Prospects of high-frequency gravimetry

A. L. Dmitriev

Abstract - The gravitational field of the Earth is assumed to be a stochastic process the wide frequency spectrum of which is conditioned by the influence of various geophysical, astrophysical and anthropogenic factors. The frequency range of fluctuations of gravity field at frequencies over 1 *Hz* has not been significantly studied yet and still remains a peculiar "Terra Incognita" of gravimetry. High-frequency changes of a free fall acceleration data are informative for understanding of the complex physical processes happening in the core and crust of the Earth. They can be used to solve practical problems such as prediction of earthquakes, exploration of minerals, as well as problems of detection and identification of massive underwater or underground artifacts.
The principles of new types of high-frequency gravimeters – the holographic ballistic gravimeter with the short length of a trajectory of a trial body and the ballistic gravimeter on the basis of freely falling mechanical rotor are considered.

Keywords - ballistic gravimeters, free fall acceleration, gravitational field of the Earth, hologram, rotor

I. Introduction

The gravitational field of the Earth is assumed to be a stochastic process the wide frequency spectrum of which is conditioned by the influence of various geophysical, astrophysical and anthropogenic factors. High sensitivity of the best modern gravimeters is achieved primarily through proper stabilization of temperature and mechanical characteristic of the equipment used and long integration time of registered signals – from tens of seconds to 24 hours [1]. Obviously, at large times of signal integration, the information about high-frequency variations of a gravitational field is lost. The frequency range of fluctuations $g_0(t)$ at frequencies over 1 *Hz* has not been significantly studied yet and still remains a peculiar "Terra Incognita" of gravimetry [2].

Meanwhile, high-frequency changes of a free fall acceleration (FFA) data are informative for understanding of the complex physical processes happening in the core and crust of the Earth.
They can be used to solve practical problems such as prediction of earthquakes, exploration of minerals, as well as problems of detection and identification of massive dynamic underwater or underground artifacts.
High-frequency (HF) gravimetry data is of a great scientific and practical importance and the development of HF-gravimetry as a new research area is inevitable.

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Such gravimeters should provide an accurate measurement of the "instantaneous" value of FFA in the frequency range from few *Hz* to thousands (and probably more) *Hz*.

For data acquisition about high-frequency fluctuations of the FFA the following experimental methods can be used
- spectral analysis of output signals of known types of ballistic and static gravimeters;
- data processing of superconducting gravimeters with the minimum mass of a trial body;
- data processing of ballistic gravimeters with very small, an order of units of mm, length of a trajectory of falling of a trial body;
- data processing of ballistic gravimeters on the basis of freely falling mechanical rotor with a horizontal axis of rotation.

The convenient modern tools of HF-gravimetry include superconducting gravimeters (SCG). Owing to a rather big proof mass, the highest frequency of variations in the gravity acceleration value registered by SCG does not exceed a few tens of *Hz*, although the frequency range of such measurements can be essentially extended after the improvement of these devices. Among HF-gravimetry measurement methods we should also mention the application of ballistic gravimeters with extremely small, for example, less than 1 mm, length of the proof mass fall trajectory.

II. A ballistic gravimeter with falled holographic grating

In a 'standard' ballistic laser gravimeter a corner reflector mounted on a free-falling trial body acts as a part of the two-beam Michelson interferometer [1]. The absolute value of the FFA is measured by counting the number of interference fringes passing in the photodetector out-plane within a present time interval. A small error of measurement in these gravimeters is achieved by using a single-frequency laser, atomic clocks, high vacuum, and with large (from several dozen seconds up the about a day) time of accumulation and averaging of measured signal. The large, several dozen centimeters, length of the falling trajectory restricts the possibility of using these laser gravimeters for measurement of high-frequency fluctuations of gravitational field in a frequency range of several hundred Hz and above.
Principle of holographic ballistic gravimeter is based on variation of the frequency of light diffracted on a moving holographic grating. Geometry of light-beam diffraction on the holographic transmission grating is shown in Fig.1.

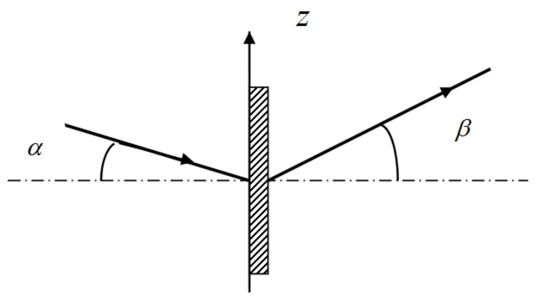
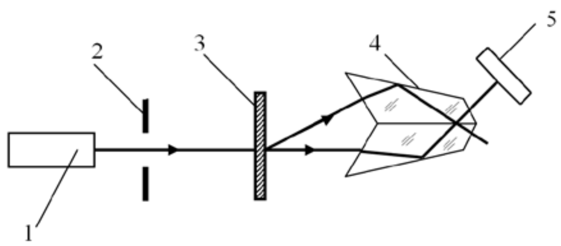


Fig.1. Light diffraction on the transmission hologram.

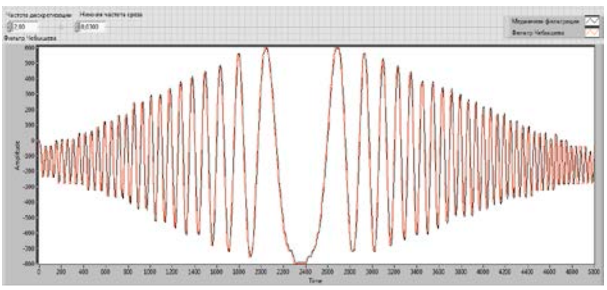
At the movement of the hologram the frequency of the diffracted light changes**.** As shown in [3], size of the acceleration g of free falling hologram is equal

$$g = \frac{\lambda}{\sin\alpha + \sin\beta}\left(\frac{\partial f}{\partial t}\right) \qquad (1)$$

and proportional to the grade of variation of the frequency f of output beam signal of hologram interferometer; here λ - wavelength and angles α, β shown in Fig. 1. The optical device of the holographic gravimeter and typical form of the registered frequency-modulated signal of beats are shown in Fig. 2 a,b.



a.



b.

Fig. 2. a. Basic optical device of the holographic gravimeter; 1 - laser, 2 – diaphragm, 3 – hologram, 4 – optical multiplexer, 5 – photo-detector.
b. Frequency-modulated output signal of beats.

In our experiments a hologram with 37% diffraction efficiency had an angular selectivity 10^0, $\alpha = 0, \beta = 37.8^0, \lambda = 632.8nm$. The hologram was mounted in the special holder (catapult).

Temporal resolution of measuring is $10^{-2}s$, error of separate (single) measurement of FAA is near 10 mGal. Using the modern methods of statistical filtration and processing of the linear frequency-modulated beat signals, it is possible to obtain the mobility and high precision of gravimetric measurements. Specific features of the holographic gravimeter is simple optics-mechanical design and very small (below 1 mm) length of the trajectory of the free-falling body.

III. Weight of Oscillator in a Variable Field of Gravitation

Ballistic gravimeters with the test body executed in the form of a mechanical rotor with a horizontal axis of rotation should also be considered as new and perspective means of HF-gravimetry. Rotary motion corresponds to two oscillatory motions of the rotor particles along the orthogonal axis of coordinates. The accelerated harmonic motion of the rotor particles on a vertical is characterized by an infinite set of time derivates. In these condition the interaction of such rotor with a non-stationary gravitational field of the Earth can have a specific, not trivial character. Such researches will promote obtaining the new data on dynamic characteristics and specific features of the gravitational field of the Earth.

Let's consider interaction of a mechanical rotor with an alternating gravitational field which is based on the gravitational analogy of the phenomenon of Faraday and Lenz's Law in electrodynamics [4-6]. According to [5,6] the change of acceleration of the gravity acting on a body, moving with acceleration $\vec{a}$ under influence of the elastic force, in the elementary (linear) approximation, is represented as

$$\Delta\vec{g}_{p,c} = -\frac{\vec{g}_0}{|\vec{g}_0|}(\vec{g}_0 \cdot \vec{a})A_{p,c} \qquad (2)$$

where symbols p, c mean passing (p) and a contrary (c), in relation to a direction of vector $\vec{g}_0$ of normal acceleration of a gravity, orientation of a vertical projection of vector $\vec{a}$ of acceleration of external forces, and factors A_p and A_c characterize a degree of change of values $\Delta\vec{g}_{p,c}$. If the massive body under action of the external, electromagnetic in nature, elastic force makes harmonious oscillations along a vertical with frequency ω and amplitude B, the average for the period $\tau = 2\pi/\omega$ of fluctuations value $\Delta\overline{g}$ of change of FFA of such mechanical oscillator is equal to the sum of average changes of FFA in movement of a body passing and contrary to vector $\vec{g}_0$,

$$\Delta\overline{g} = \Delta\overline{g}_p + \Delta\overline{g}_c \qquad (3)$$

and at constant $g_0 = |\vec{g}_0|$ it is equal

$$\Delta\overline{g} = -\frac{g_0 B\omega^2}{\pi}(A_p - A_c). \qquad (4)$$

We will note that square dependence $\Delta \bar{g} \propto \omega^2$ shows that changes of weight of the oscillator will be considerable owing to thermal fluctuations of particles of material of the oscillator which frequency is in hyper-sound area. Is shown that temperature dependence of physical weight $P(T)$ of the oscillator is represented by a formula

$$P(T) = P_0 \left[1 - \frac{C(A_p - A_c)}{\pi} \sqrt{T} \right] \quad (5)$$

where T - absolute temperature and C - the coefficient depending on elastic characteristics of material of the oscillator [4,6].

The effect of negative temperature dependence of body weight was repeatedly observed in experiments [7,8] that confirms justice of formulas (1,5).

Some interesting and deserving attention results turn out at calculations of change of weight of the mechanical oscillator which is freely falling in a variation field of gravitation.

We shall present elementary time dependence $g_0(t)$ as

$$g_0(t) = g_0(1 + \beta \sin(\Omega t + \theta)) \quad , \quad (6)$$

where Ω – frequency of changes of FFA value, β - their relative amplitude, θ - the phase. Acceleration $a(t)$ of the material point making harmonious oscillations along a vertical with amplitude B is equal to

$$a(t) = B\omega^2 \sin \omega t \quad (7)$$

where ω - frequency of oscillations.

The averages for oscillation half-cycle $\tau / 2$ of values of changes of accelerations $\Delta \bar{g}_p$ and $\Delta \bar{g}_c$ are equal to

$$\Delta \bar{g}_p = -A_p g_0 B \omega^2 \frac{2}{\tau} \int_0^{\tau/2} \sin \omega t (1 + \beta \sin(\Omega t + \theta)) dt \quad (8)$$

$$\Delta \bar{g}_c = -A_c g_0 B \omega^2 \frac{2}{\tau} \int_{\tau/2}^{\tau} \sin \omega t (1 + \beta \sin(\Omega t + \theta)) dt \quad (9)$$

The relative change of FFA of the oscillator, in view of 3, shall be presented as

$$\frac{\Delta \bar{g}}{g_0} = 4\pi A_p B F^2 f(x) \quad (10)$$

where $F = \Omega / 2\pi$, $x = \omega / \Omega$ and frequency function $f(x)$ is equal to

$$f(x) = -x^2 \left[\int_0^{\pi} \sin z (1 + \beta \sin(xz + \theta)) dz + \mu \int_{\pi}^{2\pi} \sin z (1 + \beta \sin(xz + \theta)) dz \right] \quad (11)$$

here $\mu = A_c / A_p$ and $z = \omega t$.

Examples of frequency functions $f(x, \mu, \theta, \beta)$ at various parameters μ, θ, β, and both low values of x are shown in Fig. 3.

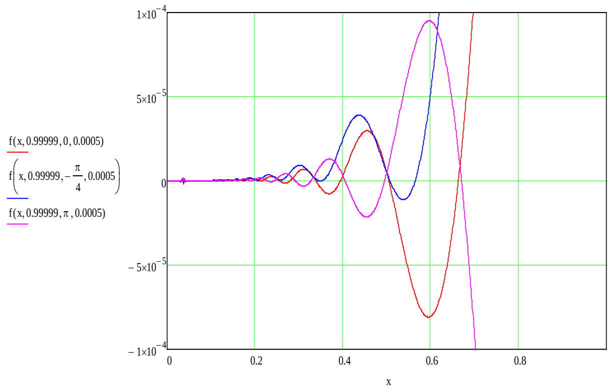


Fig. 3. Frequency functions $f(x, \mu, \theta, \beta)$ at low values of argument x; relative amplitude of fluctuations of FFA $\beta = 0.0005$.

Examples of frequency functions $f(x, \mu, \theta, \beta)$ at various parameters μ, θ, β, and both low (a) and high (b) values of x are shown in Fig. 4 and Fig. 5 a,b.

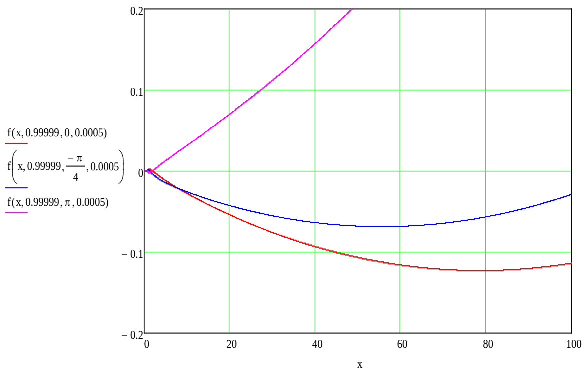
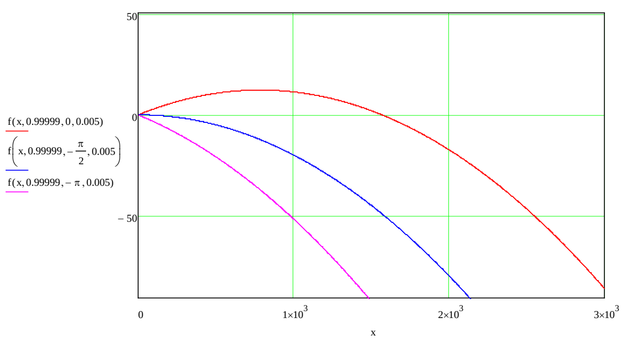
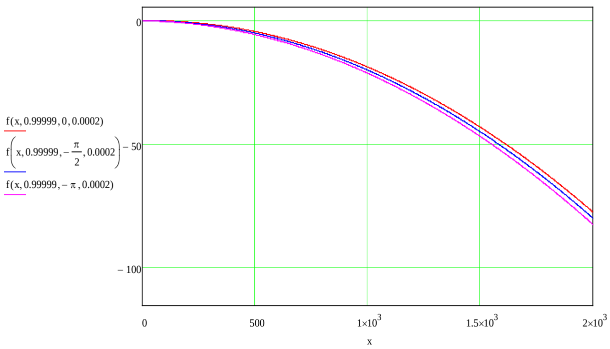


Fig. 4. Examples of frequency functions $f(x, \mu, \theta, \beta)$ at high values of argument x; relative amplitude of fluctuations of FFA $\beta = 0.0005$.



a.



b.

Fig. 5. Examples of frequency functions $f(x, \mu, \theta, \beta)$ at the high values of argument x;
a. - relative amplitude of FFA fluctuations $\beta = 0.005$,
b. - $\beta = 0.0002$.

Obviously, the sign and a general view of functions $f(x)$ essentially depend on parameters μ, θ, β. According to estimations given above, in the calculations, $\mu = 0.99999$ is assumed. The given calculated dependences show that even at small, with relative value of about the 100-th fractions of percent, amplitudes β of fluctuations in value of normal acceleration of the gravity of the Earth, the weight of mechanical oscillator can be changed appreciably.
At frequencies ω of oscillations, with an order of the frequency Ω of own FFA fluctuations, in area $x \le 1$, the weight of oscillator periodically changes with frequency, and sign and values of such changes essentially depend on a difference of phases θ of oscillations (see Fig. 3). At high ($x \succ\succ 1$) frequencies of oscillator, the monotonous dependence of average weight of oscillator on frequency of its fluctuations is taking place, with influence of phase θ being insignificant (Fig. 5. b). Such reduction of weight of oscillator at high frequencies of fluctuations will agree with negative temperature dependence of weight of bodies as the frequencies of thermal fluctuations of microparticles of solid state bodies are rather high and lie in the field of the hypersound [9].

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III. Experimental frequency dependence of free falling acceleration of rotor

In our experiment the free falling acceleration of the magnetically-, thermally- and sound-isolated container with a vacuumed aviation rotor inside it was measured [10,11]. Appearance of a rotor is shown in Fig. 6.

Fig. 6. Rotor of aviation gyroscope.

The maximal rotation frequency of a rotor is 400 *Hz*, the run out time of rotor is 22 min. Fall path length of the container is 30 mm, readout time of sample value of gravity acceleration is near 40 ms, the period of sampling is from 0.5 up to 1.0 minutes. The principle of measurements is based on photo registration of movement of the scale in form of three horizontal strings fixed on the container (Fig.7.).

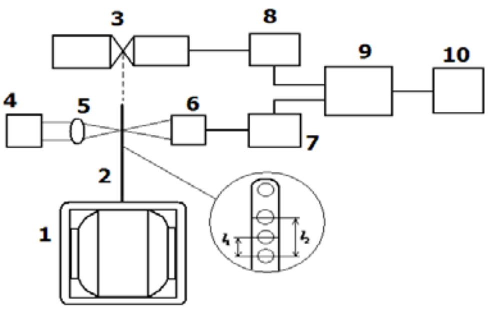

Fig. 7. Basic device of experiment. 1-falled container, 2 – scale, 3 – driver, 4 – laser, 5 – lens, 6 – photodiode, 7 – amplifier, 8 – timer, 9 – oscillograph, 10 – computer.
At the maximal falling velocity of the container equal to 60 cm/s and its dimensions of 82x82x66 mm, the joint influence of buoyancy and resistance force of air in FFA measurements did not exceed 0.1 cm/s^2. The error of some measurements of the FFA container was within the limits of 0.3-0.6 cm/s^2 and was basically determined by accuracy of readout times of registration of pulse signals in movement of the scale (near 1 microsecond).

The example of experimental frequency dependence of FFA changes $\Delta g(f)$ of the container, containing a rotor with a horizontal rotation axis, is shown (in the Fig. 8. – in Fig.8.

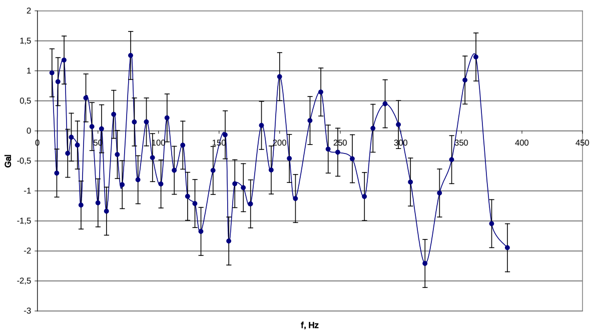

Fig.8. The frequency dependence of free falling acceleration of the container with horizontally positioned rotor; the changes of FFA (Gal) relatively to the value of FFA with the stopped **r**otor have been shown.

The value $\Delta g(0)=0$ corresponds to acceleration of free falling of the container with a motionless rotor; FFA measurements of the container with a motionless rotor were carried out till the moment when rotor got going and after its run out time, in so doing the FFA values of the container, averaged by results of 10 measurements with a motionless rotor, coincided to the accuracy of 0.05%.

Comparing Fig. 3 and Fig. 8, it can be seen that the area of steady periodic changes of FFA in Fig. 8 in a band of frequencies 200-400 Hz approximately corresponds to the area in a vicinity of value $x \approx 0.5$ in Fig. 3. Having substituted in (10) the experimental value $\Delta g / g_0 \sim 10^{-3}$, and assume then $A_p \sim 10^{-2} g_0^{-1}$, $f(x) \sim 10^{-5}$, we obtained an estimation of amplitude $B \sim 1.4\text{cm}$ of oscillator. The given size almost coincides with radius of the rotor used in experiments. At oscillation frequencies tens times higher than the frequencies F of own fluctuations of normal acceleration of the gravity (according to the given estimations, $F \sim 300/0.5 = 600Hz$) and following the suggested model, there is observed a monotonous frequency dependence of change $\Delta\bar{g}$ of average value of acceleration of free falling oscillator, with sign $\Delta\bar{g}$ being is directly determined by the difference of phases θ of fluctuations FFA and oscillator. Within the limits of applicability of formulas 6,10 there are possible both substantial growth and reduction of the average gravity working on mechanical oscillator on the part of the variable gravitational field of the Earth. Let's note that the independent measurements of high-frequency, in the range of hundreds – thousands of Hz, spectra of fluctuations of acceleration of the gravity of the Earth, executed, for example, with use of SCG, will allow to define modes of the matched fluctuations of oscillator at which the changes of its average weight can essentially surpass the ones described by formulas 6-10.

The above calculated and experimental estimations given above have an illustrative character. Nevertheless, the considered simple phenomenological model finely explains the experimental dependences and agrees with the known data of measurements of weight of accelerated moving test bodies. Experimental researches into free falling mechanical oscillators (rotors, vibrators) will allow to bring the necessary specifications into the offered models, to determine the borders of their applicability, and to prove more strictly the size parameters introduced into these models. Such researches will promote obtaining the new data on dynamic characteristics and specific features of the gravitational field of the Earth.

IV. Conclusions

1. High-frequency gravimetry – the new direction in gravitation measurements. Its purpose is research into high-frequency fluctuations of natural gravitation field of Earth
in the range of tens-hundreds Hz and more.

2. Technical approach of HF-gravimetry:
- spectral and correlation analysis of output signals and noise of known types of gravimeters;
- creation of broadband gravimeters with the minimum mass of a trial body;
- development of ballistic gravimeters with very short, about 1 mm, length of a trajectory of falling of a trial body (for example, hologram gravimeter);
- development of gravimeters with a trial body in the form of a mechanical rotor with horizontal axis of rotation and the speed of rotations about hundreds Hz;

3. Data of HF-gravimetry will allow to improve techniques of investigation of minerals, and techniques of the prevention of natural disasters (earthquakes, a tsunami and others).

Development of HF-gravimetry techniques and exploration of above-mentioned "Terra Incognita" carries significant scientific and applied values.

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