A High Precision Time-Frequency Analysis Applying To Linearity Detection

Wenxin Zhang*, Xiaojun Liu, Xiuwei Chen, Qing Liu, Guangyou Fang and Shinan Lang

***Abstract*—**This article presents a method to achieve a high precision time-frequency analysis and detect the linearity of linear frequency modulation continuous wave (LFMCW) radar system. At first, discrete time domain window is used to make the target signal to be discrete short-time pieces. The fractional Fourier transform (FRFT) is used to calculate the frequency modulation (FM) rates of every subsection. Let the short-time signal mix with a ideal linear frequency modulation (LFM) signal, the intermediate frequency (IF) signal can be obtained. The Wigner-Ville Distribution (WVD) transform can be used to estimate the time-frequency function of the IF signal. The time-frequency function of short-time signal can be calculated from the relation between the IF signal and the ideal LFM signal. This method can achieve a high precision time-frequency analysis. The simulation can show a significant resolution performance improvement over the conventional method. At last, a practical engineering application measurement is presented.

***Keywords*—**Time-Frequency Analysis; Fractional Fourier Transform; Wigner-Ville Distribution.

I. INTRODUCTION

THE signal to be processed generally has time parameters and frequency parameters. The frequency of non-stationary signals changes with time and the relation is called time-frequency function of the signal. The time-frequency function of the radar signal can be used to check the linearity of linear frequency modulation signal [1], estimating surface water flow speeds[2], radar emitter signal recognition[3], plastic landmine detection[4], heart murmurs analysis[5] and efficient analysis of phase-locked loops[6]. It is significant and necessary to do time-frequency analysis.

This work was supported by Beijing Postdoctoral Research Foundation of China(2016.ZZ-27).

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In signal processing field, the most used transformation is fast Fourier transform (FFT) and FFT can calculate the frequency of the signal. However, FFT is a holistic conversion from the time domain to the frequency domain so that it does not have time resolution. Because of the defect of FFT, Gabor presented the short-time Fourier transform (STFT) [7] in 1946 and it contains two steps: use time domain window function to choose a subsection and do FFT. Because the time length of window function is short enough to regard the segmented signal as stationary signal, we can calculate the time-frequency function through moving the window function. There is no crossing-term, but its self-term is not concentrated. When the frequency of segmented signal changes tremendously, the bandwidth of FFT is too wide and STFT cannot calculate the exact frequency of the signal. The common time-frequency analysis methods have Wigner-Ville Distribution (WVD) [8] and wavelet [9]. The STFT need to balance the resolution between time domain and frequency domain, and the WVD can avoid it, but the WVD is effected by the crossing-term and there are some ways to restrain the crossing-term [10][11]. Wavelet transformation need choose a suitable wavelet base according to the signal types, but sometimes we do not know the type of the signal.

Fractional Fourier transform (FRFT) can overcome the shortcomings of traditional STFT. Especially for chirp signal, FRFT has a good time-frequency aggregation and do not have crossing-term, so FRFT has an advantage in the time-frequency analysis of linearity detection. References [1] presents an infinitesimal method to measure the linearity of the VCO, but this method is seriously affected by noise and not very accurate. We need to use a high sampling frequency to sample the transmitting signal of FMCW radar system and the little error can lead to a big mistake. This article presents a method to measure the linearity and achieve a high resolution time-frequency analysis.

II. TIME-FREQUENCY CALCULATION

A. Rough calculation of time-frequency

The WVD of signal is defined as:

$$W_s(t,f)=\int_{-\infty}^{+\infty} s(t+\frac{\tau}{2})\cdot s^*(t-\frac{\tau}{2})\cdot \exp(-j2\pi f\tau)d\tau \qquad (2.1)$$

s(t) is analytic signal and let the integral part of equation (2.1) be expressed as:

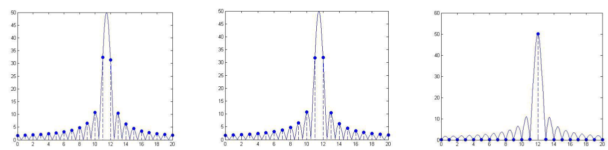
$$r_{s,s}(t,\tau) = s(t+\frac{\tau}{2}) \cdot s^*(t-\frac{\tau}{2}) \quad (2.2)$$

The equation (2.2) is an instantaneous self-correlation. The instantaneous self-correlation leads the frequency of signal double.

The non-stationary signals can be regarded as LFM signals during a pimping time[1]. If there is no noise, during a pimping time T_w the signal can be written as:

$$s_w(t)=A\cdot\exp(j2\pi(f_0\cdot t+\frac{k}{2}\cdot t^2)+j\varphi),\quad t\in[0,T_w] \quad (2.3)$$

A is the amplitude and φ is the initial phase. f_0 is the initial frequency and k is the frequency modulation (FM) rate. When we sample the signal $s_w(t)$ with sampling rate Fs and the sample points is N, the frequency resolution can be written as: $Fs/N = 1/T_w$. The discrete signal of $s_w(t)$ is $s_w(n)$, where $n = 0,1\cdots,N-1$. Let $s_w(n)$ do discrete WVD transform and there is fence effect. The fence effect is caused by the discretization of the signal and here we discuss the discrete Fourier transform (DFT) and discrete time Fourier transform (DTFT) to show the fence effect. The spectrum of DFT and DTFT are shown as Fig.1:



(a) (b) (c)

Fig. 1 The DFT and DTFT frequency spectrum

In the Fig. 1, the solid-line curves stand for the spectrum of DTFT and the solid points stand for the spectrum of DFT. Analyzing the Fig. 1, following conclusions can be summarized:

① DFT spectrum is the discrete samples of the DTFT spectrum. The frequency interval between every two DFT spectrum lines is equal to the one frequency resolution $1/T_w$. From the spectrum of DFT, we can calculate the frequency of N DFT spectrum line and the frequency can be written as: $F = n \cdot 1/T_w$, where n is the number of every DFT spectrum line and n is discrete. In digital signal processing field, researchers use the maximum DFT spectrum line of DFT to calculate the frequency of the signal.

② As the frequency varies and when the maximum DFT spectrum line is alternating, and the frequency of the signal is F1 which is approximately equal to an integral multiple of frequency resolution plusing one-half frequency resolution. The F1 can be expressed as $F1 = (M+1/2)\cdot 1/T_w$, where M is an inter and is equal to the number of the maximum spectrum line. The DFT-DTFT picture is similar to the picture (b) in Fig. 1.

③ As the frequency varies and when the DFT spectrum is similar to a pulse, the frequency of the signal is F2 which is an integral multiple of frequency resolution. The F2 can be expressed as $F2 = K \cdot 1/T_w$, where K is an inter and is equal to the number of the maximum DFT spectrum line. The DFT-DTFT picture is similar to the picture (c) in Fig. 1.

Since the frequency spectrum of the LFM signal varies against time and the number of maximum spectrum line cannot vary gradually because of fence effect. When the frequency of signal linearly increases from f_0 to $f_0 + k \cdot T_w$, the time-frequency function of WVD appears as a ladder. If $f_0 = 600Hz$ and $T_w = 0.1$, there are N sampling points and the discrete time is t: $t = (T_w/N)n, 0 \le n \le N-1$. We use WVD to calculate the frequency during T_w. When the FM rate k is bigger, the steps of the ladder is more and it is as Fig.2. When the FM rate k is smaller, the steps of the ladder is less and it is as Fig.3. In this article, the length of every jump on the horizontal axis is call time-step and the length of every jump on the vertical axis is call frequency-step.

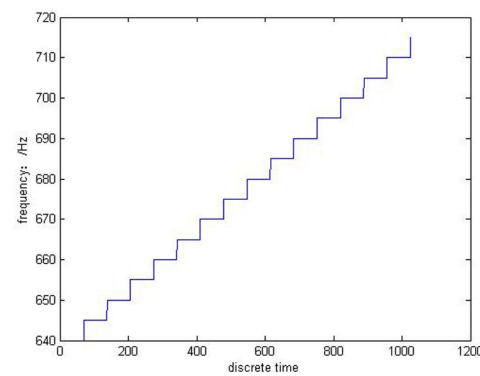


Fig.2 time-frequency analysis with bigger k

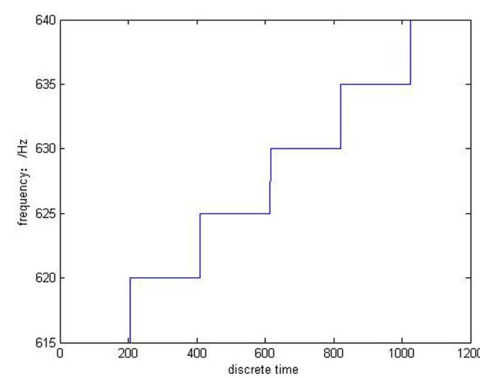


Fig.3 time-frequency analysis with smaller k

B. Precise calculation of time-frequency

The sampling rate is Fs and the sampling points are N. The frequency resolution is: $\Delta F = Fs/N = 1/T_w$. The WVD needs to do instantaneous self-correlation, and this action leads the frequency of signal double and the calculated frequency needs to be divided by two at last. After the WVD transformation we can calculate the frequency of the maximum DFT spectrum line and we can obtain the ladder frequency picture as Fig.2 and Fig.3. If there are M times jump during T_w and there are n_i discrete time points during every jump, the relation can be gotten: $N=\sum_{m=0}^{M} n_m$ and n_0 stands for the number of points before the first jump. n_i stands for the i-th time-step and is corresponding to the horizontal axis. The schematic diagram of discrete time n_i is shown in the Fig.4.

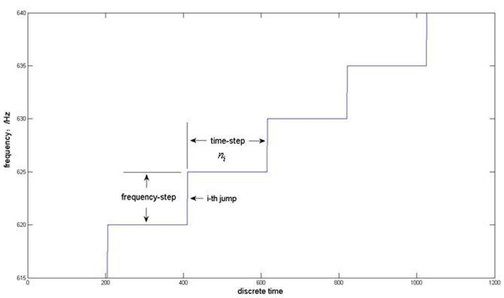


Fig. 4 time-step and frequency-step

If the number of maximum DFT spectrum line is P_i before the i-th jump and the frequency is: $F_{WVD}(P_i)=(Fs/N)\cdot P_i$. The instantaneous time of the last point before the i-th jump is $Tb_i=\sum_{m=0}^{i} n_m$.

If the number of maximum DFT spectrum line is P_{i+1} after the i-th jump and the frequency is $F_{WVD}(P_{i+1})=(Fs/N)\cdot P_{i+1}$. The instantaneous time of the first point after the i-th jump is $Ta_i=\sum_{m=0}^{i} n_m+1$.

The positions of Tbi and Tai in the time-frequency picture can be shown in the Fig. 5:

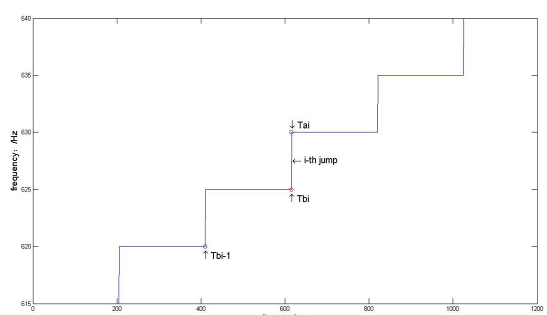


Fig. 5 positions of Tbi and Tai

If the FM rate k is positive, there is the relation: $P_{i+1}=P_i+1$ because the frequency of LFM signal is gradient change. If the FM rate k is negative there is the relation: $P_{i+1}=P_i-1$. In this article, we only analyze the situation of positive FM rate and the situation of negative FM rate can be analyzed with the same method. From the above relation, the frequency difference of WVD corresponding to every jump is equal to one frequency resolution:

$$F_{WVD}(P_{i+1})-F_{WVD}(P_i)=\Delta F \tag{2.4}$$

In this article, the WVD frequency difference of every jump is called frequency-step and is corresponding to the vertical axis as shown in the Fig.4. Therefore, in Fig.2 and Fig.3, every frequency-step of the ladder is equal to one frequency resolution. Let the frequency of the i-th jump point be equal to the mean of $F_{WVD}(P_i)$ and $F_{WVD}(P_{i+1})$:

$$F_{jump}(i)=\frac{F_{WVD}(P_i)+F_{WVD}(P_{i+1})}{2} \tag{2.5}$$

The discrete time from (i-1)-th jump to i-th jump is regarded as a period or time-step. If the number of total jumps is M and the mean time-step can be calculated by:

$$N_T=\frac{1}{M-2}\sum_{i=1}^{M-1} n_i \tag{2.6}$$

The time-frequency function of LFM signal can be written as: $F(t)=f_0+k\cdot t$ and its discrete form is:

$$F(n)=f_0+\frac{k\cdot T_w}{N-1}\cdot n\ ,\ n\in[0,N-1] \tag{2.7}$$

Taking the discrete time of i-th and (i-1)-th jumps into equation (2.7), the following equation can be obtained:

$$\left|F(Tb_i)-F(Tb_{i-1})\right|=\left|\frac{k\cdot T_w}{N-1}n_i\right| \tag{2.8}$$

If we plot the time-frequency function of WVD and the time-frequency function of LFM signal into one picture, the Fig.6 can be obtained:

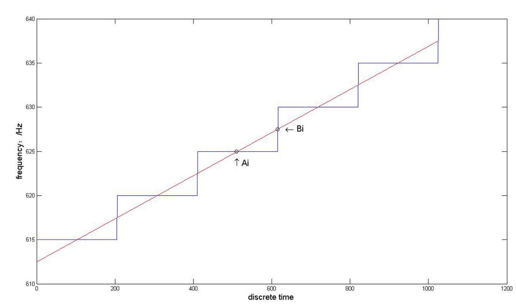


Fig.6 time-frequency function of WVD and LFM signal

In the Fig.6, the blue line stands for the time-frequency function of WVD and the red line stands for the time-frequency function of LFM signal. The two black points Ai and Bi are the intersections during the i-th jump period. The following analysis tries to estimate the frequency of all intersections Bi and linear interpolation algorithms can be used to obtain the whole time-frequency function of the signal.

Combining the equation (2.4) and (2.8), the following relation can be obtained:

$$\left|\frac{k\cdot T_w}{N-1}n_i\right| = \Delta F \tag{2.9}$$

Let the mean time-step N_T replace the n_i :

$$\left|F(N_T)\right| = \left|\frac{k\cdot T_w}{N-1}N_T\right| = \Delta F \tag{2.10}$$

$$\left|\frac{k\cdot T_w}{N-1}\right| = \frac{\Delta F}{N_T} \tag{2.11}$$

In fact, around the i-th jump point the DFT spectrum is like picture (b) in Fig. 1 and the frequency of Bi is around $\frac{P_i+P_{i+1}}{2}\cdot Fs/N$. When the discrete time is Tbi, the frequency of LFM signal is less than $\frac{P_i+P_{i+1}}{2}\cdot Fs/N$ and When the discrete time is Tai, the frequency of LFM signal is bigger than $\frac{P_i+P_{i+1}}{2}\cdot Fs/N$:

$$\begin{cases} F(\sum_{m=0}^{i} n_m) = (\frac{P_i+P_{i+1}}{2} - \Delta n1)\cdot \Delta F \\ F(\sum_{m=0}^{i} n_m + 1) = (\frac{P_i+P_{i+1}}{2} + \Delta n2)\cdot \Delta F \end{cases} \tag{2.12}$$

The $\Delta n1$ and $\Delta n2$ are tiny variables, and they satisfy the relation: $\Delta n1 \times \Delta n2 > 0$. When the FM rate k is positive, the $\Delta n1$ and $\Delta n2$ are positive. When the FM rate k is negative, the $\Delta n1$ and $\Delta n2$ are negative. The frequency difference of $F(\sum_{m=0}^{i} n_m + 1)$ and $F(\sum_{m=0}^{i} n_m)$ is:

$$\left|F(\sum_{m=0}^{i} n_m + 1) - F(\sum_{m=0}^{i} n_m)\right| = \left|(\Delta n2 + \Delta n1)\right|\cdot \Delta F \tag{2.13}$$

Combining the equation (2.7), the frequency difference of $F(\sum_{m=0}^{i} n_m + 1)$ and $F(\sum_{m=0}^{i} n_m)$ is:

$$\left|F(\sum_{m=0}^{i} n_m + 1) - F(\sum_{m=0}^{i} n_m)\right| = \left|\frac{k\cdot T_w}{N-1}\right| \tag{2.14}$$

From the equations (2.11) to (2.14), the following equation can be deduced:

$$\left|\Delta n1 + \Delta n2\right| = \frac{1}{N_T} \tag{2.15}$$

When we use WVD to calculate the frequency just as the above analysis, we can get the i-th jump frequency:

$$\begin{cases} F_{WVD}(P_i) = P_i \cdot \Delta F \\ F_{WVD}(P_{i+1}) = (P_i \pm 1)\cdot \Delta F \end{cases} \tag{2.16}$$

When the FM rate k is positive, $F_{WVD}(P_{i+1}) = (P_i + 1)\cdot \Delta F$. When the FM rate k is negative, $F_{WVD}(P_{i+1}) = (P_i - 1)\cdot \Delta F$. The following analysis is based on the positive k and the negative k is the same analysis.

The equation (2.5) also can be written as:

$$F_{jump}(i) = (\frac{2\cdot P_i + 1}{2})\cdot \Delta F \tag{2.17}$$

Because FM rate of non-stationary signal can be regarded as a constant during a short time T_w , and from the equation (2.12), the frequency of middle time of discrete time $\sum_{m=0}^{i} n_m$ and $\sum_{m=0}^{i} n_m + 1$ can be written as:

$$\begin{aligned} F(\frac{\sum_{m=0}^{i} n_m + \sum_{m=0}^{i} n_m + 1}{2}) &= \frac{F(\sum_{m=0}^{i} n_m) + F(\sum_{m=0}^{i} n_m + 1)}{2} \\ &= (\frac{2P_i + 1 + \Delta n2 - \Delta n1}{2})\cdot \Delta F \end{aligned} \tag{2.18}$$

Let the equation (2.18) minus the equation (2.17), we can get the absolute value of error:

$$error = \left|F(\frac{\sum_{m=0}^{i} n_m + \sum_{m=0}^{i} n_m + 1}{2}) - F_{jump}(i)\right| = \left|\frac{\Delta n2 - \Delta n1}{2}\right|\cdot \Delta F \tag{2.19}$$

Because $\Delta n1$ and $\Delta n2$ are positive, we can deduce the relation:

$$\left|\Delta n1 + \Delta n2\right| > \left|\Delta n2 - \Delta n1\right| \tag{2.20}$$

Combining the equations (2.15), (2.19) and (2.20), The following relation can be deduced:

$$error = \left|\frac{\Delta n2 - \Delta n1}{2}\right|\cdot \Delta F < \frac{\Delta F}{2\cdot N_T} \tag{2.21}$$

From the above analysis, if we use equation (2.17) to estimate the frequency of the jump point Bi, the error can be $2N_T$ times smaller than one frequency resolution. If there are M jump points and we can estimate the frequency of M points, and we can use linear interpolations to calculate the frequency of the rest discrete points.

Using the above method to do time-frequency analysis, this method can achieve a high precision and the precision is related to the mean time-step N_T . The equation (2.11) indicates that N_T is based on the FM rate k . When the absolute value of k is big, the N_T is small and the frequency error is relatively big. But the number M of jump points is big, which is good to calculate more frequency of jump points and is good to linear interpolation; When the absolute value of k is small, the N_T is big and the frequency error is relatively small. But the number

M of jump points is small, which is bad to calculate more frequency of jump points and is bad to linear interpolation. Therefore, the absolute value of FM rate needs a compromise value K, which can promise a relatively big N_T and enough jump points M.

C. The mixing of LFM signal

Supposing the compromised FM rate is K, the value of K is based on the system request. When the FM rate k is not equal to K, the time domain signal can be written as:

$$s_w(t) = A\cdot\exp(j2\pi(f_0\cdot t+\frac{k}{2}\cdot t^2)+j\varphi) \qquad (2.23)$$

In order to get the requisite FM rate K, we can produce a ideal chirp signal to mix with $s_w(t)$. The initial frequency of the ideal chirp signal is zero and the FM rate is (K-k): $s_{chirp}(t)=\exp(j2\pi(\frac{K-k}{2})t^2))$. Let $s_w(t)$ mix with $s_{chirp}(t)$, we can get a intermediate frequency (IF) signal $s_w^{'}(t)$ whose FM rate is about K.

$$\begin{aligned} s_w^{'}(t) &= s_w(t)\cdot s_{chirp}(t) \\ &= A\cdot\exp(j2\pi(f_0\cdot t+\frac{K}{2}\cdot t^2)+j\varphi) \end{aligned} \qquad (2.24)$$

After the mixing, the method in the section 2.2 can be used to calculate the time-frequency function $F^{'}(t)$ of $s_w^{'}(t)$ and let $F^{'}(t)$ decrease the time-frequency function of $s_{chirp}(t)$, the time-frequency function $F(t)$ of $s_w(t)$ can be gotten: $F(t)=F^{'}(t)-(K-k)t$.

Before the mixing, we need calculate the FM rate of $s_w(t)$. In this article, FRFT is used to calculate the FM rate k of $s_w(t)$.

III. FM Rate Computation

A. Definition of FRFT

Reference [12] presents a detailed instruction of FRFT and here give a brief explain. The FRFT is defined as:

$$S_\alpha(u)=\mathrm{F}^\alpha[\mathrm{s}(t)]=\int_{-\infty}^{+\infty}s(t)K_p(t,u)\,\mathrm{d}t \qquad (3.1)$$

p is the older of FRFT and the kernel of FRFT is $K_p(t,u)$:

$$K_p(t,u)=\begin{cases}\sqrt{\frac{1-j\cot\alpha}{2\pi}}\cdot\exp(\mathrm{j}\frac{t^2+u^2}{2}\cot\alpha-\mathrm{j}tu\csc\alpha)..\alpha\neq n\pi \\ \delta(t-u)..................................\alpha=2n\pi \\ \delta(t+u)............................\alpha=(2n\pm1)\pi\end{cases} \qquad (3.2)$$

δ is the impulse function and n is inter. α is the angle of rotation and $\alpha=p\pi/2$.

B. Calculation of FM rate

Supposing the primary non-stationary signal is:

$$s(t)=A\cdot\exp(j2\pi(f_0+f_1t+\frac{k_2t^2}{2}+\frac{k_3t^3}{3}....)),\ t\in[0,T] \qquad (3.3)$$

During a very short time, the non-stationary signal can be regarded as stationary signal. When we use a short-time window to cut off the signal to pieces, the time length of widow is T_w and the window move to right $(2i-1)\cdot\frac{T_w}{2}$ every piece:

$$\begin{aligned} s_i(t) &= [A\cdot\exp(j2\pi(f_0+f_1t+\frac{k_2t^2}{2}+\frac{k_3t^3}{3}....))] \\ &\cdot w(t-(2i-1)\cdot\frac{T_w}{2}) \end{aligned} \qquad (3.4)$$

If the window function is rectangle, the primary non-stationary signal can be written as: $s(t)=\sum_{i=1}^{N}s_i(t)$, $N=\frac{T}{T_w}$.

After using time domain window processing, the every piece signal can be regard as a chirp signal and the time is from zero to T_w. Because the length of the piece is very short, the power series which is bigger than three of $s_i(t)$ can be ignored and the equation (3.4) can be changed to:

$$s_i(t)=[A\cdot\exp(j2\pi(fi_0+fi\cdot t+\frac{Ki\cdot t^2}{2}))]\cdot w(t-\frac{T_w}{2}) \qquad (3.5)$$

$2\pi\cdot fi_0$ is the initial phase of $s_i(t)$ and fi is the initial frequency of $s_i(t)$. Ki is the FM rate of $s_i(t)$.

Then let the $s_i(t)$ do FRFT:

$$S_iw_\alpha(u)=\mathrm{F}^\alpha[\mathrm{s}_i(t)]=\int_{-\infty}^{+\infty}[A\cdot\exp(j2\pi(fi_0+fi_1t+\frac{Ki\cdot t^2}{2}))]\cdot w(t-\frac{T_w}{2})\cdot \sqrt{\frac{1-j\cot\alpha_i}{2\pi}}\cdot\exp((\mathrm{j}\frac{t^2+u^2}{2}\cot\alpha_i-\mathrm{j}tu\csc\alpha_i))\mathrm{d}t$$

.... (3.6)

After normalizing discrete and dimensional [13], the fast algorithm [14] can calculate the $S_iw_\alpha(u)$. The peak value of $S_iw_\alpha(u)$ can be obtained and the corresponding angle α_i also can be obtained. The FM rate can be calculated by:

$$k_i=-\cot\alpha_i\cdot Fs/T_w \qquad (3.7)$$

C. High order moments of FRFT

If we use the method of section 3.2 to calculate the FM rate k_i, it may lead some big errors because the FRFT is sensitive to noise. The high order moments of FRFT can restrain the effect of noise and can improve the anti-noise performance[15][16]. In this article, we use forth order moments to calculate the FM rate. The forth order moments of FRFT can be written as:

$$P_\alpha=\int_{-\infty}^{+\infty}S_iw_\alpha(\mathrm{u})^4(\mathrm{u}-\mathrm{m}_\alpha)^4du \qquad (3.8)$$

The m_α is first order moments of FRFT and it can be obtained by:

$$m_\alpha = \int_{-\infty}^{+\infty} S_i w_\alpha(\mathrm{u})^2 u du \tag{3.9}$$

When the SNR (signal and noise rate) is about -20 dB, we use the above two equations to calculate the forth order moments with α increasing from zero to π gradually, and the forth order moments spectrum can be obtained:

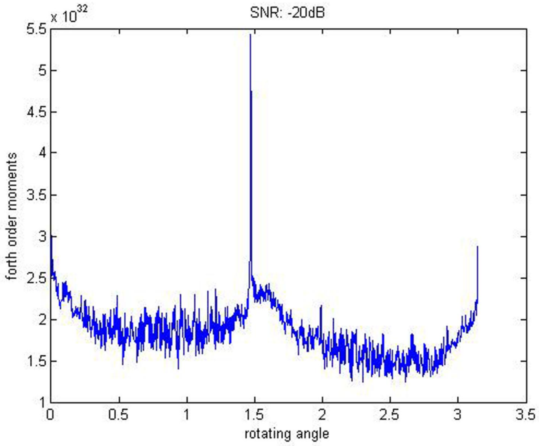


Fig.7 The forth order moments spectrum of FRFT

In the Fig.7, we can find the peak valve of forth order moments and the corresponding rotating angle also can be obtained through the peak valve. The FM rate can be calculated by the equation (3.7). If the SNR is lower, we can use higher order moments to calculate the FM rate at the cost of more calculation complexity.

IV. Simulation and Measurement

A. Simulation

The simulation uses MATLAB to achieve the proposed algorithm. In this simulation, the time-frequency function of the signal is a sine function and the center frequency is 600Hz and the max frequency variation is 500Hz/s. The signal can be written as:

$$s(t) = \cos(2\pi(600t - 500 \cdot \frac{T}{2\pi} \cdot \cos(2\pi \frac{t}{T}))) + A_n \cdot N(t) \tag{4.1}$$

In the equation (4.1), T is the length of time, N(t) is white Gaussian noise, A_n is the coefficient of white Gaussian noise and it can change the SNR by changing A_n. The time-frequency function of the signal is:

$$F(t) = 600 + 500 \sin(2\pi \frac{t}{T}) \tag{4.2}$$

The sampling rate Fs is equal to 20480 and the length of window is $T_w = \frac{T}{50}$. So the signal is sectioned to 50 pieces. When the SNR is about 5dB, the frequency estimation picture and error picture of this method is shown in Fig.8.

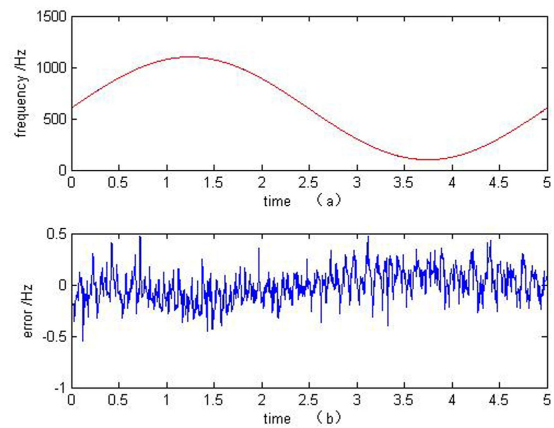


Fig.8 frequency estimation picture and error picture

In the Fig.8, picture (a) is the time-frequency estimation picture of the proposed algorithm and picture (b) is the error picture of time-frequency estimation. The error is below 0.5Hz when the SNR is about 5dB.

Moreover, the simulation also compare the error of the three algorithm: the proposed algorithm, ordinary WVD [8] and infinitesimal method [1].

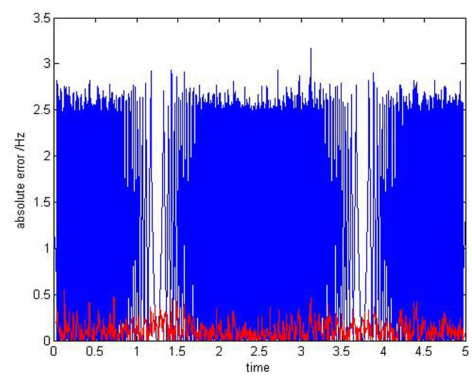


Fig.9 Ordinary WVD and the method of this article

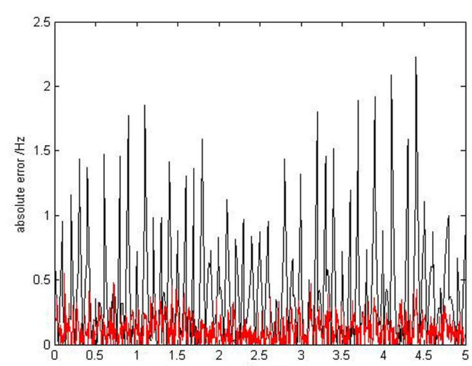


Fig.10 Infinitesimal method and the method of this article

The Fig.9 is the absolute error picture of ordinary WVD and the proposed algorithm and the blue line stands for the error of the ordinary WVD and the red line stands for the proposed algorithm.

The Fig.10 is the absolute error picture of infinitesimal method and the proposed algorithm and the black line stands for the error of the infinitesimal method and the red line stands for the proposed algorithm. From the Fig.9 and Fig.10, the time-frequency precision of the proposed algorithm is higher.

The following table 1 shows the mean square errors at different SNR:

Table 1 The mean square errors of the three algorithm

SNR(dB)	Ordinary WVD (Hz)	Infinitesimal method (Hz)	the proposed algorithm (Hz)
60	1.4247	0.4317	0.0854
50	1.4247	0.4377	0.0848
40	1.4248	0.4355	0.0855
30	1.4249	0.4512	0.0883
20	1.4253	0.4749	0.0975
15	1.4262	0.4863	0.1090
10	1.4272	0.5042	0.1192
5	1.4318	0.5746	0.1623

From the outcome of table 1, the proposed algorithm has a better precision than the ordinary WVD and the infinitesimal method when the SNR is from 60dB to 5dB.

B. Measurement

In this article, a practical linearity detection of the radar system is presented and the system chart and circuit board of the radar system is shown in Fig.11 and Fig.12.

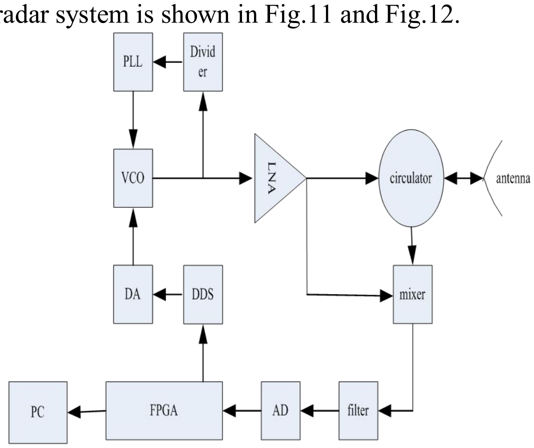


Fig.11 The system chart of the radar system

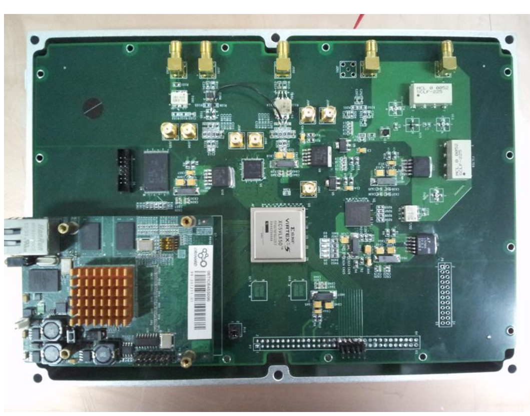


Fig.12 Circuit board of the radar system

In this system, Field Programmable Gate Array (FPGA) utilizes the Direct Digital Frequency Synthesizers (DDS) to produce a chirp baseband signal and the baseband signal controls the Vector-Controlled Oscillator (VCO) to produce the 24GHz chirp transmitting signal. The frequency divider and phase locked loop (PLL) are used to form a feedback loop and the feedback loop plays a self-correction role in this system.

The linearity of chirp baseband signal and chirp transmitting signal is very important in this system. If the linearity is not good enough, when the local oscillator signal is mixed with the echo signal and it cannot get a good IF signal. Therefore, it is necessary to detect the linearity of chirp baseband signal and chirp transmitting signal. The proposed algorithm is used to get the time-frequency function and the linearity can be detected.

The initial frequency of chirp baseband signal is about 100MHz. We use an oscilloscope to sample the signal with 500MHz sampling rate. After the analysis, the following time-frequency picture can be obtained:

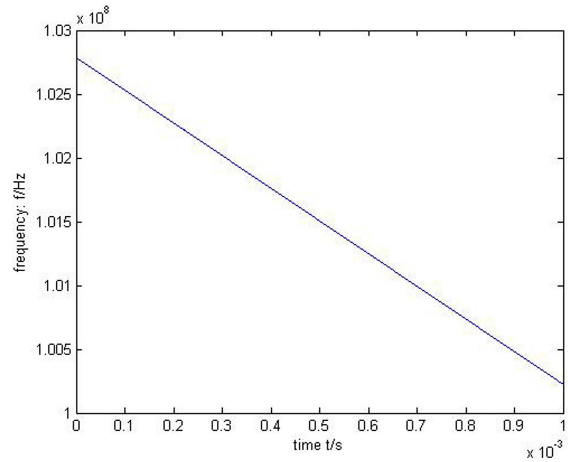


Fig.13 Time-frequency picture of chirp baseband signal

The initial frequency of the chirp transmitting signal is about 24GHz and the bandwidth is about 600MHz. The vector network analyzer produces a 24GHz signal to mix with the chirp transmitting signal and we can use an oscilloscope to sample the mixing signal with 5GHz sampling rate. After the analysis, the following time-frequency picture can be obtained:

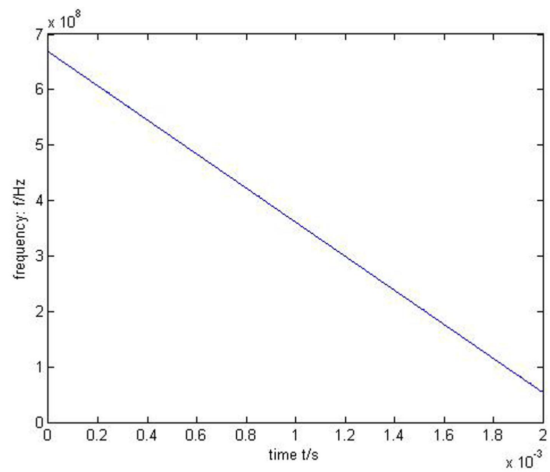


Fig.14 Time-frequency picture of chirp transmitting signal

From the Fig.13 and Fig.14, we use the proposed algorithm to estimate the time-frequency function. Many precision instruments require a very good linearity of the radar system and a bad linearity may lead big error to the results. Therefore, when we estimate the time-frequency function of the baseband signal and transmitting signal, we can calculate the linearity of the system and we also can calculate the max error brought by linearity. In a word, the linearity detection is indispensable.

V. CONCLUSION

In this article, a method of time-frequency analysis is to be submitted and this method can achieve a high precision. This method sections the signal into short-time pieces and uses FRFT to calculate the FM rates of the every piece signal. An ideal chirp signal is produced to mix with the short-time pieces signal and the IF signal with compromised FM rate can be gotten. Afterwards, let the IF signal do WVD transformation, and a jumping frequency picture can be obtained. From the frequency jumping points, we can estimate the frequency of jump points. After that, a linear interpolation is used to obtain the time-frequency function of all the piece signal. From the above way, all the time-frequency function of the signal can be estimated. The simulation proves this method has a better precision than the other two existing time-frequency analysis methods. In the measurement, a practical linearity detection of radar system is presented.

ACKNOWLEDGMENT

Wenxin Zhang is the corresponding author. The research was supported by Beijing Postdoctoral Research Foundation of China. The authors thank the referee for his or her careful reading of the paper and useful suggestions.

REFERENCES

[1] Baosong Xu, Yaqing Tu, Liangbing Liu, VCO nonlinearity detection method based on element analytic and FRFT, J. Journal of Electronic Measurement and Instrument. 05 (2008) 117-122.

[2] P.R. Kersten, R.W. Jansen, T.L. Ainsworth, J.V. Toporkov, M.A. Sletten, Estimating surface water flow speeds using time－frequency methods, IET Signal Processing, Volume 4, issue 4, 2010 , p. 406 - 412.

[3] L.B. Yang, S.S. Zhang, B. Xiao, Radar emitter signal recognition based on time-frequency analysis, IET International Radar Conference 2013, 2013, p. 0412 - 0412.

[4] Y. Sun, J. Li, Time－frequency analysis for plastic landmine detection via forward-looking ground penetrating radar, IEE Proceedings - Radar, Sonar and Navigation, Volume 150, issue 4, 2003 , p. 253 - 261.

[5] E. H. Miller, "A note on reflector arrays (Periodical style—Accepted for publication)," *IEEE Trans. Antennas Propagat.*, to be published.

[6] Sancho, S. ; Suarez, A., Efficient analysis of phase-locked loops through anovel time-frequency approach, based on twoenvelope transient formulations, Microwave Symposium Digest, 2003 IEEE MTT-S, Volume .3 ,2003, p. 2153-2156

[7] Gabor.D. Theory of communication, J. IEE, 93 (1946) 429-497.

[8] Kumar, B.V.K.V., Carroll. C.W., Performance of Wigner distribution based detection methods, Optical Engineering, 1984, Vol.23, No. 6. pp. 732-738.

[9] S.Mallat, W.L.Hwang, Singularity detection and processing with wavelets, J. IEEE Trans. On Inform. Theory, 38 (1992).

[10] Barbarossa S,A Zanalda.A combined Wigner-Ville and Hough transform for cross-terms suppression and optimal detection and parameter estimation[A].Proc ICASSP.92[C].SanFrancisco: IEEE Society,1992.173 -176.

[11] Leon Cohen.Time-frequency distributions-a review[J]. Proceedings of the IEEE,1989,77(7):941-981.

[12] Haldun M O， Orhan A M,Alper K, et al. Digital computation of the fractional Fourier transform, J. IEEE Transaction on Signal Processing, 44 (1996) 2141-2150.

[13] Zhao Xinghao, Deng Bing, Tao Ran, Dimensional normalization in the digital computation of the fractional Fourier transform. J. Transactions of Beijing Institute of Technology, 25 (2005) 360-364.

[14] Namias V. The fractional order Fourier transform and its application to quantum mechanics, J. J Inst Math App, 25 (1980) 241-265.

[15] Tatiana Alieva, Martin J.B., On fractional Fourier transform moments, IEEE signal processing letters, 7 (2000),320-323.

[16] Wang Fei, Cao Fan, Multi-component chirp signal separating and enhancing based on SVD of time-frequency space, Journal of data acquisition and processing,, 24 (2009), 767-771.