Beamforming Based on Suppression Transforming Error for Array Interpolation

Shexiang Ma, Fei Pan, Xin Meng

***Abstract*—**Aiming at the inherently problems that interpolated transformation technique can not work effectively over a large transformation area, and when the burst interference occurs to the out of transformation area, it can not be suppressed. In this paper, we propose an improved algorithm by designing the weights of interpolated array to reduce the influence of transformation errors. The key feature of this algorithm is that minimizes the error of output signal power which is caused by transforming errors and guarantees the weight is orthogonal to the transforming errors of the desired signal direction. Numerical simulations confirm the validity of the proposed algorithm. In comparing with the existing algorithms, the proposed interpolated array beamforming algorithm can suppress interference signal accurately, prevent the main lobe from shifting effectively and make the null deeper in the large transformation area. Besides, it also increases the output signal to interference plus noise ratio (SINR), and has lower complexity.

***Keywords*—**beamforming, interpolated array, null depth, transforming error

I. Introduction

With the study of array antenna, dealing with more signals with fewer elements becomes the main goal. To achieving this, in 1990s, the interpolation technology was proposed by B. Friedlander firstly [1]. The freedom of array is increased and the irregular array is transformed into uniform linear array (ULA). Since then, the interpolated array has been used in direction of arrival (DOA) estimation [2-6].

In recent years, it has become a focus in applying interpolated array to beamforming. In 1997, a arbitrary real array is transformed into several identical configuration subarrays using interpolation technology was discussed by Ta-Sung Lee, so as to coherent interferences can be suppressed by using spatial smoothing [7]. In 2007, a coherent beamforming method using interpolation technique based on uniform circular array is proposed by ZhangYang [8]. In 2010, a beamforming method based on interpolated array which is applied to conformal array was proposed by P. Yang [9]. In that paper, it makes the adaptive beamforming can be directly applied to conformal array. However, the study on the relation between transform errors and area is shown that: the interference can not be suppressed when it falls outside the transform area; besides, when the transform area is too large, it occurs to zero drift and the mainlobe offset. In other words, the 'angle-sensitive' exists in interpolated array [10].

In order to solve this, a lot of improved algorithms are proposed. In 1992, multi regions interpolated array is proposed by B. Friedlander [11]. In that paper, the transformation area is divided into several regions, but the integrated error will be introduced and the suppression performance will be degraded. The method of robust interpolated array is proposed by M. Pesavento in 2002 [12]. The main idea is to impose constraints on the transform interval. However, it is difficult to determine the threshold outside the transformation area. In the same year, the optimal transformation matrix was designed by P. Hyberg [13, 14]. Yet, the optimal conversion matrix is more complicated and higher computed. In 2016, a new approach was proposed by Hu Danting to reduce the effect of the error [15]. The key operation is that the covariance matrix of interpolated array is corrected by the error covariance matrix. But the null depth formed at the interference is shallow.

In view of this situation, we proposed the interpolated steering vector and covariance matrix are constrained by the transformation error and its covariance matrix respectively. It can not only achieve beamforming in the large angle using interpolated array, but also get better performance, deeper null and greater output signal to interference plus noise ratio (SINR) when dealing with interference signal.

II. Conventional Interpolated Array

A. Signal Model

A general real ULA with M isotropic elements and d spacing is considered. Mutual coupling is ignored. If the N radiating narrowband sourcces are observed by this array, the received signal vector can then be written as:

$$x(t) = As(t) + n(t) \tag{1}$$

where $s(t) = [s_1(t), s_2(t), \cdots s_N(t)]^T$ denotes the vector of radiated signals; $n(t) = [n_1(t), n_2(t), \cdots, n_M(t)]$ denotes the vector of noise; $A = [a(\theta_1), a(\theta_2), \cdots, a(\theta_N)]$ is defined as $M \times N$ array manifold, $a(\theta)$ represents the steering vector

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of the array in direction θ and it is given by $a(\theta)=\left[1, e^{-jkd\sin\theta}, \cdots, e^{-jk(M-1)d\sin\theta}\right]^T, k=2\pi/\lambda$.

The covariance matrix is given by

$$R=E\left[x(t)x(t)^H\right] \tag{2}$$

where $E[\bullet]$ denotes expectations, H denotes conjugation transpose.

B. The Method of Interpolated Array

The study of array signal processing found that the more elements, the better resolution when the element spacing is constant. There are several methods to increasing resolution effectively. For example: cumulant [16], linear prediction [17] and array interpolation [1]. Interpolated array can convert every array into ULA, and the transformation matrix can be processed offline, so interpolation technique is used.

Assuming that the signal falls within the region Θ and it is evenly divided as:

$$\Theta=\left[\theta_l, \theta_l+\Delta\theta, \cdots, \theta_r\right] \tag{3}$$

where, θ_l and θ_r represent the left and right boundaries of Θ respectively, $\Delta\theta$ denotes the interpolation step. Then in this region, the manifold matrix of real ULA is expressed as:

$$A=\left[a(\theta_l), a(\theta_l+\Delta\theta), \cdots, a(\theta_r)\right] \tag{4}$$

So the interpolated manifold matrix in the same area can be described as:

$$\overline{A}=\left[\overline{a}(\theta_l), \overline{a}(\theta_l+\Delta\theta), \cdots \overline{a}(\theta_r)\right] \tag{5}$$

Obviously, if we want to implement the array expansion, there is a fixed relation between the real and interpolated array, such that:

$$\min_B \left\|BA-\overline{A}\right\|_F^2 \tag{6}$$

where $\|\bullet\|_F$ denotes the Frobenius norm.

When the number of points to be interpolated are larger than the elements of real array and $\overline{A}$ is full rank, the transformation matrix can be solved as:

$$B=\overline{A}A^H\left(AA^H\right)^{-1} \tag{7}$$

The covariance matrix of interpolation array can be written as:

$$\tilde{R}=BRB^H=BAR_sA^HB^H+BR_NB^H \tag{8}$$

where R_s denotes signal autocorrelation matrix, $R_N=\sigma_n^2 I$ represents noise autocorrelation matrix. Then equation (8) can be simplified as:

$$\tilde{R}=BAR_sA^HB^H+\sigma_n^2BB^H \tag{9}$$

The noise power is estimated by snapshots in fact, so it can be expressed as:

$$\hat{\sigma}_n^2=\frac{1}{\overline{M}-N}\sum_{i=1}^{\overline{M}-N}\sigma_i^2 \tag{10}$$

Here, $\sigma_i^2\left(i=1,2\cdots\overline{M}-N\right)$ are the $\overline{M}-N$ small eigenvalues of the matrix $\tilde{R}$, $\overline{M}$ denotes the number of virtual elements.

It can be seen from the equation (9) that the Gaussian white noise is contaminated after interpolation. It is necessary to whiten the colour noise. So the whitened covariance matrix can be described as:

$$\overline{R}=\tilde{R}-\hat{\sigma}_n^2BB^H+\hat{\sigma}_n^2I \tag{11}$$

According to the actual needs, there are several adaptive beamforming algorithms, such as Least Mean Square (LMS) [18], Sample Matrix Inversion (SMI) [19], Minimum Variance Distortionless Response (MVDR) [18, 20]. Since the MVDR only needs to know the direction of the desired signal, it is used in this paper.

The desired signal is incident from θ_0, the whitened covariance matrix is used to beamformimg according to the MVDR algorithm and can be expressed as:

$$\begin{cases}\min\limits_{\overline{w}} \overline{w}^H\overline{R}\overline{w} \\ s.t.\ \overline{w}\overline{a}(\theta_0)=1\end{cases} \tag{12}$$

III. Beamforming of Interpolation Array Based on Suppression Transforming Error

A. Analysis of Error on Interpolation Transform Array

Since the interpolated points are limited, the transformation error is defined as:

$$erroe=\frac{\min\limits_B\left\|BA-\overline{A}\right\|_F}{\left\|\overline{A}\right\|_F} \tag{13}$$

where $\Delta E=BA-\overline{A}$ denotes transformation error matrix in the Θ.

It can be seen from equation (13) that transformation error is always exists and determined by interpolated step and angle range. For instance, 4 elements λ spacing real ULA was transformed into 8 elements $\lambda/2$ spacing interpolated ULA. When the transformation area is selected from $0^\circ$to$180^\circ$and step is 0.1°, the relation between transformation error and angle range is shown in Fig. 1.

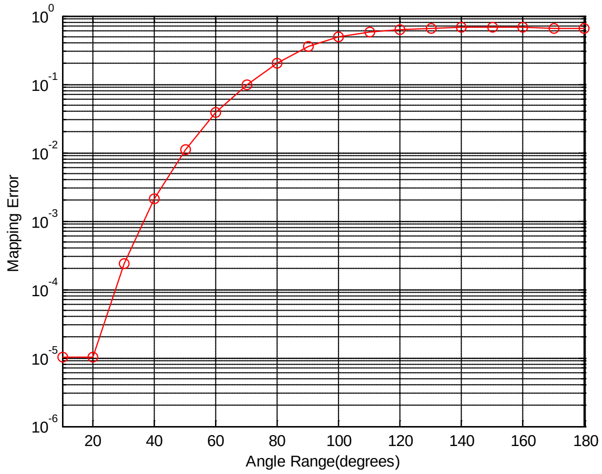

Fig. 1 Mapping error for different angle range

Fig. 1 is shown that transformation error is increased as transformation angle range increasing. When transformation angle is less than 30°, the transformation error is controlled to 10^{-3} or less. As the angle region increasing, the error is rapidly increased. When the angle is 180°, the error is greater than 0.5.

B. Beamforming of Interpolation Array Based on Suppression Transforming Error

The above analysis is shown that when transformation angle is too large, the influence of transformation error can not be neglected. In order to solve this problem, the beamforming based on suppression transforming error was proposed.

Assuming that the signal falls within the region Θ, the transformation error matrix ΔE is calculated from the formula (13), and the error of output signal can be written as:

$$y_e(t) = \bar{w}^H \Delta E \tag{14}$$

The corresponding output power is given by

$$P_e = E\left[\left|y_e(t)\right|^2\right] = E\left[\bar{w}^H R_e \bar{w}\right] \tag{15}$$

where $R_e = \Delta E \Delta E^H$ is called error covariance matrix in this paper.

In order to suppress the impact of transformation error, we minimize the error of output signal power when the weight is orthogonal to the transformation error in the desired direction. It can be described as:

$$\begin{cases} \min\limits_{\bar{w}} \bar{w}^H R_e \bar{w} \\ s.t.\ \bar{w}^H \Delta E(\theta_0) = 0 \end{cases} \tag{16}$$

Considering (12) and (16), the proposed algorithm in this paper is denoted as:

$$\begin{cases} \min\limits_{\bar{w}} \left\{(1-\alpha)\bar{w}^H \bar{R} \bar{w} + \alpha \bar{w}^H R_e \bar{w}\right\} \\ s.t.\ \bar{w}^H \bar{a}(\theta_0) = 1 \\ s.t.\ \bar{w}^H \Delta E(\theta_0) = 0 \end{cases} \tag{17}$$

Let $\hat{\bar{R}} = (1-\alpha)\bar{R} + \alpha R_e$, the above formula can be simplified as:

$$\begin{cases} \min\limits_{\bar{w}} \bar{w}^H \hat{\bar{R}} \bar{w} \\ s.t.\ \bar{w}\left[\bar{a}(\theta_0) + \Delta E(\theta_0)\right] = 1 \end{cases} \tag{18}$$

The optimal weight is calculated as:

$$\hat{\bar{w}} = \frac{\hat{\bar{R}}^{-1}\left[\bar{a}(\theta_0) + \Delta E(\theta_0)\right]}{\left[\bar{a}(\theta_0) + \Delta E(\theta_0)\right]^H \hat{\bar{R}}^{-1}\left[\bar{a}(\theta_0) + \Delta E(\theta_0)\right]} \tag{19}$$

α is defined as a constant belonging to [0,1). The larger α, the better orthogonal between weight and error, but over large α will reduce the suppression performance.

Fig. 2 shows a curve which is the eigenvalues of $\hat{\bar{R}}$ with different α. When the weight and error are orthogonal, α can't affect the small eigenvalue of $\hat{\bar{R}}$ greatly. So α is chosen as 0.3.

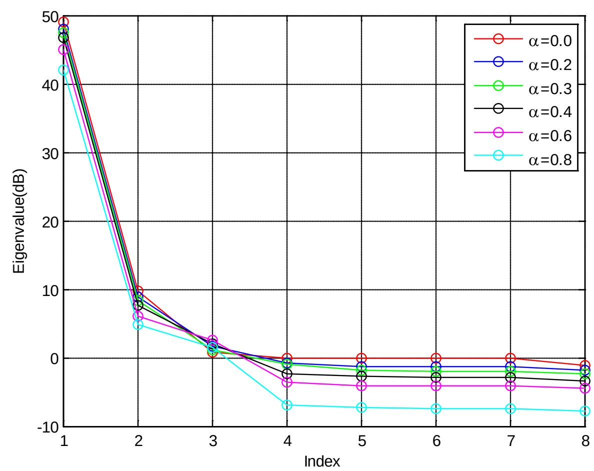

Fig. 2 Eigenvalues of $\hat{\bar{R}}$ with different α

IV. Simulation and Result Analysis

In this section, the proposed algorithm is compared with the conventional interpolated virtual array (conventional IVA) [1], multi region interpolation virtual array (multi region IVA) [11] and constraint weight interpolated virtual array (constraint weight IVA) [15]. The 4-element spacing λ ULA is extended to 8-element spacing $\lambda/2$ ULA.

A. The Situation of No Burst Interference

1) Small transformation angle

Simulation 1: analysis the beamforming performance of interpolated array. The desired signal incidents from 0°direction, add one independent interference, arriving angle is -20°. SNR=0dB, INR=40dB. The transformation areas are defined as [-30°, 0°], interpolated step is selected as 0.1, and snapshots are 200.

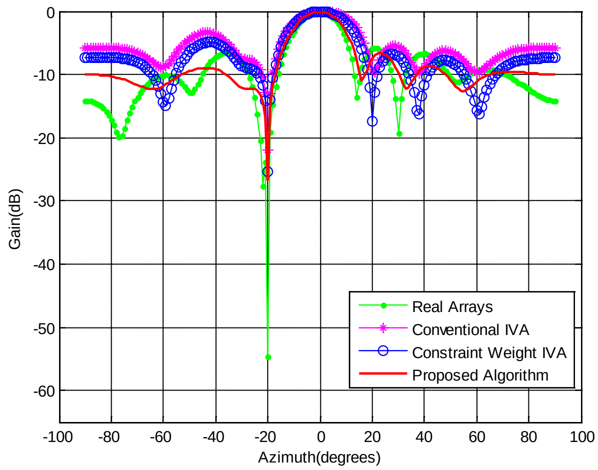

Fig.3 Analysis beamforming under small transformation angle

Fig. 3 shows that beamforming is accurately implemented by conventional IVA [1], constraint weight IVA [15] and the proposed method in this situation.

Simulation 2: analysis output SINR for different input SNR. Setting input SNR range from 0dB to 25dB. Other conditions such as simulation 1.

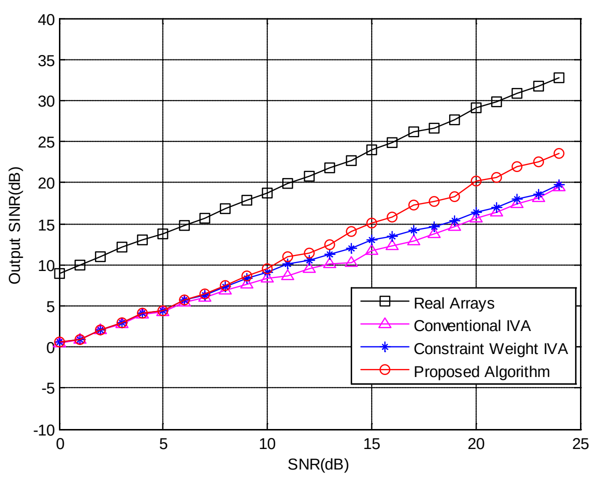

Fig.4 Analysis output SINR under small transformation angle

Fig. 4 shows that the conventional IVA [1], constraint weight IVA [15], and proposed algorithm have the same performance when the SNR is small. But the proposed algorithm becomes greater than the others when SNR is increased.

2) Small transformation angle

Simulation 3: analysis the beamforming performance of interpolated array. The desired signal incidents from 0°direction, add one independent interference, arriving angle is -20°. SNR=0dB, INR=40dB. [-90°, 90°] is divided evenly into 6 sections as the transformation area of Multi Region IVA [11]. The transformation areas of the others are defined as [-90°, 90°], step size is selected as 0.1, snapshots are 200.

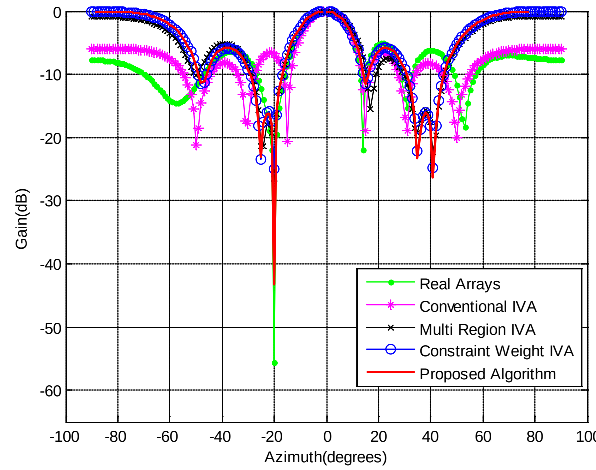

Fig.5 Analysis beamforming under large transformation angle

As shown in Fig.5, in the case of large transformation angle, the influence of transformation error can not be ignored. The interference coming from -20°can not be suppressed when conventional IVA [1] is adopted. The constraint weight IVA [15] and Multi Region IVA [11] can form a shallow null to suppress the interference. In the same situation, a deep null (about -42 dB) in the -20° is formed by the proposed algorithm. That is to say, the beamforming performance is the best than the others.

Simulation 4: analysis output SINR for different input SNR. Setting input SNR range from 0dB to 25dB. Other conditions such as simulation 3.

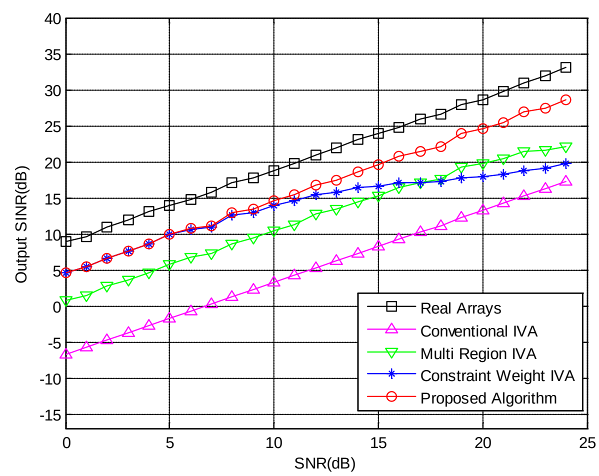

Fig.6 Analysis output SINR under large transformation angle

It can be known from the Fig. 6 that the output SINR of conventional IVA [1] is the worst, since interference can not be suppressed. Because the impact of transformation error is considered by constraint weight IVA [15] and proposed algorithm, the output SINR are better than conventional IVA [1]. Yet, with the increase of the input SNR, the proposed algorithm has obvious advantage.

B. The Situation of Burst Interference

Simulation 5: analysis the beamforming performance of interpolated array. On the basis of simulation 1, add another burst interference incident from 40°, INR=40dB. [-90°, 90°] is divided evenly into 6 sections as the transformation area of Multi Region IVA [11]. The transformation area of conventional IVA [1] is selected as [-30°, 0°], the constraint weight IVA [15] and proposed algorithm are defined as [-90°, 90°].

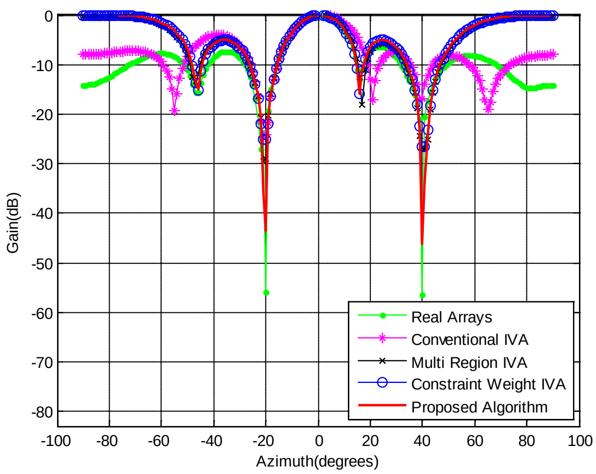

Fig.7 Analysis beamforming under burst interference

Fig. 7 shows that burst interference can not be suppressed by conventional IVA [1]. Constraint weight IVA [15] and proposed algorithm transform area are selected as [-90°, 90°], which can suppress any interference and the proposed algorithm has better suppression performance.

Simulation 6: analysis output SINR for different input SNR. Setting input SNR range from 0dB to 25dB. Other conditions such as simulation 5.

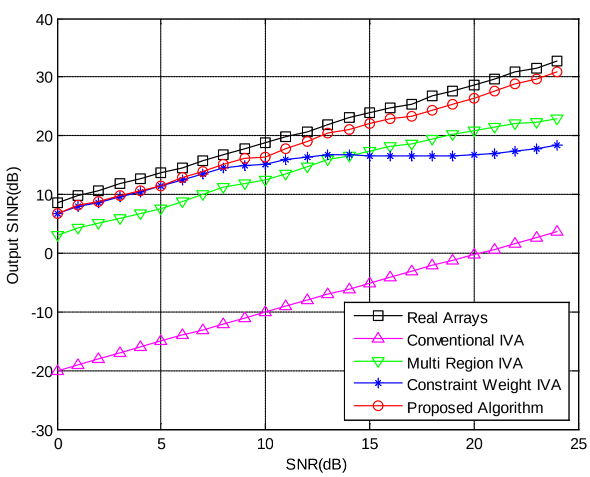

Fig.8 Analysis output SINR under burst interference

Fig. 8 shows that burst interference can not be suppressed by conventional IVA [1], so the output SINR is the worst (the maximum is about 3dB). The output SINR of constraint weight IVA [15] and the proposed algorithm are high, and the proposed algorithm is close to the 8 elements ULA (the maximum is about 31dB).

C. *analysis of beamforming performance with different interpolation step*

Simulation 7: analysis beamforming accuracy under different interpolation step. The desired signal incidents from 0°direction, add one independent interference, arriving angle is -20°. SNR=0dB, INR=40dB. The transformation area of proposed algorithm is [-90°, 90°], snapshots are 200, step are set to 0.1°and 1° respectively.

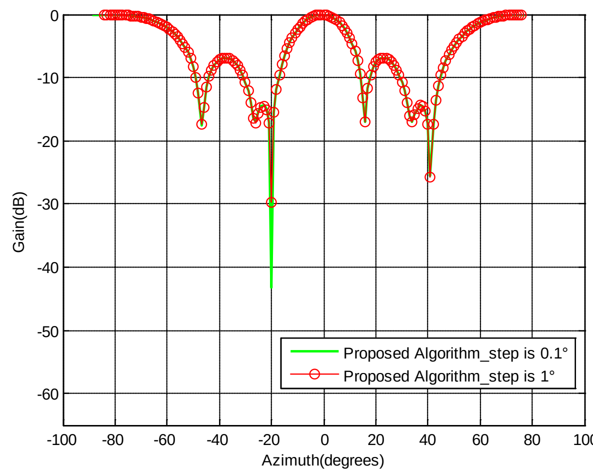

Fig.9 Analysis accuracy with different interpolation step

Fig. 9 shows that the interference also can be restrained by the proposed algorithm, but it is obvious that the depth of the nulls is shallow when the interpolation step is 1°.

Simulation 8: analysis beamforming performance under large interpolation step. On the basis of simulation 7, setting transformation area of constraint weight IVA [15] is [-90°, 90°], step size is 1°.

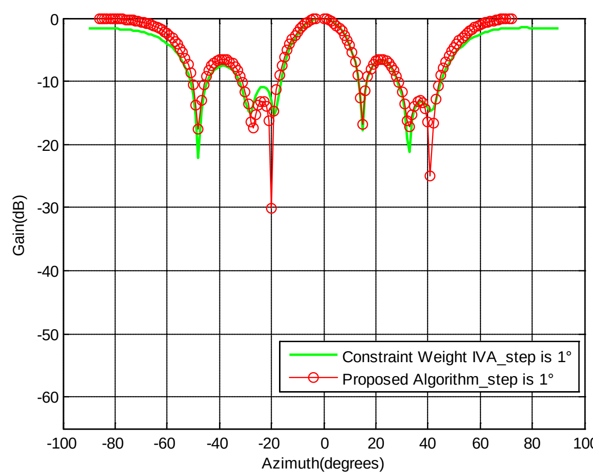

Fig.10 Analysis beamforming with large interpolation step

Fig. 10 shows that while step is 1°, the constraint weight IVA [15] aligns the desired signal with the mainlobe, but the interference can not be suppressed. Therefore, the proposed algorithm has better performance in beamforming.

V. Conclusion

In this paper we proposed a new method that minimizes the error of output power in the case of orthogonal between weights and transformation error, so that the error power contribution is minimize when the interpolated array is beamforming. Compared with the existing algorithms, it solves 'angle-sensitive'. The interference can be suppressed better; the null and the output SINR became deeper and larger.

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