Stability analysis of Discrete Predictive Sliding Mode Control for Multivariable Systems

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***Abstract*—Predictive sliding mode controller have been succefully implemented in the last 10 years overcoming the drawbacks of the traditional sliding mode control and the generalized predictive control strategies. This paper investigates the stability analysis of the new Discrete Predictive Sliding Mode Controller (DPSMC) for multivariable systems. The basic objective of the controller is to approximate the predictive sliding function vector S_{p_MIMO} to the sliding reference function vector S_{r_MIMO}, penalizing at the same time the variation in the control signal. The designed control strategy is more robust and has a chattering reduction property and a faster convergence on the system state. Finally, a numerical example is given to illustrate the effectiveness of the proposed control, in comparaison with the classical sliding mode control.**

Keywords*—Multivariable systems, Sliding Mode Control, Model Predictive Control, Predictive Sliding Mode Control, Chattering phenomenon.

I. Introduction

Many complex engineering systems are equipped with several actuators that may influence their static and dynamic behavior. Commonly, in cases where some form of automatic control is required over the system, also several sensors are available to provide measurement information about important system variables that may be used for feedback control purposes. Systems with more than one actuating control input and more than one sensor output may be considered as multivariable systems or multi-input-multi-output (MIMO). The control objective for multivariable systems is to obtain a desirable behavior of several output variables by simultaneously manipulating several input channels. withal, the presence of external disturbances, parameters uncertainties and time delays make difficult the design of an exact mathematical model and the development of a suitable control.

Research in this area continues to grow. In fact, over the last 20 years much research has been developed, particularly, in Sliding Mode Control (SMC) and in Model based Predictive Control (MPC)[1].

Sliding mode control is one of the powerful control methods for systems containing uncertainties and unknown disturbances. The first step in SMC is to define a sliding surface. At the second step, a feedback control law is designed to provide convergence of a system trajectory to the sliding surface, in finite time.

Sliding Mode Control is well used for multivariable systems [2], [3], [4], [5]. However, in spite of the robustness of the sliding mode control, the chattering phenomenon, caused by the discontinuous term of the control law, is still the main problem of the SMC which consists in a sudden and rapid variation of the control signal leading to undesirable results [1]. Many approaches have been proposed to solve this problem such as high order sliding mode control [6], [7], [8].

On the other hand, in recent years model based predictive control(MPC) has received a lot of attention in the control theory and applications. It has been successfully implemented in many industrial applications, showing good performances. The main idea behind Model Predictive Control is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The index to be optimized is a difference between the predictive system output and predictive reference sequence over the prediction horizon plus a quadratic function measuring control effort [9], [10], [11], [12], [13].

Nevertheless the control law is model dependent, so a perfect model is required to guarantee the success of MPC control strategies. Because of the finite horizon, the stability and the robustness of the process is difficult to analyze and guarantee, especially when constraints are present [14], [15].

As a solution, we have proposed in [16], [17], [18], [19], [20], [21] a controller which combine the design of SMC and MPC for single input single output systems. This combination improves the performances of the two control laws and overcome most of their specific drawbacks.

This work deals with the extension of our previous works, concerning the Discrete Predictive Sliding Mode Control (DPSMC), to multivariable systems. Moreover, it investigates the stability analysis of the proposed controller for multivariable systems.

The paper is organized as follows: Section II gives the synthesis of the classical discrete multivariable sliding mode control and the synthesis of the multivariable predictive sliding mode controller. The stability analysis is given at section III. In the following section the proposed controller is tested on a simulation example, and compared to SMC control. Finally, section V draws conclusions of the paper.

II. Discrete multivariable predictive sliding mode control

Consider a discrete multivariable system subjected to external disturbances and parameters variation, defined by [22]:

$$\begin{cases} x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)(u(k)+v(k)) \\ y(k) = Hx(k) + Du(k) \end{cases} \tag{1}$$

where:

- $x(k) \in \Re^n$ is the state vector at the instant k,
- $u(k) \in \Re^m$ is the input vector at the instant k,
- $y(k) \in \Re^p$ is the output vector at the instant k,
- $v(k) \in \Re^m$ is the disturbance input vector at the instant k,
- The matrices $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $H \in \Re^{p\times n}$ and $D \in \Re^{p\times m}$ are the nominal model matrices.
- $\Delta A \in \Re^{n\times n}$ and $\Delta B \in \Re^{n\times m}$ are the parameter uncertainties matrices.

The system (1) can be presented by the following form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + w(k) \\ y(k) = Hx(k) + Du(k) \end{cases} \tag{2}$$

with:

$$w(k) = \Delta A x(k) + \Delta B u(k) + (B+\Delta B)\, v(k) \tag{3}$$

where $w(k) \in \Re^n$.

A. Synthesis of classical discrete multivariable sliding mode control

The sliding function is defined as [23]:

$$S(k) = Cx(k) = [s_1(k) \cdots s_m(k)]^T \tag{4}$$

where the dimension of the matrix C are (m,n).
The sliding function vector is chosen in order to verify the following reaching law [5], [24]:

$$S(k+1) = \Phi S(k) - \begin{bmatrix} m_1 sign(s_1(k)) \\ m_2 sign(s_2(k)) \\ \vdots \\ m_m sign(s_m(k)) \end{bmatrix} \tag{5}$$

where Φ is a diagonal matrix with (m,m) dimension and verifying $0 \leq \Phi_{i,i} < 1$ and $m_i > 0$ for $i \in [1 \quad m]$.
and sign is the signum function defined as :

$$sign(s_i(k)) = \begin{cases} -1 & if \quad s_i(k) < 0 \\ +1 & if \quad s_i(k) > 0 \end{cases} \quad ; \quad i \in [1 \quad m]$$

Thus, using equation (5), the control law ensuring the quasi-sliding mode is calculated as follows [25]:

$$u(k) = (CB)^{-1}\left(-CAx(k) + \Phi S(k) - \begin{bmatrix} m_1 sign(s_1(k)) \\ m_2 sign(s_2(k)) \\ \vdots \\ m_m sign(s_m(k)) \end{bmatrix}\right) \tag{6}$$

(CB) is inversible.

B. Synthesis of discrete multivariable predictive sliding mode control

The principle of the Discrete Predictive Sliding Mode Controller (DPSMC) is given by the block diagram shown in Fig.1, where the primary loop is a Sliding Mode Control (SMC) and the secondary loop is a Model Predictive Control (MPC)[16], [20].

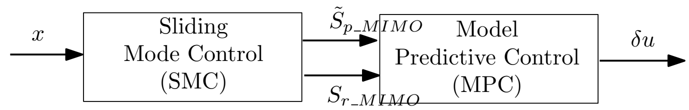

Fig. 1. DPSMC Controller bloc diagram.

The main purpose is to approximate the predictive sliding functions vector S_{p_MIMO} to the sliding reference functions vector S_{r_MIMO}, penalizing at the same time the variation in the control signal.
We consider, now, the sliding mode control problem for multivariable system (1). The objective is to design a predictive sliding mode controller taking the reaching law (5). The reference sliding mode trajectory is chosen as:

$$\begin{cases} S_r(k+1) = \Phi S_r(k) - \begin{bmatrix} m_1 sign(s_{r1}(k)) \\ m_2 sign(s_{r2}(k)) \\ \vdots \\ m_m sign(s_{rm}(k)) \end{bmatrix} \\ S_r(k) = S(k) \end{cases} \tag{7}$$

We consider that $w(k)$ is equal to null matrix.
The sliding functions vector at the instant $k+1$, $k+2$ and $k+3$ can be written as:

$$\begin{aligned} S(k+1) &= Cx(k+1) \\ &= CAx(k) + CB(u(k) - u(k-1)) + CBu(k-1) \\ &= CAx(k) + CB\delta u(k) + CBu(k-1) \end{aligned}$$

$$\begin{aligned} S(k+2) &= Cx(k+2) \\ &= CA^2x(k) + CB\delta u(k+1) + CB\delta u(k) \\ &\quad + CAB\delta u(k) + CBu(k-1) + CABu(k-1) \\ &= CA^2x(k) + CB\delta u(k+1) \\ &\quad + C(A+I)B\delta u(k) + C(A+I)Bu(k-1) \end{aligned}$$

$$\begin{aligned} S(k+3) &= Cx(k+3) \\ &= CA\left[A\left[Ax(k) + Bu(k)\right]\right] + CABu(k+1) \\ &\quad + CBu(k+2) \\ &= CA^3x(k) + CB\delta u(k+2) + C(A+I)B\delta u(k+1) \\ &\quad + C(A^2+A+I)B\delta u(k) + C(A^2+A+I)Bu(k-1) \end{aligned}$$

Then, $S(k+p)$ can be calculated as:

$$\begin{aligned} &S(k+p) = CA^p x(k) + CB\delta u(k+p-1) \\ &+C(A+I)B\delta u(k+p-2) + \cdots + C\left[\sum_{j=0}^{p-1} A^j\right] B\delta u(k) \\ &+C\left[\sum_{j=0}^{p-1} A^j\right] Bu(k-1) \end{aligned} \tag{8}$$

where:
$\delta u(k) = u(k) - u(k-1)$; I is the identity matrix with the dimension $n \times n$.

We introduce, then the predictive sliding functions vector of multivariable system S_{p_MIMO} as:

$$S_{p\_MIMO}(k+1) = \begin{bmatrix} S(k+1) \\ S(k+2) \\ \vdots \\ S(k+N) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} s_1(k+1) \\ s_2(k+1) \\ \vdots \\ s_m(k+1) \end{bmatrix} \\ \begin{bmatrix} s_1(k+2) \\ s_2(k+2) \\ \vdots \\ s_m(k+2) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} s_1(k+N) \\ s_2(k+N) \\ \vdots \\ s_m(k+N) \end{bmatrix} \end{bmatrix} \quad (9)$$

With N is prediction horizon.

Equation (9) can be described as follows::

$$S_{p\_MIMO}(k+1) = \Gamma_{MIMO}x(k) + \Omega^F_{MIMO}\Delta U(k) + \Omega^P_{MIMO}u(k-1) \quad (10)$$

where:

$$\Delta U(k) = \left[\delta u(k), \delta u(k+1), \cdots, \delta u(k+M-1), \underbrace{0 \ , \cdots, \ 0}_{m\times(N-M+1)} \right]$$

With M is control horizon.

$$\Gamma_{MIMO} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} \quad (11)$$

$$\Omega^F_{MIMO} = \begin{bmatrix} CB & 0 & \cdots & \cdots & 0 \\ C(A+I) & CB & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C(\sum_{j=0}^{M-1} A^j)B & C(\sum_{j=0}^{M-2} A^j)B & \cdots & \cdots & CB \\ C(\sum_{j=0}^{M} A^j)B & C(\sum_{j=0}^{M-1} A^j)B\cdots & \cdots & \cdots & C(A+I)B \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C(\sum_{j=0}^{N-1} A^j)B & C(\sum_{j=0}^{N-2} A^j)B & \cdots & \cdots & C(\sum_{j=0}^{N-M} A^j)B \end{bmatrix}$$

$$\Omega^P_{MIMO} = \begin{bmatrix} CB \\ C(A+I)B \\ \vdots \\ C(\sum_{j=0}^{M-1} A^j)B \\ \vdots \\ C(\sum_{j=0}^{N-1} A^j)B \end{bmatrix} \quad (12)$$

In practice, to make correction to the future predictive sliding function vector $S_{p\_MIMO}(k+p)$, we introduce the error between the sliding functions vector $S(k)$ and the predictive sliding functions vector $S(k/k-p)$. Therefore, the predictive sliding functions vector is given as follows:

$$\begin{aligned} \tilde{S}_{p\_MIMO}(k+p) &= S(k+p) + h_p e(k) \\ &= CA^p x(k) + CB\delta u(k+p-1) + C\left(A+I\right)\delta u(k+p-2) \\ &+ \cdots + C\left[\sum_{j=0}^{p-1} A^j\right]B\delta u(k) + C\left[\sum_{j=0}^{p-1} A^j\right]Bu(k-1) + h_p e(k) \end{aligned} \quad (13)$$

h_p is a correct coefficient.

The equation (13), can be given as:

$$\tilde{S}_{p\_MIMO}(k+1) = S_{p\_MIMO}(k+1) + H_p E(k) \quad (14)$$

where:

$$\begin{aligned} &\tilde{S}_{p\_MIMO}(k+1) = \left[\tilde{S}_p(k+1), \tilde{S}_p(k+2), ..., \tilde{S}_p(k+N)\right]^T \\ &H_p = diag\left[h_1 I_m, h_2 I_m, ..., h_N I_m\right] \\ &E(k) = S_v(k) - S_{mp}(k) \\ &S_v(k) = \left[S(k), S(k), ..., S(k)\right] \\ &S_{mp}(k) = \left[S(k/k-1), S(k/k-2), ..., S(k/k-N)\right]^T \end{aligned}$$

Knowing that:

$$S(k/k-p) = CA^p x(k-p) + \sum_{j=1}^{p} CA^{j-1}Bu(k-j) \quad (15)$$

The following corresponding optimization cost function is defined by:

$$\begin{aligned} j_{DPSMC} &= \sum_{j=1}^{N} q_j \left[\widetilde{S}_p(k+j) - S_r(k+j)\right]^2 \\ &+ \sum_{l=1}^{M} g_l \left[\delta u(k+l-1)\right]^2 \end{aligned} \quad (16)$$

where $S_r(k+j)$ is the sliding mode references trajectories vector, q_j and g_l are weight coefficients.

In order to simplify the synthesis of the controller, we consider $q_j = q$ and $g_l = g$. So, the following corresponding optimization cost function (16) is written by:

$$j_{DPSMC} = \sum_{j=1}^{N} q\left[\tilde{S}_p(k+j) - S_r(k+j)\right]^2 + \sum_{l=1}^{M} g\left[\delta u(k+l-1)\right]^2 \quad (17)$$

The equation (17) can be rewritten as:

$$\begin{aligned} J_{DPSMC} &= \left\|\tilde{S}_{p\_MIMO}(k+1) - S_{r\_MIMO}(k+1)\right\|^2_Q \\ &+ \left\|\Delta U(k)\right\|^2_G \\ &= \left[\Gamma_{MIMO}x(k) + \Omega^F_{MIMO}\Delta U(k) + \Omega^P_{MIMO}u(k-1) \right. \\ &\left. + H_p E(k) - S_{r\_MIMO}(k+1)\right]^T Q \left[\Gamma_{MIMO}x(k) \right. \\ &+ \Omega^F_{MIMO}\Delta U(k)) + \Omega^p_{MIMO}u(k-1) + H_p E(k) \\ &\left. - S_{r\_MIMO}(k+1)\right] + \Delta U(k)^T G \Delta U(k) \end{aligned} \quad (18)$$

where

$$\begin{aligned} &S_{r\_MIMO}(k+1) = \left[S_r(k+1), S_r(k+2), ..., S_r(k+N)\right]^T \\ &G = \left[gI_m, gI_m, ..., gI_m\right] \\ &Q = \left[qI_m, qI_m, ..., qI_m\right] \end{aligned}$$

The optimal control law can be obtained by:

$$\frac{\partial J_{DPSMC}}{\partial \Delta U(k)} = 0$$

So,

$$\Delta U(k) = -((\Omega^F_{MIMO})^T \Omega^F_{MIMO} + G)^{-1} (\Omega^F_{MIMO})^T \left[\Gamma_{MIMO} x(k) + H_p E(k) + \Omega^p_{MIMO} u(k-1) - S_{r\_MIMO}(k+1)\right] \tag{19}$$

Only the m present increment of control input signals vector are implemented, the next time increment of control signals vector $\delta u(k)$ will be calculated recursively by:

$$\delta u(k) = [1, 1, ..., 1, 0, ...0]^T \, \Delta U(k) \tag{20}$$

So, we have:

$$u(k) = u(k-1) + \delta u(k) \tag{21}$$

III. Robustness analysis

We consider the system (2) and the sliding mode function (4). The following is given:

$$\begin{aligned} S(k+1) &= Cx(k+1) \\ &= C\left[Ax(k) + Bu(k) + w(k)\right] \\ &= C\left[Ax(k) + Bu(k)\right] \\ &\quad + C\left[\Delta Ax(k) + \Delta Bu(k) + (B + \Delta B)v(k)\right] \end{aligned} \tag{22}$$

The sliding function value at time $(k+p)$ is:

$$S(k+p) = \begin{bmatrix} s_1(k+p) \\ s_2(k+p) \\ \vdots \\ s_m(k+p) \end{bmatrix} = CA^p x(k) + \sum_{j=1}^{p} CA^{j-1} Bu(k+p-j) + \sum_{i=1}^{p} CA^{i-1} w(k+p-i) \tag{23}$$

We consider then, the predictive sliding function vector of multivariable systems, $S_{p\_MIMO}(k+1)$ defined by equation (9)

Using $\Delta U(k)$ given by equation (19),we can express the predictive sliding function vector by:

$$\begin{aligned} S_{p\_MIMO}(k+1) &= \Gamma_{MIMO} x(k) + \Omega^F_{MIMO} \Delta U(k) \\ &\quad + \Omega^P_{MIMO} u(k-1) + KW(k) \end{aligned} \tag{24}$$

with:

$$K = \begin{bmatrix} C & 0 & \cdots & 0 \\ CA & C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1} & CA^{N-2} & \cdots & C \end{bmatrix}$$

and
$W(k) = [w(k), w(k+1), ..., w(k+N-1)]^T$
Or, we have:

$$\Delta U(k) = -((\Omega^F_{MIMO})^T \Omega^F_{MIMO} + G)^{-1} (\Omega^F_{MIMO})^T \times \left[\Gamma_{MIMO} x(k) + H_p E(k) + \Omega^p_{MIMO} u(k-1) - S_{r\_MIMO}(k+1)\right]$$

So:

$$S_{p\_MIMO}(k+1) = \Gamma_{MIMO} x(k) + \Omega^F_{MIMO} \left[-(\Omega^F_{MIMO})^T \times (\Omega^F_{MIMO} + G)^{-1} (\Omega^F_{MIMO})^T \left[\Gamma_{MIMO} x(k) + H_p E(k) + \Omega^p_{MIMO} u(k-1) - S_{r\_MIMO}(k+1)\right]\right] + \Omega^p_{MIMO} u(k-1) + KW(k)$$

The action of the weight coefficient matrix G is used to limit the control input ΔU. So, we can suppose that G is equal to null matrix, i.e., there is no limitation for control input ΔU.

$$\begin{aligned} S_{p\_MIMO}(k+1) &= \Gamma_{MIMO} x(k) - [\Gamma_{MIMO} x(k) + H_p E(k) \\ &\quad + \Omega^p_{MIMO} u(k-1) - S_{r\_MIMO}(k+1)]] \\ &\quad + \Omega^p_{MIMO} u(k-1) + KW(k) \\ &= S_{r\_MIMO}(k+1) - H_p E(k) + KW(k) \end{aligned} \tag{25}$$

or:

$$E(k) = S_v(k) - S_{mp}(k)$$

with:

$$S_v(k) = [S(k), S(k), ..., S(k)]_{1\times N}$$

and:

$$S_{mp}(k) = [S(k/k-1), S(k/k-2), ..., S(k/k-N)]^T$$

knowing that:

$$S(k) = CA^p x(k-p) + \sum_{j=1}^{p} CA^{j-1} Bu(k-j) + \sum_{i=1}^{p} CA^{i-1} w(k-i)$$

and

$$S(k/k-p) = CA^p x(k-p) + \sum_{j=1}^{p} CA^{j-1} Bu(k-j)$$

$$e(k) = S(k) - S(k/k-p) = \sum_{i=1}^{p} CA^{i-1} w(k-i)$$

then we can conclude that:

$$E(k) = \tilde{K}\tilde{W}(k)$$

with:

$$\tilde{K} = \begin{bmatrix} C & 0 & \cdots & 0 \\ C & CA & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C & CA & \cdots & CA^{N-1} \end{bmatrix}$$

$$\tilde{W}(k) = [w(k-1), w(k-2), ..., w(k-N)]^T$$

Because of rolling optimization, only the m present control input signal are implemented. The practical sliding mode function can be described as follows:

$$S(k+1) = [1, 1 \cdots 1, 0 \cdots 0] \left[S_r - H_p \tilde{K}\tilde{W} + KW\right] \tag{26}$$

Equation(35) can be re-written to:

$$S(k+1) = S_r(k+1) + C\left[w(k) - h_1 w(k-1)\right] \tag{27}$$

From the viewpoint of practice, usually, we choose $h_1 = 1$ [26].
Then equation(27) can be re-written to:

$$S(k+1) = S_r(k+1) + C\left[w(k) - w(k-1)\right] \tag{28}$$

Or, we have:

$$\begin{cases} S_r(k+1) = S_r(k) - \begin{bmatrix} m_1 sign(s_{r1}(k)) \\ m_2 sign(s_{r2}(k)) \\ \vdots \\ m_m sign(s_{rm}(k)) \end{bmatrix} \\ S_r(k) = S(k) \end{cases}$$

That's why:

$$S(k+1) = \Phi S_r(k) - \begin{bmatrix} m_1 \operatorname{sign}(s_{r1}(k)) \\ m_2 \operatorname{sign}(s_{r2}(k)) \\ \vdots \\ m_m \operatorname{sign}(s_{rm}(k)) \end{bmatrix} + C\left[w(k) - w(k-1)\right] \quad (29)$$

So, we have:

$$S(k+1) = \Phi S(k) - \begin{bmatrix} m_1 \operatorname{sign}(s_1(k)) \\ m_2 \operatorname{sign}(s_2(k)) \\ \vdots \\ m_m \operatorname{sign}(s_m(k)) \end{bmatrix} + C\left[w(k) - w(k-1)\right] \quad (30)$$

then we have $\forall i \in [1 \quad m]$:

$$s_i(k+1) = \Phi_{i,i} s_i(k) - m_i sign(s_i(k)) + C\left[w_i(k) - w_i(k-1)\right] \quad (31)$$

In discrete time sliding mode control, a quasi sliding mode is considered in the vicinity of the sliding surface, such that $|s(k)| < \varepsilon$ where $s(k)$ is the sliding function and ε is a positive constant called the quasi sliding mode band width [27].

The stability of the proposed control can be given by the following theorem:

Theorem 1. Consider the system (2) to which the discrete Predictive Sliding Mode Control is applied (20).

This system verifies a convergent quasi-sliding mode, if the following conditions $C1$, $C2$ and $C3$ are satisfied [28].

$C1$:

$$|s_i(k+1)| < |s_i(k)|\,, \text{if } |s_i(k)| > \varepsilon_i, \text{i} \in [\ 1 \quad m\ ] \quad (32)$$

$C2$:

$$|s_i(k+1)| < \varepsilon_i, \text{if } |s_i(k)| = \varepsilon_i, \text{i} \in [\ 1 \quad m\ ] \quad (33)$$

$C3$:

$$|s_i(k+1)| < \varepsilon_i, \text{if } |s_i(k)| < \varepsilon_i, \text{i} \in [\ 1 \quad m\ ] \quad (34)$$

where:

$\forall k$, $C[w_i(k) - w_i(k-1)]$ is bounded such that:
$|C(w_i(k) - w_i(k-1))| < w_0$, $0 < w_0 < m_i$,
and $\Phi_{i,i} > \left(\frac{m_i}{\varepsilon_i}\right)$

Proof.

Firstly, we should begin by the condition (32).
Consider the case 1: $s_i(k) > \varepsilon_i$
The difference between $s_i(k+1)$ and $s_i(k)$ is given by:

$$\begin{aligned} & s_i(k+1) - s_i(k) = (\Phi_{i,i} - 1)s_i(k) - m_i + \\ & C\left[w_i(k) - w_i(k-1)\right] \\ & < -m_i + C\left[w_i(k) - w_i(k-1)\right] \\ & < 0 \end{aligned}$$

The sum of $s_i(k+1)$ and $s_i(k)$ is given by:

$$\begin{aligned} & s_i(k+1) + s_i(k) = (\Phi_{i,i} + 1)s_i(k) - m_i \\ & +C\left[w_i(k) - w_i(k-1)\right] \\ & > \varepsilon_i + \Phi_{i,i}\varepsilon_i - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ & > \Phi_{i,i}\varepsilon_i + C\left[w_i(k) - w_i(k-1)\right] \\ & > m_i + C\left[w_i(k) - w_i(k-1)\right] \\ & > 0 \end{aligned}$$

Then:
$|s_i(k+1)| < |s_i(k)|$, if $s_i(k) < \varepsilon_i$

Consider the case 2: $s_i(k) < -\varepsilon_i$
In this case, the difference between $s_i(k+1)$ and $s_i(k)$ can be calculated as:

$$\begin{aligned} s_i(k+1) - s_i(k) &= (\Phi_{i,i} - 1)s_i(k) + m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> 0 \end{aligned}$$

The sum of $s_i(k+1)$ and $s_i(k)$ is given by:

$$\begin{aligned} s_i(k+1) + s_i(k) &= (\Phi_{i,i} + 1)s_i(k) - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< -\varepsilon_i - \Phi_{i,i}\varepsilon_i + m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< -\Phi_{i,i}\varepsilon_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< -m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< 0 \end{aligned}$$

Then:
$|s_i(k+1)| < |s_i(k)|$, if $s_i(k) < -\varepsilon_i$

Using cases 1 and 2, the condition $C1$ is verified.

Secondly, we consider the condition (33).
Consider the case 3:$s_i(k) = \varepsilon_i$
We have:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< \Phi_{i,i}\varepsilon_i - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< \varepsilon_i \end{aligned}$$

The sliding function $s_i(k+1)$ can be written as follows:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> \Phi_{i,i}\varepsilon_i - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> C\left[w_i(k) - w_i(k-1)\right] \\ &> -m_i \\ &> -\Phi_{i,i}\varepsilon_i \\ &> -\varepsilon_i \end{aligned}$$

Then:
$|s_i(k+1)| < \varepsilon_i$, if $s_i(k) = \varepsilon_i$

Consider the case 4: $s_i(k) = -\varepsilon_i$
In this case, $s_i(k+1)$ can be given by:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< C\left[w_i(k) - w_i(k-1)\right] \\ &< m_i \\ &< \Phi_{i,i}\varepsilon_i \\ &< \varepsilon_i \end{aligned}$$

Thus:
$|s_i(k+1)| < \varepsilon_i$, if $s_i(k) = -\varepsilon_i$

Using cases 3 and 4, we obtain the condition $C2$.

Finally, we consider the third condition (34).
Consider the case 5: $0 \leq s_i(k) < \varepsilon_i$
Using this condition, the sliding function $s_i(k+1)$ can be expressed as:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< \Phi_{i,i}\varepsilon_i - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< \Phi_{i,i}\varepsilon_i \\ &< \varepsilon_i \end{aligned}$$

The sliding function $s_i(k+1)$ can be given as follows:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> C\left[w_i(k) - w_i(k-1)\right] \\ &> -m_i \\ &> -\Phi_{i,i}\varepsilon_i \\ &> -\varepsilon_i \end{aligned}$$

Then:
$|s_i(k+1)| < \varepsilon_i$, if $0 \leq s_i(k) < \varepsilon_i$

Consider the case 6: $-\varepsilon_i < s_i(k) \leq 0$ The sliding function $s_i(k+1)$ can be calculated as:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> -\Phi_{i,i}\varepsilon_i + m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &> -\Phi_{i,i}\varepsilon_i \\ &> -\varepsilon_i \end{aligned}$$

We have:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + C\left[w_i(k) - w_i(k-1)\right] \\ &< C\left[w_i(k) - w_i(k-1)\right] \\ &< m_i \\ &< \Phi_{i,i}\varepsilon_i \\ &< \varepsilon_i \end{aligned}$$

$|s_i(k+1)| < \varepsilon_i$, if $-\varepsilon_i < s_i(k) \leq 0$

Using cases 1, 2, 3, 4, 5 and 6 the three conditions (32), (33) and (34) are verified. Therefore, the discrete multivariable predictive sliding mode controller is stable.

IV. Simulation Results

In order to prove the effectiveness of the proposed multivariable DPSMC, we choose to apply the classical discrete SMC and the proposed DPSMC, in the presence of constant or periodic disturbances and parameters uncertainties, to the process described by the following equation, :

$$x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)(u(k) + v(k))$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ 0.24 & 0.2 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}$$

The retained synthesis parameters are:

$$C = \begin{bmatrix} 0.6667 & 0 \\ 0 & 1 \end{bmatrix}$$

and $m_1 = 0.01$, $m_2 = 0.01$, $\Phi = [0.01 \quad 0; 0 \quad 0.01]$, $N = 10$, $M = 5$, $H_p = 0.001I(N,N)$, and $G = 0.001I(N,N)$
The sliding functions vector is given by:

$$S(k) = Cx(k) = C = \begin{bmatrix} 0.6667 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix}$$

A. *Case of constant disturbances*

In presence of constant disturbances, given as follows , the results presented in this section are obtained.

$$v(k) = \begin{bmatrix} 0.15 \\ 0.2 \end{bmatrix} \quad , \quad \forall k \geq 100$$

The parameters variation are applied at the instant $k = 300$:

$$\Delta A = 0.1 \begin{bmatrix} 5\sin(-\frac{2k\pi}{10}) & 6\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.1 \begin{bmatrix} 2\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \end{bmatrix}, \forall k \geq 300$$

The evolution of the states $x_1(k)$ and $x_2(k)$, the control inputs $u_1(k)$ and $u_2(k)$ and the sliding mode functions $s_1(k)$ and $s_2(k)$ with DPSMC and SMC are given, respectively, in figures 2 to 7.

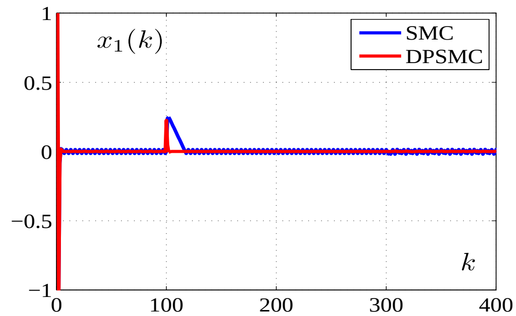

Fig. 2. Evolution of the state $x_1(k)$.

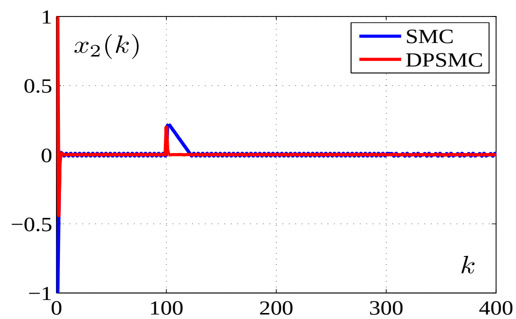

Fig. 3. Evolution of the state $x_2(k)$.

It can be seen that the performances of the DPSMC are better then the classical SMC, not only, for rejecting constant disturbances, but also, for eliminating chattering and fast convergence.
In fact, without disturbances and parameters uncertainties, the results of SMC and DPSMC are comparable. But, in presence of constant disturbances ($k \geq 100$), we find that the proposed control law ensure good performances in term of rejection of external disturbances and fast convergence.
When we add parameters uncertainties, at the instant ($k \geq 300$), the oscillation encountered, in the case of classical SMC, are reduced.

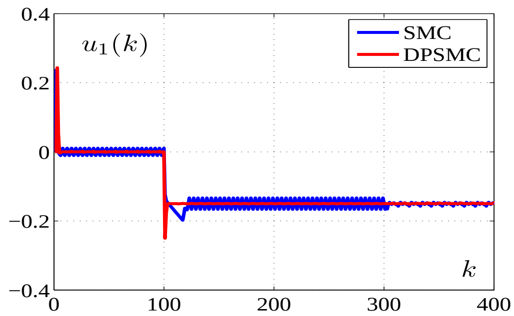

Fig. 4. Evolution of the control signal $u_1(k)$.

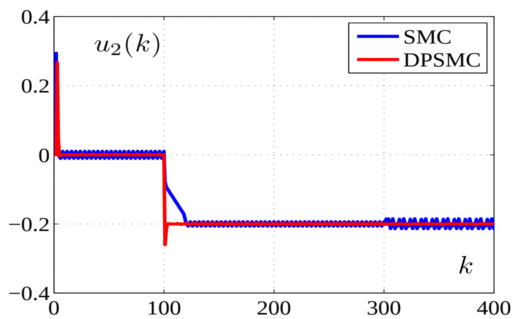

Fig. 5. Evolution of the control signal $u_2(k)$.

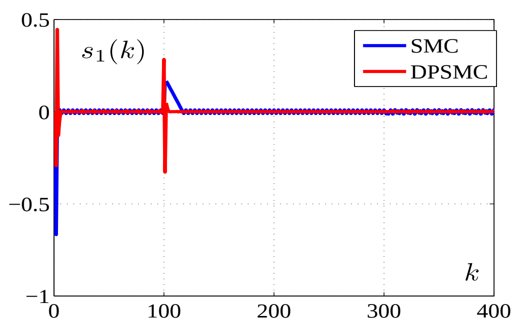

Fig. 6. Evolution of the sliding function $s_1(k)$.

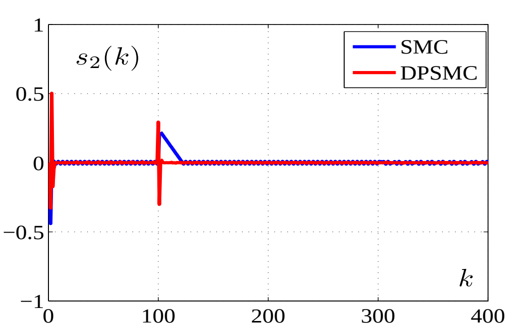

Fig. 7. Evolution of the sliding function $s_2(k)$.

B. Case of periodic disturbances

The results presented in this section are obtained with the presence of disturbances, whose evolutions are given in figures 8 and 9, and with the following parameters variation:

$$\Delta A = 0.1 \begin{bmatrix} 5\sin(-\frac{2k\pi}{10}) & 6\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.1 \begin{bmatrix} 2\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 5\sin(-\frac{2k\pi}{10}) \end{bmatrix}, \forall k \geq 300$$

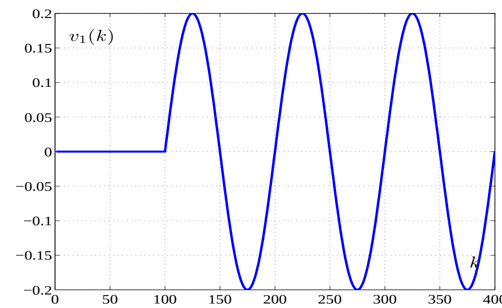

Fig. 8. Evolution of the disturbances $v_1(k)$.

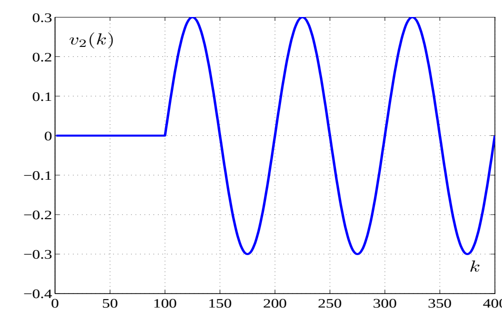

Fig. 9. Evolution of the disturbance $v_2(k)$.

The evolution of the states $x_1(k)$ and $x_2(k)$, the sliding mode functions $s_1(k)$ and $s_2(k)$ and the control inputs $u_1(k)$ and $u_2(k)$,with DPSMC and SMC are given, respectively, in figures 10 to 15.

A comparison between the DPSMC and SMC, in the case of multivariable systems, reveals that the use of the new control strategy DPSMC reduces the chattering problem effectively ($k \geq 300$).
Furthermore, the results obtained prove the capability of the proposed control law to reduce periodic disturbances ($k \geq 100$) and parameters uncertainties ($k \geq 300$).

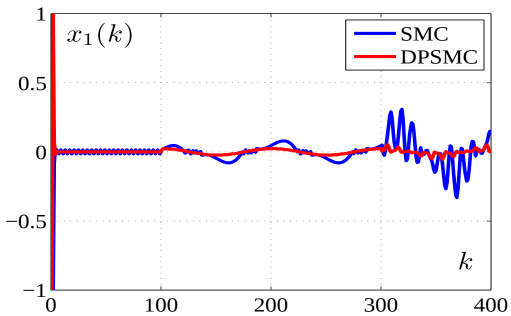

Fig. 10. Evolution of the state $x_1(k)$.

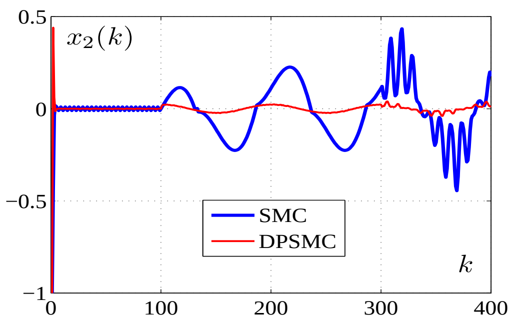

Fig. 11. Evolution of the state $x_2(k)$.

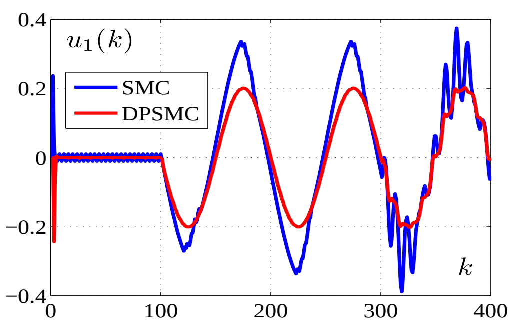

Fig. 12. Evolution of the control signal $u_1(k)$.

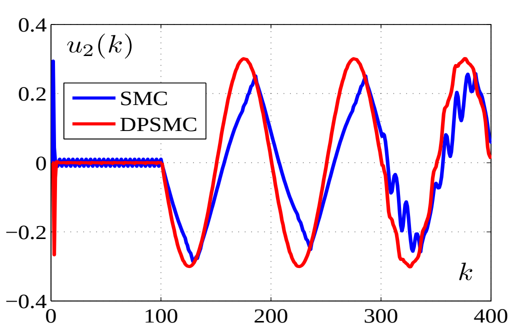

Fig. 13. Evolution of the control signal $u_2(k)$.

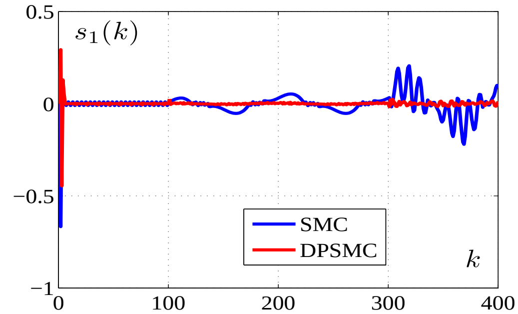

Fig. 14. Evolution of the sliding function $s_1(k)$.

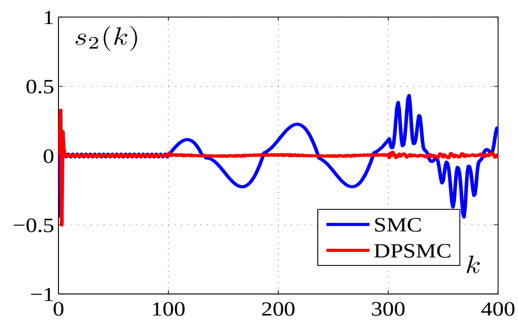

Fig. 15. Evolution of the sliding function $s_2(k)$.

V. CONCLUSION

In this paper, a discrete predictive sliding mode controller for multivariable systems was proposed. This controller combines the design technique of the SMC and MPC.
A stability analysis of the proposed controller was studied.
The method was tested on a multivariable system, and compared to the results given by the SMC controller. It is shown that mixing both control techniques, for multivariable systems, gives new controller with better robustness properties in rejecting disturbances, hard parameters variations and in eliminating the chattering problem.

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