Robustness of Control Time-delay Processes in Term of Influence Parametric Uncertainties

V. Bobál, P. Dostál, and M. Kubalčík

Abstract— This paper deals with a design of a universal and robust digital control algorithms for control of great deal processes with time-delay. Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns or some types of chemical reactors. The designed control algorithms are realized using the digital Smith Predictor (SP) based on polynomial approach – by minimization of the Linear Quadratic (LQ) criterion. For minimization of the LQ criterion is used spectral factorization principle with application of the MATLAB Polynomial Toolbox. The designed polynomial digital Smith Predictors were verified in simulation conditions. The main contribution of this paper is an experimental simulation examination of the robustness of the designed control algorithms. The robustness designed control algorithms was examined in term of influence parametric uncertainties – caused by variance of a static gain of the process model. The program system MATLAB/SIMULINK was used for these purposes.

***Keywords*—**Digital control, LQ control, Polynomial approach, Simulation of control loops, Smith Predictor, Time-delay, Robustness.

I. Introduction

TIME-delay may be defined as the time interval between the start of an event at one point in a system and its resulting action at another point in the system. Delays are also known as transport lags or dead times; they arise in physical, chemical, biological and economic systems, as well as in the process of measurement and computation. One older classification of techniques for the compensation of time-delayed processes is introduced in [1, 2] and newer overview of recent advances and open problems it is possible to find in [3].

The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robust stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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It is possible to say in present (see e. g. [4]) that "The beginning of the 21st century can be characterized as the *time-delay boom* leading to numerous important results".

When a high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy is one possible approaches for a control of time-delay processes. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by Smith [5] in 1957. This time-delay compensator (TDC) known as the Smith Predictor (SP) contained a dynamic model of the process and it can be considered as the first model predictive algorithm.

Historically, first modifications of time-delay algorithms were proposed for continuous-time (analog) controllers using some different approaches, see e.g. [6] - [9]. In industrial practice the implementation of the time-delay compensators on continuous-time technique was difficult. Therefore the Smith Predictors and its modified versions can be implemented since 1980s together with the use of microprocessors in the industrial controllers. In spite of the fact that these algorithms can be implemented in digital platforms, most of the literature analysis and synthesis time-delay systems including the robustness, disturbance rejection and the extension of suitable compensators, is focused only in the continuous-time version. The first digital time-delay compensators are presented (see e.g. in [9]).

One of possible approaches to control of processes with time-delay is digital Smith Predictor based on polynomial theory. Polynomial methods are design techniques for complex systems (including multivariable), signals and processes encountered in Control, Communications and Computing that are based on manipulations and equations with polynomials, polynomial matrices and similar objects. Systems are described by input-output relations in fractional form and processed using algebraic methodology and tools. The design procedure is thus reduced to algebraic polynomial equations [10]. Controller design consists in solving polynomial (Diophantine) equations. The Diophantine equations can be solved using the uncertain coefficient method – which is based on comparing coefficients of the same power. This is transformed into a system of linear algebraic equations [11]. Because the classical analog Smith Predictor is not suitable for control of unstable and integrating time-delay processes, the polynomial digital LQ Smith

Predictor for control of unstable and integrating time-delay processes has been designed in [12].

It is obvious that the majority processes met in industrial practice are influenced by uncertainties. The uncertainties suppression can be solved either implementation adaptive control or robust control. Some adaptive (self-tuning) modifications of the digital Smith Predictors are designed in [13] – [15]. Two versions of these controllers were implemented into MATLAB/SIMULINK Toolbox [16], [17].

Until recently, robust control and adaptive control have been viewed as two control techniques which are used for controller design in the presence of process model uncertainty (process model variations) [18].

From a robust control point of view, adaptive control is a method used for reducing the uncertainty level of the process model by recursive process model identification in closed control loops. Furthermore, the design of a robust controller deals in general with designing the controller in the presence of process uncertainties. This can be simultaneously: parameter variations (affecting low- and medium-frequency ranges) and unstructured model uncertainties (often located in high-frequency range). While in adaptive control the adaptation suppresses the parametric variations, the problem of suppressing unstructured model uncertainties remains.

The aim of this paper is the experimental examination of the robustness of control time-delay processes. Robustness is the property when the dynamic response of control closed loop (including stability of course) is satisfactory not only for the nominal process transfer function used for design but also for the entire (perturbed) class of transfer functions that express uncertainty of the designer about dynamic environment in which real controller is expected to operate. The design of robust digital controllers for systems with time delay is investigated in [19]. A particular class of digital controller is considered, namely based on the pole assignment approach.

A more comprehensive discussion of robustness is be given when design using frequency methods is considered. For root locus design, the natural measure of robustness is, in effect gain margin. One can readily compare the system gain at the desired operating point and the point(s) of onset of instability to determine how much gain change is acceptable. Just this method will be used for investigation of the robustness control time-delay processes.

The paper is organized in the following way. The general problem of a control of the time-delay systems with regard to robustness is described in Section I. The fundamental principle of digital Smith Predictor is described in Section II. Two versions of the primary polynomial LQ controller, which are components of the digital Smith Predictor, are proposed in Section III. The simulation verification of individual control-loops with their results are presented in Section IV. Section V. concludes this paper.

II. Principle of Digital Smith Predictor

The discrete versions of the SP and its modifications are more suitable for time-delay compensation in industrial practice. The block diagram of a digital SP (see [13] - [15]) is shown in Fig. 1. The function of the digital version is similar to the classical analog version.

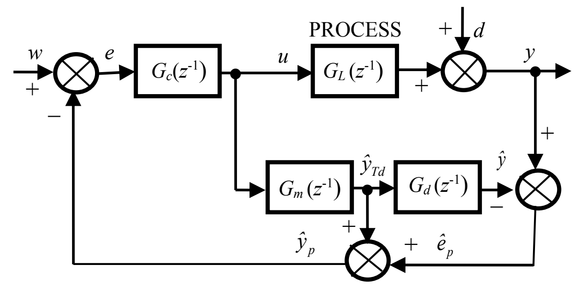

Fig. 1 Block diagram of a digital Smith Predictor

Number of higher order industrial processes can be approximated by a reduced order model with a pure time-delay. In this paper the following second-order linear model with a time-delay is considered

$$G_L\left(z^{-1}\right)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}z^{-d}=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}z^{-d} \tag{1}$$

The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period.

The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The numerator in transfer function (1) is replaced by its static gain $B(1)$, i.e. for z = 1. This is to avoid problem of controlling a model with a $B(z^{-1})$, which has non-minimum phase zeros caused by a high sampling period or fractional delay. Since $B(z^{-1})$ is not controllable as in the case of a time-delay, it is moved out of the prediction model $G_m(z^{-1})$ and is treated together with the time-delay block, as shown in Fig. 1. The difference between the output of the process y and the model including time-delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown in Fig. 1, whereas e and d are the error and the measured disturbance, w is the reference signal. The primary (main) controller $G_c(z^{-1})$ can be designed by different approaches (for example digital PID control or methods based on polynomial approach). The outward feedback-loop through the block in Fig. 1 is used to compensate load disturbances and modelling errors.

III. Design of Primary Polynomial 2DOF Controller

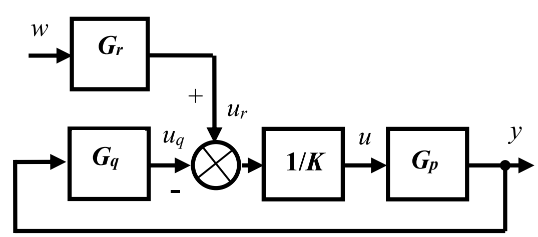

Fig. 2 Block diagram of a closed loop 2DOF control system

Polynomial control theory is based on the apparatus and methods of linear algebra. The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 2.

The controlled process is given by the transfer function in the form

$$G_p(z^{-1}) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} \tag{2}$$

where A and B are the second order polynomials. The controller contains the feedback part G_q and the feedforward part G_r. Then the digital controllers can be expressed in the form of a discrete transfer functions

$$G_r\left(z^{-1}\right) = \frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)K\left(z^{-1}\right)} \tag{3}$$

$$G_q\left(z^{-1}\right) = \frac{Q\left(z^{-1}\right)}{P\left(z^{-1}\right)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{\left(1+p_1 z^{-1}\right)\left(1-z^{-1}\right)} \tag{4}$$

where $K\left(z^{-1}\right) = 1 - z^{-1}$

According to the scheme presented in Fig. 2 and equations (2) – (4) it is possible to derive a polynomial Diophantine equation for computation of feedback controller parameters as coefficients of the polynomials Q and P

$$A(z^{-1})\mathrm{K}(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \tag{5}$$

where $D\left(z^{-1}\right)$ is the characteristic polynomial.

Asymptotic tracking of the reference signal w is provided by the feedforward part of the controller which is given by solution of the following polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right) + B(z^{-1})R(z^{-1}) = D(z^{-1}) \tag{6}$$

For a step-changing reference signal value, polynomial $D_w(z^{-1}) = 1 - z^{-1}$ and S is an auxiliary polynomial which does not enter into the controller design. Then it is possible to derive the polynomial R from equation (6) by substituting $z = 1$

$$R = r_0 = \frac{D(1)}{B(1)} \tag{7}$$

The 2DOF controller output is given by

$$u\left(k\right) = \frac{r_0}{K\left(z^{-1}\right)P\left(z^{-1}\right)} w\left(k\right) - \frac{Q\left(z^{-1}\right)}{K\left(z^{-1}\right)P\left(z^{-1}\right)} y\left(k\right) \tag{8}$$

Two primary polynomial LQ controllers are derived in this paper using minimization of LQ criterion [20]. For the minimization procedure is used spectral factorization by means of the MATLAB Polynomial Toolbox 3.0 [21].

A. *Minimization of LQ Criterion Using Variable u(k)*

In the first case the linear quadratic control methods try to minimize the quadratic criterion by penalization of the quadrat controller output $u(k)$

$$J = \sum_{k=0}^{\infty} \left\{ \left[w(k) - y(k) \right]^2 + q_u \left[u(k) \right]^2 \right\} \tag{9}$$

where q_u is the so-called penalization constant, which gives the rate of the controller output on the value of the criterion (where the constant at the first element of the criterion is considered equal to one). In this paper, criterion minimization will be realized through the spectral factorization for an input-output description of the system

$$A\left(z\right) q_u A\left(z^{-1}\right) + B\left(z\right) B\left(z^{-1}\right) = D\left(z\right) \delta D\left(z^{-1}\right) \tag{10}$$

where δ is a constant chosen so that $d_0 = 1$.

Spectral factorization of polynomials of the first and the second degree can be computed simply by an analytical way [12], [22]; the procedure for higher degrees must be performed iteratively. Although $A(z^{-1})$ and $B(z^{-1})$ are the second degree polynomials (spectral factorization (10) can be computed by an analytical way), the MATLAB Polynomial Toolbox is used for this computation. The factorized polynomial $D(z^{-1})$ must be also of second degree

$$D_2\left(z^{-1}\right) = 1 + d_{21} z^{-1} + d_{22} z^{-2} \tag{11}$$

For computation of the spectral factorization (10) was used in this paper file *spf.m* by command

d = spf(a*qu*a' + b*b') (12)

It is obvious that by using of the spectral factorization, only two parameters d_{21} and d_{22} of the second degree polynomial $D_2(z^{-1})$ (11) can be computed. This approach is applicable only for control of processes without time-delay (out of Smith Predictor). The primary controller in the digital Smith Predictor structure requires usage of the fourth degree polynomial

$$D_4\left(z^{-1}\right) = 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} + d_4 z^{-4} \tag{13}$$

in equations (5) and (6). The polynomial $D_2(z^{-1})$ (11) has two different real poles α, β or one complex conjugated pole $z_{1,2} = \alpha \pm j\beta$ (in the case of oscillatory systems). These poles must be included into polynomial $D_4(z^{-1})$ (13) and other two poles γ, λ are user-defined real poles. A suitable pole assignment was designed for both types of the processes in [12].

Then the digital 2DOF controller (8) can be expressed in the form

$$\begin{aligned}&\left[1+\left(p_1-1\right)z^{-1}-p_1z^{-2}\right]u\left(k\right)\\&\quad=r_0w\left(k\right)-\left(q_0+q_1z^{-1}+q_2z^{-2}\right)y\left(k\right)\end{aligned} \tag{14}$$

where

$$r_0=\frac{1+d_1+d_2+d_3+d_4}{b_1+b_2} \tag{15}$$

and parameters q_0, q_1, q_2 are computed from (5). The primary 2DOF controller output is given by

$$\begin{aligned}u\left(k\right)&=r_0w\left(k\right)-q_0y\left(k\right)-q_1y\left(k-1\right)-q_2y\left(k-2\right)\\&\quad+\left(1-p_1\right)u\left(k-1\right)+p_1u\left(k-2\right)\end{aligned} \tag{16}$$

B. Minimization of LQ Criterion Using Increment Δu(k)

In the second case the linear quadratic control methods try to minimize the quadratic criterion by penalization of the square incremental value of controller output $\Delta u(k)$

$$J=\sum_{k=0}^{\infty}\left\{\left[w(k)-y(k)\right]^2+q_u\left[\Delta u(k)\right]^2\right\} \tag{17}$$

Equation (10) for computation of the spectral factorization changes into

$$\begin{aligned}&\left(1-z\right)A\left(z\right)q_u\left(1-z^{-1}\right)A\left(z^{-1}\right)\\&\quad+B\left(z\right)q_uB\left(z^{-1}\right)=D\left(z\right)\delta D\left(z^{-1}\right)\end{aligned} \tag{18}$$

It is obvious that after arrangement and substitution the first term of the left side (18) has this form

$$\left(1+a_{s1}z+a_{s2}z^2+a_{s3}z^3\right)q_u\left(1+a_{s1}z^{-1}+a_{s2}z^{-2}+a_{s3}z^{-3}\right) \tag{19}$$

where

$$A_s\left(z^{-1}\right)=1+a_{s1}z^{-1}+a_{s2}z^{-2}+a_{s3}z^{-3} \tag{20}$$

and

$$a_{s1}=a_1-1;\quad a_{s2}=a_2-a_1;\quad a_{s3}=-a_3. \tag{21}$$

Because (20) is the third degree polynomial whose parameters and poles α, β and γ it is impossible to compute by an analytical way, MATLAB Polynomial Toolbox 3 was used for their computation using command (12).

The characteristic polynomial is the sixth degree polynomial in this case

$$D_6\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+d_4z^{-4}+d_5z^{-5}+d_6z^{-6} \tag{22}$$

Spectral factorization (18) gives three optimal parameters of polynomial (20) and then it is possible to write characteristic polynomial (26) as a combination of polynomial (23) and product root of factors in positive power of variable z

$$D_6\left(z\right)=\left(z^3+a_{s1}z^2+a_{s2}z+a_{s3}\right)\left(z-\lambda\right)\left(z-\mu\right)\left(z-\nu\right) \tag{23}$$

where λ, μ, ν are user-defined real poles. After modification (23) the characteristic polynomial is in the following form

$$D_6\left(z\right)=z^6+d_1z^5+d_2z^4+d_3z^3+d_4z^2+d_5z+d_6 \tag{24}$$

After comparison of (23) and (24) it is possible to obtain expressions for computation of individual parameters of polynomial (24)

$$\begin{aligned}d_1&=a_{s1}-\left(\lambda+\mu+\nu\right)\\d_2&=a_{s2}-a_{s1}\left(\lambda+\mu+\nu\right)+\lambda\mu+\lambda\nu+\mu\nu\\d_3&=a_{s3}-a_{s2}\left(\lambda+\mu+\nu\right)-a_{s1}\nu\left(\lambda\mu+\lambda\nu+\mu\nu\right)-\lambda\mu\nu\\d_4&=-a_{s3}\left(\lambda+\mu+\nu\right)+a_{s2}\left(\lambda\mu+\lambda\nu+\mu\nu\right)-a_{s1}\lambda\mu\nu\\d_5&=a_{s3}\left(\lambda\mu+\lambda\nu+\mu\nu\right)-a_{s2}\lambda\mu\nu\\d_6&=-a_{s3}\lambda\mu\nu\end{aligned} \tag{25}$$

Then the 2DOF controller design consists of determination of parameters (25) of polynomial (24) using command (12) from the Polynomial Toolbox and solution of the Diophantine equation for computation of feedback controller parameters

$$A_s(z^{-1})K(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D_6(z^{-1}) \tag{26}$$

where

$$\begin{aligned}&K\left(z^{-1}\right)=1-z^{-1};\quad P\left(z^{-1}\right)=1+p_1z^{-1}+p_2z^{-2};\\&Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2}+q_3z^{-3}\end{aligned} \tag{27}$$

and from expression (7)

$$r_0=\frac{1+d_1+d_2+d_3+d_4+d_5+d_6}{b_1+b_2} \tag{28}$$

The primary 2DOF controller output is given by

$$u(k) = r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) + (p_1 - p_2) u(k-2) - p_2 u(k-3) \quad (29)$$

IV. Simulation Verification and Results

A simulation verification of the designed control algorithms was performed in MATLAB/SIMULINK environment. The robustness of individual control loops was experimental investigated by a change of the static gain K of the nominal process model. From the point of view of the robust theory it is possible to consider these experiments on behalf of the gain margin determination by the parametric uncertainty influence.

Three types of process models were chosen for simulation experiments. Consider the following continuous-time transfer functions (nominal continuous-time models):

1) Stable non-oscillatory $G_1(s) = \dfrac{2}{(s+1)(4s+1)} e^{-8s}$

2) Stable oscillatory $G_2(s) = \dfrac{2}{4s^2 + 2s + 1} e^{-8s}$

3) Non-minimum phase $G_3(s) = \dfrac{2(1-5s)}{(s+1)(4s+1)} e^{-8s}$

Let us now discretize them a sampling period $T_0 = 2$ s. The discrete nominal models of these transfer functions are

$$G_{N1}(z^{-1}) = \frac{0.4728z^{-1} + 0.2076z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \quad (30)$$

$$G_{N2}(z^{-1}) = \frac{0.6806z^{-1} + 0.4834z^{-2}}{1 - 0.7859z^{-1} + 0.3679z^{-2}} z^{-4} \quad (31)$$

$$G_{N3}(z^{-1}) = \frac{-1.0978z^{-1} + 1.7783z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \quad (32)$$

The individual simulation experiments are realized subsequently: the static gain K=2 was increased as far as the control closed-loop was in the stability boundary (no damping oscillation was achieved).

A. Control Using Primary Controller (16)

1) *Experiments with stable non-oscillatory model (30) G_1:*

For these experiments the penalization factor was chosen as q_u = 2. The characteristic polynomial $D_4(z)$ is given by

$$D_4(z) = z^4 - 1.1461z^3 + 0.4409z^2 - 0.00652z + 0.0032$$

with individual poles

$\alpha = 0.3796;\quad \beta = -0.7419;\quad \gamma = 0.1;\quad \lambda = 0.5.$

The control courses of the process output and controller output for the nominal model $G_{N1}(z^{-1})$ are shown in Fig. 3. The pole map of the control nominal model $G_{N1}(z^{-1})$ is shown in Fig. 4.

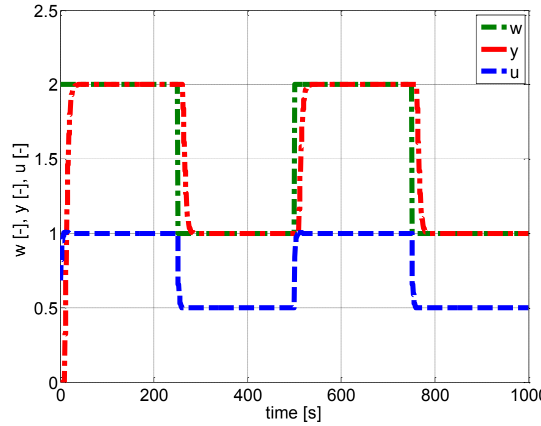

Fig. 3 Control of nominal model $G_{N1}(z^{-1})$, $K = 2$

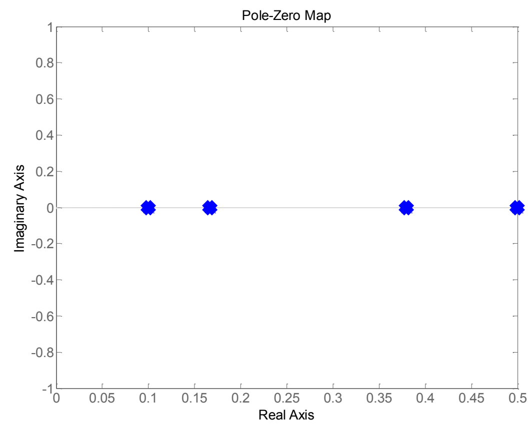

Fig. 4 Pole map of control nominal model $G_{N1}(z^{-1})$

Perturbed models:

$$K = 3: \quad G_{P1}(z^{-1}) = \frac{0.7092z^{-1} + 0.3114z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \quad (33)$$

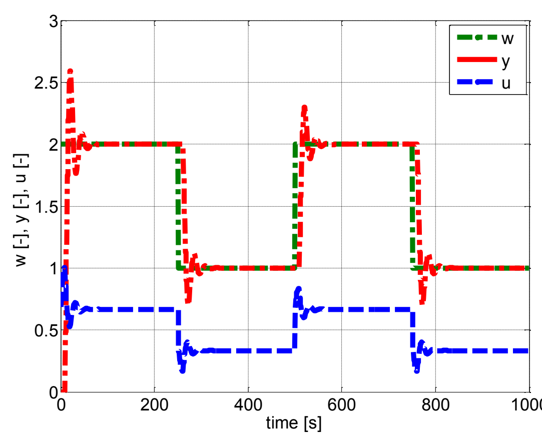

Fig. 5 Control of perturbed model $G_{P1}(z^{-1})$, $K = 3$

$$K = 4: \quad G_{P2}\left(z^{-1}\right) = \frac{0.9456z^{-1} + 0.4153z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \qquad (34)$$

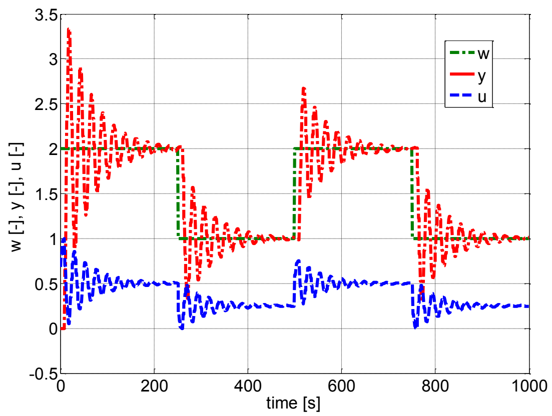

Fig. 6 Control of perturbed model $G_{P2}\left(z^{-1}\right)$, $K = 4$

$$K = 4.4: \quad G_{P3}\left(z^{-1}\right) = \frac{1.0402z^{-1} + 0.4568z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \qquad (35)$$

The control courses of the process output and controller output for the individual perturbed models are shown in Figs. 5 - 7. It is obvious from Fig. 7 that for the static gain $K = 4.4$ is the closed-loop control on the stability boundary. It is obvious from these Figs. that approximate interval of the robust stability of nominal model $G_{N1}\left(z^{-1}\right)$ is for the static gain $K = \left(2,\ 4.4\right)$.

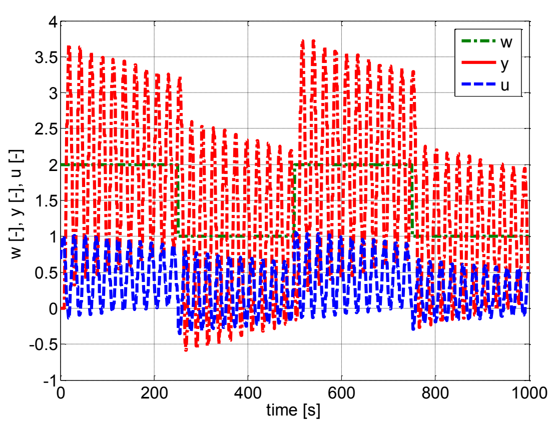

Fig. 7 Control of perturbed model $G_{P3}\left(z^{-1}\right)$, K = 4.4

2) *Experiments with stable oscillatory model (31) G_2:*

For these experiments the penalization factor was chosen as q_u = 1. The characteristic polynomial $D_4\left(z\right)$ is given by

$$D_4\left(z\right) = z^4 - 0.8902z^3 + 0.3911z^2 - 0.1147z + 0.0083$$

with individual poles

$$\alpha, \beta = 0.1451 \pm 0.3890i; \quad \gamma = 0.1; \quad \lambda = 0.5.$$

The control courses of the process output and controller output for the nominal model $G_{N2}\left(z^{-1}\right)$ are shown in Fig. 8. The pole map of the control nominal model $G_{N2}\left(z^{-1}\right)$ is shown in Fig. 9.

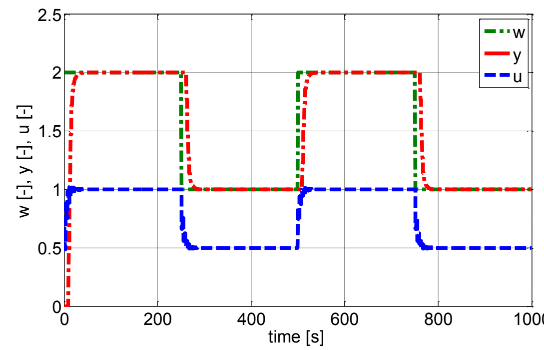

Fig. 8 Control of nominal model $G_{N2}\left(z^{-1}\right)$, $K = 2$

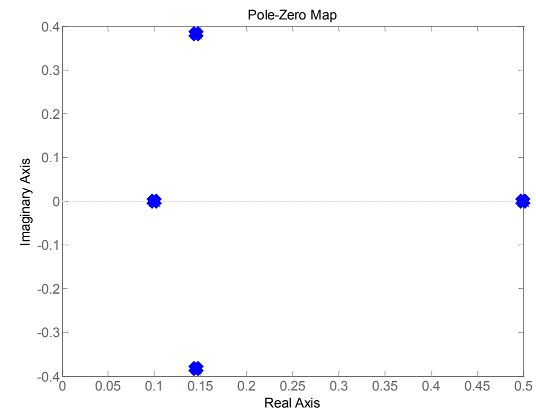

Fig. 9 Pole map of control nominal model $G_{N2}\left(z^{-1}\right)$

The experimental examination of robustness was realized just as in the case of the model $G_{N1}\left(z^{-1}\right)$. It was demonstrated that the approximate interval of the robust stability of the nominal model is for the static gain $K = \left(2,\ 5\right)$.

3) *Experiments with non-minimum phase model (32) G_3:*

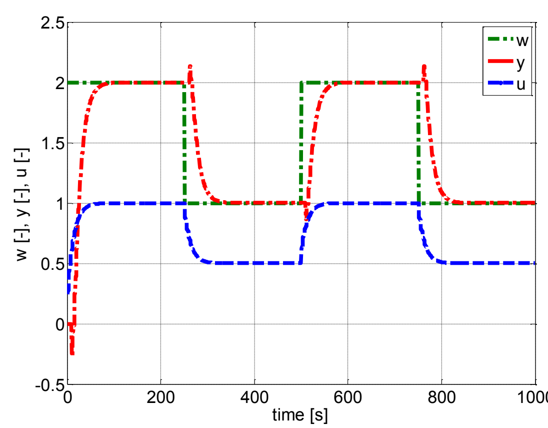

Fig. 10 Control of nominal model $G_{N3}\left(z^{-1}\right)$, $K = 2$

For these experiments the penalization factor was chosen as q_u = 1. The characteristic polynomial $D_4\left(z\right)$ is given by

$$D_4(z) = z^4 - 1.4973z^3 + 0.6449z^2 - 0.0653z + 0.0015$$

with individual poles
$\alpha = 0.0320;\quad \beta = 0.6153;\quad \gamma = 0.1;\quad \lambda = 0.75$.

The control courses of the process output and controller output for the nominal model $G_{N3}(z^{-1})$ are shown in Fig. 10. The pole map of the control nominal model $G_{N3}(z^{-1})$ is shown in Fig. 11.

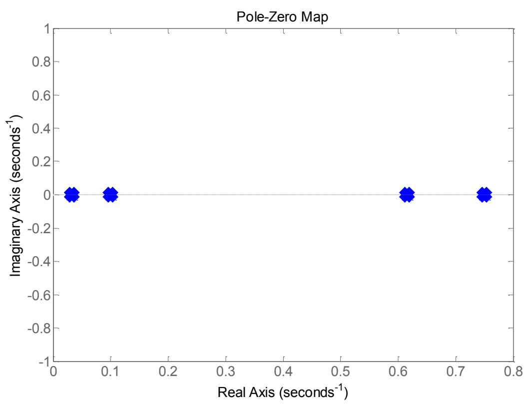

Fig. 11 Pole map of control nominal model $G_{N3}(z^{-1})$

The experimental examination of robustness was realized just as in the case of the model $G_{N1}(z^{-1})$. It was demonstrated that the approximate interval of the robust stability of the nominal model is for the static gain $K = (2,\ 3.7)$.

B. Control Using Primary Controller (29)

1) *Experiments with stable non-oscillatory model (30) G_1:*

For these experiments the penalization factor was chosen as $q_u = 2$. The characteristic polynomial $D_6(z)$ is given by

$$D_6(z) = z^6 - 2.1438z^5 + 1.8185z^4 - 0.7769z^3 + 0.1654z^2 - 0.0165z + 0.0006$$

with individual poles
$\alpha, \beta = 0.4561 \pm 0.2867i;\ \gamma = 0.1316;\ \lambda = 0.1;\ \mu = 0.2;\ \nu = 0.8$.

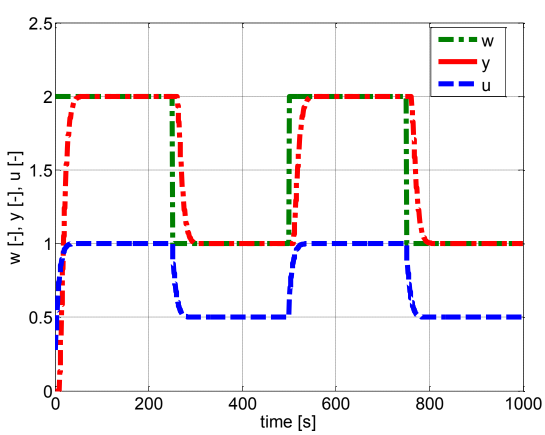

Fig. 12 Control of nominal model $G_{N1}(z^{-1})$, $K = 2$

The user-defined poles λ, μ, ν were chosen in order that the control closed-loop did not oscillate. The control courses of the process output and controller output for the nominal model $G_{N1}(z^{-1})$ are shown in Fig. 12. The pole map of the control nominal model $G_{N1}(z^{-1})$ is shown in Fig. 13.

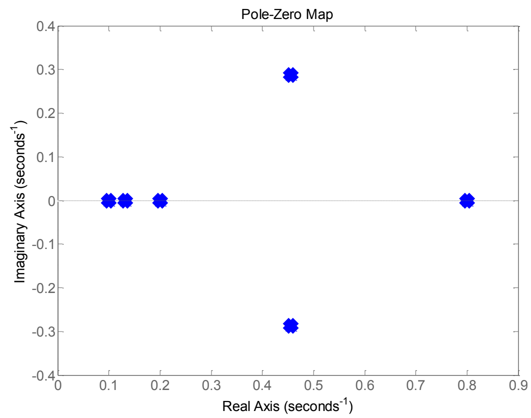

Fig. 13 Pole map of control nominal model $G_{N1}(z^{-1})$

The control courses of the process output and controller output for the individual perturbed models are shown in Figs. 14 - 16. It is obvious from Fig. 16 that for the static gain $K = 7$ is the control closed-loop on the stability boundary. It is obvious from these Figs. that the approximate interval of the robust stability of the control nominal model $G_{N1}(z^{-1})$ is for the static gain $K = (2,\ 7)$.

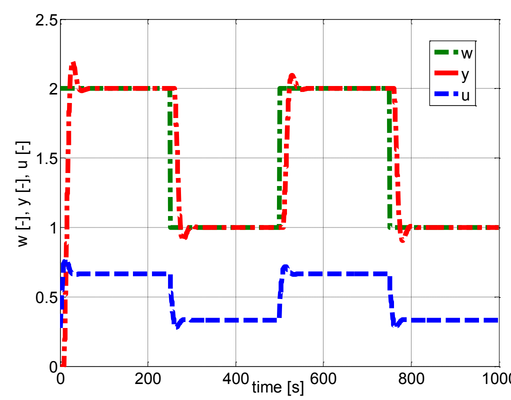

Fig. 14 Control of perturbed model $G_{\Delta P1}(z^{-1})$ - $K = 3$

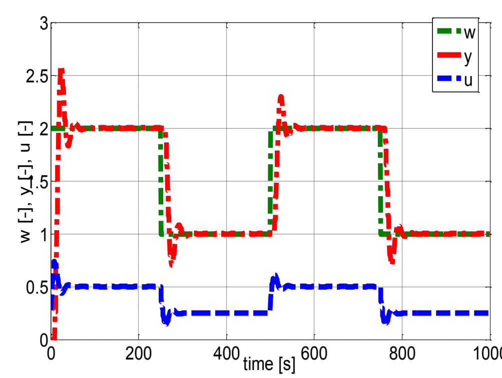

Fig. 15 Control of perturbed model $G_{\Delta P2}(z^{-1})$ - $K = 4$

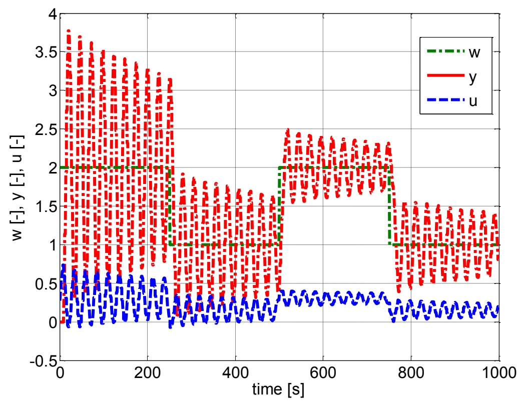

Fig. 16 Control of perturbed model $G_{\Delta P3}\left(z^{-1}\right)$ - $K = 7$

2) *Experiments with stable oscillatory model (31) G_2:*

For these experiments the penalization factor was chosen as $q_u = 1$. The characteristic polynomial $D_6(z)$ is given by

$$D_6(z) = z^6 - 1.3136z^5 + 0.9496z^4 - 0.4291z^3 + 0.1082z^2 - 0.0132z + 0.0006$$

with individual poles

$\alpha, \beta = 0.1848 \pm 0.5001i;\ \gamma = 0.3441;\ \lambda = 0.1;\ \mu = 0.2;\ \nu = 0.3$.

The control courses of the process output and controller output for the nominal model $G_{N2}\left(z^{-1}\right)$ are shown in Fig. 17. The pole map of the control nominal model $G_{N2}\left(z^{-1}\right)$ is shown in Fig. 18.

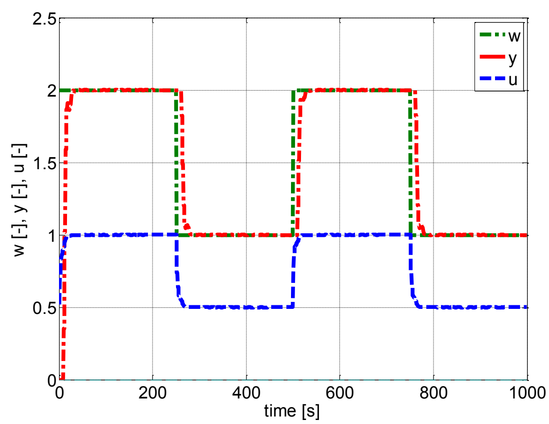

Fig. 17 Control of nominal model $G_{N2}\left(z^{-1}\right)$, $K = 2$

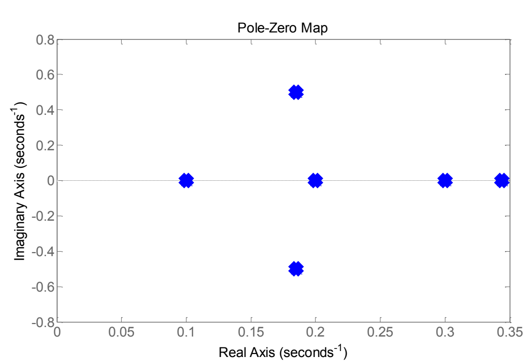

Fig. 18 Pole map of control nominal model $G_{N2}\left(z^{-1}\right)$

The experimental examination of robustness was realized just as in the case of the model $G_{N1}\left(z^{-1}\right)$. It was demonstrated that the approximate interval of the robust stability of the nominal model is for the static gain $K = (2,\ 4.6)$.

3) *Experiments with non-minimum phase model (32) G_3:*

For these experiments the penalization factor was chosen as $q_u = 1$. The characteristic polynomial $D_6(z)$ is given by

$$D_6(z) = z^6 - 2.0579z^5 + 1.6738z^4 - 0.6884z^3 + 0.1526z^2 - 0.0180z + 0.0010$$

with individual poles

$\alpha, \beta = 0.1201 \pm 0.1071i;\ \gamma = 0.6176;\ \lambda = 0.3;\ \mu = 0.4;\ \nu = 0.5$.

The control courses of the process output and controller output for the nominal model $G_{N3}\left(z^{-1}\right)$ are shown in Fig. 19. The pole map of the control nominal model $G_{N3}\left(z^{-1}\right)$ is shown in Fig. 20.

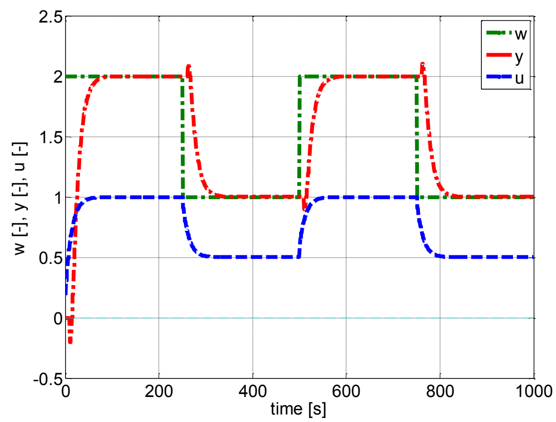

Fig. 19 Control of nominal model $G_{N3}\left(z^{-1}\right)$, $K = 2$

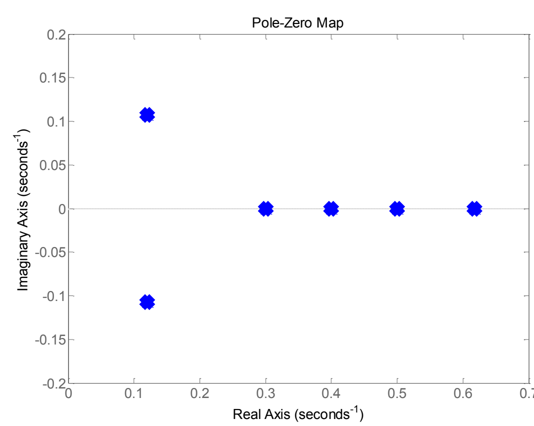

Fig. 20 Pole map of control nominal model $G_{N3}\left(z^{-1}\right)$

The experimental examination of robustness was realized just as in the case of the model $G_{N1}\left(z^{-1}\right)$. It was demonstrated that the approximate interval of the robust stability of the nominal model is for the static gain $K = (2,\ 4.6)$.

The experimental examination of robustness properties of designed controller algorithms is demonstrated in the Table I. It is necessary to take consideration that robustness properties are dependent not only on dynamical types of individual processes but also on choice of user-defined poles, sampling period, size of parameter q_u. Also parametric uncertainties of time constants influence the robustness of control closed-loops.

TABLE I
EXPERIMENTAL EXAMINATION OF ROBUST PROPERTIES

Nominal Model	Robust Interval K Criterion (9)	Robust Interval K Criterion (17)
G_{N1} (30)	(2, 4.4)	(2, 7.0)
G_{N2} (31)	(2, 5.0)	(2, 4.6)
G_{N3} (32)	(2, 3.7)	(2, 5.3)

V. CONCLUSION

The paper presents an experimental simulation investigation of robust algorithms for control of time-delay systems. The MATLAB Polynomial Toolbox 3.0 is used for design of the polynomial digital Smith Predictor. The primary controllers of the digital Smith Predictor are based on minimization of the LQ criterion using spectral factorization. Two types of minimization of LQ criterions have been designed. In criterion (9) it is minimized a square of the controller output $u(k)$ – controller (17). In criterion (16) it is minimized a square of the increment value of the controller output $\Delta u(k)$ – controller (29). Simulation experiments demonstrated the influence of static gain K (parametric uncertainty) on the course of control variables (robustness of the control closed-loop). From comparison of both methods it is evident that minimization criterion (9) leads to faster courses of control variables. However the control closed-loop is in the stability boundary for a lower value K as in the case of minimization criterion (16) – except of the control closed-loop with stable oscillatory process (31). In this case the robust intervals are nearly identical using controllers (17) and (29). However minimization criterion (16) leads to quieter courses of control variables with their smaller oscillations for grater values of static gain K. The controller (29) is more conservative and robust than controller (16). From simulation experiments it is evident that both control algorithms are relative simply and they are suitable for application for control in real-time conditions. Designed universal Smith Predictors were verified by control of a laboratory heat exchanger [23]. The real-time experiments confirmed that both designed LQ Smith Predictors are able to cope with given control problem [24].

REFERENCES

[1] O'Dwyer, "A Classification of techniques for the compensation of time delayed processes. Part 1: Parameter optimised controllers," in *Proc. of the 3rd IMACS/IEE International Multiconference on Circuits, Systems, Communications and Computers*, Athens, Greece, 1999, pp. 176-186.

[2] O'Dwyer, "A Classification of techniques for the compensation of time delayed processes. Part 2: Structurally optimised controllers," in *Proc. of the 3rd IMACS/IEE International Multiconference on Circuits, Systems, Communications and Computers*, Athens, Greece, 1999, pp. 187-198.

[3] J. P. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, pp. 1667-1694, 2003.

[4] E. Fridman, *Introduction to Time-Delay Systems. Analysis and Control.* Basel: Birkhäuser, 2014.

[5] O. J. Smith, "Closed control of loops," *Chem. Eng. Progress*, vol. 53, pp. 217-219, 1957.

[6] R. Matušů, and R. Prokop, "Control of Periodically Time-Varying Systems with Delay: An Algebraic Approach vs. Modified Smith Predictors," *WSEAS Transactions on Systems*, vol. 9, No. 6, pp. 689-702, 2010.

[7] L. Pekař, "A Ring for Description and Control of Time-Delay Systems," *WSEAS Transactions on Systems*, vol. 11, No. 4, pp. 571-585, 2012.

[8] M. A. Pakzad, "Kalman Filter Design for Time Delay Systems," *WSEAS Transactions on Systems*, vol. 11, No. 4, pp. 551-560, 2012.

[9] J. E. Normey-Rico and E. F. Camacho, *Control of Dead-time Processes*. London: Springer-Verlag, 2007.

[10] M. Šebek and M. Hromčík. "Polynomial Design Methods*," International Journal of of Robust and Nonlinear Control*, vol. 17, pp. 679-681, 2007.

[11] V. Kučera, "Diophantine equations in control a survey," Automatica, vol. 29, pp. 1391-1375, 1993.

[12] V. Bobál, P. Chalupa, P. Dostál, and M. Kubalčík, "Digital control of unstable and integrating processes," *International Journal of Circuits, Systems and Signal Processing*, vol. 8, pp. 424-432, 2014.

[13] C. C. Hang, K. W. Lim and B. W. Chong, "A dual-rate adaptive digital Smith predictor," *Automatica*, vol. 20, pp. 1-16, 1989.

[14] C. C. Hang, H. L. Tong and K. H. Weng. *Adaptive Control*, North Carolina: Instrument Society of America, 1993.

[15] V. Bobál, P. Chalupa, P. Dostál and M. Kubalčík, "Design and simulation verification of self-tuning Smith predictor," *International Journal of Mathematics and Computers in Simulation*, vol. 5, no. 4, pp. 342-351, 2011.

[16] V. Bobál, P. Chalupa, J. Novák, J. and P. Dostál, "MATLAB Toolbox for CAD of self-tuning of time-delay processes," in *Proc. of the International Workshop on Applied Modelling and Simulation*, Roma, 2012, pp. 44 – 49.

[17] V. Bobál, P. Chalupa, and J. Novák, "Toolbox for CAD and Verfication of Digital Adaptive Control Time-Delay Systems," Available from http://nod32.fai.utb.cz/promotion/Software_OBD/Time_Delay_Tool.zip

[18] I. D. Landau, "From robust control to adaptive control," *Control Engineering Practice*, vol. 7, pp.1113-1124, 1999.

[19] I. D. Landau, "Robust digital control of systems with time-delay (the Smith predictor revisited)," *International Journal of Control*, vol. 62, pp. 325-347, 1995.

[20] M. Šebek and V. Kučera, "Polynomial approach to quadratic tracking in discrete linear systems," *IEEE Trans. Automatic. Control*, vol. AC-27, pp. 1248-1250, 1982.

[21] M. Šebek, "Polynomial Toolbox for MATLAB, Version 3.0.", PolyX, Prague, Czech Republic, 2014.

[22] V. Bobál, J. Böhm, J. Fessl, and J. Macháček, *Digital Self-tuning Controllers: Algorithms, Implementation and Applications*. London: Springer-Verlag, 2005.

[23] L. Pekař, R. Prokop, and P. Dostálek, "Circuit heating plant model with internal delays," *WSEAS Transactions on Systems and Control*, vol. 8, pp. 1093-1104, 2009.

[24] V. Bobál, M. Kubalčík, P. Dostál, and S. Talaš, "Use of polynomial approach for control of heat exchanger," in *Proc. of the 20th International Conference on Process Control*, Štrbské pleso, Slovak Republic, 2015, pp. 170-175.