

Mathematical and Physical modelling of the dynamic electrical bioimpedance

Georgios Giannoukos, Mart Min

Abstract—In this overview a mathematical and physical model of the dynamic electrical bioimpedance is presented. The properties of dielectric materials, the dependence of dielectric permittivity on frequency as well as the expression of the complex permittivity for different theory approaches such as Debye, Cole – Cole, Cole – Davidson and Havriliak – Negami are also given. In addition the Fricke–Morse model, the Cole impedance model and the Debye model for biological tissue are analyzed.

Keywords—Conductivity, dielectric materials, impedance, permittivity.

I. INTRODUCTION

When an electric field (EF) can be maintained with zero or almost zero power losses in a material, then it is characterized as a dielectric or electrical insulator. In fact, the dielectric is not an ideal insulator, because a number of electrons can pass through the material. Because of the processes of changing the polarity in the material, part of the electrical energy is lost as heat. Dielectric material is one that has the ability to store energy when an external EF is applied. When a constant voltage is applied across the parallel plates of a capacitor with a dielectric between them, then more energy is stored. This means that the dielectric increases the ability of energy storage in the capacitor, eliminating some of the charges of the electrodes that would help in increasing the EF strength between the plates. The measured capacity with dielectric depends on the electrical loads of the material and is related to the dielectric constant. The following equation applies:

$$C = \varepsilon'_r C_o \Rightarrow \varepsilon'_r = \frac{C}{C_o} \quad (1)$$

where C and C_o are the capacities with a dielectric and vacuum, respectively, and ε'_r is the relative dielectric permittivity of the material. Dielectric materials used in the manufacture of capacitors, are the most suitable for preventing the creation of electrical currents through their mass while at the same time maintaining the voltage in the different parts of the electrical devices.

The relative dielectric constant of an insulating material

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should be close to 1 while for a dielectric it can generally be up to 10. The terms dielectric and insulator are often used almost synonymously, but with the former we focus on the physical properties of the material while with the latter its use in practical applications.

The electric force between the charges in a material is given by the following formula (Coulomb's law):

$$F = \frac{1}{4\pi\varepsilon_o} \frac{Q_1 Q_2}{r^2} \quad (2)$$

where $\varepsilon_o = 8.854 \cdot 10^{-12}$ F / m is the electric permittivity of free space.

If material is inserted between the charges, power is reduced according to the formula:

$$F = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{r^2} \quad (3)$$

where ε is the absolute permittivity of the medium, which is given as $\varepsilon = \varepsilon_r \cdot \varepsilon_o$, and ε_r is the relative permittivity of the medium. The dimensions of ε and ε_o are F/m in SI, but ε_r is a dimensionless number greater or equal to 1 in the case of the vacuum.

The capacitance of a capacitor consisting of two parallel plates is given by the equation:

$$C = \frac{\varepsilon_o A}{d} \quad (4)$$

where A is the area of each plate and d is the distance between them.

The presence of a dielectric material between the plates reduces the electrical field between the plates due to the existence of a polarized field in the opposite direction within the material in which case the above equation becomes:

$$C = \frac{\varepsilon A}{d} \Rightarrow C = \frac{\varepsilon_r \varepsilon_o A}{d} \quad \text{or} \quad C = \varepsilon_r C_o \quad (5)$$

For an ideal dielectric material ε is a real number but in practice for any dielectric material there is also an imaginary component that is associated with the dielectric loss because of the poor conductivity of the material which is why ε is called absolute complex permittivity which is also denoted by ε^* and describes the total interaction of the dielectric materials with the variable electrical field. In this case the

following equation is used: $\epsilon = \epsilon' - j\epsilon''$

(6)

II. RELATIVE COMPLEX PERMITTIVITY

In practice the relative complex permittivity is used instead of the absolute complex permittivity because it is dimensionless and takes simple numerical values. It is equal to the absolute complex permittivity divided by the electrical permittivity of free space, so:

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \left(\frac{\epsilon'}{\epsilon_0} \right) - j \left(\frac{\epsilon''}{\epsilon_0} \right) = \epsilon'_r - j\epsilon''_r$$

(7)

where $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$ is the electrical permittivity of free space.

ϵ' is the real part of the relative complex permittivity which is related to the stored energy within the medium.

ϵ'' is the imaginary part of the relative complex permittivity which is related to the dissipation (or loss) of energy within the medium.

The ϵ'_r is an indication of how much energy can be stored in the material by the applied EF and is called relative permittivity. For the vacuum $\epsilon'_r = 1$ while for gaseous dielectric materials $\epsilon'_r \approx 1$, but for most liquid and solid insulators: $1 \leq \epsilon'_r \leq 10$. For semiconductors it is usually $10 \leq \epsilon'_r \leq 20$ and for metals $\epsilon'_r \rightarrow \infty$ because there are no dielectrics. The ϵ''_r is related to the loss of energy in the medium and is an indication of how polar loose the material is to the external imposed EF. Loose material in a frequency occurs when the polarization mechanism of the material is able to follow the ER changes applied to the material. The ϵ''_r is always a positive quantity much smaller than the ϵ'_r .

Another symbol of the ϵ_r is the ϵ_r^* or k_r^* in which case $k_r^* = k' - jk''$ where $k' = \epsilon'_r$ and $k'' = \epsilon''_r$.

III. DEPENDENCE OF DIELECTRIC PERMITTIVITY ON FREQUENCY

The dependence of the frequency of dielectric permittivity is associated with the mechanisms of polarization which take place in each frequency range. At low frequencies all mechanisms are present. Increasing the frequency and reaching the microwave region (105-1010Hz), the permanent dipoles, due to inertia, can monitor changes in the field and align with it. The polarization orientation stops and the dielectric permittivity declines. Also some energy no longer goes to the circuit but is absorbed by the material indicating the existence of losses. The curve ϵ'' in this region takes the form of a resonance curve.

At higher frequencies in the infrared the mechanism of ionic polarization stops while in the UV the mechanism of electronic polarization stops.

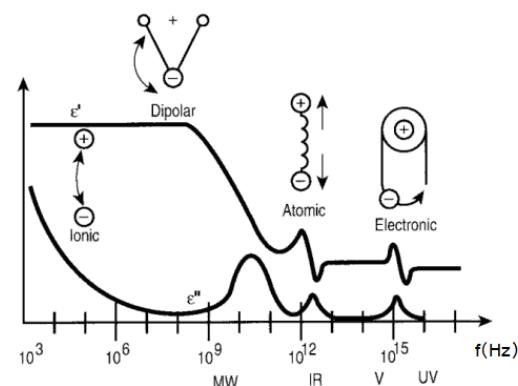


Fig.1: ϵ' and ϵ'' change with the frequency [1]

The real part of dielectric permittivity ϵ' varies between a maximum value ϵ_s , corresponding to static fields or very low frequencies and a minimum value ϵ_∞ , corresponding to very high frequencies. The imaginary part ϵ'' is related to losses and has a maximum at the resonance frequency.

The ϵ' and ϵ'' are given by Debye's equations:

$$\epsilon' = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2 \tau^2} \quad \epsilon'' = \omega \tau \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2 \tau^2} \quad (8)$$

Where τ is the relaxation time of the material

From the above equations we get:

$$\begin{aligned} \epsilon - \epsilon_\infty &= \epsilon' - \epsilon_\infty - j\epsilon'' = \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2 \tau^2} - j\omega \tau \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2 \tau^2} \Rightarrow \\ (\epsilon' - \epsilon_\infty)^2 + \epsilon''^2 &= (\epsilon' - \epsilon_\infty)(\epsilon_s - \epsilon_\infty) \Rightarrow \\ \epsilon'^2 + \epsilon''^2 &= \epsilon'(\epsilon_s - \epsilon_\infty) - \epsilon_s \epsilon_\infty \end{aligned} \quad (9)$$

The above equation in a diagram representing a circle has its center on the axis ϵ' at the point $(\epsilon_s + \epsilon_\infty)/2$. The diagram shown in the figure below is a graph called Cole-Cole.

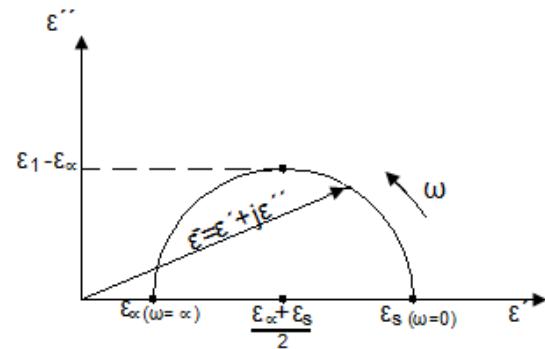


Fig.2: Cole-Cole graph

A dielectric material subjected to a field can be visualized at any frequency from an equivalent circuit of a capacitance and a resistance in series or parallel. For materials that exhibit dielectric losses the parallel equivalent is usually considered more suitable.

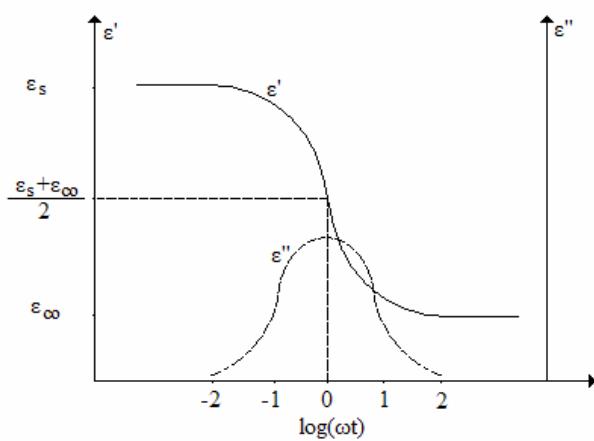


Fig.3: Real and imaginary part of the dielectric permittivity for a Debye mechanism

As shown in the figure above, in a frequency range around the value $\omega = 1 / \tau$ we observe a peak in ϵ'' which is because in this region energy losses are maximized due to the frequencies ω and $1/\tau$ being comparable. The maximum peak located at the frequency $\omega_{\max} = 1/\tau$

We also see a step in ϵ because in much smaller frequencies than ω_{\max} the dipoles have time to follow the changes in the field whereas at much higher frequencies they don't thus they are not involved in the polarization which is why at low frequencies $\epsilon' = \epsilon_s$ and at high frequencies $\epsilon' = \epsilon_\infty$.

Debye's dispersion equations describe a relaxation process which is characterized by a single relaxation time and each relaxation process is described separately. The experimental results are consistent with the theoretical in the case of polar liquids. However, when studying systems in condensed matter, due to interactions between atoms and molecules, several relaxation times appear and the experimental data deviates from Debye's theory. For a description of the experimental data different theories are used such as the Cole-Cole, Cole-Davidson and Havriliak-Negami because they take into account different types of distribution of relaxation times [2]. The expression of the complex permittivity for each approach (theory Debye [3], Cole - Cole [4], Cole - Davidson [5], Havriliak - Negami [6]) is given by the following relations:

$$\text{Debye: } \epsilon = \epsilon_\infty + \frac{(\epsilon_s - \epsilon_\infty)(1 - j\omega\tau)}{1 + \omega^2\tau^2} \quad (10)$$

$$\text{Cole-Cole: } \epsilon = \epsilon_\infty + \frac{(\epsilon_s - \epsilon_\infty)}{1 + (j\omega\tau)^{1-\alpha}}, \quad 0 \leq \alpha \leq 1, \quad (11)$$

where α is a parameter and the distribution of relaxation times is symmetrical around τ . If $\alpha=0$ then it gives the Debye equation

$$\text{Cole-Davidson: } \epsilon = \epsilon_\infty + \frac{(\epsilon_s - \epsilon_\infty)}{(1 + j\omega\tau)^\beta}, \quad 0 \leq \beta \leq 1, \quad (12)$$

the distribution of relaxation times is not symmetrical and β is a parameter which determines the shape. If $\beta=1$ then it gives the Debye equation.

Havriliak - Negami [7],[8],[9]:

$$\epsilon = \epsilon_\infty + \frac{(\epsilon_s - \epsilon_\infty)}{[1 + (j\omega\tau)^{1-\alpha}]^\beta}, \quad 0 \leq \alpha \leq 1 \quad \text{and} \\ 0 \leq (1-\alpha)\beta \leq 1, \quad (13)$$

if $\beta=1$ it gives the Cole-Cole equation, if $\alpha=0$ the Cole-Davidson equation and if $\beta=1$ and $\alpha=0$ the Debye. [10]

IV. DIELECTRIC SPECTROSCOPY

In the above analysis we assumed that the electrical field which is applied to the dielectric is sinusoidal with stable frequency. Dielectric spectroscopy studies the change in dielectric properties of the material with time and frequency. Inside in an isotropic and homogeneous dielectric the polarization density vector P and the intensity E of the EF have the same direction and are linked in the equation [11, 12]:

$$P = \chi \epsilon_0 E \quad (14)$$

where χ is the electrical susceptibility of the material. The χ indicates the degree of all kinds of polarizing of a dielectric and is dimensionless (for the vacuum it is equal to zero). The $\epsilon_0 = 8.85419 \cdot 10^{-12} \text{ As/Vm}$, is the dielectric constant of the vacuum.

The electric displacement D is determined by the overall positive or negative electric charge per unit area induced in the corresponding electrode. The electrical charge of the two electrodes is the origin of all the electrical field lines. In the area between the electrodes without a dielectric, the electrical displacement is parallel to the electric field E and is connected to it by the equation $D = \epsilon_0 E$. When there is an isotropic dielectric between the electrodes then the electrical displacement increases by the polarization density P and the equation becomes $D = \epsilon_0 E + P$ [13, 14]. This happens because in each electrode part of the charge creates the electric field E while the remaining charge compensates polarization charges of the dielectric.

Considering the above equations we get:

$$D = \epsilon_0 E + \chi \epsilon_0 E = (1 + \chi) \epsilon_0 E \quad (15)$$

Thus in a time constant electrical field the electrical

displacement D is proportional to E and for isotropic dielectric materials the vectors P and D are parallel to E. The proportionality factor $(1 + \chi)\epsilon_0$ is the dielectric permittivity ϵ of the material and the $1+\chi$ is the relative dielectric permittivity ϵ_r . In this case the use of complex equations is not necessary.

When the electrical field is time varying E(t) then the D in the vacuum immediately follows changes in the field D(t) = $\epsilon_0 E(t)$. The charge density at the electrodes is determined by the displacement current, which is derived from the voltage source and equals dQ / dt , where Q is the total charge on each electrode.

When the vacuum is replaced by an isotropic dielectric material then the electric displacement D is:

$$D(t) = \epsilon_0 E(t) + P(t) \quad (16)$$

The time dependence of P(t) is not the same as E(t) because the polarization of the dielectric is not directly related to the applied field, but it has a time delay which is different for the various types of polarization. To find the relationship between the field and the polarization we will try to determine a time-varying function of the electrical susceptibility $\chi = \chi(t)$ and hence the relative dielectric permittivity $\epsilon_r = 1 + \chi(t)$ applies to the equations:

$$P(t) = \chi(t) \epsilon_0 E(t) \quad (17)$$

$$\text{and } D(t) = (1 + \chi(t)) \epsilon_0 E(t) \quad (18)$$

It is assumed that a temporal changing EF is applied: $E(t) = E_0 u(t - t_0)$ where $u(t - t_0)$ is the step function of the temporal change. Initially the field is zero and in t_0 the applied EF has a constant value E_0 , which is maintained for $t > t_0$.

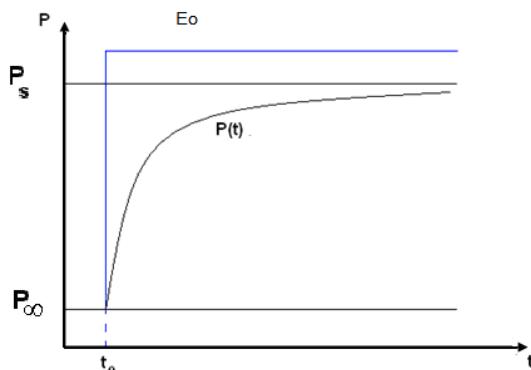


Fig.4: The temporal variation of the polarization of the dielectric as a result of stepping electric field.

In this case the polarization P(t) of the dielectric is associated with the time-dependent susceptibility of $\chi(t)$ according to the equation:

$$\frac{P(t)}{E_0} = \epsilon_0 \chi(t) u(t - t_0) \quad (19)$$

That is, the time-dependence of the polarization follows that of susceptibility $\chi(t)$, which a characteristic function of the material, and the magnitude of the polarization is

proportional to the applied field.

The function of polarization and susceptibility is generally distinguished into three time domains. In principle there are very fast processes of polarization (mainly electronic) that can be considered to be affected in a very short time close to zero. Simultaneously with the application of the field there is an instantaneous polarization $P(t = t_0)$, denoted P_∞ because it is performed at very high speed. This part of the polarization function cannot be recorded with the usual measuring equipment. After a long period of time, the polarization eventually becomes constant and takes the value P_s . Considering these two extreme values, the polarization can be given by the equation:

$$P(t) = P_\infty + (P_s - P_\infty) g(t - t_0) \quad (20)$$

where $g(t)$ is a dimensionless, monotone increasing function (characteristic of the material) which determines the way the polarization goes from the baseline value P_∞ to the final value P_s .

Using the indices s and ∞ for the respective values of χ , the polarization can be written as follows:

$$P(t) = \epsilon_0 [\chi_\infty + (\chi_s - \chi_\infty) g(t - t_0)] E_0 \Rightarrow \quad (21)$$

$$P(t) = \epsilon_0 \chi(t) E_0$$

where

$$\chi(t) = \chi_\infty + (\chi_s - \chi_\infty) g(t - t_0) \quad (22)$$

If $1 + \chi_s = \epsilon_s \Rightarrow \chi_s = \epsilon_s - 1$ and $1 + \chi_\infty = \epsilon_\infty \Rightarrow \chi_\infty = \epsilon_\infty - 1$ then the above equations become:

$$P(t) = \epsilon_0 [(\epsilon_\infty - 1) + (\epsilon_s - \epsilon_\infty) g(t - t_0)] E_0 \quad (24)$$

and

$$\chi(t) = (\epsilon_\infty - 1) + (\epsilon_s - \epsilon_\infty) \cdot g(t - t_0) \quad (25)$$

According to equation $D = (1 + \chi) \epsilon_0 E$ then we obtain

$$D(t) = \epsilon_0 [\epsilon_\infty + (\epsilon_s - \epsilon_\infty) g(t - t_0)] E_0 \quad (26)$$

Where

$$\epsilon(t) = 1 + \chi(t) = \epsilon_\infty + (\epsilon_s - \epsilon_\infty) g(t - t_0) \quad (27)$$

is the time depending relative permittivity of the dielectric material. All of the above are valid only for the simple case of stepping

V. ELECTRICAL BIOIMPEDANCE

Electrical bioimpedance [15] describes how a living organism responds to an externally applied electrical current [19]. It can be defined as the impedance of biological specimens. It is a measure of the difficulty of the flow of electrical current through the tissues.

A biological tissue can be modeled from a structural viewpoint as the grouping of a number of elements called cells, which are immersed in an ionic medium (Na^+ , K^+ , Ca^{2+} , Cl^-) called extracellular fluid which also contains proteins and it can be divided into plasma and interstitial fluid. We can consider any biological tissue as an electrolyte

because of the ions it contains.

Inside the cell membrane there is also a fluid (cytosol or intracellular fluid) and ion concentration in the intracellular environment, which is where the body's metabolic processes take place. It also contains organelles and the nucleus of the cell.

The cell membrane consists of proteins and phospholipids forming a bilayer lipid membrane. Each monolayer has very small electrical conductance (10^{-6} A/V) and for that reason we can consider it as a dielectric material. Also there are ion gates in the membrane which control the ion conductance (which is the inverse of the resistance).

For these reasons we can model the cell membrane as a two plated capacitor connected to a resistor in parallel (C_m - R_m). The intracellular medium of the cell also behaves as a resistor (R_i).

Taking into account the ions in the extracellular fluid we can add another resistor (R_e) to represent the extracellular fluid to the equivalent circuit [16].

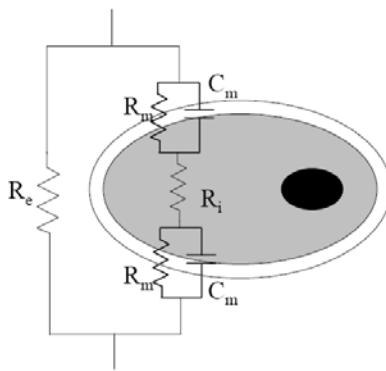


Fig.5: Equivalent electrical circuit of a tissue

By simplifying the above circuit taking into account that the two R_m resistors are connected in series and also the capacitors C_m , then the circuit becomes:

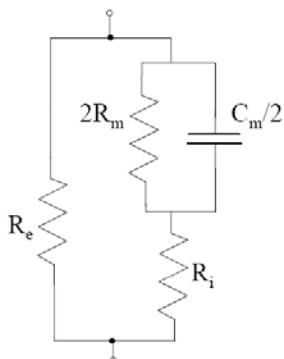


Fig.6: Simplified equivalent electrical circuit of a tissue

As aforementioned, the conductance of the membrane is very low in which case R_m takes a very high value. So the above circuit is further simplified [17] (this is the Fricke-Morse model [18]):

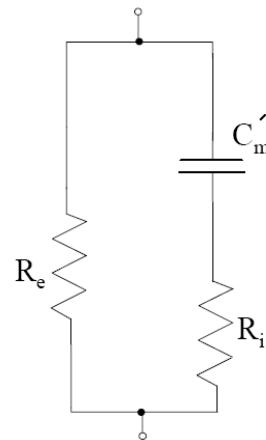


Fig.7: Fricke-Morse model

The impedance of the above circuit is equal to:

$$Z = \frac{R_e(1 + jR_iC\omega)}{1 + jC\omega(R_i + R_e)} \quad (28)$$

At low frequencies most of the current flows around the cell and only a little goes through the cell the membrane impedance is very high. If in equation (28) $\omega \rightarrow 0$ then $Z = R_e$ due to the fact that.

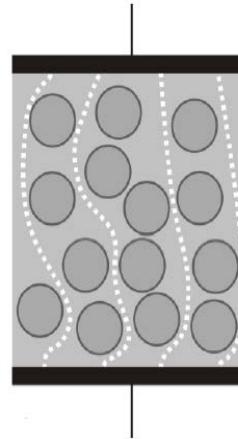


Fig.8: Passage of low frequency currents through a cell suspension or tissue

At high frequencies the current flows through both the extracellular and intracellular fluid because the membrane capacitance doesn't act as an impediment.

$$Z_c = \frac{1}{jC\omega} \rightarrow 0$$

When $\omega \rightarrow \infty$ then $Z = \frac{R_e R_i}{R_e + R_i}$ thus R_e/R_i and

$$(29)$$

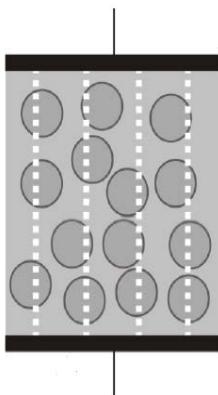


Fig.9: Passage of high frequency currents through a cell suspension or tissue

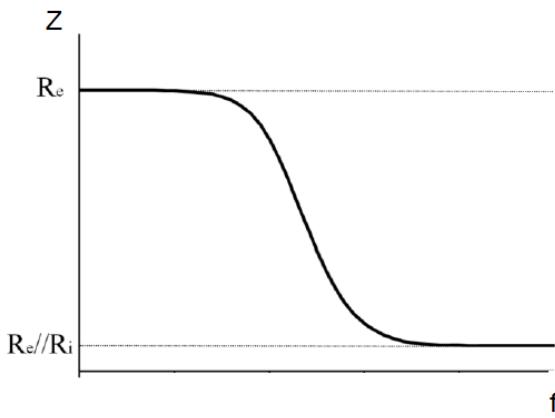


Fig.10: Impedance vs frequency in a tissue

The conductivity and the permittivity of tissue are frequency dependent [19]. This phenomenon is called dispersion [20, 21, 22, 23, 24] of which there are four types: α, β, δ and γ .

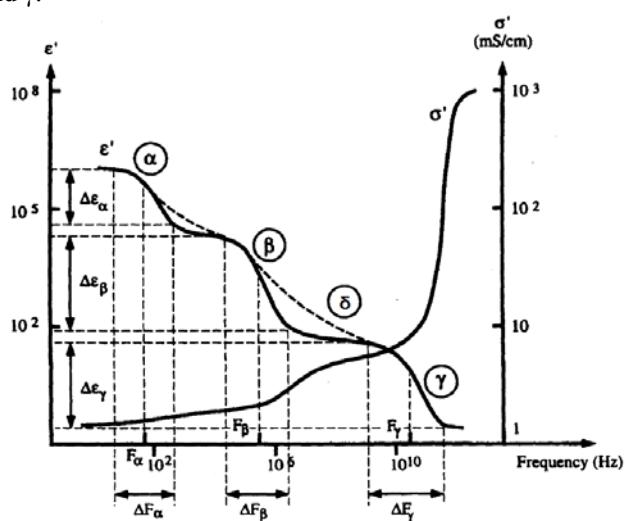


Fig.11: Frequency dispersions [22]

The Fricke-Morse model is not very accurate so the Cole

impedance model was proposed for tissue. The Cole empirical equation [20] is expressed by the following equation:

$$Z = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau)^\alpha} \quad (30)$$

where: Z is the complex impedance, R_0 is the resistance at zero frequency, R_∞ is the resistance when $f \rightarrow \infty$ (only resistive parts), ω is the angular frequency, τ is the characteristic relaxation time constant and α is a parameter with values between 0 and 1.

For example if $\alpha=1$ we obtain Fricke-Morse model

In the Fricke-Morse model we substitute the capacitance (Debye model) with the Constant Phase Element (CPE) [25] which is described as an imperfect capacitor which is frequency dependent. The impedance of the CPE is:

$$Z_{CPE} = \frac{1}{(j\omega C)^\alpha}, \quad (31)$$

and when $\alpha=1$ the CPE behaves as an ideal capacitor.

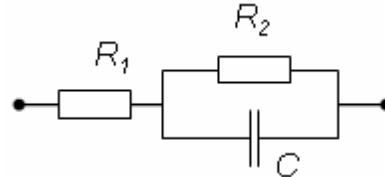


Fig.12: Debye model for tissues

By applying the Cole equation to this model we obtain:
 $R_0=R_1+R_2$, $R_\infty=R_1$ and $\tau=R_2C$.

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