

Adaptive Subspace Predictive Control and Its Application in 2-CSTR System

Baoshan Cao, Xiaosuo Luo

Abstract—A new adaptive subspace predictive control method is proposed for the problem that the control performance is ineffective when conventional control methods are applied in the processes of complex chemical production, and the chemical processes have the nonlinear and time-varying characteristics. We get the subspace predictors through input and output data by the subspace identification method. The subspace predictors are used as the prediction model to design the predictive controller directly. The predictive controller is to get the control sequence which can be obtained by minimizing the cost function and the control input is calculated from the control sequence. Based on the advantages of the integrated rolling window and the recursive identification method, the adaptive strategy of updating the subspace predictors is given. Furthermore, the decision coefficient is introduced to filter the bad data and the problem of data inaccuracy is well solved in the proposed method. The effectiveness of the proposed control method is verified by the simulation test of 2-CSTR process control system.

Keywords—Subspace identification, Predictive control, Adaptive strategy, 2-CSTR

I. INTRODUCTION

With the development of science and technology, chemical production processes are becoming more and more complex, the use of conventional control methods, such as PID control, have been difficult to get a satisfactory control performance [1]. Moreover, the chemical production processer has the nonlinear and time-varying characteristics, resulting in poor performance of the controlled processes, product quality declining [2]. Therefore, the control workers have been looking for an effective solution to chemical production problems. The model predictive control is a kind of algorithm which is produced in the field of industrial process control in the 1970s. It has a profound engineering background and theoretical significance. It is proved that the model predictive control is very suitable for chemical production engineering and has been

widely used in chemical production engineering [3]. The subspace predictors obtained in the subspace identification can be directly used as the prediction model output to form the subspace predictive control method [4-5]. Instead of the system state space model in detail, the concept of the controller is designed. When this method is applied to the complex chemical production processes, it can reflect its unique advantages.

Conventional subspace predictive control method uses a fixed linear model to design the controller, but in the chemical production processes, there are strong nonlinear and time-varying characteristics, making the control performance of this method is not ideal [6]. The characteristics of subspace identification makes the subspace identification method very suitable for the design of adaptive predictive controller, and uses the online updating subspace predictors to realize the data driving adaptive predictive control strategy. At present, the online subspace adaptive identification is basically divided into two ways: One is the recursive identification method, the process is achieved of tracking changes by using different weighting of new sampling data and historical data, which is characterized with the operation of the process of modeling data gathering is increasing; Another online identification method is the rolling window method, which is characterized by maintaining the size of the modeling data set and removing the earliest data in the modeling window when new data arrives. Paper [7] lists the recursive adaptive predictive control methods; Paper [8] lists the rolling window adaptive predictive control method. In this paper, the advantages of the two methods are combined to keep the size of the modeling dataset unchanged. At the same time, the recursive QR decomposition method in [9] to realize the online updating of the R array, and then update the subspace predictor online. The coefficient determines whether to add new data, which can effectively filter the bad data. And finally, as a prediction model, the subspace predictor is used to design adaptive subspace predictor controller, and we make the controller apply to 2-CSTR system temperature control, and an attractive control performance is achieved.

II. SUBSPACE PREDICTIVE CONTROL

Consider the following linear discrete-time invariant system form:

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (1)$$

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$$y_k = Cx_k + Du_k + e_k \tag{2}$$

where $u_k \in \mathbb{R}^l$ is the system input measurement value, $y_k \in \mathbb{R}^m$ is the system output measurement value, $x_k \in \mathbb{R}^n$ is the system process state; $e_k \in \mathbb{R}^m$ is the smooth, zero mean white noise message sequence; K is the stable state Kalman gain; (A, B, C, D) is the corresponding dimension of the system matrices.

Assuming the sampling time $k \in \{1, 2, \dots, 2i + j - 1\}$, we can construct the i row j column Hankel matrices of system input u_k , output y_k , noise e_k :

$$\begin{aligned}
 U_p &= \begin{bmatrix} u_1 & u_2 & \dots & u_j \\ u_2 & u_3 & \dots & u_{j+1} \\ \vdots & \vdots & \ddots & \vdots \\ u_i & u_{i+1} & \dots & u_{i+j-1} \end{bmatrix}, \\
 U_f &= \begin{bmatrix} u_{i+1} & u_{i+2} & \dots & u_{i+j} \\ u_{i+2} & u_{i+3} & \dots & u_{i+j+1} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i} & u_{2i+1} & \dots & u_{2i+j-1} \end{bmatrix} \\
 Y_p &= \begin{bmatrix} y_1 & y_2 & \dots & y_j \\ y_2 & y_3 & \dots & y_{j+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_i & y_{i+1} & \dots & y_{i+j-1} \end{bmatrix}, \\
 Y_f &= \begin{bmatrix} y_{i+1} & y_{i+2} & \dots & y_{i+j} \\ y_{i+2} & y_{i+3} & \dots & y_{i+j+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{2i} & y_{2i+1} & \dots & y_{2i+j-1} \end{bmatrix} \\
 E_p &= \begin{bmatrix} e_1 & e_2 & \dots & e_j \\ e_2 & e_3 & \dots & e_{j+1} \\ \vdots & \vdots & \ddots & \vdots \\ e_i & e_{i+1} & \dots & e_{i+j-1} \end{bmatrix}, \\
 E_f &= \begin{bmatrix} e_{i+1} & e_{i+2} & \dots & e_{i+j} \\ e_{i+2} & e_{i+3} & \dots & e_{i+j+1} \\ \vdots & \vdots & \ddots & \vdots \\ e_{2i} & e_{2i+1} & \dots & e_{2i+j-1} \end{bmatrix}
 \end{aligned} \tag{3}$$

where p and f represent ‘past’ and ‘future’ respectively. The predictive output can be obtained by iterating from (1) - (2):

$$Y_f = \Gamma_i X_f + H_i U_f + H_i^s E_f \tag{6}$$

where $\Gamma_i \in \mathbb{R}^{im \times n}$ is the generalized observable matrix, $H_i \in \mathbb{R}^{im \times il}$ and $H_i^s \in \mathbb{R}^{im \times im}$ is the lower triangular matrices. They are shown in the following respectively:

$$\begin{aligned}
 \Gamma_i &= \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix}, H_i = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \dots & D \end{bmatrix}, \\
 H_i^s &= \begin{bmatrix} I_m & 0 & \dots & 0 \\ CK & I_m & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}K & CA^{i-3}K & \dots & I_m \end{bmatrix}
 \end{aligned} \tag{7}$$

The optimal predictive output \hat{Y}_f of Y_f can be obtained by orthogonal projection of the row space of Y_f to the row space

of $\begin{bmatrix} W_p \\ U_f \end{bmatrix}$:

$$\hat{Y}_f = Y_f / \begin{pmatrix} W_p \\ U_f \end{pmatrix} = L_w W_p + L_u U_f \tag{8}$$

where W_p is the input and output data matrix for the past, that is $W_p = [Y_p^T \ U_p^T]^T$. L_w and L_u represent the state and the deterministic input subspace predictor matrix respectively, which can be obtained by the following QR decomposition and SVD decomposition.

Using QR decomposition:

$$\begin{bmatrix} W_p \\ U_f \\ Y_f \end{bmatrix} = R^T Q^T = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix} \tag{9}$$

where R is the next triangular matrix, Q is the orthogonal matrix. Implement orthogonal projection (8):

$$[L_w \ L_u] = [R_{31} \ R_{32}] \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix}^\dagger \tag{10}$$

where the superscript ‘ \dagger ’ represents the Moore-Penrose generalized inverse, L_u and L_w will be obtained and used to predict the design of the controller.

Consider the following predictive control objective cost function:

$$J = \sum_{k=1}^{N_2} (r_{t+k} - \hat{y}_{t+k|t})^2 + \sum_{j=1}^{N_u} \lambda (\Delta u_{t+k-1})^2 \quad (11)$$

$$= (r_f - \hat{y}_f)^T (r_f - \hat{y}_f) + \Delta u_f^T (\lambda I) \Delta u_f$$

where N_2 is the prediction horizon, N_u is the control horizon, λ is the control effect of the weighting factor, r_{t+k} is the set value of future time $t+k$. The first column of \hat{Y}_f is used to predict the future output, where we use the incremental form:

$$\hat{y}_f = Fy_t + L_w^c(1:N_2m,:) \Delta w_p + S_{N_2, N_u} \Delta u_f \quad (12)$$

where $\hat{y}_f^c = [y_{t+1} \ \dots \ y_{t+N_2}]^T$, $F = [I_m \ \dots \ I_m]^T$,

$$S_{N_2, N_u} = L_u(1:N_2m, 1:N_u l) \begin{bmatrix} I_l & 0 & \dots & 0 \\ I_l & I_l & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_l & I_l & \dots & I_l \end{bmatrix}, \quad L_w^c \text{ is}$$

constructed by L_w :

$$L_w^c(m(k-1)+1:mk,:) = \sum_{i=1}^k L_w(m(i-1)+1:mi,:) \quad (13)$$

where $1 \leq k \leq N_2$. Substitute equation (12) into (11) and obtain the control law:

$$\Delta u_f = (S_{N_2, N_u}^T S_{N_2, N_u} + \lambda I)^{-1} S_{N_2, N_u}^T (r_f - Fy_t - L_w^c(1:N_2m,:) \Delta w_p) \quad (14)$$

The first value Δu_t of Δu_f is the control input, and then recalculate the control input in the next time, so the control input of t time is:

$$u_t = u_{t-1} + \Delta u_t \quad (15)$$

III. ADAPTIVE STRATEGY

At present, most chemical processes have strong nonlinear and time-varying characteristics, so that the performance of the controller is greatly reduced by the traditional fixed model design. Therefore, the adaptive method based on the on-line updating model has been very widely concerned and gradually is applied to the chemical processes. The adaptive predictive control method has been a lot of successful applications in the chemical processes [10]. Based on the paper [9], this paper proposes an online subspace adaptive identification method, which draws the advantages of the rolling window method and keeps the size of the rolling window unchanged. The recursive identification method is used to realize the data update at the

same time. However, the disturbance and noise lead a certain time data and real data to match the larger error, this data is called bad data. So we introduce decision coefficient to check the new data, filter bad data, eliminate bad data on the adverse effects of the system, and then update subspace predictors online to achieve data-driven adaptive predictive control strategy.

Since the subspace predictors are obtained from the R matrix, the recursive algorithm updates the R matrix on-line, and then obtains the update prediction model and obtains the control input. Assuming the current time is t , the input and output data

Hankel matrix is $A = \begin{bmatrix} W_p(t) \\ U_f(t) \\ Y_f(t) \end{bmatrix}$, the earliest data is the first

column of the data matrix $b = \begin{bmatrix} w_p(1) \\ u_f(1) \\ y_f(1) \end{bmatrix}$, give $c = \begin{bmatrix} w_p(t+1) \\ u_f(t+1) \\ y_f(t+1) \end{bmatrix}$

to a set of new input and output data at $t+1$ time, to keep the size of the rolling window unchanged, it is necessary to remove b in the A and add c to obtain the input and output Hankel

matrix $D = \begin{bmatrix} W_p(t+1) \\ U_f(t+1) \\ Y_f(t+1) \end{bmatrix}$ in $t+1$ time. The QR decomposition

of A is:

$$A = R^T(t) Q^T(t) = \begin{bmatrix} R_{11}(t) & 0 & 0 \\ R_{21}(t) & R_{22}(t) & 0 \\ R_{31}(t) & R_{32}(t) & R_{33}(t) \end{bmatrix} \begin{bmatrix} Q_1^T(t) \\ Q_2^T(t) \\ Q_3^T(t) \end{bmatrix}$$

$$= \begin{bmatrix} R_{11}(t) Q_1^T(t) \\ R_{21}(t) Q_1^T(t) + R_{22}(t) Q_2^T(t) \\ R_{31}(t) Q_1^T(t) + R_{32}(t) Q_2^T(t) + R_{33}(t) Q_3^T(t) \end{bmatrix} \quad (16)$$

The QR decomposition of D can be obtained:

$$D = R^T(t+1) Q^T(t+1)$$

$$= \begin{bmatrix} R_{11}(t+1) Q_1^T(t+1) \\ R_{21}(t+1) Q_1^T(t+1) + R_{22}(t+1) Q_2^T(t+1) \\ R_{31}(t+1) Q_1^T(t+1) + R_{32}(t+1) Q_2^T(t+1) + R_{33}(t+1) Q_3^T(t+1) \end{bmatrix} \quad (17)$$

Due to $[A:c] = [b:D]$, we can obtain $[A:c][A:c]^T = [b:D][b:D]^T$, then:

$$AA^T + cc^T = bb^T + DD^T \quad (18)$$

According to (16)-(17), we can obtain

$$R_{11}(t+1) = \text{chol}\left(R_{11}(t)R_{11}^T(t) + w_p(t+1)w_p^T(t+1) - w_p(1)w_p^T(1)\right) \quad (19)$$

where chol is Cholesky decomposition. Then it is used to calculate the elements, which are used to solve the $R(t+1)$:

$$R_{21}(t+1) = \left[R_{21}(t)R_{11}^T(t) + u_f(t+1)w_p^T(t+1) - u_f(1)w_p^T(1)\right] \left[R_{11}^T(t+1)\right]^{-1} \quad (20)$$

$$R_{31}(t+1) = \left[R_{31}(t)R_{11}^T(t) + y_f(t+1)w_p^T(t+1) - y_f(1)w_p^T(1)\right] \left[R_{11}^T(t+1)\right]^{-1} \quad (21)$$

$$R_{22}(t+1) = \text{chol}\left(R_{21}(t)R_{21}^T(t) + R_{22}(t)R_{22}^T(t) + u_f(t+1)u_f^T(t+1) - u_f(1)u_f^T(1) - R_{21}(t+1)R_{21}^T(t+1)\right) \quad (22)$$

$$R_{32}(t+1) = \left[R_{31}(t)R_{21}^T(t) + R_{32}(t)R_{22}^T(t) + y_f(t+1)u_f^T(t+1) - y_f(1)u_f^T(1) - R_{31}(t+1)R_{21}^T(t+1)\right] \left[R_{22}^T(t+1)\right]^{-1} \quad (23)$$

The Eqs. (19)-(23) are substituted into the (10), the subspace predictors are obtained in $t+1$ time:

$$\begin{bmatrix} L_w(t+1) & L_u(t+1) \end{bmatrix} = \begin{bmatrix} R_{31}(t+1) & R_{32}(t+1) \end{bmatrix} \begin{bmatrix} R_{11}(t+1) & 0 \\ R_{21}(t+1) & R_{22}(t+1) \end{bmatrix}^{-1} \quad (24)$$

Then, the control input u_{t+1} of the $t+1$ time can be obtained according to the step in above part, the new input and output data is measured at the next time, and the R matrix of the next time is recursively calculated according to the above steps. The subspace predictors are updated online to realize the adaptive Strategy.

We check the new data by the determination coefficient, define ε as the coefficient of determination in:

$$\varepsilon = 1 - \frac{\delta(y_{t+1} - \hat{y}_{t+1})}{\delta(y_{t+1})} \quad (25)$$

where δ is the variance operator, y_{t+1} and \hat{y}_{t+1} are the actual output and the prediction output of $t+1$ time respectively. When no new data is added, the determination coefficient is

$$\varepsilon_p = 1 - \frac{\delta(y_{t+1} - \hat{y}_{t+1}^p)}{\delta(y_{t+1})} \quad (26)$$

When new data is added, the determination coefficient is

$$\varepsilon_f = 1 - \frac{\delta(y_{t+1} - \hat{y}_{t+1}^f)}{\delta(y_{t+1})} \quad (27)$$

By comparing the size of ε_p and ε_f , we decide whether to add new data. When $\varepsilon_p \geq \varepsilon_f$, do not add new data and the control input is continued to use at the time of t . When $\varepsilon_p < \varepsilon_f$, the new data is added and the control input is calculated by using the updated subspace predictors.

IV. SIMULATION EXAMPLE

The CSTR system is a kind of typical chemical production process with nonlinear and time-varying characteristics. Various control methods have been widely used in CSTR, and the predictive control is one of the successful methods [11-12]. 2-CSTR system, derived from the CSTR system, can effectively

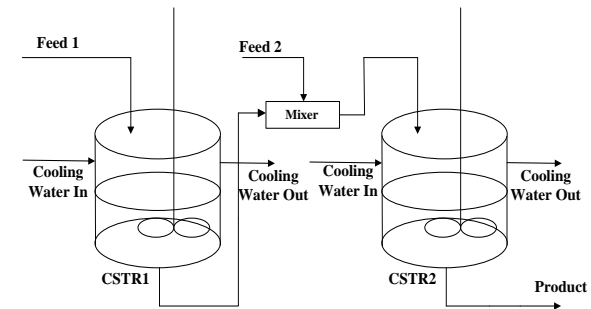


Fig. 1 2-CSTR plant

improve the quality of the device. The final product can be more pure through the two reactor exothermic reaction. At present, 2-CSTR currently has a variety of structures [13-14], taking paper [15] for example, the system structure is shown in Fig. 1.

The volume of the two reactors is kept constant, and the system of nonlinear differential equations is obtained as follows:

$$\begin{cases} \frac{dx_1}{dt} = -K_1x_1 + \frac{Q_{I1}}{V_1}(C_{I1} - x_1) \\ \frac{dx_2}{dt} = \Delta HK_1x_1 + \frac{Q_{I1}}{V_1}(T_{I1} - x_2) - \frac{U_{a1}}{V_1}(x_2 - x_3) \\ V_{J1} \frac{dx_3}{dt} = Q_{CW1}(T_{CW1} - x_3) + U_{a1}(x_2 - x_3) \\ \frac{dx_4}{dt} = -K_2x_4 + \frac{Q_{I2}}{V_2}(C_{I2} - x_4) + \frac{Q_{I1}}{V_2}x_1 \\ \frac{dx_5}{dt} = \Delta HK_2x_5 + \frac{Q_{I2}}{V_2}(T_{I2} - x_5) - \frac{U_{a2}}{V_2}(x_5 - x_6) + \frac{Q_{I1}}{V_2}x_2 \\ V_{J2} \frac{dx_6}{dt} = Q_{CW2}(T_{CW2} - x_6) + U_{a2}(x_5 - x_6) \end{cases} \quad (28)$$

where $K_1 = K_0 \exp(-E/Rx_2)$, $K_2 = K_0 \exp(-E/Rx_5)$. The six states of the system indicate the product concentration of the reactor 1: $x_1 = C_{O1}$, the product temperature: $x_2 = T_{O1}$, the

coolant effluent temperature: $x_3 = T_{CW01}$, the product concentration of the reactor 2: $x_4 = C_{O_2}$, the product temperature: $x_5 = T_{O_2}$, and the coolant effluent temperature: $x_6 = T_{CW02}$. The quality of the product to be finally obtained by the system is determined by the concentration of the product in the reactor. However, the concentration of the product in the actual product is difficult to measure on-line. It is often choose the temperature of the reactor to control the concentration. The goal is to keep the temperature inside the two reactors tracking the setpoint. Inputs $u = [Q_{I1}, Q_{I2}]^T$, represent two feed flow; outputs $y = [T_{o1}, T_{o2}]^T$, represent the temperature inside the two reactors.

When the input Q_{I1} 20% step change occurs, the system output open-loop response is shown in Fig. 2. It is indicated that the system has a strong nonlinear characteristic. In order to verify whether the subspace predictor identification model obtained by this method is matched with the actual model, the Gaussian random signal of Q_{I1} and Q_{I2} is taken as input to obtain the output signal of the system. The sampling number is set to $N = 1000$, The sampling time is set to 1s, take the first 400 data as the verification data. In order to objectively compare the data, the prediction error of the [16] is introduced:

$$\varepsilon = 100 \frac{1}{m} \sum_{c=1}^m \left[\frac{\sum_{k=1}^N ((y_k)_c - (y_k^p)_c)^2}{\sum_{k=1}^N ((y_k)_c)^2} \right] \% \quad (29)$$

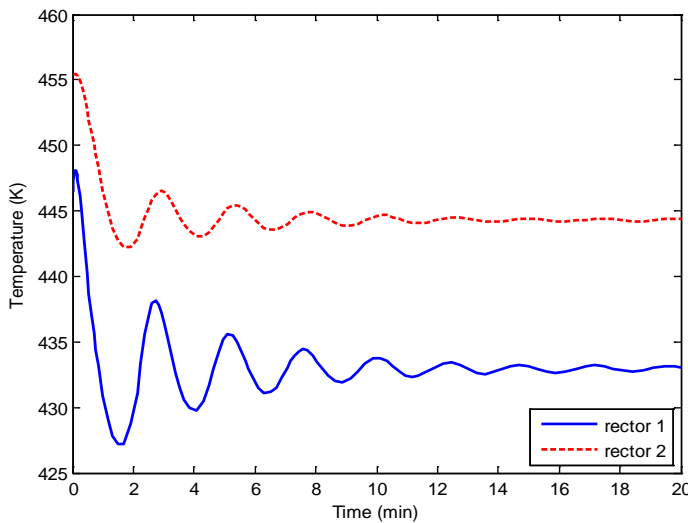


Fig. 2 Open loop response diagram of 2-CSTR system output

where the output y_k^p is one step prediction output. After the calculation, $\varepsilon = 1.583$ can be drawn. Therefore, the

identification model has good prediction ability to the system output.

The adaptive subspace predictive controller (ASPC) is designed based on the above identification model. $N = 1500$ is selected as the controller parameter, the sampling time $T_s = 1s$, the prediction horizon $N_2 = 9$, the control horizon $N_u = 3$ and the weighting matrices $Q = I_{18}$, $R = 0.1 * I_6$. To highlight the effectiveness of the algorithm, we use the linear state-space model predictive controller (SSMPC) [17] and the PID controller as the comparison. The SSMPC parameters are the same as that of the ASPC. The PID controller selects the optimal fixed gains and takes the first 1000 samples to carry out the tracking experiment. The tracking and comparison of T_{o2} is shown in Fig. 3. For the sake of clarity, the prediction error ξ is conducted to verify the output performances:

$$\xi = \sqrt{\frac{\sum_{i=1}^N (y_i - y_i^p)^2}{\sum_{i=1}^N (y_i)^2}} * 100 \quad (30)$$

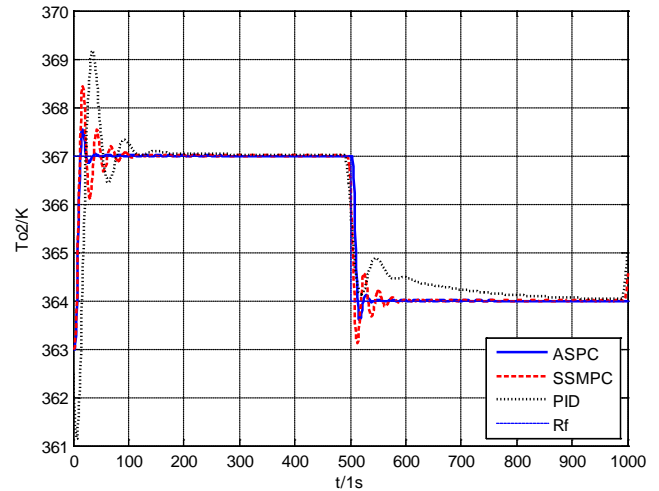


Fig. 3 Tracking comparison performance of T_{o2}

Table 1. The prediction error of the methods

Control method	ASPC	SSMPC	PID
Prediction error ξ	0.0815	0.1097	0.2136

where y_i and y_i^p are the values of the reference and process output at instant moment i , respectively. The results presented in Fig. 3 are summarized in Table 1. It can be seen that ASPC in the control performance is better than SSMPC and PID controller.

V. CONCLUSION

In this paper, a direct adaptive subspace predictive controller is designed. The real-time subspace predictors are obtained by the on-line identification algorithm to achieve the adaptive purpose, and then the system is controlled by the predictive control method. The controller is successfully applied to the simulation test of six-state 2-CSTR system. The simulation results verify the effectiveness of the controller. The primary contribution of this article is the development of a new solution of subspace predictive control ensuring adaptation of the complex chemical processes.

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