Adaptive Waveform Selection Based on Relative Value Iteration

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***Abstract*—**Traditional radar lacks flexibility, and modern science and technology promote the progress of radar technology. Modern intelligent radar should transmit different waveforms in different working conditions. In this paper, we set up radar detection model based on range-Doppler resolution cell and make analysis of matched filtering. We introduce reward theory and establish stochastic dynamic programming model of waveform selection. In order to overcome the shortcoming of backward recursion method, we propose a relative value iteration method. In simulation part, we compare fixed waveform, relative value iteration method and the optimal waveform selection scheme. Simulation results show that the method we proposed has lower tracking errors. Meanwhile, relative value iteration method approaches the optimal waveform selection scheme. Finally, a summary of the full paper is presented.

***Keywords*—**intelligent radar, adaptive waveform selection, relative value iteration.

I. Introduction

THE history of radar can be traced back to modern electromagnetic theory. Radar was first used in military, and nowadays the military is still driving the development of radar. Radar now enjoys wide range of applications. On the other hand, traditional radar systems are lack of flexibility to the complex environment. With the rising complexity of electromagnetic environment, the interference of communications is becoming more and more serious. Adaptive waveform selection is very necessary so that modern radar can be more flexible to transmit waveform in different working conditions.

The core problem of adaptive waveform selection is adaptive model and its solution method. Many researchers have done lots of work in the related field.

In 2006, Simon Haykin proposed cognitive radar. As a new framework and advanced form of radar system, it may intelligently interrogate a propagation channel with all available knowledge [1]. The problem of adaptive waveform selection is studied for target tracking by a multistatic radar system. It is formed by a dedicated transmitter and multiple receivers. In order to minimize the tracking mean square error, the authors minimize the tracking cost function, which is obtained using the Cramer-Rao lower bound of radar estimates [2]. The authors focus on the latter line of research. A survey and new results are presented, which is pertained to adaptation for waveform selection. Especially, the authors propose a golden standard for adaptive methods which is satisfied for a simple adaptive law [3]. Recently the cognitive radar is becoming an area of vigorous research. The performance of cognitive radar is superior to the traditional fixed waveform radar. A novel wind driven optimization technique-based waveform selection scheme is proposed [4]. In a multistatic radar system, the authors propose a method which performs adaptive waveform selection and joint target tracking at all the receivers. Diffusion Kalman filter is utilized and mean-square tracking error is minimized [5]. In order to strive for tracking error minimization for cognitive radar, a dynamic waveform selection method is presented, which is based on measurement extraction cell and resolution cell [6]. A method is proposed to reduce the effect of these signals and show a procedure to obtain the minimum number of samples that is needed to neglect the effect of the brain activity. The method requires the obtention of the optimum waveform for the applied current to minimize the variance of the electric potential estimation [7]. The authors describe specific digital beam-forming methods and prototype of the data acquisition system. Moreover, the detailed description of the beam-forming and reception algorithms explains the possible estimation criteria for given system [8]. In order to improve the range and velocity resolution, a new method for adaptive design of orthogonal frequency-hopping waveforms is proposed [9]. In order to reduce the minimum mean square error of estimation, a suboptimal wide sense stationary-uncorrelated-based waveform design scattering target impulse response model is proposed [10]. The authors research on radar waveform selection methods for the track of accelerating targets. An interacting multiple model framework is utilized to minimize cost functions [11]. However, the above methods lack flexibility and difficult to adapt to the changing working conditions.

In this paper, after matched filtering analysis, adaptive waveform selection problem is viewed as stochastic dynamic programming. Then relative value iteration method is proposed to solve the problem.

II. Radar Detection Model And Matched Filtering

For a radar measuring a target, Doppler frequency and range are very essential. Besides, two orthogonal space angles are also important.

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A radar resolution cell can be envisioned containing a certain four-dimensional hypervolume. We assume that different targets will fall in different resolution cells and different measurements are independent. So when each target occupying a resolution disjoint cell, the radar can make no-interference measurements.

For example, if a radar trys to measure targets resolved in Doppler frequency, matched filters is necessary. Radar can separate targets resolved in the range coordinate.

However, if a radar wants to measure targets in angle coordinates, that is almost impossible. According to the theory of radar detection, we usually make a independent consideration to angle resolution. So their resolution properties are different.

So a radar resolution cell can be envisioned. Defining resolution utlizes two-dimensional hypervolume. Doppler frequency is utlized to make a distinction between moving targets and stationary targets. Fig. 1 is sketch map of Doppler and range.

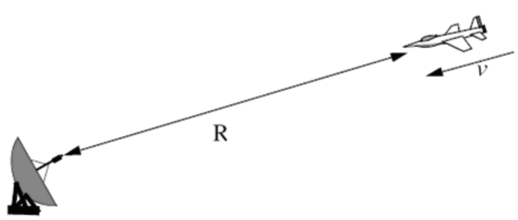

Fig. 1. Sketch map of Doppler and range

In a new model of waveform selection, range-Doppler-based resolution cell is defined. Range resolution, which is denoted as ΔR, can describe a radar's ability to find targets.

We usually design the opertation of radar between a maximum range $R_{\max}$, and minimum range $R_{\min}$. ΔR is the minimum resolution distance.

$$N = (R_{\max} - R_{\min})/\Delta R \quad (1)$$

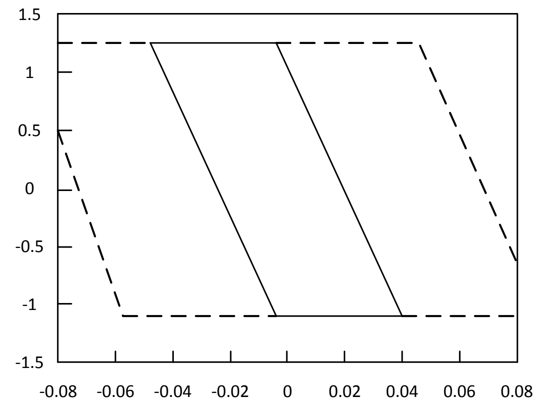

Fig. 2. Resolution cell and corresponding parallelogram

We usually design waveform to achieve the performance of either good range resolution or good Doppler. However, the two performances are contradictory.

Considering the adaptive waveform selection problem, defining a cost function is very necessary. A cost function describes the cost of selecting a transmitted waveform to optimize transmission. Fig. 2 is resolution cell and corresponding parallelogram.

In range-Doppler space, the area is divided into several grids. The cells in Doppler are indexed by $\upsilon = 1,...,M$, while the cells in range are indexed by $\tau = 1,...,N$. A number of targets are possible existed. So

$$C_{NM}^{0} + C_{NM}^{1} + C_{NM}^{2} + ... + C_{NM}^{NM-1} + C_{NM}^{NM} = 2^{NM} \quad (2)$$

Radar scene is defined as 2^{NM}. Related parameters are space χ, model state $X_k = x$ where $x \in \chi$, the measurement variable Y_k, and the control variable u_k.

In other words, u_k represents waveform the radar will transmit.

Let $a_{x'x}$ be the state transition probability

$$a_{x'x} = P(x_{k+1} = x' \mid x_k = x) \quad (3)$$

Let $b_{x'x}$ be the measurement probability

$$b_{x'x}(u_k) = P(Y_{k+1} = x' \mid X_k = x, u_k) \quad (4)$$

$s(k)$ is the transmit baseband signal, while $r(k)$ is the receive baseband signal. ν_0 denotes frequency shift. According to matched filtering theory, the impulse response can be expressed as

$$h(k) = s^{*}(-k)e^{j2\pi\nu_0 k} \quad (5)$$

The output can be expressed as

$$x(k) = \int s^{*}(\lambda - k)e^{-j2\pi\nu_0(\lambda - k)}r(\lambda)d\lambda \quad (6)$$

We consider two situations.

When there is no target exists, the random variable $x(\tau_0)$ is complex Gaussian, whose mean is zero mean and variance is also zero. Assume ξ denotes energy of the transmit pulse, so

$$\sigma_0^2 = E\left\{x(\tau_0)x^*(\tau_0)\right\} = 2N_0\xi \tag{7}$$

When the target exists, the random variable has zero mean. The variance can be expressed as

$$\begin{aligned}\sigma_1^2 &= E\left\{x^*(\tau_0)x(\tau_0)\right\} \\ &= \sigma_0^2(1+2\frac{\sigma_A^2\xi^2}{\sigma_0^2}A(\tau_0-\tau,\nu_0-\nu))\end{aligned} \tag{8}$$

where $A(\tau,\nu)$ is ambiguity function.

In the circumstance that a target appears in cell (τ,υ), suppose its true position obeys uniform distribution

$$P_d = \frac{1}{|A|}\int_{(\tau_a,\upsilon_a\in A)} e^{-\frac{D}{2\sigma_0^2(1+\frac{2\sigma_A^2\xi^2}{\sigma_0^2}A(\tau_0-\tau,\nu_0-\nu))}} d\tau_a d\upsilon_a \tag{9}$$

where A is the resolution cell centered on (τ,υ).

III. Solution Method of Adaptive Waveform Selection

Different targets will fall in different resolution cells, so different measurements are independent and do not interfere with each other. When each target occupying a resolution disjoint cell, the radar can make no-interference measurements.

Assume π is a waveforms sequence. We can use π for the decision-making process. For a given beam, define $\pi = \{u_0, u_1, ..., u_K\}$. Different waveforms sequence can be acquired in different working conditions. Suppose γ denotes discount factor. So

$$V_k(X_k) = E[\sum_{k=0}^{K}\gamma^k R_k(X_k,u_k)] \tag{10}$$

where $R_k(X_k,u_k)$ is the reward obtained. Then our goal is to seek out the optimal sequence π^*

$$V^*(X_k) = \max_{\pi} E[\sum_{k=0}^{K}\gamma^k R_k(X_k,u_k)] \tag{11}$$

In fact, knowledge of the actual state is usually unavailable. According to optimization principle, we can use conditional density of the state instead of X_k. The solution of (11) has the same π^* with the following formula

$$V^*(\mathbf{p}_0) = \max_{\pi} E[\sum_{k=0}^{K}\gamma^k R_k(\mathbf{p}_k,u_k)] \tag{12}$$

In the above formula, $\mathbf{p}$ is a sufficient statistic for the true state X_k. $\mathbf{p}_0$ denotes priori probability density. When the controls and the measurements are known, $\mathbf{p}_k$ denotes conditional density.

So the issue converts to

$$\max_{\pi} E[\sum_{k=0}^{K}\gamma^k R_k(\mathbf{p}_k,u_k)] \tag{13}$$

This is the intelligent radar's adaptive model of waveform selection. $\mathbf{p}_k$ and R_k should be determined.

Refreshment of $\mathbf{p}_k$ is given by the following formula

$$\mathbf{p}_{k+1} = \frac{\mathbf{BAp}_k}{\mathbf{1'LAp}_k} \tag{14}$$

where $\mathbf{A}$ is state transition matrix, and $\mathbf{B}$ is state diagonal matrix.

Under policy π, rewrite the earnings

$$G_k(\mathbf{p}_k) = E\{\sum_{k'=k}^{K-1} R_{k'}(\mathbf{p}_{k'},u_{k'}) + R_K(\mathbf{p}_K) \mid \mathbf{p}_k\} \tag{15}$$

$G_k(\mathbf{p}_k)$ is the whole earnings. If we use dynamic programming algorithm, V_k can be expressed as recursively using method

$$V_k(\mathbf{p}_k) = R_k(\mathbf{p}_k,u_k) + E\{V_{k+1}(\mathbf{p}_{k+1}) \mid \mathbf{p}_k\} \tag{16}$$

Calculating $G_k(\mathbf{p}_k)$ is difficult, so our solution method is to establish a relationship between the above two formulas. DP is a useful solution method.

Obviously, $G_K(\mathbf{p}_K) = V_K(\mathbf{p}_K) = R_K(\mathbf{p}_K)$.

Then $V_k(\mathbf{p}_k)$ can be expressed as

$$\begin{aligned}&V_k(\mathbf{p}_k) = R_k(\mathbf{p}_k,u_k) + \\ &E\{E[\sum_{k'=k+1}^{K-1} R_{k'}(\mathbf{p}_{k'},u_{k'}) + R_k(\mathbf{p}_K) \mid \mathbf{p}_{k+1}] \mid \mathbf{p}_k\}\end{aligned} \tag{17}$$

According the discrete characteristics, we can get

$$\begin{aligned}&E\{E[G_{k'} \mid \mathbf{p}_{k+1}] \mid \mathbf{p}_k\} \\ &= \sum_{g\in G} gP(G_{k'} = g \mid \mathbf{p}_k) \\ &= E[G_{k'} \mid \mathbf{p}_k]\end{aligned} \tag{18}$$

Considering formula (17) and formula (18), it is obvious that

$$\begin{aligned} V_k(\mathbf{p}_k) &= E[\sum_{k'=k}^{K-1} R_{k'}(\mathbf{p}_{k'}, u_{k'}) + R_k(\mathbf{p}_K) \mid \mathbf{p}_k] \\ &= G_k(\mathbf{p}_k) \end{aligned} \tag{19}$$

We propose a backward recursion to calculate $V_k^{\pi}(\mathbf{p}_k)$. Our goal is to seek out the best policy π. The expression can be expressed as

$$G_k^*(\mathbf{p}_k) = \max_{\pi \in \prod} G_k^{\pi}(\mathbf{p}_k) \tag{20}$$

Assume the policy space is infinite, we can solve the problem through the method of making solutions to the optimality equations.

$$\begin{aligned} V_k(\mathbf{p}_k) &= \max_{u_k}(R_k(\mathbf{p}_k, u_k) + \\ &\gamma \sum_{\mathbf{p}' \in \mathbf{P}} P(\mathbf{p}' | \mathbf{p}_k, u_k) V_{k+1}(\mathbf{p}')) \end{aligned} \tag{21}$$

As $V_k(\mathbf{p}_k) \geq G_k^*(\mathbf{p}_k)$ and $V_k(\mathbf{p}_k) \leq G_k^*(\mathbf{p}_k)$, so $V_k(\mathbf{p}_k)$ equal to $G_k^*(\mathbf{p}_k)$.

Formula (21) is also the probability form of classical dynamic programming. Use the method to solve the related problem, we can get optimal solution. This is the optimal adaptive waveform selection scheme. However, the efficiency of this method is low. We propose the relative value iteration method.

When the value function converges much more slowly than the optimal policy, relative value iteration is more useful in these problems. In most iteration, relative value iteration grows steadily, where we are more interested in the convergence of the difference. In relative value iteration, all the values start the same-rate increasing, so any state can be picked out. Then its value can be subtracted from all the other states.

The algorithm is described as follows:

a.Choose some $v^0 \in V$

b.Choose a base state $\mathbf{p}^*$ and a tolerance ε

c.Let $w^0 = v^0 - v^0(\mathbf{p}^*)e$

d.Let $s = 1$

e.Let $v^s = Mw^{s-1}$

f.Set $w^s = v^s - v^s(\mathbf{p}^*)e$

g.If $sp\left(v^s - v^{s-1}\right) < \left(1-\gamma\right)\varepsilon / \gamma$, jump to step i

h. If $sp\left(v^s - v^{s-1}\right) \geq \left(1-\gamma\right)\varepsilon / \gamma$, jump to step e and f

i.Set $u^{\varepsilon} = \arg\max_{u \in U}\left(R\left(u\right) + \gamma P^{\pi} v^s\right)$

Using this method, we can get solution of adaptive waveform selection. In the next section, we will compare this method to fixed waveform and the optimal scheme.

IV. SIMULATIONS

First, we explain the necessity of waveform selection. We simulate measurement probability under Swerling III and Swerling IV with different cumulative pulse numbers and different SNR.

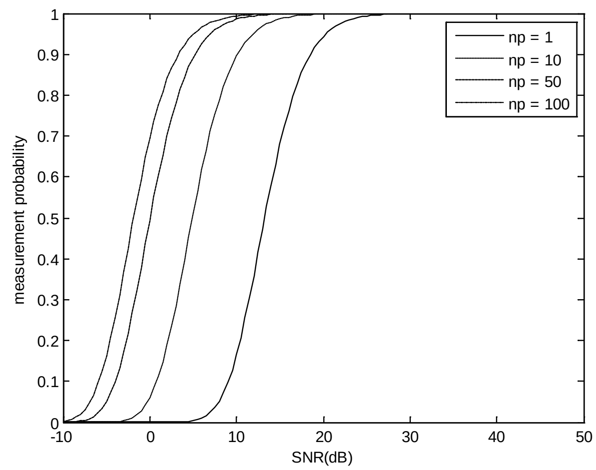

Fig. 3. Measurement probability under Swerling III

Fig. 3 is measurement probability under Swerling III. The measurement probability of target varies with the changes of signal to noise ratio and pulse width. The specific change values can be obtained from the figure. So according to the true signal to noise ratio to select the appropriate pulse width, it is both not only to achieve the desired detection probability, but also to save energy.

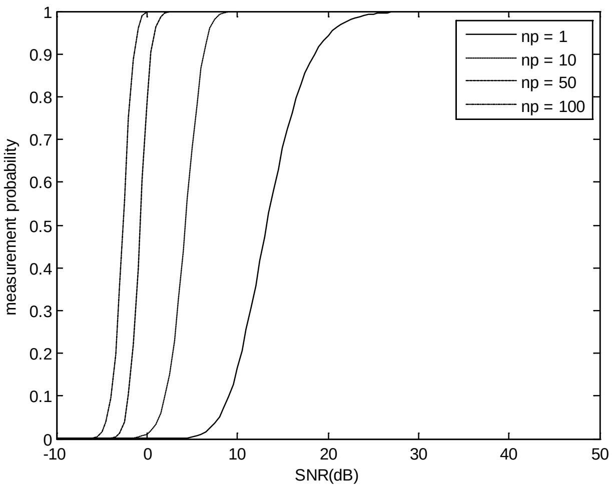

Fig. 4. Measurement probability under Swerling IV

Fig. 4 is measurement probability under Swerling IV. We can obtain a similar conclusion with fig. 3. The measurement probability of target varies with the changes of signal to noise ratio and pulse width, and it is independent of target type and wave mode.

Fig. 3 and fig. 4 show that in order to get greater measurement probability, it is important to make a selection of waveform. Pulse energy and duration are two key factors. The balance of the two key factors can help to obtain greater measurement probability. This is the necessity of waveform selection

Second, we consider a situation in which state space is 4×4. Assume the discount factor $\gamma = 0.9$. We adopt 5 different kinds of waveforms. We provide state transition matrix and the distribution of target.

$$\mathbf{A} = \begin{bmatrix} 0.96 & 0.02 & 0.01 & 0.01 \\ 0.01 & 0.93 & 0.03 & 0.02 \\ 0.02 & 0.03 & 0.95 & 0.02 \\ 0.01 & 0.02 & 0.01 & 0.95 \end{bmatrix} \quad (22)$$

Reward function is adopted with linear form

$$R(\mathbf{p}, u) = \mathbf{pp}' - 1 \quad (23)$$

$E(-R)$ denotes tracking errors, which is also the uncertainty of state estimation. Table 1 is measurement probabilities of 5 waveforms.

Table 1. Measurement probabilities of 5 waveforms

	1	2	3	4	5
x=1 x'=1,2,3,4	0.97 0.01 0.01 0.01	0.96 0.01 0.02 0.01	0.94 0.02 0.03 0.01	0.96 0.01 0.01 0.02	0.95 0.02 0.01 0.02
x=2 x'=1,2,3,4	0.01 0.01 0.96 0.02	0.02 0.95 0.01 0.02	0.02 0.02 0.01 0.95	0.01 0.02 0.96 0.01	0.01 0.97 0.01 0.01
x=3 x'=1,2,3,4	0.01 0.02 0.01 0.96	0.01 0.01 0.01 0.97	0.02 0.96 0.01 0.97	0.97 0.01 0.01 0.01	0.02 0.01 0.96 0.01
x=4 x'=1,2,3,4	0.96 0.01 0.01 0.02	0.02 0.96 0.01 0.01	0.01 0.02 0.95 0.02	0.03 0.95 0.01 0.01	0.04 0.94 0.01 0.01

Fig. 5 is curve of tracking errors. The curve shows that with the increase of time, the tracking errors are decreasing. Fixed waveform, relative value iteration and the optimal waveform selection scheme is all changing with time.

Compared to the relative value iteration method we proposed, the fixed waveform has greater tracking errors. That means relative value iteration method will reduce tracking errors.

The uncertainty of locating targets is cut down, too. At the same time, relative value iteration method approaches the optimal waveform selection scheme.

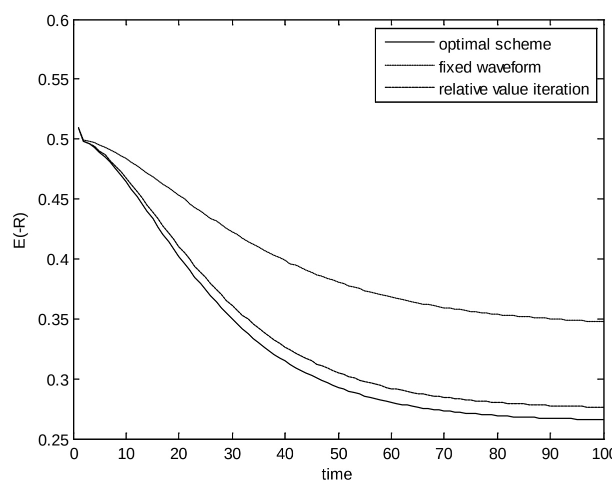

Fig. 5. Curve of tracking errors

V. Conclusions

To obtain better goals, intelligent radar can persistently inquiry for the working condition information at any time. Adaptive waveform selection is a hot issue in the related research field.

In this paper, after analysis of radar detection model and matched filtering, we set up stochastic dynamic programming model of waveform selection. Backward recursion method can obtain the optimal waveform, however, the efficiency is low. The relative value iteration method we proposed is more useful in the problems that the value function converges much more slowly than the optimal policy. In simulation, we explain the necessity of waveform selection under Swerling III and Swerling IV. Curve of tracking errors shows that the method we proposed has lower tracking errors, and it approaches the optimal waveform selection scheme at the same time.

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