Mathematical methods to locate touch points using laser optic modules

Sang-Young Cho

***Abstract*—**An interactive whiteboards are used in a variety of settings, including in classrooms, in corporate board rooms, in training rooms, in broadcasting studios, and others. Interactive whiteboard implementation is heavily dependent on touchscreen technologies. Existing technologies are suffering from high manufacturing cost, low resolution, screen size scalability when the display screen size is over 100 inches. In this paper, we introduce a new interactive whiteboard system that uses laser modules. The operational principal of the system is similar to that of the camera-based optic system. However, our system positions touches more accurately by using laser emitter and detector, and its operation is simpler due to the absence of image processing. We derived several expressions based on trigonometry calculation to operate the whiteboard including locating laser sensors, screen positioning, calibration, distortion detection, and point locating. With the simplest form of expressions, the operating hardware has little burden to consume software time.

***Keywords*—**calibration, laser sensor, touchscreen, interactive whiteboard.

I. Introduction

An interactive whiteboard is a large interactive display in the form factor of a whiteboard. It can either be a standalone touchscreen computer used independently to perform tasks and operations, or a connectable apparatus used as a touchpad to control computers from a projector. They are used in a variety of settings, including classrooms at all levels of education, in corporate board rooms and work groups, in training rooms for professional sports coaching, in broadcasting studios, and others [1].

Interactive whiteboard implementation is heavily dependent on touchscreen technologies such as resistive, surface acoustic wave, capacitive, infrared grid, infrared acrylic projection, and optical imaging [2]. A touchscreen is an input and output device normally layered on the top of an electronic visual display of an information processing system. A user can give input or control the information processing system through simple or multi-touch gestures by touching the screen with a special stylus or one or more fingers [3].

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S.-Y. Cho is with the Division of Computer & Electronic Systems Engineering, Hankuk University of Foreign Studies, Yongin, Gyeonggi, 17035, South Korea (corresponding author to provide phone: +82-10-3331-4298; e-mail: sycho@hufs.ac.kr).

Resistive technology is the first generation of touchscreens [4]. It uses two transparent electrically resistive layers. When the user touches the screen, the two layers are connected at the touch point and work as voltage dividers, and the touch location is then calculated. The technology has disadvantages of poor durability and optical quality, and lack of multi-touch. In capacitive-based touch panels, electrodes are arranged as rows and columns and are separated by an insulating material [5]. Capacitive touch panels are most commonly used in smartphones because they support multitouch without altering the visibility and transparency of the display. It suffers from high manufacturing cost when the screen size is large. In surface acoustic wave schemes, the touch position is detected by acoustic waves [6]. It has lower cost, better optical performance, higher durability, and easier integration compared to capacitive method but it requires heavy force for touch and bezel to cover reflectors. In the infrared grid scheme, two adjacent sides of a touch screen are equipped with light emitting diodes, which face photodetectors on the opposite sides, forming an infrared grid pattern [7]. This technology is not suitable for interactive whiteboard applications because of inadequate resolution, slow speed, and limited touch object size.

In camera-based optical touchscreen, a peripheral backlight is provided via IR LEDs in the corners of the screen with a retroreflector around the periphery of the screen. As a result of the retroreflectors, light is radiated from the edges of the screen across the surface of the screen. Cameras are placed in two or more corners of the screen; when a finger touches the screen, the peripheral light is blocked and a shadow is seen by the cameras. Then touch position is calculated with image processing and trigonometry [8]. It has advantage of lower cost for larger touch screen but suffers from low resolution. Currently, there are many researches for human-machine interaction beyond 2-D touch interface [9].

An interactive whiteboard can use a touchscreen computer. However, touchscreen technique has some drawbacks such as high-cost and implementation difficulty for more than 100-inch displays [10]. Therefore, cheap and scalable touchscreen methods have been used for interactive whiteboard implementation. Commercially available large-scale touchscreen technologies are optical methods and ultrasonic methods. Optical methods include optical image sensor method and infrared grid method.

In this paper, we introduce a new interactive whiteboard system that uses laser modules. The module consists of spinning motor, laser LED, photo detector, and some optic systems. The whiteboard hardware system is composed of optic sensing board, reflective bar, and display. The principle of our system is similar to the camera-based optic scheme. However, our system positions touches more accurately due to the laser emitter and simpler due to the absence of image processing. We suggest software methods based on trigonometry calculation to operate the whiteboard including locating laser sensors, screen positioning, calibration, distortion detection, and point locating.

This paper organized as follows. In Section 2, we introduce the new interactive whiteboard system and explain the operation principals. Section 3 derives several expressions for the whiteboard system operations such as installation, distortional detection, touch locating, and calibration. The paper concludes with Section 4.

II. Touchscreen Method Using Laser Optic Modules

A. Overall system

Fig. 1 shows the overall view of the touchscreen methods using laser optic modules. The main component is optic module that contains two laser sensors. Three reflective bars are installed left, right, and down sides of the display plane.

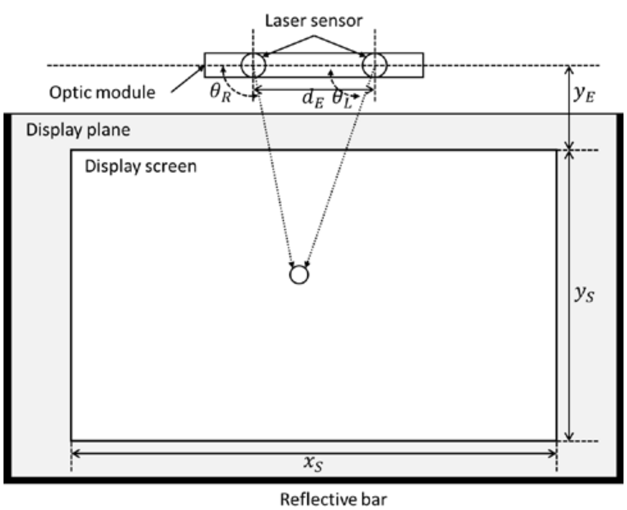

Fig. 1 The overview of the laser sensor touchscreen

Each laser sensor has a red laser emitter, a light detector, and a 7200-rpm motor that rotates 45-degree prism counterclockwise. Fig. 2 shows the laser sensor operations. The rotating prism reflects the light of the laser emitter so that the light beam rotates along the reflective bar and non-reflection path. The reflected light from the bar can reach the light detector if there is no touch on the display plane. The light detector generates high signal if it detects light. The detector generates low signal when the emitted light goes through the non-reflection path or when a touching object absorbs and scatters the light. As shown in Figure 2, the light detector generates a continuous square wave if there is no touch. The period of the square wave is the time of one rotation of the motor. However, if there is a screen touch, then the wave has a hole and we can measure the time from the start of high signal to the hole. By simple calculation, we can get a geometrical angle between the line of the start location of the left reflective bar and the laser sensor and the line of the touching object and the laser sensor. We can calculate the position of the touching object with two right and left laser sensors by applying trigonometric method [5].

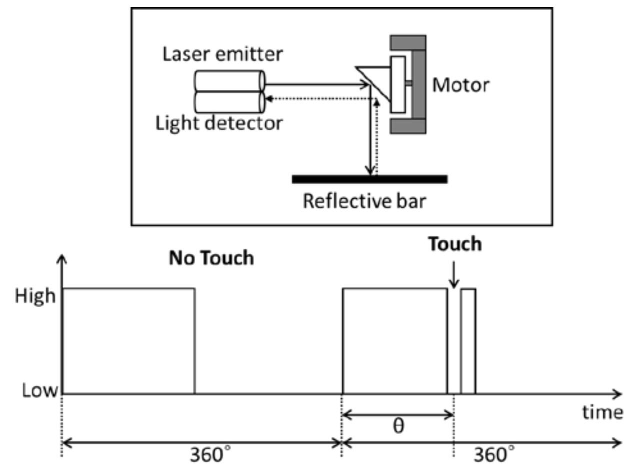

Fig. 2 Internal structure of laser sensor and operation

B. Obstacles of Whiteboard Based on Laser Sensors

We assume that the whiteboard system uses a beam projector and the optic module and the display plane are installed on a wall. Because the entire whiteboard system requires individual parts to be installed separately, it is not easy to install it correctly in a predetermined position. The Optic module may not be parallel to the x-axis of the display plane or its center may not position at the center of the display plane. The larger the display plane, the more error it produces. The size of screen image shot by the actual projector cannot be determined in advance, and after the installation, it is necessary to adjust the shape of the projected image.

In order to calculate the touch point by applying trigonometry, it is necessary to know the exact sensor position, the distance between the sensors, and the size of the horizontal and vertical dimensions of the actual screen. The geometrical values of the whiteboard can be obtained through actual measurements after installation of the system, but it is not easy work because every measurement may contain errors also. So, the whiteboard system requires a simple and accurate method to get the geometrical values of the system.

III. Geometry Calculation of Installed Whiteboard System

In order to determine each position of a whiteboard system on the 2-D plane, a distance unit and a reference point is required for calculation of each point. We can know only the resolution of the display screen exactly because it is set by the computer that originates the image for projecting on the screen. When an image is projected by the projector, the size of the pixels constituting the image is changed by the position and direction of the projector. In this paper, we derive the formula for the two cases to obtain the position values of the whiteboard system. One is to determine the size of the display screen projected by the projector in a situation where the distance between the sensors of the optic module is correctly known. Through this, the distortion of the image of the projector can be verified with respect to the optic module. The other is to locate the laser sensor of the optic module when the image projected by the projector has the correct magnification. This makes it possible to check the installation error of the optic module for the projector image.

A. Geometric Position of Display Screen

We describe how to find the positions of the four corner points of the diplay screen by the projector, assuming that the optic module and display plane are correctly installed. Finding the positions of the four points can verify that the projector is installed and adjusted correctly. Fig. 3 shows the geometric values for a general expression for obtaining the touch location when touching the screen.

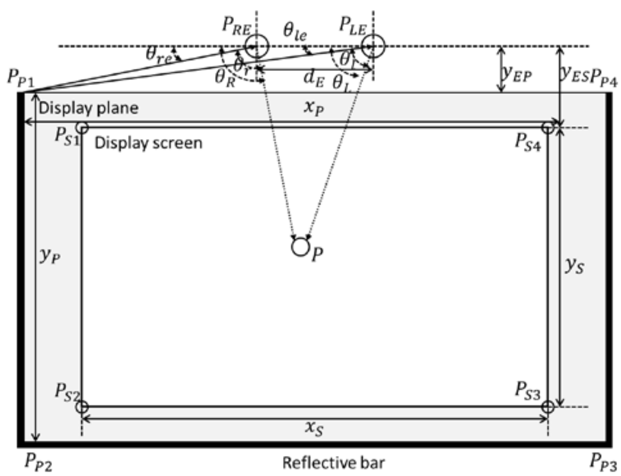

Fig. 3 Geometrical values for touch position calculation

If someone touches the P position on the display screen, θ_r and θ_l can be measured by the optic module. Using these angle degrees, we can find the positon of the point P. To facilitate calculation, we calculate θ_R and θ_L by adding the fixed values of θ_{re} and θ_{le}. θ_{re} is the angle between the straight line connecting the position of the right laser sensor P_{RE} and the point P_{P1} where the reflective bar starts and the straight line of the optic module. θ_{le} is the angle between the straight line connecting the position P_{LE} of the left laser sensor and the point P_{P1} and the straight line of the optic module. θ_{re} and θ_{le} can be calculated with the pre-determined values. The height from P_{P1} to the optic module is y_{EP} and the horizontal distance from P_{P1} to P_{RE} is $x_P/2 - d_E/2$. Then θ_{re} is $arctan(y_{EP} / (x_P/2 - d_E/2))$ and θ_{le} is obtained in the same way.

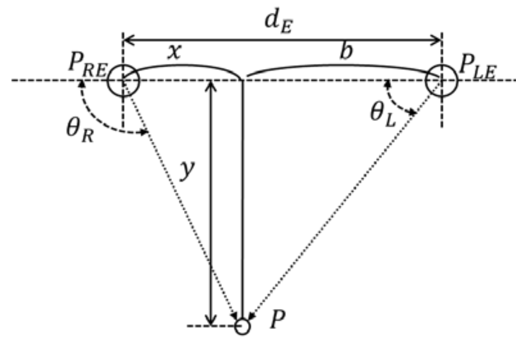

Fig. 4 Finding a touch position using optic module

We derive expressions for the position of a touch point P using θ_R and θ_L in Fig. 4. Though finding the lengths of x and y is well known from trigonometry application, we are to use a fast expression when a floating-point unit (FPU) is available. $x = y/tan(\pi - \theta_R) = -ycot(\theta_R)$ and $b = ycot(\theta_L)$. $d_E = x + b = y(cot(\theta_L) - cot(\theta_R))$. Therefore, $y = d_E/(cot(\theta_L) - cot(\theta_R))$ and $x = -ycot(\theta_R)$. These expressions uses only five FPU operations.

The actual positions of the four points (P_{S1}, P_{S2}, P_{S3}, P_{S4}) shown in Fig. 3 created by the projector can be obtained easily by touching each of the four points and measuring the corresponding angles of each point. Using the position information, we can check the distortion of the screen based on the optic module: x-y distortion and resolution distortion.

B. Geometric Position of Laser Sensors

Due to installation difficulties of optic module, it is required to adjust the position of the optic module according to the display screen. Even though we install the optic module geometrically correctly, the optic module may not behave ideally because the optic module contains mechanical parts and produces behavioral errors. We assume that projector screen is correctly positioned and resolution magnification is exactly proportional to the pixel size of original image. In this case, we know the resolution of a display screen image and the pixel size can be used as a distance unit. We are to find the laser sensors location using three touch points.

Fig. 5 is for finding the locations of two laser sensors. We explain the method to find the location of the left laser sensor and the same method can be applied to the right laser sensor. We assumed that the image resolution is known a priori. If the image resolution is FHD, then x_S is 1920 pixels and y_S is 1080 pixels. The three points (P_1, P_2, P_3) are displayed with fixed positions. Let the distance between P_1 and P_2 be the same to the distance between P_2 and P_3 with a pixel size units. By touching each point we can get the measured angle of the point. Then we obtain θ_r by subtracting the measured angle of P_1 from the measured angle of P_2. Similarly, θ_l is obtained with the measured angles of P_2 and P_3.

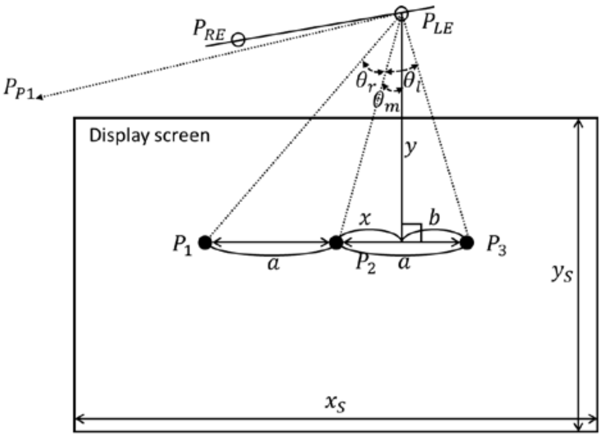

Fig. 5 Geometrical values for laser sensor locations

In order to find the position of the left laser sensor, a perpendicular is drawn from P_{LE} to the straight line connecting P_1 and P_3 and let the intersection point be *A*. Let *x* be the distance from *A* to P2 and let *y* be the distance from *A* to P_{LE}. We can know the position of P_{LE} if we can calculate *x* and *y* with θ_r, θ_l, and *a*. From the values of Fig. 5, we get the following expression (1), (2), and (3).

$$tan\ \theta_m = x/y \quad (1)$$

$$tan\,(\theta_r + \theta_m) = (a + x)/y \quad (2)$$

$$tan\,(\theta_l - \theta_m) = (a - x)/y \quad (3)$$

By applying (1) to (2) and (3), we can obtain (4).

$$tan\,(\theta_r + \theta_m) - \tan\theta_m = tan\,(\theta_l - \theta_m) + \tan\theta_m \quad (4)$$

Using the tangent addition theorem, (4) can be transformed as (5) and (6).

$$\frac{tan\ \theta_r + tan\ \theta_m}{1 - tan\ \theta_r\ tan\ \theta_m} - \tan\theta_m = \frac{tan\ \theta_l - tan\ \theta_m}{1 + tan\ \theta_l\ tan\ \theta_m} + \tan\theta_m \quad (5)$$

$$\frac{tan\ \theta_r + tan\theta_r\ tan^2\theta_m}{1 - tan\ \theta_r\ tan\ \theta_m} = \frac{tan\ \theta_l + tan\theta_l\ tan^2\theta_m}{1 + tan\ \theta_l\ tan\ \theta_m} \quad (6)$$

Then, $tan\ \theta_m$ can be obtained by (7) and θ_m is calculated with (8).

$$tan\,\theta_m = \frac{tan\,\theta_l - tan\,\theta_r}{2\ tan\ \theta_r\ tan\ \theta_l} \quad (7)$$

$$\theta_m = arctan\,\left(\frac{tan\,\theta_l - tan\,\theta_r}{2\,tan\,\theta_r\ tan\,\theta_l}\right) \quad (8)$$

Finally, the values of *x* and *y* can be obtained using (9) and (10) that are transformed from (1) and (2).

$$y = \frac{a}{tan(\theta_r + \theta_m) - tan\,\theta_m} \quad (9)$$

$$x = y\ tan\ \theta_m \quad (10)$$

Using the position information of two laser sensors, we can verify the correctness of optic module installation or can adjust the sensor installation.

C. Finding x-y Distortion Ratio

Distortion may occur in the display screen generated by a beam projector. Here we deal with distortions that occur under the assumption that the display screen maintains a rectangular shape. If there is no distortion in the screen image, the shape of the screen is displayed at the ratio of the resolution of the screen. However, if the size of the screen changes in the x-axis or y-axis, the size of the pixel in the x or y direction will change. In this case, an error may occur in the calculation of the position of the touch point, and it is necessary to detect the error and control the beam projector to produce correct screen images. When the correction is impossible, the touch position can be more accurately obtained through some calculation compensation according to the degree of distortion. Since the size of the display screen generated by the beam projector is determined by the installation and adjustment of the projector, the absolute size is meaningless and the aspect ratio of the horizontal and vertical directions of screen is important. Therefore, we assume that the screen is displayed correctly in the horizontal direction of the screen in accordance with the resolution. We show how to calculate the ratio of vertical size based on the horizontal size of a pixel by measuring the optic module.

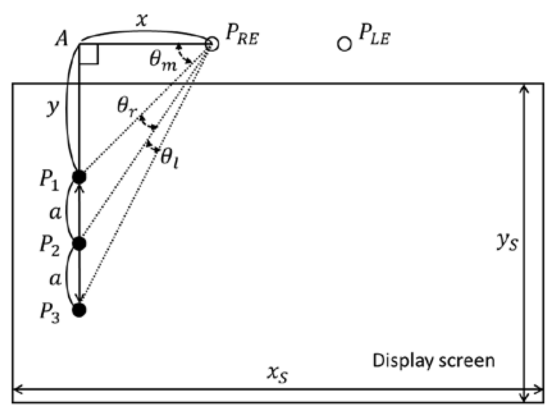

Fig. 6 Finding x-y distortion using three vertical points

Fig. 6 shows the three points (P_1, P_2, P_3) and related values for x-y distortion measuring. Three points are displayed at the same interval *a* in the vertical direction on the screen. The θ_r and θ_l values are measured by touching the three points. The measured value of *a* is obtained by using the The θ_r and θ_l values and the previously known *x* value. If the measured value is smaller than the displayed number of pixels, the screen is reduced in the *y*-axis direction. If it is larger, the screen is enlarged.

From the values of Fig. 6, we get the following expression (11), (12), and (13).

$$tan\ \theta_m = y/x \quad (11)$$

$$tan\,(\theta_r + \theta_m) = (a + y)/x \quad (12)$$

$$tan\,(\theta_l + \theta_r + \theta_m) = (2a + y)/x \quad (13)$$

By applying (11) to (12) and using the tangent addition theorem, we can obtain (14).

$$\frac{\tan\theta_r+\tan\theta_r\tan^2\theta_m}{1-\tan\theta_r\tan\theta_m}=a/x \quad (14)$$

Similarly, we can get (15) from (13).

$$\frac{\tan(\theta_l+\theta_r)+\tan(\theta_l+\theta_r)\tan^2\theta_m}{1-\tan(\theta_l+\theta_r)\tan\theta_m}=2a/x \quad (15)$$

From (14) and (15), (16) is obtained.

$$\frac{2\tan\theta_r}{1-\tan\theta_r\tan\theta_m}=\frac{\tan(\theta_l+\theta_r)}{1-\tan(\theta_l+\theta_r)\tan\theta_m} \quad (16)$$

Then, $\tan\theta_m$ can be obtained by (17) and θ_m is the arctangent of (17).

$$\tan\theta_m=\frac{2\tan\theta_r-\tan(\theta_l+\theta_r)}{\tan\theta_r\tan(\theta_l+\theta_r)} \quad (17)$$

Finally, the measured a is obtained using (18).

$$a=x\left(\tan(\theta_r+\theta_m)-\tan\theta_m\right) \quad (18)$$

By diving the number of pixels of a used to display with the measured a of (18), we can get the x-y distortion ratio of the display screen.

D. Distortion Check at Corners of Display Screen

Distortion of the display screen is prone to occur in the corner area of the display screen. The degree of distortion of the corner area can be verified by comparing the position of a laser sensor obtained by using touch points at the center area of the screen with the position using touch points of the corner areas.

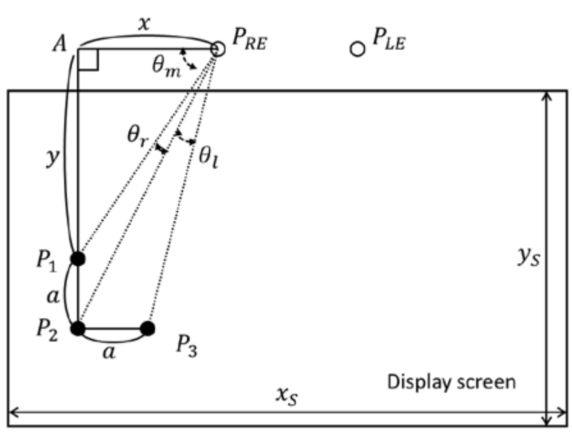

Fig. 7 Checking distortion using three points at corner areas

Fig. 7 shows how to locate a sensor in corner areas. In this case, expressions for position of the right laser sensor are derived, and a position of the left laser sensor can be obtained by the same method.

To find the location of the right laser sensor, we use three points (P_1, P_2, P_3) that form an right angle. The distance a is known and two angles θ_r and θ_l are measured by touching the points and simple calculation. Using the known and measured values, we derives an expression for θ_m, x, and y.

Let $\theta_t=\theta_r+\theta_l$. Then, we get the following (19), (20), and (21).

$$\tan\theta_m=y/x \quad (19)$$

$$\tan(\theta_r+\theta_m)=(y+a)/x \quad (20)$$

$$\tan(\theta_t+\theta_m)=(y+a)/(x-a) \quad (21)$$

By applying (19) to (20) and using the tangent addition theorem, we can obtain (22).

$$\frac{\tan\theta_r(1+\tan^2\theta_m)}{1-\tan\theta_r\tan\theta_m}=a/x \quad (22)$$

From (21), by using the tangent addition theorm, we can obtain (23).

$$(\tan\theta_t+\tan\theta_m)(x-a)=(1-\tan\theta_t\tan\theta_m)(y+a) \quad (23)$$

By dividing (2) with x and applying (19), we get (24).

$$\frac{\tan\theta_t(1+\tan^2\theta_m)}{1+\tan\theta_t+\tan\theta_m-\tan\theta_t\tan\theta_m}=a/x \quad (24)$$

We get (25) for $\tan\theta_m$ using (22) and (24).

$$\tan\theta_m=\frac{\tan(\theta_l+\theta_r)}{\tan\theta_r}-\tan(\theta_l+\theta_r)-1 \quad (25)$$

Then, x and y are (26) from (19) and (20).

$$x=\frac{a}{\tan(\theta_r+\theta_m)-\tan\theta_m},\ y=x\tan\theta_m \quad (26)$$

IV. Conclusion

The P-capacitive, infrared grid, and camera-based optic technologies are mainly used for interactive whiteboards with a screen size over 65 inches, and touch screens are usually implemented using LCD panels. However, these methods have disadvantages in that the larger the screen size, the higher the manufacturing cost or the lower screen resolution.

The method using the laser optical module introduced in this paper has the advantage that only two laser sensors are used regardless of the size of the screen and the position of the touch point can be accurately obtained even when the screen is enlarged. Because LCD panels over 100 inches are very expensive, large interactive whiteboards are usually assembled using projectors.

When using a projector for interactive whiteboard, it is not easy to install the projector and touch apparatus precisely because of its large screen size. Also, the characteristics of components may change during use and the initial settings may be not adequate to the changed situation, so that new settings may be adjusted. Software for the installation and management of interactive whiteboards is needed to automate these tasks.

In this paper, we derived expressions for extracting the information needed for management through minimum computation based on the angle data measured by the optical module. The position of the laser sensor can be located only by touching three times at the predetermined positions on the display screen, and distortion of the screen can be checked and corrected using the derived expressions.

The screen using a projector is prone to have various distortions, and in many cases, the screen is not guaranteed linearity in the calculation of touch points due to the characteristics of the optical module containing the mechanical parts. In this case, it is necessary to divide the entire screen into sub-screens having similar characteristics, and then set the reference calculation points in each sub-screen. A more sophisticated calibration technique for more accurate touch point calculations should be developed.

REFERENCES

[1] Interactive whiteboard, [Online] https://en.wikipedia.org/wiki/Interactive_whiteboard

[2] G. Walker, "A review of technologies for sensing contact location on the surface of a display," *Journal of the Society for Information Display*, vol. 20, no. 8, pp. 413–440, Aug. 2012.

[3] Touch screen, [Online] https://en.wikipedia.org/wiki/Touchscreen.

[4] C. J. William and H. S. George, "Discriminating contact sensor," U.S. Patent 3 911 215 A, Oct. 1975.

[5] G. Barrett and R. Omote, "Projected-capacitive touch technology," *Inf. Display Mag.*, vol. 26-3, pp. 16–21, 2010.

[6] K. North and H. D'Souza, "Acoustic pulse recognition enters touch-screen market," *Information Display*, pp. 22–25, Dec. 2006.

[7] A. Butler, S. Izadi, and S. Hodges, "Side Sight: Multi-touch interaction around small devices," in *Proc. 21st Annu. ACM Symp. User Interface Softw. Technol.*, pp. 201-204, 2008.

[8] G. Walker, "Camera-based optical touch technology," *Information Display*, vol. 26, no. 3, pp. 30–34, Mar. 2010.

[9] A. Nathan and S. Gao, "Interactive Displays: The Next Omnipresent Technology," *Proc. Of the IEEE*, vol. 104, no. 8, pp. 1503-1507, Aug. 2016.

[10] S. Chun, I.-S. Koo, "Beam projector calibration System based on Zigbee," *The Journal of the Institute of Internet, Broadcasting and Communication*, vol. 11, no. 2, pp. 13-19, 2011.

[11] Trigonometry, [Online] https://en.wikipedia.org/wiki/Trigonometry.

Sang-Young Cho received his B.S. degree in Control and Instrumentation from Seoul National University, Korea, in 1988, and M.S. and Ph.D. degrees in Electrical Engineering from KAIST, Daejeon, Korea, in 1990 and 1994, respectively. He is currently a full professor with the Division of Computer and Electronics Systems Engineering in Hankuk University of Foreign Studies. His research interests include embedded system, computer architecture, and software optimization.

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