Filtering properties of periodically grounded multiconductor power lines

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***Abstract*—** This paper deals the effect of the periodical grounding of the shield wire in multiconductor power lines. The grounding of the shield wire is usually realized in order to prevent direct lightning of the power wire or the mitigate the induced overvoltages. However, this operation introduces a filtering property in the frequency behavior of the transmission line, which can be clearly observed in the characteristic impedance. It is worth to observe that, even if the grounding is performed just on the shield wire, the filtering property can be observed also on the phase wires. The paper also shows a methodology to deal with problems mixing concentrated and distributed parameters equations.

***Keywords*—**Transmission lines, characteristic impedance, Non-symmetric Algebraic Riccati Equation, ladder network.

I. Introduction

POWER transmission lines are essential components of all the modern electric and electronic systems. Its analysis is a classical electromagnetic topic [1-3] and is included in several academic courses. In these cases, the transmission line is usually modeled as an infinite set of conductor of different cross-section, eventually interfering with some media. This configuration easily allows to define and evaluate some *per unit length* parameters (inductance, capacitance, resistance) and to model the transmission line in terms of equivalent circuits. In this model, an essential parameter is represented by the characteristic impedance of the line, defined as the value of impedance to use as termination of the transmission line in order to make it looking like an infinite one, i.e. there are no reflection at the termination.

In some practical situation, some conductors may be grounded in order to improve the line performance [4-5]. In power lines, the shield wire may be periodically connected to ground, aiming at intercepting the (direct) lightning strokes and avoiding the fault of the line due to an excessive overvoltage on the power wires. In distribution lines, periodically grounded shield wires may be still useful reducing induced voltages from external electromagnetic fields caused by indirect lightning [6].

In these cases, the periodical grounding has an additional consequence in the line behavior, that is a filtering effect at specific predictable frequencies. This aspect, not adequately discussed in literature, can positively enhance the line performance if the grounding is properly designed. Moreover, it is possible to observe that, even if just the shield wire is grounded, the filtering effect occurs on the power conductor too.

This aspect can be clearly observed in the frequency behavior of the characteristic impedance. However, even this parameter is affected by the presence of the periodical grounding and a proper method to compute has to be adopted, involving the solution of a Non-symmetric Algebraic Riccati Equation (NARE) [7-8]. This kind of equation occurs in several mathematical and technical problems, so the proposed solution can be useful for a broad class of problems [9-17].

Finally yet importantly, the considered model involves both distributed and concentrated parameters elements. This is a complex situation, appearing in different kind of problems [18-24], and the proposed method is valuable by this point of view, too.

II. Computation of the Characteristic Impedance

The reference problem is shown in Fig. 1: a multiconductor transmission line (MTL) has several conductors, and part of them are periodically connected to ground through a specific resistance.

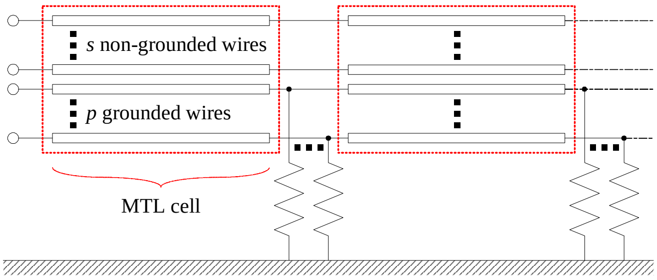

Fig. 1 Scheme of the MTL cells with the periodical grounding.

In general, each part of transmission line between two grounding points can be modelled as an identical MTL cell.

In the next sections, we will discuss about the computation of the characteristic impedance of periodically grounded transmission lines in different cases and adopting different approaches.

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A. Ungrounded single transmission line

In this section, just to help the reader for further comparisons, we recall the classical model of ungrounded transmission line, in case of just one wire. Depending on the length of the line, it is possible to use a lumped parameters approximation or a distributed parameters model. The scheme is represented in Fig. 2, not considering the presence of the grounding resistor R_g.

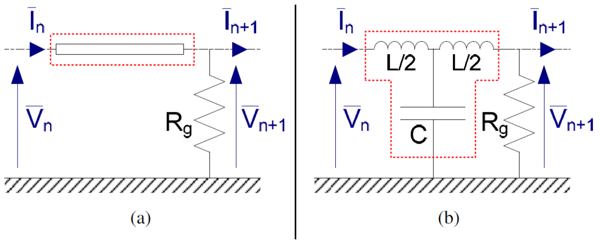

Fig. 2 Scheme of the single conductor transmission line: (a) problem scheme and (b) lumped parameters approximation.

1) Lumped parameters model

In case of lumped parameters approximation, the transmission line can be represented with a T model as show in Fig. 2-b, the horizontal impedances representing the line inductance (L) and the vertical one representing the line capacitance to ground (C). In this model, the characteristic impedance is well-known, frequency independent and equal to $\dot{Z}_0 = \sqrt{L/C}$.

2) Distributed parameters model

In case of distributed parameters model, the lumped parameters L and C are substituted with the per-unit-length inductance l and capacitance c, however the characteristic impedance is still a frequency independent value equal to $\dot{Z}_0 = \sqrt{l/c}$.

B. Grounded single transmission line

Let us consider now the presence of the grounding resistor R_g. as effectively shown in Fig. 2.

1) Lumped parameters approximation

In case of lumped parameters approximation, the L and C elements and the grounding resistor R_g can be described through a chain matrix (1), where $\varpi = \omega\ell / c$ is the normalized frequency, ω is the angular frequency, ℓ is the length of a single TL cell and c is the speed of the light in the free space, $\dot{Z}_0$ is the characteristic impedance of the ungrounded cell. It's worth remembering that, being the TL lossless, the $lc = 1/c^2$.

$$\begin{cases} V_{n+1} = \left(1-\varpi^2/2\right)V_n - j\varpi\dot{Z}_0\left(1-\varpi^2/4\right)I_n \\ I_{n+1} = -\left(\dfrac{1-\varpi^2/2}{R_g} + j\dfrac{\varpi}{\dot{Z}_0}\right)V_n + \\ \qquad + \left(1 + j\varpi\dot{Z}_0\left(\dfrac{1-\varpi^2/4}{R_g} + j\dfrac{\varpi/2}{\dot{Z}_0}\right)\right)I_n \end{cases} \tag{1}$$

Since the network is semi-infinite, then $V_n = \dot{Z}_C I_n$ and $V_{n+1} = \dot{Z}_C I_{n+1}$, being $\dot{Z}_C$ the researched characteristic impedance of the whole network. By substituting these relationships into (1), whit some tricky manipulation the following non-linear algebraic equation is found:

$$\left(1-\varpi^2 + j\varpi R_g/\dot{Z}_0\right)\dot{Z}_C^2 - j\varpi\dot{Z}_0\left(2-\varpi^2\right)\left(\dot{Z}_C + R_g\right) = 0 \tag{2}$$

whose solution is

$$\dot{Z}_C = \frac{j\omega\dot{Z}_0\left(2-\varpi^2\right)}{2\left(1-\varpi^2 + j\varpi R_g/\dot{Z}_0\right)} \pm \\ \pm \frac{\sqrt{j\varpi\dot{Z}_0\left(2-\varpi^2\right)\left(2-\varpi^2 + j2\varpi R_g/\dot{Z}_0\right)\left(j\varpi\dot{Z}_0 + 2R_g\right)}}{2\left(1-\varpi^2 + j\varpi R_g/\dot{Z}_0\right)} \tag{3}$$

In order to have the real part of $\dot{Z}_C$ as positive, since the network is passive, it is necessary to choose the positive sign in (3) for $\varpi \le 1$.

This solution is frequency dependent and a-periodical. It is quite easy to observe that for R_g going to infinity, the solution is again the one in sub-section II.A.1.

2) Distributed parameters model

Even if we consider a distributed parameters model, a similar can be adopted. The chain matrix (1) becomes

$$\begin{cases} V_{n+1} = \cos(\varpi)V_n - j\dot{Z}_0\sin(\varpi)I_n \\ I_{n+1} = -\left(\dfrac{\cos(\varpi)}{R_g} + j\dfrac{\sin(\varpi)}{\dot{Z}_0}\right)V_n + \\ \qquad + \left(j\dfrac{\dot{Z}_0}{R_g}\sin(\varpi) + \cos(\varpi)\right)I_n \end{cases} \tag{4}$$

By making the same assumptions on the relationships between the voltages and the currents as in the previous sub-section, whit some tricky manipulation the following new second-order equation is found:

$$\dot{Z}_C^2\left(\dot{Z}_0\cos(\varpi)+jR_g\sin(\varpi)\right)-j\left(\dot{Z}_C+R_g\right)\dot{Z}_0^2\sin(\varpi)=0 \tag{5}$$

whose solution is

$$\dot{Z}_C=\frac{j\dot{Z}_0^2\pm\sqrt{4j\dot{Z}_0R_g\cot(\varpi)-\left(\dot{Z}_0^2+4R_g^2\right)}}{2\left(Z_0\cot(\varpi)+jR_g\right)} \tag{6}$$

In order to have the real part of $\dot{Z}_C$ as positive, the negative sign in (3) has to be chosen for $(0.5+2n)\le\varpi/\pi\le(1.5+2n)$, the positive otherwise, with n = 1, 2, …

It is easy to observe that for R_g going to infinity, the solution is again the one in sub-section II.A.2.

The solution (6) is periodical and very different from the (3) in terms of general behavior. This means that, even if the lumped parameters approximation and distributed parameters model have the same characteristic impedance in the ungrounded case, there is a relevant difference in case of grounding. Since both the models represents the same network, this means that the lumped parameters approximation can't be adopted to describe periodically grounded lines, even in the case where the lumped model itself is acceptable (i.e. the line is short enough).

C. Grounded multiconductor transmission line

Let us consider now the most complex case, that is to say the multiconductor transmission line, as case in which just some (eventually all) the conductors are periodically connected to the ground [25-27]. So let us consider that the transmission line has *m* conductors, the first *s* conductors being ungrounded and the next *p* conductors being periodically grounded.

The problem can be formulated considering at first the chain matrix of just the ungrounded MTL, that here is

$$\begin{cases}\overline{\mathbf{V}}_{n+1}=\cos(\varpi)\overline{\mathbf{V}}_n-j\sin(\varpi)\dot{\mathbf{Z}}_0\overline{\mathbf{I}}_n\\ \overline{\mathbf{I}}_{n+1}=-j\sin(\varpi)\dot{\mathbf{Z}}_0^{-1}\overline{\mathbf{V}}_n+\cos(\varpi)\overline{\mathbf{I}}_n\end{cases} \tag{7}$$

Then, it is possible to introduce some modal voltages and currents in order to simplify the next calculations and the analysis of the system. In our case we consider the modal voltages $\tilde{\mathbf{V}}=\mathbf{T}_v\overline{\mathbf{V}}$ and $\tilde{\mathbf{I}}=\mathbf{T}_i\overline{\mathbf{I}}$, being $\mathbf{T}_i=\mathbf{T}_v^{-1}=\mathbf{1}\sqrt{c\ell}$. By applying these transformations to (7), the chain matrix in the modal domain just becomes:

$$\begin{cases}\tilde{\mathbf{V}}_{n+1}=\cos(\varpi)\tilde{\mathbf{V}}_n-j\sin(\varpi)\tilde{\mathbf{I}}_n\\ \tilde{\mathbf{I}}_{n+1}=-j\sin(\varpi)\tilde{\mathbf{V}}_n+\cos(\varpi)\tilde{\mathbf{I}}_n\end{cases} \tag{8}$$

Regarding the grounding, it's chain matrix can be expressed as

$$\begin{cases}\overline{\mathbf{V}}_{n+1}=\overline{\mathbf{V}}_n\\ \overline{\mathbf{I}}_{n+1}=-\mathbf{G}\overline{\mathbf{V}}_n+\overline{\mathbf{I}}_n\end{cases} \tag{9}$$

where **G** is a an *m*x*m* matrix where all the elements are zero but the last *p* ones on the main diagonal, namely $G_{i,i}=1/R_g$ with i = *s*+1, …, m. In the modal domain it becomes just

$$\begin{cases}\tilde{\mathbf{V}}_{n+1}=\tilde{\mathbf{V}}_n\\ \tilde{\mathbf{I}}_{n+1}=-\tilde{\mathbf{G}}\tilde{\mathbf{V}}_n+\tilde{\mathbf{I}}_n\end{cases} \tag{10}$$

being $\tilde{\mathbf{G}}=\mathbf{T}_v^{-1}\mathbf{G}\mathbf{T}_v$. So the chain matrix of the grounded MTL cell can be obtained in the modal domain just multiplying the chain matrixes, that is to say

$$\begin{cases}\tilde{\mathbf{V}}_{n+1}=\cos(\varpi)\tilde{\mathbf{V}}_n-j\sin(\varpi)\tilde{\mathbf{I}}_n\\ \tilde{\mathbf{I}}_{n+1}=-\left[\cos(\varpi)\tilde{\mathbf{G}}+j\sin(\varpi)\mathbf{1}\right]\tilde{\mathbf{V}}_n+\left[j\sin(\varpi)\tilde{\mathbf{G}}+\cos(\varpi)\mathbf{1}\right]\tilde{\mathbf{I}}_n\end{cases} \tag{11}$$

Now, introducing a characteristic impedance matrix in the modal domain and imposing anyway that $\tilde{\mathbf{V}}_n=\tilde{\mathbf{Z}}_C\tilde{\mathbf{I}}_n$ and $\tilde{\mathbf{V}}_{n+1}=\tilde{\mathbf{Z}}_C\tilde{\mathbf{I}}_{n+1}$, that it is possible to find the following equation

$$\tilde{\mathbf{Z}}_C\left(\cos(\varpi)\tilde{\mathbf{G}}+j\sin(\varpi)\mathbf{1}\right)\tilde{\mathbf{Z}}_C-j\sin(\varpi)\left(\tilde{\mathbf{Z}}_C\tilde{\mathbf{G}}+\mathbf{1}\right)=\mathbf{0} \tag{12}$$

or, for $\varpi\neq k\pi$, with *k* = 0, 1, …, in the more compact form

$$\tilde{\mathbf{Z}}_C\left(\mathbf{1}-j\cot(\varpi)\tilde{\mathbf{G}}\right)\tilde{\mathbf{Z}}_C-\tilde{\mathbf{Z}}_C\tilde{\mathbf{G}}-\mathbf{1}=\mathbf{0} \tag{13}$$

This equation, whose appearance recalls (5), is a Nonsymmetric Algebraic Riccati Equations (NARE) with complex coefficients, the worst cases among the Riccati equations [28]. It is not possible to find a closed form solution of such a problem and numerical solutions have to be found. Hereafter we discuss some efficient methods.

1) Iterative methods

Iterative methods are widely used in literature for the solution of non-linear problems and some of them are also applied to the solution of NAREs [29-30], mainly based on the fixed-point or the Netwon-Raphson alghoritm.

In order to reduce possible convergence problem, it is convenient to consider the real and complex part of the characteristic impedance (namely $\tilde{\mathbf{Z}}_C=\tilde{\mathbf{R}}_C+j\tilde{\mathbf{X}}_C$) and to divide the (12) as well, obtaining the two following quadratic equations:

$$\begin{aligned}&\cos(\varpi)\left(\tilde{\mathbf{R}}_C\tilde{\mathbf{G}}\tilde{\mathbf{R}}_C-\tilde{\mathbf{X}}_C\tilde{\mathbf{G}}\tilde{\mathbf{X}}_C\right)\tilde{\mathbf{Z}}_C\\&\quad+\sin(\varpi)\left(\tilde{\mathbf{R}}_C\tilde{\mathbf{X}}_C+\tilde{\mathbf{X}}_C\tilde{\mathbf{G}}-\tilde{\mathbf{X}}_C\tilde{\mathbf{R}}_C\right)=\mathbf{0}\end{aligned} \tag{13}$$

$$\cos(\varpi)\left(\tilde{\mathbf{X}}_C\tilde{\mathbf{G}}\tilde{\mathbf{R}}_C - \tilde{\mathbf{R}}_C\tilde{\mathbf{G}}\tilde{\mathbf{X}}_C\right)\tilde{\mathbf{Z}}_C + \sin(\varpi)\left(\tilde{\mathbf{R}}_C\tilde{\mathbf{R}}_C - \tilde{\mathbf{X}}_C\tilde{\mathbf{X}}_C - \tilde{\mathbf{R}}_C\tilde{\mathbf{G}} - \mathbf{1}\right) = \mathbf{0} \quad (14)$$

For both fixed-point and Netwon-Raphson methods, at low frequency the ungrounded solution can be a good starting point for the iterations. Then, if a frequency sweep is performed, adopting as starting point the solution found at the previous frequency step and adopting a not so large step frequency, it is possible to solve the problem. However, this approach requires, in order to find the characteristic impedance at a specific frequency, to perform anyway a sweep that, in some cases may be highly resource consuming.

2) Decomposition methods

Decomposition methods [31-32] are based on the study of the Hamiltonian matrix associated to the NARE (13). The methods introduce some more mathematical difficulties, however allow to directly compute the solution at any frequency, without requiring iterations or the determination of initial points.

It is possible to find a solution of (13) by studying the Hamiltonian matrix associated to such an equation, that can be written as:

$$\mathrm{H} = \begin{pmatrix} \tilde{\mathbf{G}} & \mathbf{1} - \mathrm{j}\cot(\varpi)\tilde{\mathbf{G}} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}. \quad (15)$$

It can be found that (15) can be always diagonalized for $\varpi \neq \mathrm{k}\pi$, by applying normal diagolization techniques or, in some more complex situation, requiring to the Jordan decomposition, the Schur decomposition or other special techniques [33-39]. Then, by analyzing the eigenvalues of (15), it can be found that there are:

- *p* distinct complex eigenvalues with phase between 0 and π;
- *p* distinct complex eigenvalues with phase always between 0 and $-\pi$;
- the real eigenvalue +1 with algebraic multiplicity *s*;
- the real eigenvalue –1, with algebraic multiplicity *s*.

Then, it is possible to sort the eigenvectors' matrix placing in the first *s* columns the ones corresponding to the eigenvalue –1, then the *p* eigenvectors corresponding to

- the complex eigenvalues with positive phase if $\mathrm{k} \leq \varpi \leq 0.5 + \mathrm{k}$, for *k* = 0, 1, …;
- the complex eigenvalues with negative phase if $0.5 + \mathrm{k} \leq \varpi \leq 1 + \mathrm{k}$, for *k* = 0, 1, …;

and finally the remaining eigenvectors.

Let us define with **U** the corresponding eigenvector matrix, sorted according to these rules. It can be written as

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{U}_{21} & \mathbf{U}_{22} \end{pmatrix} . \quad (16)$$

where each sub-matrix is *m*x*m*. It is also found that $\mathbf{U}_{11}$ is nonsingular, and the solution of (13) is

$$\dot{\mathbf{Z}}_C = -\mathbf{U}_{21}\mathbf{U}_{11}^{-1} . \quad (17)$$

Inverting the eigenvalues and eigenvectors of **H**, in some cases other solutions are found, in other cases the procedure doesn't lead to a solution since the sub-matrix $\mathbf{U}_{11}$ reveals to be not inventible. Among all the possible solution of (13), the one obtained with the previously described procedure is the only one physical solution.

It's worth to observe that the result does not depend on the order of the eigenvalues in the sub-matrix $\mathbf{U}_{11}$. So, once the good set of eigenvectors is found, any permutation in the first *m* columns of the matrix leads to the same solution.

III. Filtering effect in the periodically grounded transmission line

As previously stated, the analysis of the characteristic impedance is one of the best way to provide evidence of the filtering effect of the periodically grounded transmission line.

So, let us consider the practical case of a 220 kV power line with 3 power conductors (numbered from 1 to 3) and 2 shield wires (numbered 4 and 5), as in Fig. 3 [40]. The power conductors' cross section is 150 mm^2, the shield wires one is 50 mm^2. In our investigation, we consider a distance between the grounding points of ℓ = 100 m.

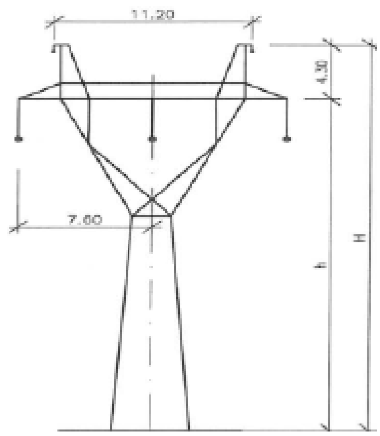

Fig. 3 Typical supporting tower used in existing 220 kV overhead transmission lines (h=26,5 m, H=30,8 m).

In Fig. 4 we show, as example, the real and imaginary parts of the term $\dot{\mathbf{Z}}_{(55)}$ (equal to $\dot{\mathbf{Z}}_{(44)}$ due to symmetry reasons) of the characteristic impedance matrix in one period, that is to say the self-impedance of one of the shield wires.

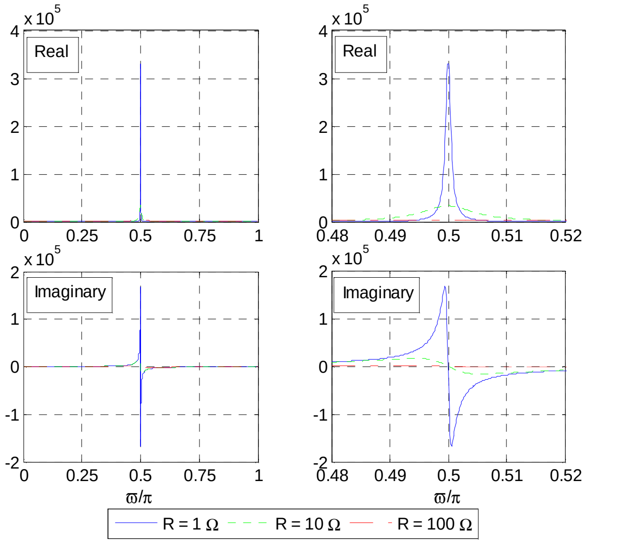

Fig. 4 Coefficient $\dot{\mathbf{Z}}_{(55)}$: plot over an entire period (left hand side) and zoom in (right hand side), for different values of the grounding resistance. Distance ℓ = 100 m.

As expected, the real part of the term $\dot{\mathbf{Z}}_{(55)}$ has a maximum for $\varpi = 0.5 + k$, with k = 0,1,…, that is to say for $f = 480 \cdot (0.5 + k)$ kHz, while for the same frequency the imaginary part vanishes. As for a filter, for lower values of the grounding resistance, the peak value increases while the width of the peak decreases.

In Figs 5 and 6 we show for a comparison the real and imaginary parts of the term $\dot{\mathbf{Z}}_{(11)}$ (equal to $\dot{\mathbf{Z}}_{(33)}$) due to the same symmetry reasons) and $\dot{\mathbf{Z}}_{(22)}$ of the characteristic impedance matrix in one period, that is to say the self-impedance of the power conductors.

The terms $\dot{\mathbf{Z}}_{(11)}$ and $\dot{\mathbf{Z}}_{(22)}$ exhibits the same behaviour of $\dot{\mathbf{Z}}_{(55)}$, an it is possible to verify that it is common to all the terms of characteristic impedance matrix, also outside the diagonal. Although the height of the peak is much lower than in $\dot{\mathbf{Z}}_{(55)}$, such a behaviour anyway confirms that the filtering effect produced by the periodical grounding appears on all the conductors.

This aspect can be also observed in terms of voltages.

IV. 4 Conclusion

In this paper, the analysis of multiconductor transmission lines has been discussed with particular reference to the effect of the periodical grounding.

At first, it has been shown how the periodical grounding affects the calculation of the characteristic impedance, leading to the computation of a non-linear algebraic equation. Then, an efficient method useful to solve the equations has been shown, being efficient and accurate. The proposed method is quite general and can be used for similar problems.

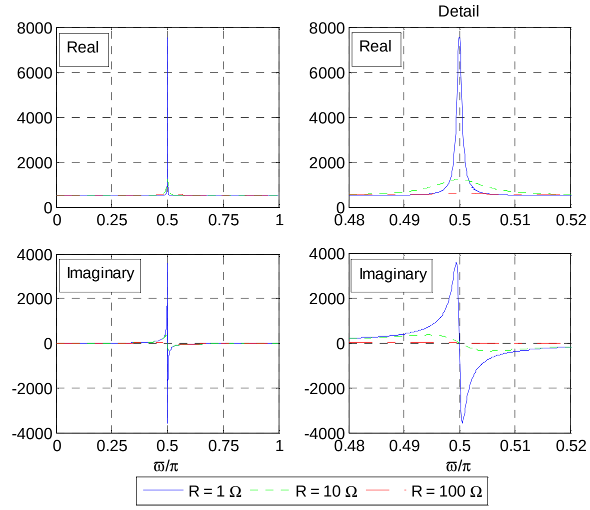

Fig. 5 Coefficient $\dot{\mathbf{Z}}_{(11)}$: plot over an entire period (left hand side) and zoom in (right hand side), for different values of the grounding resistance. Distance ℓ = 100 m.

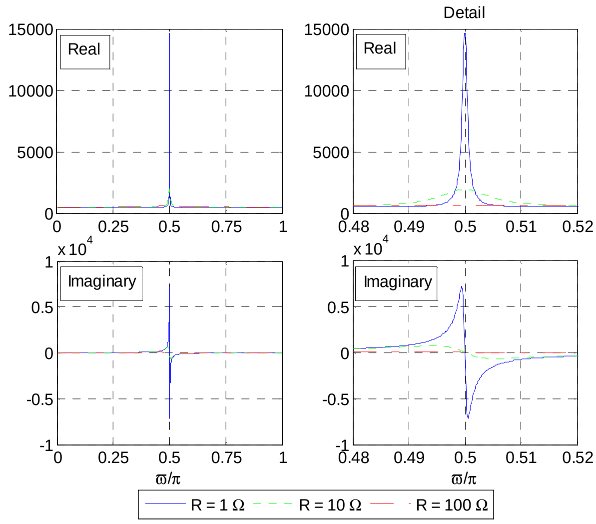

Fig. 6 Coefficient $\dot{\mathbf{Z}}_{(22)}$: plot over an entire period (left hand side) and zoom in (right hand side), for different values of the grounding resistance. Distance ℓ = 100 m.

Then, thanks to numerical results of practical cases, the filtering effect of the transmission line due to the periodical grounding has been shown. Both the frequency behavior of the transmission line and voltages in time domain have been shown and discussed.

It's worth observing that filtering effect of the line depends by the distance between the poles, while the ground resistance of the shield wire is usually designed to be as small as possible. However, in case there is flexibility in choosing the position of the (grounded) poles and/or the grounding resistance, it is possible to design these two parameters properly in order to have some wished filtering effect of the line, that may cancel or lower undesired frequencies.

REFERENCES:

[1] C. R. Paul, *Analysis of Multiconductor Transmission Lines*, Wiley, 2008.

[2] C. R. Paul, *Introduction to Electromagnetic Compatibility*, 2nd edition, Wiley, 2006.

[3] A. Andreotti, D. Assante, V.A. Rakov, L. Verolino, "Electromagnetic Coupling of Lightning to Power Lines: Transmission-Line Approximation versus Full-Wave Solution", *IEEE Trans. on EMC*, n° 53 (2), pp. 421-428, 2011.

[4] J.J. Grainger, W.D. Stevenson, *Power system analysis*, McGraw-Hill, 1994.

[5] L. Grcev, "Time- and Frequency-Dependent Lightning Surge Characteristics of Grounding Electrodes", *IEEE Trans. on Power Delivery*, vol. 24 (4), pp. 2186-2196, 2009.

[6] A. Piantini, "Lightning protection of overhead power distribution lines", in *Proc. 29th International Conference on Lightning Protection*, pp. 1-29, June 2008.

[7] S. Bittanti, A.J. Laub, J.C. Willems, *The Riccati Equation*, Springer Science & Business Media, 2012.

[8] G. Freiling, "A survey of nonsymmetric Riccati equations", *Linear Algebra and its Applications*, Vol. 351-352, pp. 243-270, 2002.

[9] V. Zeidan, "The Riccati equation for optimal control problems with mixed state-control constraints: necessity and sufficiency", *SIAM Journal on Control and Optimization*, vol. 32(5), 1297-1321, 1994.

[10] D.L Jaggard, J.C. Liu, "The matrix Riccati equation for scattering from stratified chiral spheres", *Antennas and Propagation, IEEE Transactions on*, vol. 47(7), 1201-1207, 1999.

[11] C. Cesarano, C. Fornaro, "Operational identities on generalized two-variable chebyshev polynomials", *International Journal of Pure and Applied Mathematics*, vol. 100, p. 59-74, 2014.

[12] C.. Cesarano, "Integral representations and new generating functions of Chebyshev polynomials", *Hacettepe Journal of Mathematics and Statistics*, vol. 44, p. 541-552, 2015.

[13] J. Saak, P. Benner, "Efficient numerical solution of the LQR-problem for the heat equation", *PAMM*, vol. 4(1), 648-649, 2004.

[14] C. Cesarano, "Identities and generating functions on Chebyshev polynomials", *Georgian Mathematical Journal*, vol. 19, p. 427-440, 2012.

[15] L. Placidi, "A variational approach for a nonlinear 1-dimensional second gradient continuum damage model", *Continuum Mechanics and Thermodynamics*, vol. 27, p. 623-638, 2015.

[16] L. Placidi, "A variational approach for a nonlinear one-dimensional damage-elasto-plastic second-gradient continuum model", *Continuum Mechanics and Thermodynamics*, in press.

[17] L. Placidi, U. Andreaus, A. Della Corte, T. Lekszycki, "Gedanken experiments for the determination of two-dimensional linear second gradient elasticity coefficients", *Zeitschrift fur Angewandte Mathematik und Physik*, in press.

[18] Q.S. Li, "Classes of exact solutions for buckling of multi-step non-uniform columns with an arbitrary number of cracks subjected to concentrated and distributed axial loads", *International journal of engineering science*, vol. 41(6), 569-586, 2003.

[19] U. Andreaus, B. Chiaia, L. Placidi, "Soft-impact dynamics of deformable bodies", *Continuum Mechanics and Thermodynamics*, vol. 25, p. 375-398, 2013.

[20] C. Cennamo, B. Chiaia, V. De Biagi, L. Placidi, "Monitoring and compartmentalized structures", *Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 95, p. 638-648, 2015.

[21] A.I. Egorov, L.N. Znamenskaya, "Observability of elastic oscillations of the network with distributed and concentrated parameters on free boundaries", *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, vol. 16(5), 76-81, 2010.

[22] U. Andreaus, P. Baragatti, L. Placidi, "Experimental and analytical investigations of the forced response of a cantilever beam contacting a deformable and dissipative obstacle under harmonic excitation", *International Journal of Non-Linear Mechanics*, in press.

[23] B. Chiaia, O. Kumpyak, L. Placidi, V. Maksimov, "Experimental analysis and modelling of two-way reinforced concrete slabs over different kinds of yielding supports under short-term dynamic loading", *Engineering Structures*, vol. 96, p. 88-99, 2015.

[24] U. Andreaus, L. Placidi, G. Rega, "Microcantilever dynamics in tapping mode atomic force microscopy via higher eigenmodes analysis", *Journal of Applied Physics*, vol. 113, 2013.

[25] D. Assante, "Propagation of lightning pulses along transmission lines", *Recent Researches in Electric Power and Energy Systems*, pp. 211-216, 2013.

[26] D. Assante, C. Fornaro, "Voltage Calculation in Periodically Grounded Multiconductor Transmission Lines", *International Journal of Circuits, Systems and Signal Processing*, (8), pp. 54-60, 2014.

[27] A. Andreotti, D. Assante, L. Verolino, "Characteristic Impedance of Periodically Grounded Lossless Multiconductor Transmission Lines and Time-Domain Equivalent Representation", *IEEE Transactions on EMC*, 56 (1), pp. 221-230, 2014.

[28] LZ. Lu, "Newton iterations for a non-symmetric algebraic Riccati equation", *Numer. Linear Algebra Appl.*, n. 12, pp. 191-200, 2005.

[29] D.A. Bini, B. Iannazzo, F. Poloni, "A fast Newton's method for a nonsymmetric algebraic Riccati equation", *SIAM J. Matrix Anal. Appl.*, Vol. 30, No. 1, pp. 276-290, 2008.

[30] X.X. Guo, Z.Z. Bai, "On the minimal nonnegative solution of nonsymmetric algebraic Riccati equation", *J. Comput. Math.*, n. 23, pp. 305-320, 2005.

[31] W.F. Arnold, A.J. Laub, "Generalized eigenproblem algorithms and software for algebraic Riccati equations", *Proceedings of the IEEE*, Vol. 72 (12), pp. 1746-1754, 1984.

[32] A. Laub, "A Schur method for solving algebraic Riccati equations", *IEEE Trans. on Automatic Control*, Vol. 24 (6), pp. 913-921, 1979.

[33] A. Ferrante, M. Pavon, S. Pinzoni, "Asymmetric Algebraic Riccati Equation: A homeomorphic parametrization of the set of solutions", *Linear Algebra and its Applications*, Vol. 329, pp. 137-156, 2001.

[34] C. Cesarano, C. Fornaro, L. Vazquez, "A note on a special class of Hermite polynomials", *International Journal of Pure and Applied Mathematics*, vol. 98, p. 261-273, 2015.

[35] C. Cesarano, "Generalized Chebyshev polynomials", *Hacettepe Journal of Mathematics and Statistics*, vol. 43, p. 731-740, 2014.

[36] C. Cesarano, G.M. Cennamo, L. Placidi, "Humbert Polynomials and Functions in Terms of Hermite Polynomials Towards Applications to Wave Propagation", *WSEAS Transactions on Mathematics*, vol. 13, p. 595-602, 2014.

[37] C. Cesarano, G.M. Cennamo, L. Placidi, "Operational methods for Hermite polynomials with applications", *WSEAS Transactions on Mathematics*, vol. 13, p. 925-931, 2014.

[38] G. Dattoli, S. Lorenzutta, P.E. Ricci, C. Cesarano, "On a Family of Hybrid Polynomials", *Integral Transforms and Special Functions*, vol. 15, p. 485-490, 2004.

[39] G. Dattoli, S. Lorenzutta, C. Cesarano, "Generalized Polynomials and New Families of Generating Functions", *Annali dell'Università di Ferrara. Sezione 7: Scienze Matematiche*, vol. XLVII, p. 57-61, 2001.

[40] M. Loboda, K. Lenarczyk, "Correlation between recorded CG lightning discharges and shut-downs of selected HV overhead power transmission lines in Poland", *International Coneference of Lightning Protection*, Shanghai, October 2014.

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