Comparison of two methods for determination of instantaneous state of dynamical system with LCLC circuit

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***Abstract*—** The paper deals with chosen analytical and numerical methods which make possible to estimate instantaneous state of dynamical system in any time instant. Analytical model of the LCTLC filer uses Laplace-Carson transformation with complex operator p. The method described in the paper using transient component separation makes it possible to use steady state- and transient components to generate of total time waveforms of chosen output state variables or other quantities. The steady state component is created using response of AC input voltage during the first one half-period. Worked-out simulation experiment results are compared to common numerical solution done in Matlab/Simulink environment using discrete type of dynamical model of the filter system which is modelled and analyzed by second method for determination of instantaneous state of discrete dynamical system Theoretical analysis, computer simulation, and experimental verification are given in the paper.

***Keywords*—**Circuit analysis, modelling and simulation, Laplace-Carson transform, state-space equation, non-harmonic supply, linear discrete control

I. Introduction

CONCEPTIONS of resonant converters greatly expanded into the various spheres of industry and consumer applications. Generally known switched mode power supplies as well as for power converters, to target the highest switching frequency together with the highest efficiency that is possible. If will be increased both phenomes together, simultaneously the power density increases. In order to reach the satisfactory electrical parameters and behaviour of converter, it is necessary to utilize new concepts of its main circuit [1]. In every industrial and consumer application became energy efficiency, power density and harmonic current emissions a main qualitative indicators of power semiconductor converters. LCTLC resonant converter belongs between generally known topologies used in various applications. Basically, the multi-element topologies are based on serial and parallel connection of accumulation components. Their combinations along with high frequency transformer creates varies topologies with individual specific properties [2]. Analysing of resonant systems may help to improve the design and final properties of devices.

The paper was supported from R&D operational program Centre of excellence of power electronics systems and materials for their component No OPVaV-2008/2.1/01-SORO ITMS 26220120003, and also from Slovak Grant Agency VEGA by the grant No 1/0928/15.

II. Theoretical aspects of used methods

Generally, we can use analytical and/or numerical solution for the analysing and investigating of dynamical system. Consequently, we have to create either continuous or discrete dynamical model of the system. The first one is well known in the state-space form [1],[5]

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}.\boldsymbol{x}(t) + \boldsymbol{B}.\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}(t)$ is vector of state variables, $\boldsymbol{u}(t)$ exciting vector, $\boldsymbol{A}$ and $\boldsymbol{B}$ are system matrices. Vector of output quantities is expressed as

$$\boldsymbol{y}(t) = \boldsymbol{C}.\boldsymbol{x}(t) + \boldsymbol{D}.\boldsymbol{u}(t) \tag{1a}$$

By numerical integration of (1) we obtain discrete form of dynamical model

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_{\Delta}\boldsymbol{x}_k + \boldsymbol{G}_{\Delta}\boldsymbol{u}_k \tag{2}$$

More detailed description of discrete model is given in **B.** subchapter later.

A. Analytical method using steady-state and transient components under non-harmonic supply

Using method of operator calculus Laplace or Laplace-Carson ($\mathcal{L}$-$\mathcal{C}$), respectively, transform [6],[7]

$$U_1(p) = \frac{U_1^{T/2}(p)}{1+\mathrm{e}^{-pT/2}} \tag{3}$$

where operator voltage during one half-period is in Laplace-Carson

$$U_1^{T/2}(p) = U(1-\mathrm{e}^{-pT_s}) \quad \text{or} \quad u_1^{\mathrm{T}/2}(\mathrm{s}) = U\frac{(1-\mathrm{e}^{-sT_s})}{s} \tag{4}$$

using Laplace transform.

Generally, the current response on such a voltage consists of steady state and transient components [8]

$$F(p) = U\frac{(1-\mathrm{e}^{-pT_s})}{1+\mathrm{e}^{-pT/2}}\frac{A(p)}{B(p)} \tag{5}$$

The response of any state variable during one half-period of transient phenomenon can be obtain and evaluated just by roots of polynomial of denominator $B(p)$; and $T_s = T/2$

$$F^{T/2}(p) = U\left(1 - \mathrm{e}^{-pT/2}\right)\frac{A(p)}{B(p)} \tag{6}$$

Transient component during one half-period

$$F_{trans}^{T/2}(p) = U\frac{\left(1 - \mathrm{e}^{-pT/2}\right)}{1 + \mathrm{e}^{-pT/2}}\frac{A(p)}{B(p)} \tag{7}$$

Steady state component during one half-period can be simply obtain by their subtracting

$$F_{steady}^{T/2}(p) = F^{T/2}(p) - F_{trans}^{T/2}(p) \tag{8}$$

Now, after inverse L-C transformation the steady state component for one half-period

$$f_{steady}^{T/2}(t) = f^{T/2}(t) - f_{trans}^{T/2}(t) \tag{9}$$

And

$$f(t) = f_{trans}(t) \pm \left[f^{T/2}(t) - f_{trans}^{T/2}(t)\right] \tag{10}$$

Such methodology makes it possible to separate both components similarly as in the DC linear circuit [5],[8].

B. Method for determination of instantaneous state of discrete dynamical system

As mentioned above the numerical integration using, Euler-, Runge-Kutta-, Taylor expansion methods of (1) are giving [3],[5],[9]

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}_\Delta \boldsymbol{x}_k + \boldsymbol{G}_\Delta \boldsymbol{u}_k \tag{11}$$

where $\boldsymbol{x}(k)$ is vector of state variables in discrete form, $\boldsymbol{u}(k)$ exciting vector, k is order of calculation, $\boldsymbol{F}$ and $\boldsymbol{G}$ are system matrices discretized by integration step Δ.
Determination of $\boldsymbol{F}_\Delta$, $\boldsymbol{G}_\Delta$ matrices is possible to provide by using of:

- analytical method (suitable for low order system);
- numerical methods depending on their type:
  Euler direct explicit method ($h \equiv \Delta$):

$$\boldsymbol{x}(h) = (\boldsymbol{1} + h.\boldsymbol{A})\boldsymbol{x}_0 + (h.\boldsymbol{B})\boldsymbol{u}_0$$

$$\rightarrow\ \boldsymbol{F}_\Delta = \boldsymbol{1} + h.\boldsymbol{A}, \qquad \boldsymbol{G}_\Delta = h.\boldsymbol{B}. \tag{11a}$$

Euler indirect implicit method:

$$\boldsymbol{x}(h) = \mathrm{inv}(\boldsymbol{1} - h.\boldsymbol{A})\boldsymbol{x}_0 + \mathrm{inv}(\boldsymbol{1} - h.\boldsymbol{A})h.\boldsymbol{B}.\boldsymbol{u}_0$$

$$\rightarrow\ \boldsymbol{F}_\Delta = \mathrm{inv}(\boldsymbol{1} - h.\boldsymbol{A}), \boldsymbol{G}_\Delta = \mathrm{inv}(\boldsymbol{1} - h.\boldsymbol{A})h.\boldsymbol{B}.$$

Taylor expansion:

$$\boldsymbol{x}(h) = e^{\boldsymbol{A}.h} + \int_0^h e^{\boldsymbol{A}.(h-t)}\boldsymbol{B}.\boldsymbol{u}_0 \mathrm{d}t \ \dots$$

$$= \sum_{i=0}^{n}\frac{(\boldsymbol{A}.h)^i}{i!}\boldsymbol{x}_0 + h\sum_{i=0}^{n}\frac{(\boldsymbol{A}.h)^i}{(i+1)!}\boldsymbol{B}.\boldsymbol{u}_0$$

$$\rightarrow\ \boldsymbol{F}_\Delta = \sum_{i=0}^{n}\frac{(\boldsymbol{A}.h)^i}{i!}, \boldsymbol{G}_\Delta = h\sum_{i=0}^{n}\frac{(\boldsymbol{A}.h)^i}{(i+1)!}\boldsymbol{B}.$$

- Z-transformation method;
- method of experiment when state variables $\boldsymbol{x}(h)$ and $\boldsymbol{F}_\Delta$, $\boldsymbol{G}_\Delta$ can be obtain at discrete time $\Delta \equiv h$.

Then, by gradual sovereign and generalization /mathematical induction/ we get [xx], [xx]:

$$\boldsymbol{x}_1 = \boldsymbol{F}_\Delta{}^1\boldsymbol{x}_0 + \boldsymbol{F}_\Delta{}^0\boldsymbol{G}_\Delta u_0 = \boldsymbol{G}_\Delta u_0 \tag{12a}$$

$$\boldsymbol{x}_2 = \boldsymbol{F}_\Delta{}^2\boldsymbol{x}_0 + \boldsymbol{F}_\Delta{}^1\boldsymbol{G}_\Delta u_0 + \boldsymbol{F}_\Delta{}^0\boldsymbol{G}_\Delta u_1$$

$$\boldsymbol{x}_3 = \boldsymbol{F}_\Delta{}^3\boldsymbol{x}_0 + \boldsymbol{F}_\Delta{}^2\boldsymbol{G}_\Delta u_0 + \boldsymbol{F}_\Delta{}^1\boldsymbol{G}_\Delta u_1 + \boldsymbol{F}_\Delta{}^0\boldsymbol{G}_\Delta u_2$$

$$\boldsymbol{x}_k = \boldsymbol{F}_\Delta{}^k\boldsymbol{x}_0 + \boldsymbol{F}_\Delta{}^{k-1}\boldsymbol{G}_\Delta u_0 + \boldsymbol{F}_\Delta{}^{k-2}\boldsymbol{G}_\Delta u_1 + + \boldsymbol{F}_\Delta{}^{k-(k-1)}\boldsymbol{G}_\Delta u_{k-1} + \boldsymbol{F}_\Delta{}^{k-k}\boldsymbol{G}_\Delta u_k \tag{12b}$$

Thus

$$\boldsymbol{x}_k = (\boldsymbol{F}_\Delta)^k\boldsymbol{x}_0 + \boldsymbol{G}_\Delta\sum_{l=k-1}^{0}\boldsymbol{F}_\Delta{}^l\left\{\boldsymbol{u}_{k-(l+1)}\right\} \tag{13}$$

III. Modeling and simulation of LCLC filter circuit

Using A method of steady-state and transient components

Considering, the basic scheme of LCTLC inverter *Fig. 1* and equivalent scheme of LCLC filter circuit, *Fig. 2* [12].

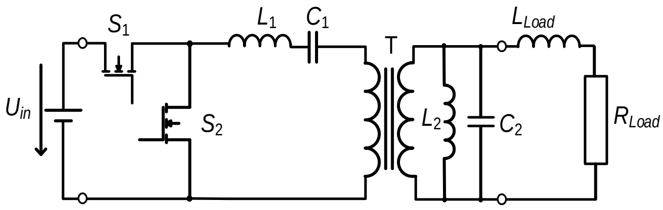


Fig. 1 Basic schematic of LCTLC inverter

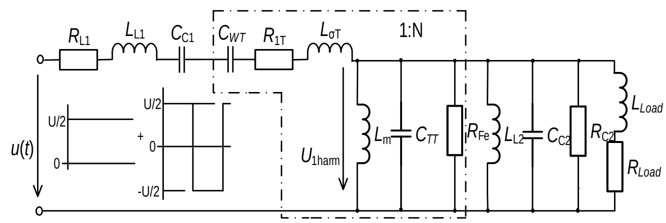


Fig. 2 Equivalent scheme of LCTLC inverter with HF transformer

Under *resistive-inductive* load the impedance of series and parallel part of the LCTLC filter is defined by the following equations

$$|Z_N| = \sqrt{R_{sload}{}^2 + (\omega L_{sload})^2} = \frac{U_{outN}^2}{P_{outN}},$$

$$Z_1(\omega) = R_1 + \mathrm{j}\left(\omega L_1 - \frac{1}{\omega C_1}\right) = \tag{14a}$$

$$= \frac{R_1}{|Z_N|}|Z_N| + \mathrm{j}|Z_N|q_{N1}\left(f_{rel} - \frac{1}{f_{rel}}\right) =$$

$$= r_{1N}|Z_N| + \mathrm{j}|Z_N|q_{N1}\left(f_{rel} - \frac{1}{f_{rel}}\right) =$$

$$= |Z_N|\left[r_1 + \mathrm{j}q_{N1}\left(f_{rel} - \frac{1}{f_{rel}}\right)\right]$$

where

$$R_1 = R_{L1} + R_{1T} + R_{C1};\ L_1 = L_{L1} + L_{\sigma T};\ \frac{1}{C_1} = \frac{1}{C_{C1}} + \frac{1}{C_{WT}}$$

and $R_1$is resistance of series part of the filter, and its relative value $r_{1N} = \frac{R_1}{|Z_N|}$; R_{L1} - resistance of series filter coil; R_{1T} - resistance of primary winding of the transformer; R_{C1} - resistance of series filter capacitor; L_{L1} – inductance of series filter coil; $L_{\sigma T}$ – leakage inductance of the transformer; C_{C1} - capacitance of series capacitor; C_{WT} - capacitance of between windings of the transformer; $f_{rel} = \frac{\omega}{\omega_{res}}$ - is relative frequency; Z_N – nominal impedance.

Similarly for nominal admittance for parallel *resistive-inductive* load

$$|Y_N| = \sqrt{\left(\frac{1}{R_{pload}}\right)^2 + \left(\frac{1}{\omega L_{pload}}\right)^2} = \frac{P_{outN}}{U_{outN}^2}$$

and similarly for $\frac{|Z_2(\omega)|}{|Z_{\mathrm{N}}(\omega)|}$ with little help of $Y_2(\omega)$

$$Y_2(\omega) = \frac{1}{R_2} + \frac{1}{R_{load}} + j\left(\omega C_2 - \frac{1}{\omega L_2}\right) =$$

$$= \frac{1}{r_2}|Y_N| + \frac{1}{r}|Y_N| + j|Y_N|q_{N2}\left(f_{rel} - \frac{1}{f_{rel}}\right) =$$

$$= |Y_N|\left[\left(\frac{1}{r_2} + \frac{1}{r}\right) + jq_{N2}\left(f_{rel} - \frac{1}{f_{rel}}\right)\right] \qquad (14b)$$

where

$$\frac{1}{R_2} = \frac{1}{R_{Fe}} + \frac{1}{R_{L2}} + \frac{1}{R_{C2}};\ \ C_2 = C_{C2} + C_{TT};\ \ \frac{1}{L_2} = \frac{1}{L_m} + \frac{1}{L_{L2}}$$

$R_2$is resistance of paralel part of the filter, and its relative value $r_{2N} = \frac{1}{R_2|Y_N|}$; R_{Fe} – resistance of transformer ferromagnetic; R_{L2} – resistance of parallel filter coil; R_{C2} - resistance of parallel filter capacitor; C_{C2} - capacitance of parallel capacitor; C_{TT} - capacitance of between turns of the windings of the transformer and $\frac{1}{r_{loadN}} = \frac{1}{R_{load}|Y_N|}$ – relative value of load conductance.

At the beginning will be defined simple resistive load. So, for pure resistive R-load, by application of Kirchhoff laws we get

$$u_1(t) - R_1 i_{L1}(t) - L_1\frac{\mathrm{d}i_{L1}(t)}{\mathrm{d}t} - u_{C1}(t) - u_{C2}(t) = 0 \quad (15a)$$

$$i_{L1}(t) - i_{L2}(t) - C_2\frac{\mathrm{d}u_{C2}(t)}{\mathrm{d}t} - \frac{1}{R_1}u_{C2}(t) - \frac{1}{R_l}u_{C2}(t) = = 0 \qquad (15b)$$

Then, operator calculus expression using L-C transformation under zero initial condition

$$U_1(p) - I_{L1}(p)\left[R_1 + pL_1 + \frac{1}{pC_1}\right] - U_{C2}(p) = 0 \quad (16a)$$

$$I_{L1}(p) - U_{C2}(p)\left[\left(\frac{1}{R_2} + \frac{1}{R_l}\right) + \frac{1}{pL_2} + pC_2\right] \qquad (16b)$$

Since $U_{C2}(p) \equiv U_2(p)$ and $\left[R_1 + pL_1 + \frac{1}{pC_1}\right] = Z_1(p)$, $\left[\left(\frac{1}{R_2} + \frac{1}{R_l}\right) + \frac{1}{pL_2} + pC_2\right] = Y_2(p)$ it can be written

$$U_2(p) = U_1(p)\frac{1}{1 + Z_1(p)Y_2(p)} = U_1(p)F(p) \qquad (17)$$

where $\frac{1}{1+Z_1(p)Y_2(p)}$ is the operator transfer function of the system.

Introducing

$$L_1 = L_2 = L;\ C_1 = C_2 = C;\ {}^1/_{LC} = \omega_r^2;\ r = {}^{R_1}/_{\omega_r L_1};\ g = {}^{R_2}/_{1/\omega_r C_2} = \omega_r R_2 C_2;\ z = 1/R_l$$

as designed by [13] (p.u.) values, then

$$U_2(p) =$$

$$= U_1(p)\frac{\omega_r^2 p^2 \ldots}{p^4 + [r + (g+z)]\omega_r p^3 + [3 + r(g+z)]\omega_r^2 p^2 + } \ldots$$

$$\ldots \frac{\ldots}{+ [r + (g+z)]\omega_r^3 p + \omega_r^4} =$$

$$= U_1(p)\frac{\theta p^2}{\alpha p^4 + \beta p^3 + \gamma p^2 + \delta p^1 + \varepsilon p^0} =$$

$$= U\left(1 - e^{-pT/2}\right)\frac{A(p)}{B(p)} \qquad (18)$$

where

$\alpha = \omega_r^0$; $\beta = [r + (g+z)]\omega_r$; $\gamma = [3 + r(g+z)]\omega_r^2$; $\delta = [r + (g+z)]\omega_r^3$; $\varepsilon = \omega_r^4$; $\theta = \omega_r^2$.

Thus

$$\frac{\theta p^2}{\alpha p^4 + \beta p^3 + \gamma p^2 + \delta p^1 + \varepsilon p^0} = \frac{U_2(p)}{U_1(p)} \qquad (19)$$

is the voltage transfer function in Laplace operator form.

Supposing complex conjugated roots of denominator polynomial in form (let $\omega_r = 1$)

$$p_{1,2} = (-a \pm \omega.\mathrm{i}) \ \text{ and } \ p_{3,4} = (-x \pm \omega.\mathrm{i}) \qquad (20)$$

For $r = g = 0.05$; $z = 1 >>$ control vector $c = [1;\ 1.1;\ 3.0525;\ 1.1;\ 1]$ (real elements, nominal load) the roots are

$$p_{1,2} = (-0.3887 \pm 1.5027\mathrm{i}) \ \text{ and }$$
$$p_{3,4} = (-0.1613 \pm 0.6238\mathrm{i}) \qquad (20a)$$

For $r = g = 0$; $\mathrm{z} = 0 \gg$

$\gg$ control vector $\mathrm{c} = [1;\ 0.0;\ 3.00;\ 0.0;\ 1]$ (ideal elements, zero load) the roots are

$$p_{1,2} = (0.0000 \pm 1.6180\mathrm{i}) \ \text{ and }$$
$$p_{3,4} = (-0.0000 \pm 0.6180\mathrm{i}) \qquad (20b)$$

Based on the transfer function (obtained from operator form eq.(19) the bode diagram in Matlab environment has been created, Fig. 3.

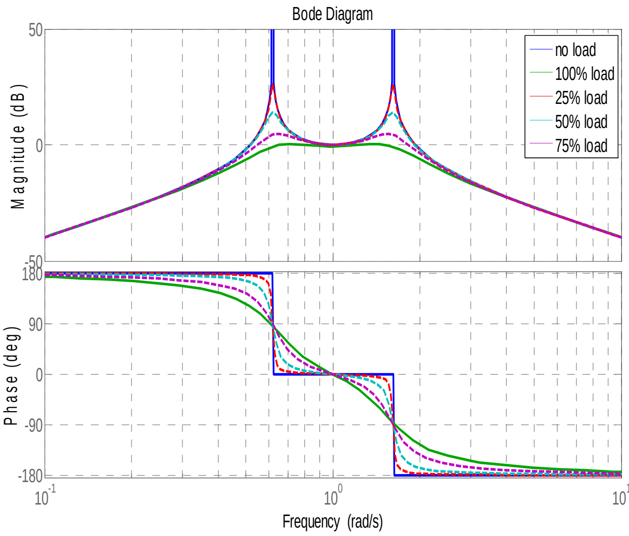


Fig. 3 Bode diagram of LCLC resonant filter

It is important that $\omega_{1,2}$ and $\omega_{3,4}$ are frequencies when input impedance features zero values, and voltage transfer by infinite values.

The inverse Laplace transform can be worked out [1],[8]

$$u_{0-T/2}(t) = \sum_{k=1}^{N} u_1(t)\frac{A(p_k)}{p_k B'(p_k)}\mathrm{e}^{p_k t} \tag{21}$$

since ${A(0)}/_{B(0)} = 0$.

$$u_{trans}(t) = \sum_{k=1}^{N} U\frac{1-\mathrm{e}^{-p_k T/2}}{1+\mathrm{e}^{-p_k T/2}}\frac{A(p_k)}{p_k B'(p_k)}\mathrm{e}^{p_k t} \tag{22}$$

Then

$$u_{steady}(t) = u_{0-T/2}(t) - u_{trans}(t). \tag{23}$$

Regarding the complex conjugated roots the members of

$$\frac{A(p_k)}{p_k B'(p_k)} \text{ will give } |R_k|\mathrm{e}^{j\rho_k} \tag{24}$$

and, similarly

$$\frac{1-\mathrm{e}^{-p_k T/2}}{1+\mathrm{e}^{-p_k T/2}} \text{ will give } |T_k|\mathrm{e}^{j\tau_k} \tag{25}$$

So

$$\frac{A(p_1)}{p_1 B'(p_1)} = |R_1|\mathrm{e}^{j\rho_1}, \qquad \frac{A(p_2)}{p_2 B'(p_2)} = |R_2|\mathrm{e}^{j\rho_2}, \quad \text{etc.} \tag{26}$$

It is possible to show

$$|R_1|e^{j\rho_1}e^{(-a+j.b)\omega_r} + |R_2|e^{j\rho_2}e^{(-a-j.b)\omega_r} = = 2|R_1|e^{-a\omega t}\cos(\rho_1 + b\omega_r t) \tag{27}$$

and

$$|R_3|\mathrm{e}^{j\rho_3}\mathrm{e}^{(-x+j.y)\omega_r} + |R_4|\mathrm{e}^{j\rho_4}\mathrm{e}^{(-x-j.y)\omega_r} = = 2|R_3|\mathrm{e}^{x\omega t}\cos(\rho_3 + y\omega_r t) \tag{28}$$

Similarly

$$\frac{1-\mathrm{e}^{-\frac{p_1 T}{2}}}{1+\mathrm{e}^{-\frac{p_1 T}{2}}} = |T_1|e^{j\tau_1}, \qquad \frac{1-\mathrm{e}^{-p_2 T/2}}{1+\mathrm{e}^{-p_2 T/2}} = |T_2|\mathrm{e}^{j\tau_2}, \quad \text{etc.} \tag{29}$$

And also

$$|T_1|\mathrm{e}^{j\tau_1}e^{(-a+j.b)\omega_r} + |T_2|\mathrm{e}^{j\tau_2}e^{(-a-j.b)\omega_r} = = 2|T_1|\mathrm{e}^{-a\omega t}\cos(\tau_1 + b\omega_r t) \tag{30}$$

$$|T_3|\mathrm{e}^{j\tau_3}e^{(-x+j.y)\omega_r} + |T_4|\mathrm{e}^{j\tau_4}e^{(-x-j.y)\omega_r} = = 2|T_3|\mathrm{e}^{-x\omega t}\cos(\tau_3 + y\omega_r t) \tag{31}$$

Then, for output voltage

$$u_2^{T/2}(t) = 2U\{|R_1|\mathrm{e}^{-a\omega t}\cos(\rho_1 + b\omega_r t) + |R_3|\mathrm{e}^{x\omega t}\cos(\rho_3 + y\omega_r t)\} \tag{32}$$

$$u_{trans}(t) = 2U\{|R_1||T_1|\mathrm{e}^{-a\omega t}\cos(\tau_1 + \rho_1 + b\omega_r t) + |R_3||T_3|\mathrm{e}^{-x\omega t}\cos(\tau_3 + \rho_3 + b\omega_r t)\} \tag{33}$$

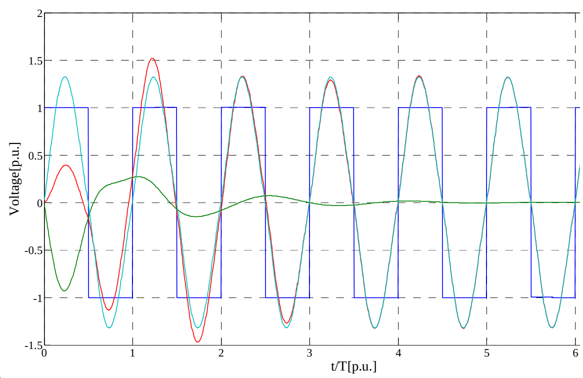
Steady state component for one half-period

$$u_{steady}^{T/2}(t) = u_2^{T/2}(t) - u_{trans}^{T/2}(t) \tag{34}$$

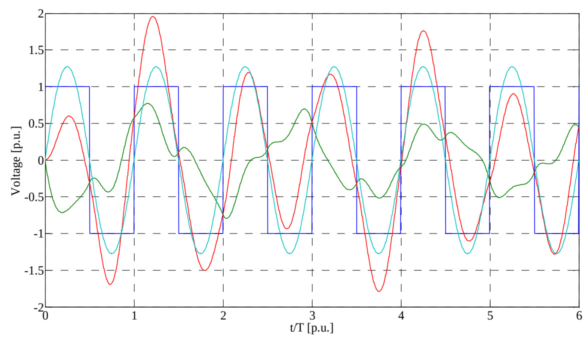
And

$$u_2(t) = u_{trans}(t) \pm \left[u_2^{T/2}(t) - u_{trans}^{T/2}(t)\right] \tag{35}$$

Results of simulation are shown in Fig. 4a,b.



a)



b)

Fig. 4 Time waveforms of the model for real parameters a) Z_{load}=100 %(nominal value) and b) ideal parasite-less with Z_{load}=0 %

Important, when ideal elements, zero load i.e. $r = g = 0$; $z = 0$ >> c = [1; 0.0; 3.00; 0.0; 1] then $\mathrm{e}^{-a\omega t} = 0$; $\mathrm{e}^{-x\omega t} = 0$ thus $u_{trans}(t)$ will never be zero.

Using B method for determination of instantaneous state of discrete dynamical system

For calculation $\boldsymbol{x}_k = (\boldsymbol{F}_\Delta)^k \boldsymbol{x}_0 + \boldsymbol{G}_\Delta \sum_{l=k-1}^{0} \boldsymbol{F}_\Delta{}^l \{\boldsymbol{u}_{k-(l+1)}\}$ at any time instant $\tilde{t} = k.\Delta$ we need, at first, to know $\boldsymbol{F}_\Delta, \boldsymbol{G}_\Delta$ and $\boldsymbol{x}_0, \boldsymbol{u}_k$.

Then continuous dynamic state space model yields the for following system equations [19]

$$\frac{\mathrm{d}i_{L1}(t)}{\mathrm{d}t} = -\frac{R_1}{L_1} i_{L1}(t) - \frac{1}{L_1} u_{C1}(t) - \frac{1}{L_1} u_{C2}(t) + \frac{1}{L_1} u(t) \tag{36a}$$

$$\frac{\mathrm{d}i_{L2}(t)}{\mathrm{d}t} = \frac{1}{L_2} u_{C2}(t) \tag{36b}$$

$$\frac{\mathrm{d}u_{C1}(t)}{\mathrm{d}t} = \frac{1}{C_1} i_{L1}(t) \tag{36c}$$

$$\frac{\mathrm{d}u_{C2}(t)}{\mathrm{d}t} = \frac{1}{C_1} i_{L1}(t) - \frac{1}{C_2} i_{L2}(t) - \left(\frac{1}{R_2} + \frac{1}{R_l}\right)\frac{1}{C_2} u_{C2}(t). \tag{36d}$$

Thus for parameters designed by [13]

$$L_1 = L_2 = L = 0.1, C_1 = C_2 = C = 5 \text{ x } 10^{-3}, R_l = 1, r \equiv R_1 = 0.01, g \equiv \frac{1}{R_2} = 0.01$$

and elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices:

$$a_{11} = -\frac{r}{L} = -0.1;\ a_{12} = 0;\ a_{13} = -\frac{1}{L} = -10;\ a_{14} = = -\frac{1}{L} = -10;$$

$$a_{21} = a_{22} = a_{23} = 0; a_{24} = \frac{1}{L} = 10;$$

$$a_{31} = \frac{1}{C} = 200;\ \ a_{32} = a_{33} = a_{34} = 0;$$

$$a_{41} = \frac{1}{C_1}; a_{42} = -\frac{1}{C_2}; a_{43} = 0;\ a_{44} = -\left(g + \frac{1}{R_l}\right)\frac{1}{C} = -\frac{1}{RC} = -202\,;$$

$$b_{11} = \frac{1}{L} = 10;\ b_{12} \div b_{44} = 0.$$

The $\boldsymbol{F}_\Delta$ and $\boldsymbol{G}_\Delta$ will be

$$\boldsymbol{F}_\Delta = [\boldsymbol{E} - \Delta.\boldsymbol{A}];\ \boldsymbol{G}_\Delta = \Delta.\boldsymbol{B} \tag{37}$$

and $\boldsymbol{u}_k = [u_k; 0; 0; 0]^T$ where $u_k = \sqrt{2} U_{DC} \sin\left[\mathrm{fix}\left(4\frac{\Delta}{T}k\right)\frac{\pi}{2} + \frac{\pi}{4}\right]$, $\boldsymbol{E}$ is unit matrix and $\Delta = 10^{-4}$.

Then

$$\boldsymbol{F}_\Delta = \begin{bmatrix} 1.0000 & 0 & 0.0010 & 0.0010 \\ 0 & 1.0000 & 0 & -0.0010 \\ -0.0200 & 0 & 1.0000 & 0 \\ -0.0200 & 0.0200 & 0 & 1.0201 \end{bmatrix}$$

$$\boldsymbol{G}_\Delta = \begin{bmatrix} 0.0010 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

If k goes from 0 to 2 160 (*6 T*) the result will be as in Fig. 5.

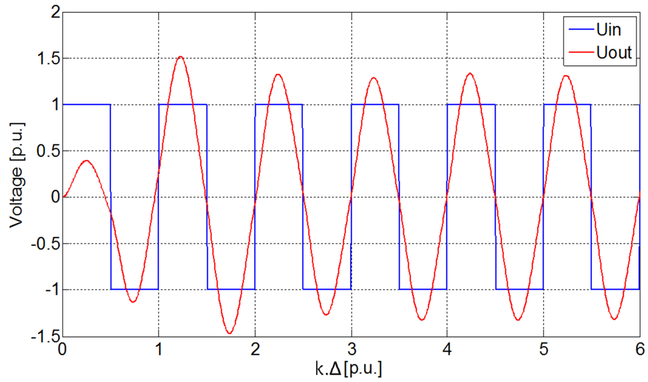


Fig. 5 Time waveforms of the discrete dynamic state space model

By comparison of voltage waveforms in Fig. 5 and Fig. 4a we can conclude that are in very good agreement.

Finally, there is shown the experimental verification at steady state.

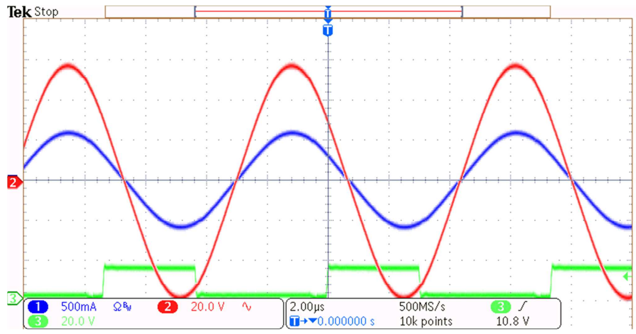


Fig. 6 Experimental time waveforms of the output quantities(current - blue, voltage – red) at steady state under nominal loading

By comparison of voltage waveform to those in Fig. 5 and Fig. 4a at steady state we can conclude again that is in good agreement.

IV. Conclusion

The method of steady-state and transient components (A), and method of determination of instantaneous state of discrete dynamical system (B) have been introduced. Both methods make possible to calculate the system response to input non-harmonic voltage signal at any continuous or discrete time instant (t, or $k.\Delta$, respectively) regardless if input voltage is continuous or discrete impulse one. Comparison of simulation results of both methods is possible from Fig. 4a and Fig. 5 – the results are practically identical. Main difference between methods can be seen in used approaches. Since the method A deals with evaluation of the roots of denominator of transfer voltage function, the method (B) works directly - without the evaluation. If we need to know the poles due to behaviour of the discrete impulse system, so the inverse Z-transform using residua lemma should be used [10], [14]. The mentioned in the paper methods make it possible to solve any dynamical state of the system such as step changes of the load (switch on/off), step changes of the switching frequency (+/-), short circuiting of the filter circuit, etc. Analyzing of resonant filter system may help to improve the design and final properties of devices.

ACKNOWLEDGMENT

The paper was supported from R&D operational program Centre of excellence of power electronics systems and materials for their component No OPVaV-2008/2.1/01-SORO ITMS 26220120003, and also from Slovak Grant Agency VEGA by the grant No 1/0928/15.

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