Error Analysis of Sensor Geometric Factor for the Multi-node Cooperative Localization Accuracy

Yao Fan

***Abstract*—**The localization accuracy is very important for the multi-node cooperative localization system. In the localization system, the geometrical factor has a great influence on the localization result. In order to solve this problem, the error analysis of the sensor geometry factor for multi-node cooperative localization accuracy is researched in this paper. By using the total differential method to analyze the time difference equation, the intrinsic relations of the localization error, the measurement error of signal arrival time, the localization error of measurement platform and the sensors layout is determined, the localization accuracy distribution of different sensors layout is analyzed through the simulation experiments. Experiments show that the method in this paper provides an important theoretical basis for improving the multi-node cooperative localization accuracy.

***Keywords*—**geometric factor, multi-node cooperative, total differential, localization accuracy.

I. INTRODUCTION

THE multi-node cooperative localization is the extension of the single sensor node localization[1], it can expand the scope of the monitoring area and improve the recognition capability for the vibration source at the long distance. The technology has been widely used not only in bridge monitoring, environmental monitoring, regional monitoring, but also in military surveillance, battlefield detection and so on [2]. The localization accuracy mainly is related to the measurement factor and the geometric factor [3-4].

In recent years, due to the continuous improvement of measurement technology, the influence of the measurement factor to localization accuracy was by constantly decreased [5], however the research of the sensor geometric factor for the multi-node cooperative localization accuracy is less. It plays a vital role for achieving the more accurate localization in multi-node cooperative system how to choose the sensors layout.

This paper researches the influence of the sensor geometry factor on multi-node cooperative localization accuracy. By using the total differential method to analyze the time difference equation, the intrinsic relations of the localization error, the measurement error of signal arrival time, the position error of measuring platform and the sensors layout is determined, the localization accuracy distribution of the different sensor layout is determined through the simulation experiments.

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II. TIME DIFFERENCE OF ARRIVAL PRINCIPLE

Each node uses three (or more) known coordinates of vibration sensor to sampling and collect the vibration source in multi-node cooperative localization system. The time difference value of the vibration source and the different sensors is obtained according to the principle of time delay, and the vibration source is on the two groups of hyperbolic line which three sensors (or more) as the focus, the intersection of two groups of hyperbolic line is the actual location of the vibration source according to the hyperbola principle in the mathematics [6]. Take three sensors as an example and it is done the time difference location by using the hyperbolic theory, its orientation diagram as shown in figure 1.

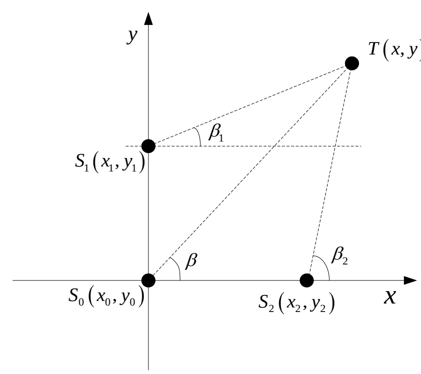


Fig.1. The sensor layout of time difference localization

Figure 1 includes the primary sensor $S_0(x_0, y_0)$, the two secondary sensors $S_1(x_1, y_1)$ and $S_2(x_2, y_2)$, β, β_1, β_2 respectively are the azimuthal angle of each sensor and the target, assuming the location of the measured target is $T(x, y)$, R_0, R_1, R_2 respectively are the distance between the vibration source and the sensor $S_0(x_0, y_0)$, $S_1(x_1, y_1)$, $S_2(x_2, y_2)$, the distance expressions is:

$$\begin{cases} R_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2} \\ R_1 = \sqrt{(x-x_1)^2 + (y-y_1)^2} \\ R_2 = \sqrt{(x-x_2)^2 + (y-y_2)^2} \end{cases} \tag{1}$$

The time difference ΔT_1 and ΔT_2 can be obtained by TDOA measurement algorithm, ΔT_1 is the time difference of S_0 and S_1, ΔT_2 is the time difference of S_0 and S_2, equation (1) can be converted into equation (2).

$$\begin{cases} R_0 - R_1 = \sqrt{(x-x_0)^2 + (y-y_0)^2} - \sqrt{(x-x_1)^2 + (y-y_1)^2} = v \cdot \Delta T_1 \\ R_0 - R_2 = \sqrt{(x-x_1)^2 + (y-y_1)^2} - \sqrt{(x-x_2)^2 + (y-y_2)^2} = v \cdot \Delta T_2 \end{cases} \tag{2}$$

To solve the binary quadratic equations, the location of the target $T(x, y)$ can be calculated.

III. Influence Analysis of the Sensor Geometric Factor on Localization Accuracy

A. The sensor geometric factor

The influence of the geometric factor on the target localization accuracy is described by using the Geometrical Dilution of Precision (GDOP)[7], it is an important index which it is used to measure the size of the positioning error. The geometric factor includes these factors, for example, the distance between the target and each sensor, the sensor layout and the measurement error. GDOP expression is:

$$GDOP = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{3}$$

Among formula (3), σ_x and σ_y respectively are the localization error which the localization result on the x axis and y axis.

B. Total differential localization accuracy algorithm

In the passive time difference localization, the localization system needs more sensors to participate in the coordination localization, it is objectively caused the diversity of error sources and the complexity of system building [8], the geometric localization accuracy is obtained through solving the localization differential equations, it can be analyzed that the influence of the sensor layout and the measurement error on the localization result.

According to the three sensors localization principle [9-10] in the time difference localization, formula (2) can be changed into the following forms:

$$\begin{cases} \Delta_1 = \sqrt{(x-x_0)^2 + (y-y_0)^2} - \sqrt{(x-x_1)^2 + (y-y_1)^2} \\ \Delta_2 = \sqrt{(x-x_0)^2 + (y-y_0)^2} - \sqrt{(x-x_2)^2 + (y-y_2)^2} \end{cases} \tag{4}$$

Among formula (4), the primary sensor is $S_0(x_0, y_0)$, two secondary sensors are $S_1(x_1, y_1)$ and $S_2(x_2, y_2)$, the measured target is $T(x, y)$, R_0, R_1 and R_2 respectively are the distance between the vibration source and the sensor $S_0(x_0, y_0)$, $S_1(x_1, y_1)$, $S_2(x_2, y_2)$, Δ_1 and Δ_2 respectively are the distance differences between S_0 and S_1, S_0 and S_2. Solving the partial differential for x, y in formula (4), it can get the under formula:

$$\begin{cases} d\Delta_1 = \left(\frac{x-x_0}{\sqrt{(x-x_0)^2+(y-y_0)^2}} - \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right) dx \\ \quad + \left(\frac{y-y_0}{\sqrt{(x-x_0)^2+(y-y_0)^2}} - \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right) dy \\ d\Delta_2 = \left(\frac{x-x_0}{\sqrt{(x-x_0)^2+(y-y_0)^2}} - \frac{x-x_2}{\sqrt{(x-x_2)^2+(y-y_2)^2}} \right) dx \\ \quad + \left(\frac{y-y_0}{\sqrt{(x-x_0)^2+(y-y_0)^2}} - \frac{y-y_2}{\sqrt{(x-x_2)^2+(y-y_2)^2}} \right) dy \end{cases} \tag{5}$$

Among formula (5), the localization error components of the target are dx and dy, and the measurement errors of time difference are $d\Delta_1$ and $d\Delta_2$.

The figure 1 shows that β is the azimuth angle of the primary sensor and the target source, β_i is the azimuth angle of the secondary sensor and the target source. The azimuth angle formula (6) is concluded.

$$\begin{cases} \cos\beta_i = \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} \\ \sin\beta_i = \frac{y-y_i}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \end{cases} \tag{6}$$

In order to facilitate to calculate for formula (5), formula (5) is represented by the product of matrix, and the azimuth angle formula (6) is brought into formula (5), then it can be simplified into the formula (7).

$$\begin{bmatrix} d\Delta_1 \\ d\Delta_2 \end{bmatrix} = \begin{bmatrix} \cos\beta - \cos\beta_1 & \sin\beta - \sin\beta_1 \\ \cos\beta - \cos\beta_2 & \sin\beta - \sin\beta_2 \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} \tag{7}$$

Set $dX = [dx \;\; dy]^T$, $dZ = [d\Delta_1 \;\; d\Delta_2]^T$, $H = \begin{bmatrix} \cos\beta - \cos\beta_1 & \sin\beta - \sin\beta_1 \\ \cos\beta - \cos\beta_2 & \sin\beta - \sin\beta_2 \end{bmatrix}$, the formula (7) can be represented as formula (8).

$$dZ = HdX \tag{8}$$

Then

$$dX = H^{-1}dZ \tag{9}$$

Formula (10) can be obtained according to the inverse matrix calculating method of the second order matrix:

$$H^{-1}=\frac{1}{|H|}\begin{bmatrix} 2\cos\left(\frac{\beta+\beta_2}{2}\right)\sin\left(\frac{\beta-\beta_2}{2}\right) & -2\cos\left(\frac{\beta+\beta_1}{2}\right)\sin\left(\frac{\beta-\beta_1}{2}\right) \\ 2\sin\left(\frac{\beta+\beta_2}{2}\right)\sin\left(\frac{\beta-\beta_2}{2}\right) & -2\sin\left(\frac{\beta+\beta_1}{2}\right)\sin\left(\frac{\beta-\beta_1}{2}\right) \end{bmatrix} \quad (10)$$

The $|H|$ can be represented as:

$$|H|=4\sin\left(\frac{\beta-\beta_1}{2}\right)\sin\left(\frac{\beta-\beta_2}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right) \quad (11)$$

The formula (9) can be transformed into formula (12).

$$\begin{bmatrix} dx \\ dy \end{bmatrix}=\begin{bmatrix} \frac{\cos\left(\frac{\beta+\beta_2}{2}\right)}{2\sin\left(\frac{\beta-\beta_1}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)} & -\frac{\cos\left(\frac{\beta+\beta_1}{2}\right)}{2\sin\left(\frac{\beta+\beta_2}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)} \\ \frac{\sin\left(\frac{\beta+\beta_2}{2}\right)}{2\sin\left(\frac{\beta-\beta_1}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)} & -\frac{\sin\left(\frac{\beta+\beta_1}{2}\right)}{2\sin\left(\frac{\beta-\beta_2}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)} \end{bmatrix}\begin{bmatrix} d\Delta_1 \\ d\Delta_2 \end{bmatrix} \quad (12)$$

Set $M=\frac{\cos\left(\frac{\beta+\beta_2}{2}\right)}{2\sin\left(\frac{\beta-\beta_1}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)}$, $N=-\frac{\cos\left(\frac{\beta+\beta_1}{2}\right)}{2\sin\left(\frac{\beta+\beta_2}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)}$,

$P=\frac{\sin\left(\frac{\beta+\beta_2}{2}\right)}{2\sin\left(\frac{\beta-\beta_1}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)}$, $Q=-\frac{\sin\left(\frac{\beta+\beta_1}{2}\right)}{2\sin\left(\frac{\beta-\beta_2}{2}\right)\sin\left(\frac{\beta_2-\beta_1}{2}\right)}$

Formula (12) can be abbreviated to:

$$dX=\begin{pmatrix} M & N \\ P & Q \end{pmatrix}dZ \quad (13)$$

According to the covariance matrix of position error, there is:

$$P_{dX}=\begin{pmatrix} \sigma_x^2 & \sigma_x\sigma_y \\ \sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}=E\left[dXdX^T\right] \quad (14)$$

Put formula (13) to bring into formula (14), it can be abbreviated to:

$$P_{dX}=\begin{pmatrix} M & N \\ P & Q \end{pmatrix}\left\{E\left[dZdZ^T\right]\right\}\begin{pmatrix} M & N \\ P & Q \end{pmatrix}^T \quad (15)$$

In formula (15), $E\left[dZdZ^T\right]=diag\left\{\sigma_{\Delta_1}^2,\sigma_{\Delta_2}^2\right\}$, $\sigma_{\Delta_1}^2$ and $\sigma_{\Delta_2}^2$ are the measurement error of distance difference, the covariance matrix P_{dX} is obtained through the matrix rule, it is as follows:

$$P_{dX}=\begin{bmatrix} M^2\sigma_{\Delta_1}^2+N^2\sigma_{\Delta_2}^2 & MP\sigma_{\Delta_1}^2+NQ\sigma_{\Delta_2}^2 \\ MP\sigma_{\Delta_1}^2+NQ\sigma_{\Delta_2}^2 & P^2\sigma_{\Delta_1}^2+Q^2\sigma_{\Delta_2}^2 \end{bmatrix} \quad (16)$$

In 2D plane, $GDOP=\sqrt{trace\left[P_{dX}\right]}$, it is the root of the sum of squares for localization error.

So the geometrical dilution of precision of the localization error is:

$$GDOP=\sqrt{M^2\sigma_{\Delta_1}^2+N^2\sigma_{\Delta_2}^2+P^2\sigma_{\Delta_1}^2+Q^2\sigma_{\Delta_2}^2} \quad (17)$$

Put M, N, P, Q to bring into formula (17), it can be abbreviated to the formula (18).

$$GDOP=\sqrt{\frac{\sigma_{\Delta_1}^2}{4\sin^2\left(\frac{\beta_2-\beta_1}{2}\right)\sin^2\left(\frac{\beta-\beta_1}{2}\right)}+\frac{\sigma_{\Delta_2}^2}{4\sin^2\left(\frac{\beta_2-\beta_1}{2}\right)\sin^2\left(\frac{\beta-\beta_2}{2}\right)}} \quad (18)$$

Through the analysis of the formula (18), the GDOP value of localization results is greater, the localization error is the greater and the accuracy is lower. At the same time, the influence of the different target azimuth on the final measurement error also is very big ,when $\beta_2=\beta_1$, $\beta_2=\beta$, $\beta_1=\beta$, the azimuth angle of each sensor and the target source are all same , the localization error is infinity, it will not be able to locate at this time. At the same time as we can see that the measurement error of the distance between each sensor and the distance size between the sensors all can affect the accuracy of measurement

IV. Simulation Experiment

By the analysis of the total differential positioning accuracy algorithm, the factors, include the geometric layout、 the target source location and the time difference measurement error, all influence the localization accuracy of the positioning system. In order to determine the inner relationship of the geometric accuracy and the site measurement error, the target azimuth angle and the sensor layout, the simulation experiment was done to calculate the GDOP value.

To evaluate the performance of the algorithm, the simulation experiment was done by using the total differential positioning accuracy algorithm.

A. Linear sensor layout simulation

The simulation analysis of the geometric accuracy for linear sensor array was done to research the relationship of the localization accuracy of sensor and the sensor layout.

Select three sensors (x_0,y_0), (x_1,y_1), (x_2,y_2), its coordinates respectively were $(0,0)$, $(0,-100)$, $(0,100)$, the vibration source coordinates was (x,y), the measurement error of each sensor site is 0.5 m, the standard deviation of time measurement error are equal, they all are 0.1 ms, the propagation speed of wave on the surface is 130 m/s.

Brought the parameters into GDOP geometric accuracy formula (18) to do the simulation, it can get the three-dimensional image of the source coordinate (x,y) taking continuous values in a fixed interval, it as shown in figure 2.

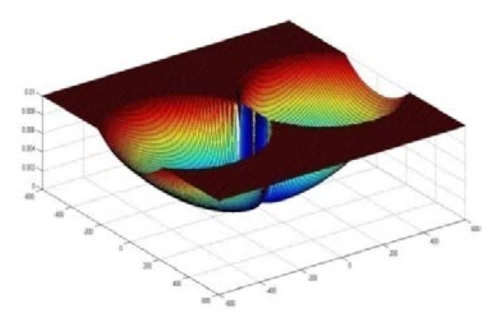


Fig.2. Geometric accuracy distribution

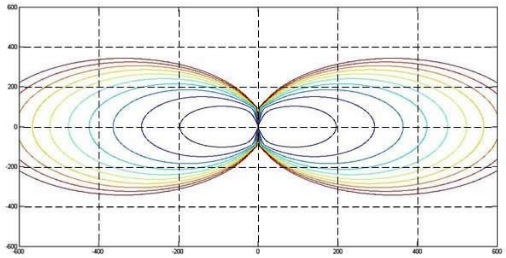


Fig.3. Precision error distribution

Transverse projection was done for section in the Z semi-axis, we made the judgment for the error value which is obtained by GDOP geometric precision formula, the Z axis values is the threshold that the system set, it is unable to locate when GDOP value is greater than the threshold, the GDOP distribution is shown in figure 3.

The figure 3 shows that the localization accuracy was the highest for linear sensor layout in the midline vertical direction, the accuracy error was very big highest on the extension position of the three sensors, so the linear sensor array was not able to adopted, it was unable to locate when the target was located on the sensor attachment and extension position, it should choose to the sensor layout which the azimuth angle between target and sensor was larger.

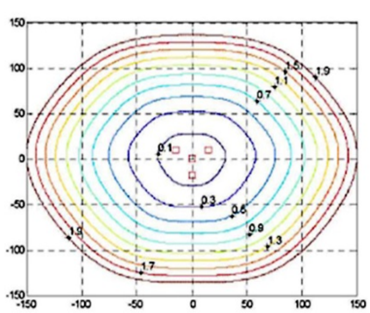
B. Four kinds of sensors layout simulation

Common sensor site layout including Y layout, the diamond layout，the ladder layout and the parallelogram layout , this paper respectively simulated for the four kinds of typical sensor layout. The location of the four sensor layout coordinates as shown in table 1.

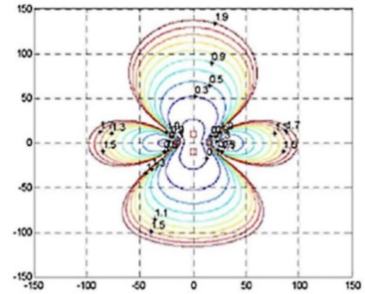
The accuracy error distribution of the four sensor layout was as shown in figure 4. Figure 4(a), figure 4(b), figure 4(c) and figure 4(d) respectively were on behalf of the accuracy error distribution of the Y layout, the diamond layout、 the ladder layout and the parallelogram layout.

Tab. 1. Four kinds of sensor layout coordinate

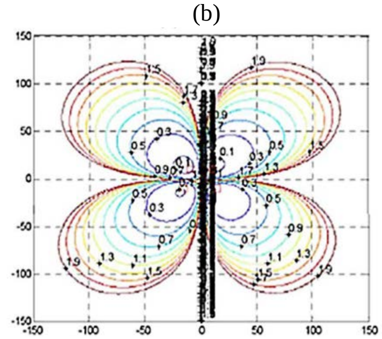
Layout / Coordinate	Y layout	Diamond layout	Ladder layout	Parallelogram layout
(x_0,y_0)	(0,0)	(0,0)	(0,0)	(0,0)
(x_1,y_1)	(100,50)	(100,0)	(100,0)	(50,0)
(x_2,y_2)	(-100,50)	(50,50)	(20,60)	(30,50)
(x_3,y_3)	(0,-100)	(50,-50)	(80,60)	(80,50)



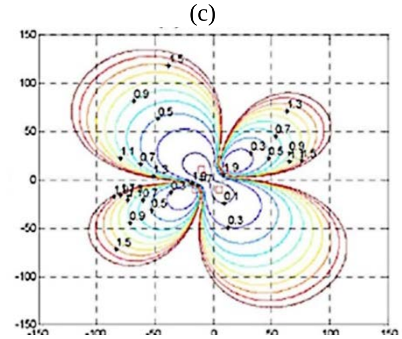
(a)



(b)



(c)



(d)

Fig.4. The accuracy error distribution of four sensor layout

Through the above you can see, the accuracy error distribution of Y layout was more uniform, it was the circular distribution of (x_0, y_0) as center, the accuracy increased when the distance between the target source and the primary sensor was bigger; the accuracy was high in $\pm 50^\circ$ area around y axis for the diamond layout and the localization error was bigger in other area; the positioning error was bigger in $\pm 15^\circ$ area of the x axis and y axis for the ladder layout and the accuracy was high in other area; the accuracy was high in $\pm 40^\circ$ area of the target source and the main diagonal for the parallelogram layout and the accuracy was low in $\pm 30^\circ$ area of the target source and the other diagonal.

In the multi-node cooperative localization system, the localization accuracy distribution of Y layout was more uniform and the accuracy was high; the accuracy distribution of different direct was nonuniform when the sensor layout was the diamond array, the ladder array or the parallelogram preach array, but the positioning accuracy was higher than Y array in a fixed direction. Therefore, it should be used Y array when the target direction was uncertainty; it should be choose the sensor layout which the positioning accuracy is highest when the target direction is certainty according to the target direction and the regional environment

V. Conclusion

This paper research the influence of the sensor geometry factor on multi-node cooperative localization accuracy. By using the total differential method to analyze the time difference equation, these factors, include the sensor layout, the target source location, the azimuth angle of sensors and the target, can produce an effect on the localization accuracy. This paper respectively simulate for the five kinds of typical sensor layout including the linear layout, the Y layout, the diamond layout, the ladder layout and the parallelogram layout. Simulation experiments prove that:

1) The linear layout is unable to locate when the target was located on the sensor attachment and extension position, it should choose to the sensor layout which the azimuth angle between target and sensor was larger.
2) It should be used the Y array when the target direction is uncertainty; it should be choose the sensor layout which the positioning accuracy is highest when the target direction is certainty according to the target direction and the regional environment.

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