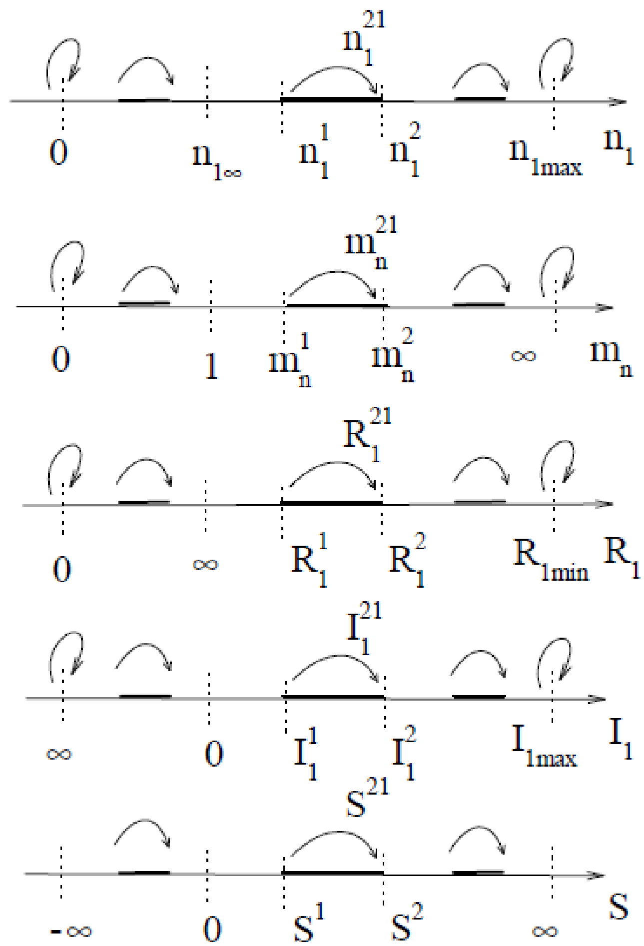


Fig.6 Conformity of different regime parameters

Similarly to [9], we can obtain the analogous expression for the change n_1^{21} of the transformation ratio so that the following relationships are performed

$$n_1^{21} = \frac{m_n^{21} - 1}{m_n^{21} + 1}, \;\; m_n^{21} = \frac{n_1^{21} + 1}{1 - n_1^{21}}. \tag{12}$$

For this purpose, we make substitution of variables that to use ready expressions [9]. Let us introduce the value

$$\tilde{n}_1 = 2\overline{n}_1 - 1. \tag{13}$$

The conformity of these variables is shown in Fig. 7. According to [9], the change $\tilde{n}_1^{21}$ has the form

$$\tilde{n}_1^{21} = \frac{\tilde{n}_1^2 - \tilde{n}_1^1}{1 - \tilde{n}_1^2 \tilde{n}_1^1}.$$

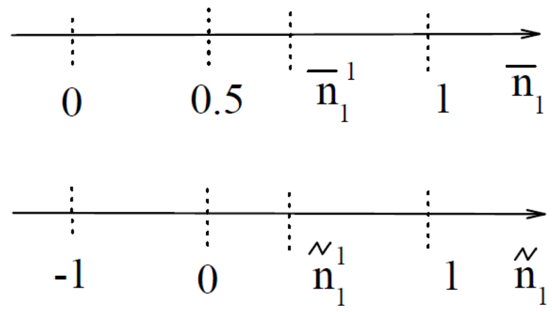


Fig.7 Conformity of the normalized transformation ratio $\overline{n}_1$ and a new variable $\tilde{n}_1$

Using substitution of variables (13), we get

$$\tilde{n}_1^{21} = \frac{\overline{n}_1^2 - \overline{n}_1^1}{\overline{n}_1^2 + \overline{n}_1^1 - 2\overline{n}_1^2\overline{n}_1^1} = n_1^{21}. \tag{14}$$

In this expression the changes of the variables are equal to among themselves

$\tilde{n}_1^{21} = n_1^{21}$.

There are the following bases for this equality. Linear expression (13) preserves a cross ratio and, consequently, the regime change m_n^{21}. On the other hand, the change of the transformation ratio n_1^{21} is expressed by regime change (12).

Using (14), we obtain the subsequent value n_1^2 of the transformation ratio

$$\overline{n}_1^2 = \frac{\overline{n}_1^1(1+n_1^{21})}{1+n_1^{21}(2\overline{n}_1^1 - 1)}. \tag{15}$$

There is a group transformation. Moreover, if the initial value $\overline{n}_1^1 = 1$, then the subsequent value $\overline{n}_1^2 = 1$ regardless of the value n_1^{21}. Therefore, movement of a point is realized in hyperbolic geometry [14].

Similarly to the above, let us consider the cross ratio for the load resistance R_1. Using the dependence $R_1(n_1)$ by Fig.5, we demonstrate the conformity of the variables R_1, n_1 in Fig.6. The cross ratio for the initial point R_1^1, relatively to the base points $0, R_{1\min}$, and a unit point $R_1 = \infty$ has the form

$$m_R^1 = (0\ R_1^1\ \infty\ R_{1\min}) = \frac{n_1^1}{1 - 4\dfrac{R_i}{R_1^1}\dfrac{(U_{1=})^2}{U_0^2}}. \tag{16}$$

Expression (16) equals the corresponding cross ratio for the conductance $Y_1 = 1/R_1$ and load current $I_1 = U_{1=}/R_1$. The following equality takes place

$$m_R = (m_n)^2. \tag{17}$$

This expression leads to identical values if we use the hyperbolic metric to determine the regime value as the distance

$$S = Ln\, m_R = 2Ln\, m_n. \tag{18}$$

This distance values are also shown in Fig.6. The base points $0, R_{1\min}$ correspond to infinitely large distance.

Similarly to (10), the cross ratio m_R^{21}, which corresponds to a regime change $R_1^1 \to R_1^2$, has the view

$$m_R^{21} = (0\ R_1^2\ R_1^1\ R_{1\min}) = \frac{R_1^2\,(R_1^1 - R_{1\min})}{R_1^1\,(R_1^2 - R_{1\min})}. \tag{19}$$

We can introduce the change R_1^{21} of the load resistance by the following expression

$$m_R^{21} = \frac{1+R_1^{21}}{1-R_1^{21}}. \tag{20}$$

Further, we use the normalized values

$$\overline{R}_1^2 = \frac{R_1^2}{R_{1\min}},\quad \overline{R}_1^1 = \frac{R_1^1}{R_{1\min}}.$$

Similarly to (14), we get

$$R_1^{21} = \frac{\overline{R}_1^2 - \overline{R}_1^1}{\overline{R}_1^2 + \overline{R}_1^1 - 2\overline{R}_1^2\overline{R}_1^1}. \tag{21}$$

Then, *there is a strong reason to introduce the changes of the load resistance R_1^{21} and the transformation ratio n_1^{21} as expressions (21), (14).*

The validity of such definitions for changes is confirmed by the following expression similar to initial expression (6)

$$R_1^{21} = \frac{2n_1^{21}}{1+(n_1^{21})^2}. \tag{22}$$

Using (21), we obtain the subsequent value R_1^2 of the load resistance

$$\overline{R}_1^2 = \frac{\overline{R}_1^1(1+R_1^{21})}{1+R_1^{21}(2\overline{R}_1^1 - 1)}. \tag{23}$$

As well as (15), if the initial value $\overline{R}_1^1 = 1$, then the subsequent value $\overline{R}_1^2 = 1$ regardless of the value R_1^{21}.

Thus, *the concrete kind of a circuit and character of regime imposes the requirements to definition of already system parameters.*

Therefore, arbitrary and formal expressions for the regime parameters are excluded.

A.2 Example

Let the circuit parameters be given as follows

$U_0 = 5$, $R_i = 1$, $U_{1=} = 2.5$.

Herein after, the value dimensions are not specified.

The initial and subsequent value of the load resistance

$R_1^1 = 2.0$, $R_1^2 = 1.25$.

The minimum value of load resistance (3)

$$R_{1\min} = 4R_i \frac{(U_{1=})^2}{U_0^2} = 1.$$

The maximum value of transformation ratio (4)

$$n_{1\max} = 2\frac{U_{1=}}{U_0} = 1.$$

Transformation ratio (5)

$$n_{1\infty} = \frac{U_{1=}}{U_0} = 0.5.$$

The values of the transformation ratio by quadratic equation (7)

$n_1^1 = 0.585$, $n_1^2 = 0.691$.

The normalized values are equal to these ones.

Cross ratio (8) for the initial regime

$$m_n^1 = (\,0 \;\; n_1^1 \;\; n_{1\infty} \;\; n_{1\max}\,) = \frac{0.585}{1-0.585} = 1.41.$$

Cross ratio (10) for the regime change

$$m_n^{21} = (\,0 \;\; n_1^1 \;\; n_{1\max}\,) = \frac{n_1^2(n_1^1 - n_{1\max})}{n_1^1(n_1^2 - n_{1\max})} = \frac{2.236}{1.41} = 1.581.$$

Change (14) of the transformation ratio

$$n_1^{21} = \frac{\overline{n}_1^2 - \overline{n}_1^1}{\overline{n}_1^2 + \overline{n}_1^1 - 2\overline{n}_1^1\overline{n}_1^2} = \frac{0.106}{0.467} = 0.226.$$

Now, we consider the cross ratio for the load resistance R_1.

Cross ratio (16) for the initial regime

$$m_R^1 = \frac{1}{1 - 4\dfrac{R_i(U_{1=})^2}{R_1^1 U_0^2}} = \frac{1}{1 - 4\dfrac{6.25}{2\cdot 25}} = 2.0.$$

Let us check equality (17)

$m_R^1 = (\,m_n^1)^2 = 1.41^2 = 2.0$.

Cross ratio (19) for the regime change

$$m_R^{21} = (\,0 \;\; R_1^2 \;\; R_1^1 \;\; R_{1\min}\,) = \frac{R_1^2(R_1^1 - R_{1\min})}{R_1^1(R_1^2 - R_{1\min})} = 2.5.$$

The equality takes place also

$m_R^{21} = (\,m_n^{21})^2 = 1.581^2 = 2.5$.

Change (21) of the load resistance

$$R_1^{21} = \frac{\overline{R}_1^2 - \overline{R}_1^1}{\overline{R}_1^2 + \overline{R}_1^1 - 2\overline{R}_1^1\overline{R}_1^2} = 0.429.$$

Equality (22)

$$R_1^{21} = \frac{2n_1^{21}}{1 + (n_1^{21})^2} = \frac{2\cdot 0.226}{1 + 0.051} = 0.429.$$

B. Stabilization of voltage of two loads

Let us consider a circuit with two loads in Fig.1. Variation of one of loads leads to mutual change of stabilization regimes for both loads. Therefore, it is necessary to change the transformation ratios n_1, n_2 in coordination. We will obtain the required relationships.

Equation (2), taking into account the second load power, becomes as

$$\frac{R_i}{R_1}U_1^2 + \frac{R_i}{R_2}U_2^2 + \left(U - \frac{U_0}{2}\right)^2 = \frac{U_0^2}{4}. \tag{24}$$

By definition,

$$n_1 = \frac{U_1}{U}, n_2 = \frac{U_2}{U}. \tag{25}$$

Using (24), (6), and (25), we get the system of equations

$$\begin{cases} U_1 = \dfrac{n_1 U_0}{1 + \dfrac{R_i}{R_1}n_1^2 + \dfrac{R_i}{R_2}n_2^2} \\ U_2 = \dfrac{n_2 U_0}{1 + \dfrac{R_i}{R_1}n_1^2 + \dfrac{R_i}{R_2}n_2^2}. \end{cases} \tag{26}$$

It follows that

$$\begin{cases} \dfrac{R_i}{R_1}n_1^2 + \dfrac{R_i}{R_2}n_2^2 - \dfrac{n_1 U_0}{U_1} + 1 = 0 \\ \dfrac{R_i}{R_1}n_1^2 + \dfrac{R_i}{R_2}n_2^2 - \dfrac{n_2 U_0}{U_2} + 1 = 0. \end{cases} \tag{27}$$

By definition (25)

$$n_2 = n_1 \frac{U_2}{U_1}. \quad (28)$$

Substituting this expression in the first equation of system (27), we get

$$n_1^{\,2} R_i \left[\frac{1}{R_1} + \frac{1}{R_2} \left(\frac{U_2}{U_1} \right)^2 \right] - \frac{n_1 U_0}{U_1} + 1 = 0. \quad (29)$$

The expression in the square brackets is the total resistance (conductance) of both loads relatively to the first load

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \left(\frac{U_2}{U_1} \right)^2 = Y_T. \quad (30)$$

If, for example, the load voltages are equal to among themselves, $U_2 = U_1$, then these loads are connected in parallel and

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = Y_T.$$

Finally, we get the expression

$$n_1^{\,2} \frac{R_i}{R_T} - \frac{n_1 U_0}{U_1} + 1 = 0. \quad (31)$$

This expression corresponds to (7) and the dependence $R_T(n_1)$ coincides with Fig.5.

Therefore, for given load resistances, we determine: total resistance (30), the transformation ratio n_1 (as the solution of (31)), and the value n_2 (by (28) or by the second equation of (27)).

Also, we must to check stability conditions (3), (4). In this case, these conditions have the form

$$\begin{cases} R_{T\min} = 4R_i \dfrac{(U_{1=})^2}{U_0^{\,2}} \\ n_{1\max} = 2\dfrac{U_{1=}}{U_0} \\ n_{2\max} = 2\dfrac{U_{2=}}{U_0} \end{cases}. \quad (32)$$

Further, it is possible to use the above idea of hyperbolic geometry in the case of one load.

III. Given Voltage for the First Variable Load and Voltage Regulation of the Second Given Load

Let us consider the general case for operating regime of a circuit in Fig.1. Let the first load voltage $U_{1=}$ be given invariable and resistance of this load be changed. Moreover, the first load resistance can be both positive $R_1 > 0$ and negative $R_1 < 0$. At the same time, the second constant load resistance is positive, $R_2 > 0$.

For example, pulse regulators in Fig.8 correspond to the positive load $R_1 > 0$ and regulators in Fig.9 conform to the negative load $R_1 < 0$.

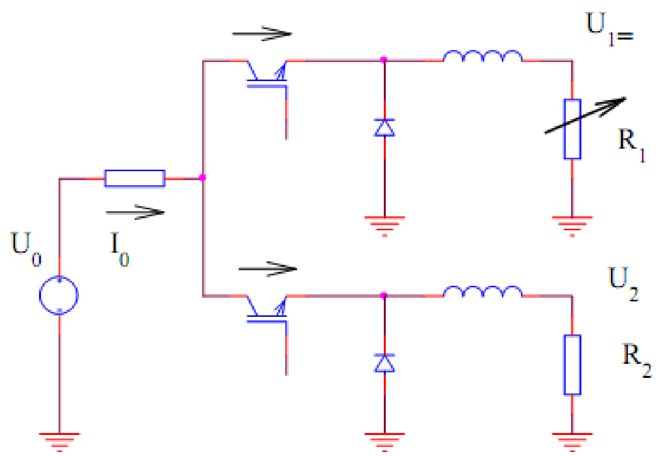


Fig.8 Power supply system with invariable values $U_{1=}$ and R_2

At first, we consider expression (24) in coordinates U_1, U_2, U with parameter R_1.

If $R_1 > 0$, expression (24) represents a sphere (ellipsoid) similarly to the circle in Fig.2. Both loads consume energy; the voltage source U_0 gives energy.

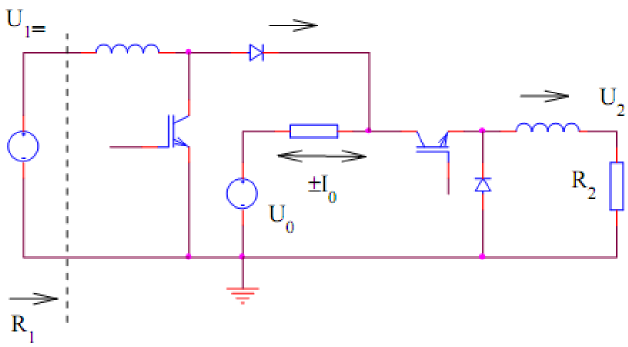
If $R_1 < 0$, this expression realizes one-sheeted hyperboloid in Fig.9(b). The first load, as a constant voltage source, gives energy. In addition, the voltage source U_0, as a storage energy module, can consume and give back energy. The direction of current flow I_0 determines these regimes.

For different values R_1, expression (24) represents a bunch of spheres or hyperboloids. If $R_1 = \infty$, as open circuit regime, surface (24) degenerates into cylinder.

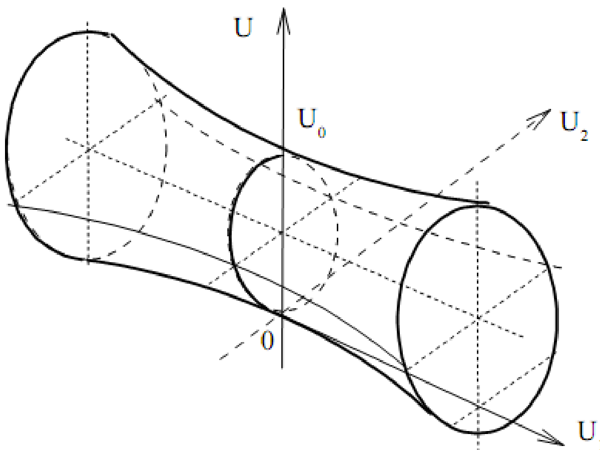
Let us now return to the general case for operating regime; that is,

$$U_1 = U_{1=}, \; R_2 = const.$$

For realization of this regime, it is necessary to change the transformation ratios n_1, n_2 in coordination.



(a)



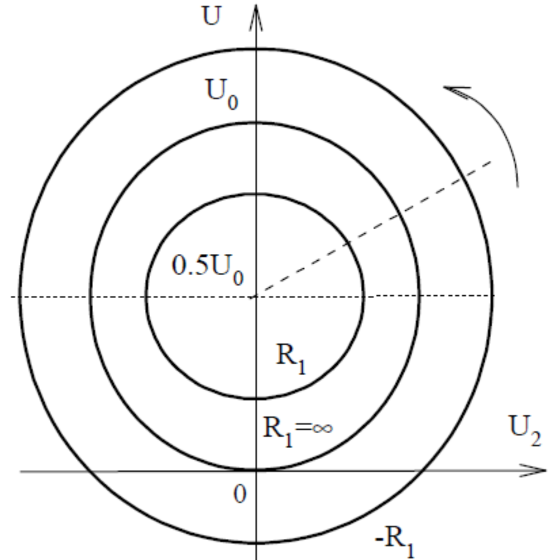
(b)

Fig.9 Power supply system with voltage source $U_{1=}$ - (a) and its geometrical model - (b)

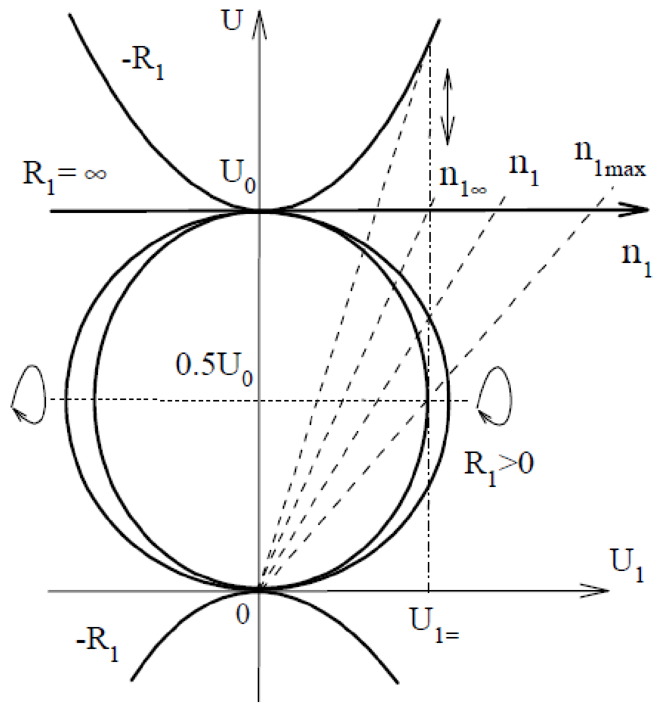
At it is, the plane with parameter $U_{1=}$ intersects the bunch of spheres and hyperboloids. As a result of this section, the bunch of circles in coordinates U_2, U are obtained, as it is shown in Fig.10(a). In this case, expression (24) has the form

$$\frac{R_i}{R_2}U_2^2+\left(U-\frac{U_0}{2}\right)^2=\frac{U_0^2}{4}-\frac{R_i}{R_1}U_{1=}^2 . \tag{33}$$

The second member of this equation is a radius of circle for the given value R_1. It is possible to consider the change of the voltage U_2 as a rotation of radius-vector. This rotation determines the movement of a point along the line $U_{1=}$ in coordinates U_1, U in Fig.10(b); this figure at $U_2=0$ is analogous to Fig.2. In the given case, the bunch of these surfaces rotates around the diameter $U=U_0/2$, as it is shown by closed arrows. Also, the transformation ratios n_1, n_2 are resulted by the stereographic projection of sphere's



(a)



(b)

Fig.10 Bunch of circles at $U_{1=}$ - (a) and bunch of curves at $U_2=0$ - (b)

points on the tangent plane or conformal plane [15]. The axes n_1, n_2 are superposed in Fig.10(b).

Further, we use the first equation of system (27)

$$\frac{R_i}{R_1}n_1^2+\frac{R_i}{R_2}n_2^2-\frac{n_1U_0}{U_1}+1=0 . \tag{34}$$

This equation circumscribes the trajectories of change n_1, n_2 for different values R_1, as it is shown in Fig.11. These trajectories are characteristic for a conformal plane [15].

If $R_1 > 0$, the bunch of circles with parameters R_1^1, R_1^2 is obtained. For $R_1 = \infty$, equation (34) corresponds to a parabola

$$\frac{R_i}{R_2} n_2^2 - \frac{n_1 U_0}{U_1} + 1 = 0 \ . \tag{35}$$

The case $R_1 < 0$ conforms to a hyperbola. For limit values $R_1 = 0$ and $n_1 = 0$, the hyperbola degenerates and coincides with the axis n_2.

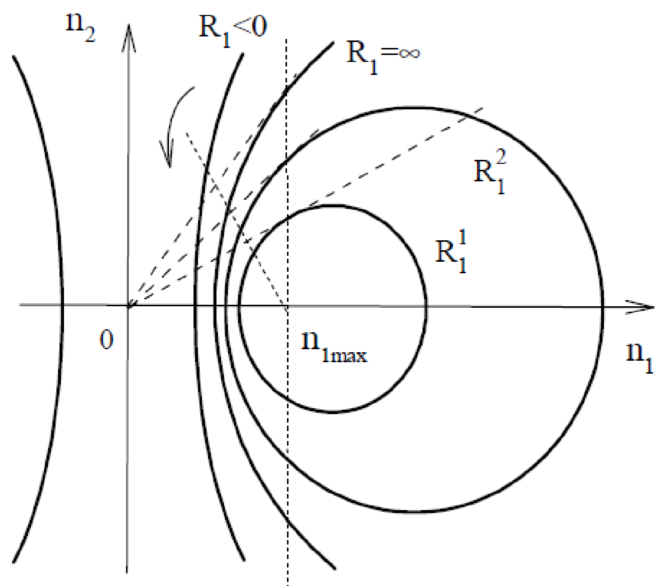


Fig.11 Trajectories of change n_1, n_2 for different values R_1

The rotation of radius-vector in the plane $U_2\, U$ determines the analogous rotation in the plane $n_1\, n_2$ with the centre $n_{1\max}$. This centre corresponds to the minimum load resistance $R_{1\min}$.

Let us found these values $n_{1\max}$, $R_{1\min}$. We assume the value $n_2 = 0$ in expression (34). The obtained quadratic equation has the view

$$\frac{R_i}{R_1} n_1^2 - \frac{n_1 U_0}{U_1} + 1 = 0 \ . \tag{36}$$

The roots coincide for the load resistance and

$$R_1 = R_{1\min} = 4 R_i \frac{U_{1=}^2}{U_0^2} .$$

Using (36), we get

$$n_{1\max} = 2\frac{U_{1=}}{U_0} .$$

These values coincide with (3) and (4) for one load.

Next, we display the voltage value U_2 on the trajectories in Fig.11.Using definition (25), we obtain the relationship

$$U_2 = \frac{n_2}{n_1} U_{1=} .$$

Therefore, the voltage value U_2 is directly proportional to the voltage value U_1 at a constant value n_2 / n_1; that is, we have a straight line, which intersects the bunch of circles with parameters R_1 in two points. Then, the tangent line to the circle (curve) determines the point of maximum voltage value $U_{2\max}$ and the voltage $U = U_0 / 2$.

In this case, we get

$$U_{2\max} = \frac{U_0}{2} \sqrt{\frac{R_2}{R_i}} \sqrt{1 - \frac{R_{1\min}}{R_1}} ,$$
$$n_{2\max} = \sqrt{\frac{R_2}{R_i}} \sqrt{1 - \frac{R_{1\min}}{R_1}} . \tag{37}$$

It is interestingly to note that all these tangent lines to the curves correspond to the value $n_{1\max}$. Therefore, the operating area of the transformation ratio is limited by the value $n_1 \le n_{1\max}$. So, we must decrease the next value n_1 (for the next regulator switching period) by some rule. In this sense, we come to hyperbolic geometry.

A. *Use of hyperbolic geometry*

The straight line $n_{1\max}$ in the plane $n_2\, n_1$ is the line of infinitely. Therefore, geometry of the half plane $n_2, n_1 \le n_{1\max}$ in Fig.11and normalized half plane in Fig.12(a) corresponds to Poincare's model of hyperbolic geometry for the half plane $g_2, g_1 \ge 0$ with projective coordinates in Fig.12(a) and orthogonal coordinates in Fig.12(b) [14].

In Poincare's model of hyperbolic geometry, the half-rounds with the given resistance R_1 intersect the axes g_2 orthogonally. Let us introduce this geometry. To do this, it is necessary to change the variables $n_2(g_1, g_2)$, $n_1(g_1, g_2)$ so that all curves of the plane $n_2\, n_1$ are converted into circles of the plane $g_2\, g_1$.

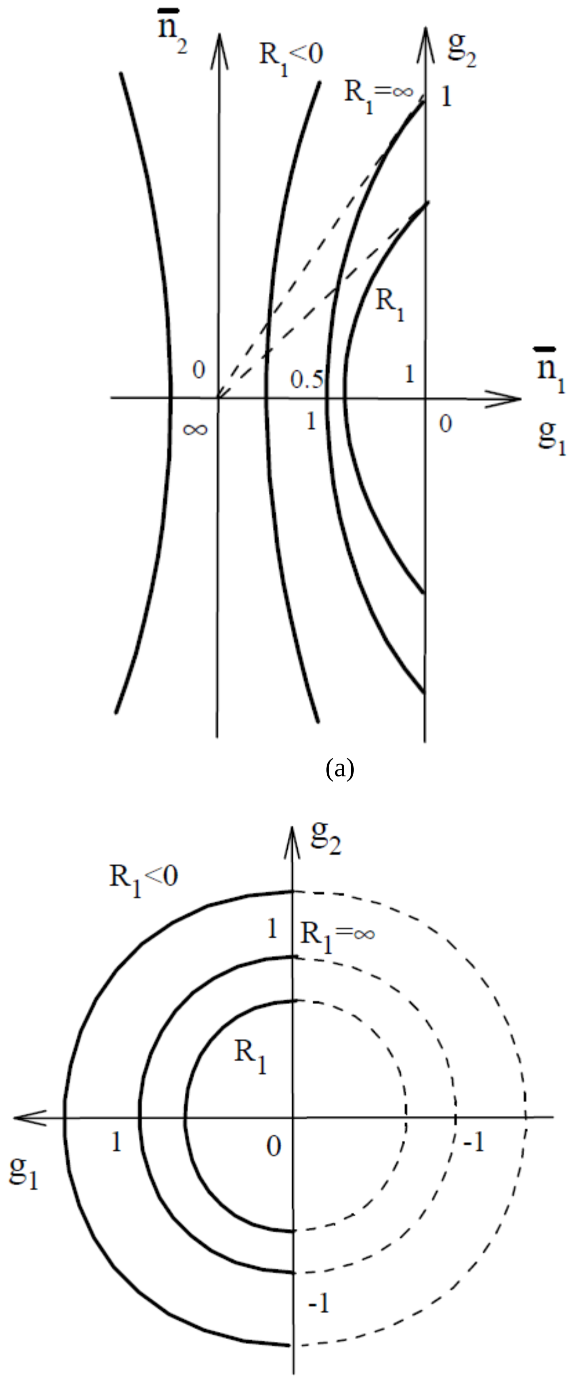


Fig.12 Poincare's model of hyperbolic geometry: superposed half planes $\bar{n}_2, \bar{n}_1 \leq 1$ and $g_2, g_1 \geq 0$ - (a); half plane with orthogonal coordinates $g_2, g_1 \geq 0$ - (b)

Further, we use the normalized values of the transformation ratios

$$\bar{n}_1 = \frac{n_1}{n_{1\max}}, \; \bar{n}_2 = \frac{n_2}{n_{2ref}}.$$

As a scale value n_{2ref}, we can use a circle with some characteristic value of the parameter R_1. The resistance value $R_1 = \infty$ can be such a characteristic value.

Using (35) and value $n_{1\max}$, we get

$$n_{2ref} = \sqrt{\frac{R_2}{R_i}}. \tag{38}$$

It is possible to represent expression (34) in the normalized form

$$\frac{R_{1\min}}{R_1}\bar{n}_1^2 + \bar{n}_2^2 - 2\bar{n}_1 + 1 = 0. \tag{39}$$

The required change of variables has the view

$$\bar{n}_1 = \frac{1}{1+g_1}, \; \bar{n}_2 = \frac{g_2}{1+g_1}. \tag{40}$$

Let us check the offered expressions. In this case, equation (39) transforms into the equation of circle

$$g_1^2 + g_2^2 = 1 - \frac{R_{1\min}}{R_1} = r_1^2. \tag{41}$$

The second member of this equation is a radius squared for the given value R_1. We can term the variables g_1, g_2 as hyperbolic transformation ratios. This geometric model allows to use a cross ratio for determination of regimes and their change.

B. *Regime change for the given load resistance* R_1

For clarity, let us consider the half-rounds with a parameter R_1^1 in Fig.13. Let points C_{1g}, D_{1g} be the points of an initial and subsequent regime. Then, the cross ratio, which corresponds to the regime change $C_{1g} \to D_{1g}$, has the form similar to (10)

$$m_g^{DC} = (A_{1g}\; D_{1g}\; C_{1g}\; F_{1g}) = \frac{D_{1g} - A_{1g}}{D_{1g} - F_{1g}} \div \frac{C_{1g} - A_{1g}}{C_{1g} - F_{1g}}. \tag{42}$$

The points A_{1g}, F_{1g} are base ones. The coordinates of all the points $A_{1g}, D_{1g}, C_{1g}, F_{1g}$ are given by complex numbers as follows

$$g^{A1} = g_2^{A1} + j0, \; g^{D1} = g_2^{D1} + jg_1^{D1},$$
$$g^{C1} = g_2^{C1} + jg_1^{C1}, \; g^{F1} = g_2^{F1} + j0.$$

In particular, the radius of half-rounds (41) defines the coordinates g_2^{A1}, g_2^{F1}, that is,

$$g_2^{A1} = -r_1, \; g_2^{F1} = r_1.$$

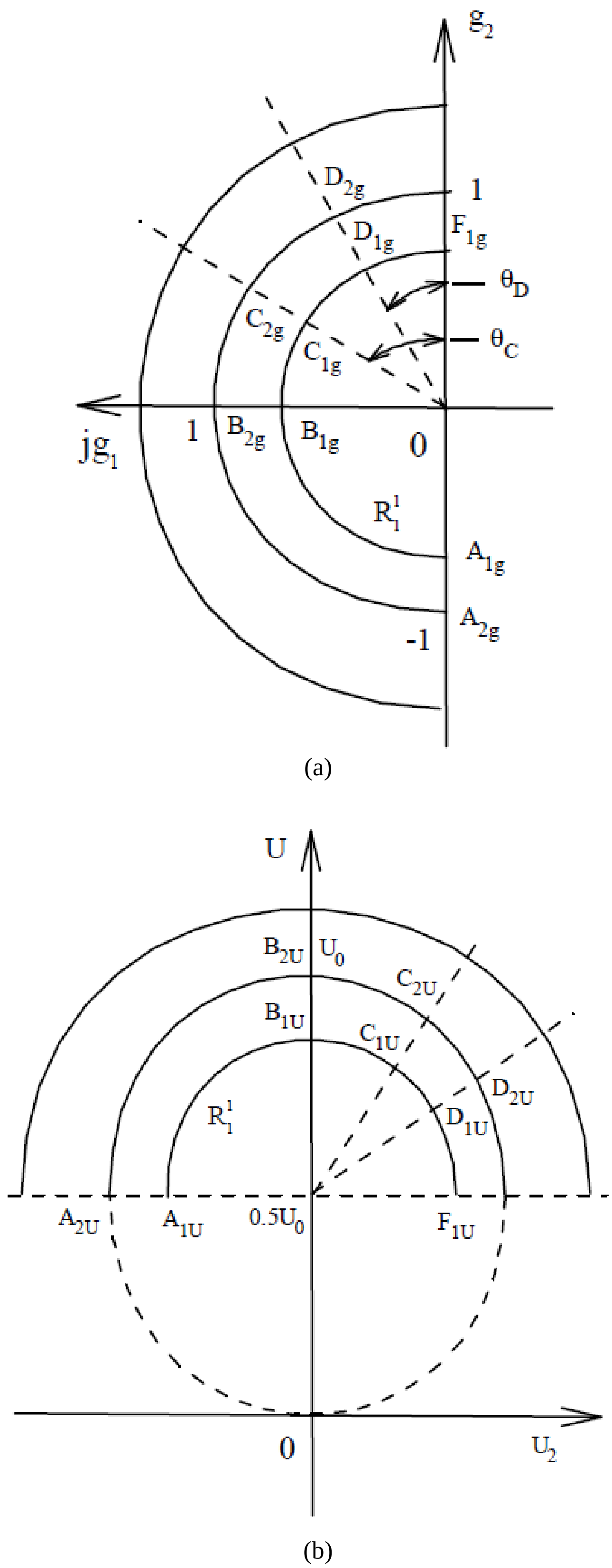


Fig.13 Regime change for Poincare's model of hyperbolic geometry: half plane $g_2, g_1 \geq 0$ - (a); half plane $U_2, U \geq 0.5U_0$ - (b)

For Poincare's model of hyperbolic geometry by Fig.13(a), cross ratio (42) looks like

$$m_g^{DC} = \frac{tg\theta_C}{tg\theta_D} = \frac{r_1 - g_2^{C1}}{g_1^{C1}} \div \frac{r_1 - g_2^{D1}}{g_1^{D1}}. \tag{43}$$

Using (41), we get

$$\left(m_g^{DC}\right)^2 = \frac{r_1 - g_2^{C1}}{r_1 + g_2^{C1}} \div \frac{r_1 - g_2^{D1}}{r_1 + g_2^{D1}}. \tag{44}$$

This expression gives the subsequent value

$$\frac{g_2^{D1}}{r_1} = \frac{\dfrac{g_2^{C1}}{r_1} + g_2^{DC}}{1 + \dfrac{g_2^{C1}}{r_1} g_2^{DC}}, \tag{45}$$

where the change of the hyperbolic transformation ratio is introduced as

$$g_2^{DC} = \frac{\left(m_g^{DC}\right)^2 - 1}{\left(m_g^{DC}\right)^2 + 1}. \tag{46}$$

This change corresponds to the points C_{2g}, D_{2g} of the half-rounds with parameter R_1^2 and so on.

We can obtain the expression for the subsequent value of the transformation ratios n_1, n_2. To do this, it is necessary to apply to (45) the inverse change of variables relatively to (40)

$$g_1 = \frac{1 - \overline{n}_1}{\overline{n}_1}, \quad g_2 = \frac{\overline{n}_2}{\overline{n}_1}. \tag{47}$$

But difficult formulas are obtained. Therefore, using (40), we can directly calculate the subsequent value of transformation ratios n_1, n_2.

Let us compare the half-rounds in the plane $g_2\ g_1$ with the half-rounds in the plane U_2, U in Fig.13(b). The points A_{1U}, F_{1U} are base ones. The points C_{1U}, D_{1U} correspond to the initial and subsequent regime. It is possible to conclude that the half plane $U_2, U \geq 0.5U_0$ is also hyperbolic geometry model. Therefore, similarly to (44), the regime change has the view

$$m_U^{DC} = (-U_{2\max}\ U_2^{D1}\ U_2^{C1}\ U_{2\max}) = \\ = \frac{U_2^{D1} + U_{2\max}}{U_2^{D1} - U_{2\max}} \div \frac{U_2^{C1} + U_{2\max}}{U_2^{C1} - U_{2\max}}. \tag{48}$$

Also, the following equality takes place

$$m_U^{DC} = \left(m_g^{DC}\right)^2 .$$

Using (48), we obtain the subsequent value of the voltage

$$\frac{U_2^{D1}}{U_{2\max}} = \frac{\frac{U_2^{C1}}{U_{2\max}} + U_2^{DC}}{1 + \frac{U_2^{C1}}{U_{2\max}} U_2^{DC}}, \tag{49}$$

where the change of the voltage is introduced

$$U_2^{DC} = \frac{m_U^{DC} - 1}{m_U^{DC} + 1} = g_2^{DC} . \tag{50}$$

It now follows that

$$\frac{U_2^{D1}}{U_{2\max}} = \frac{g_2^{D1}}{r_1} . \tag{51}$$

C. Example

Let the circuit parameters be given as follows

$$U_0 = 5 ,\ R_i = 1,\ R_2 = 2 ,\ U_{1=} = 2.5 ,$$
$$R_{1\min} = 1,\ n_{1\max} = 1 .$$

Scale value (38)

$$n_{2ref} = \sqrt{\frac{2}{1}} = 1.414 .$$

The initial regime, point C_1, is set by the first load resistance $R_1^1 = 1.25$ and second load voltage $U_2^{C1} = 0.707$.
Using (24), we get

$$U = \begin{cases} 3.5 \\ 1.5 \end{cases} .$$

Further, we use the voltage value $U^{C1} = 3.5$ because this value is greater than $0.5U_0 = 2.5$.
Maximum values (37)

$$U_{2\max} = \frac{5}{2}\sqrt{2}\sqrt{1 - \frac{1}{1.25}} = 2.5\sqrt{2}\sqrt{0.2} = 1.581 ,$$

$$n_{2\max} = \sqrt{2}\sqrt{0.2} = 0.6324 .$$

Radius (41)

$$r_1 = \sqrt{0.2} = 0.447 .$$

Transformation ratios (25)

$$n_1^{C1} = \frac{2.5}{3.5} = 0.714 ,\ n_2^{C1} = \frac{0.707}{3.5} = 0.202 .$$

The normalized transformation ratios

$$\overline{n}_1^{C1} = 0.714 ,\ \overline{n}_2^{C1} = \frac{0.202}{1.414} = 0.143 .$$

Hyperbolic transformation ratios (47)

$$g_1^{C1} = \frac{1 - 0.714}{0.714} = 0.4 ,\ g_2^{C1} = \frac{0.143}{0.714} = 0.2 .$$

Let us check expression (41)

$$0.4^2 + 0.2^2 = 0.2 .$$

Further, we are verifying the regime change for the given load resistance R_1.
Let the subsequent regime, point D_1, be given by the second load voltage $U_2^{D1} = 1.414$ and voltage $U^{D1} = 3$.
Therefore, we have

$$\overline{n}_1^{D1} = \frac{2.5}{3} = 0.833 ,\ \overline{n}_2^{D1} = \frac{1.414}{3 \cdot 1.414} = 0.333 ,$$

$$g_1^{D1} = \frac{1 - 0.833}{0.833} = 0.2 ,\ g_2^{D1} = \frac{0.333}{0.833} = 0.4 .$$

Regime change (43)

$$m_g^{DC} = \frac{\sqrt{0.2} - 0.2}{0.4} \div \frac{\sqrt{0.2} - 0.4}{0.2} = 2.618 = \sqrt{6.8528} .$$

Regime change (44)

$$\left(m_g^{DC}\right)^2 = \frac{\sqrt{0.2} - 0.2}{\sqrt{0.2} + 0.2} \div \frac{\sqrt{0.2} - 0.4}{\sqrt{0.2} + 0.4} = 6.8528 .$$

The change of hyperbolic transformation ratio (46)

$$g_2^{DC} = \frac{6.8528 - 1}{6.8528 + 1} = 0.7453 .$$

Subsequent value (45)

$$\frac{g_2^{D1}}{r_1} = \frac{\frac{0.2}{\sqrt{0.2}} + 0.7453}{1 + \frac{0.2}{\sqrt{0.2}} \cdot 0.7453} = 0.8944 = \frac{0.4}{\sqrt{0.2}}.$$

Regime change (48)

$$m_U^{DC} = \frac{1.414 + 1.581}{1.414 - 1.581} \div \frac{0.707 + 1.581}{0.707 - 1.581} = 6.8528.$$

Equality (51)

$$\frac{1.414}{1.581} = \frac{0.4}{0.4472} = 0.8944.$$

IV. Conclusion

Non-Euclidean geometrical interpretation of graphical charts defines the load regime parameters and describes the movement of an operating point along the regulation trajectories.

Obtained expressions can be generalized for three or more loads.

From the methodological point of view, the presented approach is applied for a long time in other scientific areas, as mechanics (the principles of special relativity), biology (the principles of age changes of plants and organisms).

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