Face classification based on PCA by using the centroid of a triangle

W. Klongdee, and W. Ieosanurak

***Abstract*—** This paper focuses centroid of a triangle for a face classification. We propose a simple, fast, uncomplicated and effective classification method for a face grayscale image based on principal component analysis (PCA) in grayscale of face images. Any triangle is generated from three points, which are obtained from the combination of m (a number of image per class) distinct points taken from the same class. The classification criteria is minimum the distance between the tested image and the centroid of the triangle. The proposed method tests on the Grimace and faces94 databases. The recognition rate is compared with the nearest neighbor (NN), the nearest feature line (NFL), the shortest feature line segment (SFLS), the restricted nearest feature line with ellipse (RNFL), and the nearest and the center of ellipse (NCE). The proposed algorithm shows high performance and it has recognition rate over 90%. Moreover, we compare time spent on the experiment of the proposed algorithm and other algorithms. We found that the time of the proposed algorithm is less than other algorithms.

***Keywords*—**face classification, triangle, centroid, Euclidean distance.

I. Introduction

THERE are several methods for feature extraction and face classification. An example method for feature extractions are PCA [1], LDA [2], ICA [3] and others methods [4-13]. An example algorithms for face classification are nearest neighbour (NN) [14], the nearest feature line (NFL) [15], the nearest feature centre (NFC) [16], extended nearest feature line (ENFL) [17], the shortest feature line segment (SFLS) [18], the restricted nearest feature line with ellipse RNFLE [19], the nearest feature midpoint (NFM) [20], The nearest and the center of ellipse (NCE) [21] and others methods [22-24].

Some face classifications in this paper are follow: Cover and Hart proposed the nearest neighbor (NN) classifier, which is a simple nonparametric classification; in the meantime, Li and Lu improved the nearest neighbor, which is called the nearest feature line (NFL). Furthermore, Zhou, Zhang and Wang extended the nearest feature line, which calculates the product of the distance between the test and the two points in a training set. Likewise, Han, Han and Yang investigated the shortest feature line segment (SFLS): the SFLS used circle and tried to find the shortest feature line segment. The restricted nearest feature line with ellipse (RNFL) is proposed by Feng, Pan, and Yan, the RNFL improved the miss classification of NFL and it used the ellipse to restrict the feature line. The nearest and the center of ellipse (NCE) is presented by Ieosanurak and Klongdee, NCE uses the center of ellipse to find the distance between the tested image and the center. The ellipse is created from 3 points of the same class, then class that the minimum of the distance is class of the tested image.

In this paper, we want to improve a method for classification which is simple, fast, uncomplicated and effective classification by a triangle, so we propose a new classification method which is classified by minimum the distance between the tested image and the centroid of the triangle.

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II. Literature Review

A. The nearest feature line (NFL)

$\overline{x_i^c x_j^c}$ represents the line which is passing through x_i^c and x_j^c called a feature line (FL) of the class C_{NFL}, as shown in Fig. 1. Define x_p as the projection point of x which can be calculated by

$$x_p = x_i^c + t\left(x_j^c - x_i^c\right), \tag{1}$$

where $t = \frac{\left(x_p - x_i^c\right)^T \left(x_j^c - x_i^c\right)}{\left(x_j^c - x_i^c\right)^T \left(x_j^c - x_i^c\right)}$.

The distance between the test image x and the feature line $\overline{x_i^c x_j^c}$ can be calculated by

$$d\left(x, \overline{x_i^c x_j^c}\right) = \left|\left|x - x_p\right|\right|_2, \tag{2}$$

where $\left|\left|\cdot\right|\right|_2$ is the Euclidean distance.

The classification decision called the nearest feature distance can be defined as follow:

$$C_{\mathrm{NFL}} = \arg\min_c \left\{ \min_{1 \le i < j \le m} d\left(x, \overline{x_j^c x_i^c}\right) \right\} \tag{3}$$

for $c = 1,2,3,\dots,N_c$ and C_{NFL} is the class of the tested image.

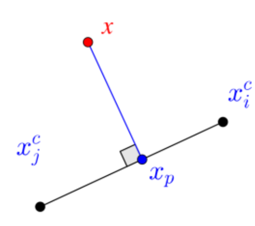

Fig. 1 the nearest feature line

B. *The shortest feature line segment (SFLS)*

This method finds the shortest feature line segment which is classified by the test, which is inside a circle of class C_{SFLS} as shown in Fig. 2. The creation of the circle form two training z_i^c, z_j^c, the distance metric of SFLS can be calculated by

$$d\left(x, x_i^c x_j^c\right) = ||x_i^c - x_j^c|| \,, \tag{4}$$

The classification decision can be defined as follow:

$$C_{\mathrm{SFLS}} = \arg\min_{c} \left\{ \min_{i,j} d\left(x, x_i^c x_j^c\right) \right\} \tag{5}$$

for $c = 1,2,3, \ldots, N_c$. Where C_{SFLS} is the class of the test image.

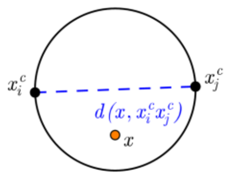

Fig. 2 the metric of the shortest feature line segment

C. *The restricted nearest feature line with ellipse (RNFL)*

The main idea of this method, which uses ellipse to restrict the feature line. Define x_i^c, x_j^c as foci of any ellipse like Fig. 3, and a_0 as the ratio between the length of ellipse major axis and the length of the center to either focus.

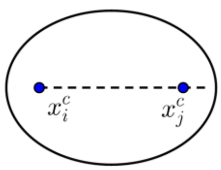

Fig. 3 x_i^c, x_j^c be foci of the ellipse

The classification decision can be defined as follows:

1. If the test (x) is inside the ellipse which is shown in Fig. 4 (Left), the distance between the tested image and the feature line $\overline{z_i^c z_j^c}$ is as follow:

$$\mathrm{d}\left(\mathrm{x}, \overline{\mathrm{x}_i^c \mathrm{x}_j^c}\right) = \left|\left|\mathrm{x} - \mathrm{x}_{ijp}^c\right|\right|, \tag{6}$$

where x_{ijp}^c represents the projection of x on the feature line $\overline{x_i^c x_j^c}$.

2. If the test (x) is outside the ellipse which is shown in Fig. 4 (Right), the distance between the test image and the feature line is as follow:

$$\mathrm{d}\left(\mathrm{x}, \overline{\mathrm{x}_i^c \mathrm{x}_j^c}\right) = \min\{||\mathrm{x} - \mathrm{x}_i^c||, \left|\left|\mathrm{x} - \mathrm{x}_j^c\right|\right|\}, \tag{7}$$

The test image is classified into class C_{RNFLE}

$$C_{\mathrm{RNFLE}} = \arg\min_{c} \left\{ \min_{1 \le i < j \le m} \mathrm{d}\left(\mathrm{x}, \mathrm{x}_i^c \mathrm{x}_j^c\right) \right\}, \tag{5}$$

where $\mathrm{c} = 1,2,3, \ldots, \mathrm{N_c}$.

D. *The nearest and the center of ellipse (NCE)*

The main idea of this method, which uses the center of ellipse to find the distance between the tested image and the center.

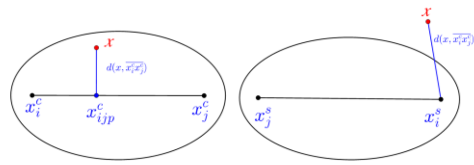

Fig. 4 the left figure shows the test image is inside the ellipse and the right figure shows the test image is outside the ellipse

The ellipse is generalized by the three points, which are obtained from the combinations of m distinct points taken of the same class, that is the ellipse is created from z_i^c, z_j^c and z_l^c. Without loss of generality, setting $\mathrm{d}(\mathrm{z}_i^c, \mathrm{z}_j^c) = \mathrm{diam}(\{z_i^c, z_j^c, z_l^c\})$ and $\mathrm{diam}(A) = \max\{d(u,v) | u, v \in A\}$ The center (h,k) is the midpoint of the z_i^c and z_j^c while $a_{c_{ijl}} = \frac{1}{2} d(z_i^c z_j^c)$ and

$$b_{c_{ijl}} = \left\| \sqrt{\frac{\left(-(x_l - h)\sin\theta + (y_l - k)\cos\theta\right)^2}{1 - \frac{\left((x_l - h)\cos\theta + (y_j - k)\sin\theta\right)^2}{a_{c_{ijl}}^2}}} \right\|_2$$

where $z_l = (x_l, y_l)$ and $|| \quad ||_2$ is Euclidean distance.

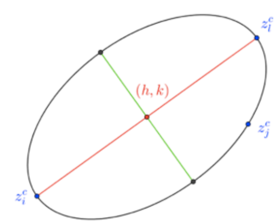

Fig. 5 the nearest feature line

The classification decision can be defined as follows:

If the test image (z) is inside the c_{ijk} ellipse and $z_c = [h\ k]^T$ represents the center of the c_{ijk} ellipse, the distance between the test and the center of the c_{ijk} ellipse is as follow:

$$\mathrm{d}\left(\mathrm{z}, \overline{\mathrm{z}_i^c \mathrm{z}_j^c \mathrm{z}_k^c}\right) = \left|\left|\mathrm{z} - \mathrm{z_c}\right|\right|_2, \tag{12}$$

If the tested image (z) is outside the c_{ijk} ellipse, or three points lie on the same line, the distance between the tested image and each point in three points of the ellipse is as follow:

$$\mathrm{d}\left(\mathrm{z}, \overline{\mathrm{z}_i^c \mathrm{z}_j^c \mathrm{z}_k^c}\right) = \min\left\{||\mathrm{z} - \mathrm{z}_i^c||_2, \left|\left|\mathrm{z} - \mathrm{z}_j^c\right|\right|_2, ||\mathrm{z} - \mathrm{z}_k^c||_2\right\}, \tag{13}$$

The test image is classified into class C_{NCE},

$$C_{\mathrm{NCE}} = \arg\min_{c} \left\{ \min_{1 \le i < j < k \le m} \mathrm{d}\left(\mathrm{z}, \overline{\mathrm{z}_i^c \mathrm{z}_j^c \mathrm{z}_k^c}\right) \mid \mathrm{c} = 1,2,3, \ldots, \mathrm{N_c} \right\}. \tag{14}$$

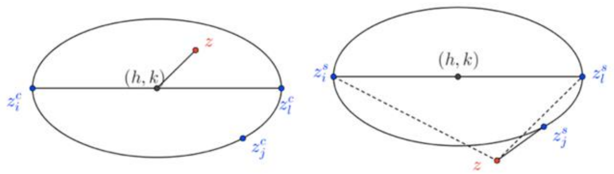

Fig. 5 The left figure shows the test image is inside the ellipse and the right figure shows the test is outside the ellipse

III. THE PROPOSED ALGORITHM

In this section, we proposed an algorithm for classification by using the centroid of a triangle. Before we classify the face image, we use the principal component analysis (PCA) to find a subset of the principle component in a set of training faces; then we project faces into the principal components space which can be gathered the feature vectors (x_i).

The shortest distance with the centroid of the triangle (SDC)

For this algorithm, each class needs at least three points. The x_i^c, x_j^c and x_k^c are the feature vectors from Principal Components Analysis (PCA) [1], which also are the three points training set from the same class c, $1 \le i < j < k < m, 1 \le c \le N_c$: N_c and m represent the number of the class and the number of images per class, respectively.

The propose algorithm or SDC based on PCA can be elucidate as follows:

Step 1. Input the training image set and read the image which is denoted by Y_i^c.

Step 2. Transform Y_i^c into a new column matrix as Z_i^c and use PCA in order to get the feature vector as x_i^c.

Step 3. The triangle is generalized by three points, which are obtained from the combination of m distinct points taken of the same class, that is the triangle is created from x_i^c, x_j^c and x_k^c. Define O_{ijk}^c as the centroid of the triangle defined by

$$O_{ijk}^c = \frac{1}{3}(x_i^c + x_j^c + x_k^c) \qquad (15)$$

Step 4. Input the tested image and read image which is denoted by x and $d(x, O_{ijk}^c)$ as the distance between the tested image and the centroid of the triangle described by

$$d(x, O_{ijk}^c) = \left\| x - O_{ijk}^c \right\|_2, \qquad (16)$$

where $\| \quad \|_2$ is Euclidean distance.

Step 5. The tested image is classified into class C_{SDC} as following form

$$C_{\mathrm{SDC}} = \arg\min_c \left\{ \min_{1 \le i < j \le m} d(x, O_{ijk}^c) \right\}. \qquad (17)$$

The recognition rate is calculated by

$$\text{The recognition rate} = \frac{N_{cr}}{N_{test}} \times 100\% \qquad (17)$$

where $\mathrm{N_{cr}}$ is number of correct recognition of the tested face images and $\mathrm{N_{test}}$ is total number of tested images.

IV. EXPERIMENTAL RESULTS

In this section, we present the experimental results of the nearest feature line (NFL), the shortest feature line segment (SFLS), the restricted nearest feature line with ellipse (RNFL), the nearest and the center of ellipse (NCE) and the proposed algorithm (SDC). The databases are used by Grimace and Faces94 as in [7]. Before we verify class of face image from various algorithms, the training image and tested image are transformed as column vector and we use PCA for feature extraction. For each database, the data is divided into 2 sets, the training set and the tested image. The training set is divided into 0.95, 0.9, 0.85, …, 0.6 proportion and the remaining is the tested set. The recognition rate is shown in Table 1,3 and 5. Time spent on the experiment is shown in Table 2,4 and 6. Moreover, we calculate percentage relative error of SDC and other algorithms which are shown in Table 7,8 and 9.

Fig. 7-9 show Time spent on the experiment results in another form as bar graph to see the results of the experiment easier. Fig. 10-12 show Percentage Relative Error of Time spent on the experiment results in another form as graph to see the results of the experiment easier.

Percentage Relative Error of Time spent on the experiment is define by

$$\frac{x-y}{\max(x,y)},$$

where x is Time spent on the experiment of SDC algorithm and y is time spent on the experiment of NFL, RNFLE, SFLS and NCE algorithms.

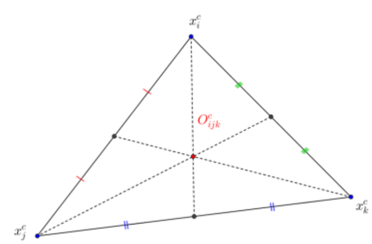

Fig. 6 O_{ijk}^c be the centroid of the triangle class c

Table 1. The recognition rate of Grimace database with various algorithms

Grimace	The recognition rate				
Proportion	NFL	RNFLE	SFLS	NCE	SDC
0.95	33.33	94.44	94.44	94.44	94.44
0.90	58.33	47.22	97.22	97.22	97.22
0.85	27.78	31.48	94.44	94.44	94.44
0.80	22.22	38.89	100.00	100.00	98.61
0.75	13.33	16.67	98.89	98.89	97.78
0.70	9.26	14.81	97.22	99.07	97.22
0.65	14.29	8.73	96.83	96.03	96.83
0.60	9.72	11.81	97.22	97.92	96.53

Table 1 shows the recognition rate of NFL, RNFLE, SFLS, NCE and SDC algorithms. The recognition rate of NCE algorithm is better than other algorithms.

Table 2 shows time spent on the experiment of NFL, RNFLE, SFLS, NCE and SDC algorithms. The minimum time spent on the experiment is SDC algorithm and the maximum time spent on the experiment is NCE algorithm.

Table 2. Time spent on the experiment with various algorithms for Grimace database

Grimace	**Time spent on the experiment (second)**				
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**	**SDC**
0.95	0.83	1.15	0.37	7.78	0.66
0.90	1.52	1.46	0.44	13.48	0.36
0.85	2.02	1.98	0.56	15.82	0.45
0.80	2.72	2.23	0.85	15.03	0.50
0.75	3.85	2.48	0.77	14.78	0.57
0.70	4.93	2.87	0.84	13.39	0.62
0.65	3.41	2.87	0.88	12.65	0.66
0.60	3.88	2.89	0.91	11.20	0.68

Table 3. The recognition rate of Faces94 (female) database with various algorithms

Faces94 (female)	**the recognition rate**				
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**	**SDC**
0.95	63.16	94.74	94.74	94.74	94.74
0.90	21.05	28.95	97.37	97.37	97.37
0.85	15.79	24.56	94.74	94.74	94.74
0.80	14.47	23.68	94.74	94.74	94.74
0.75	8.42	27.37	95.79	95.79	95.79
0.70	7.02	25.44	93.86	94.74	95.61
0.65	9.02	21.05	93.23	92.48	92.48
0.60	5.92	21.05	94.74	95.39	92.11

Table 3 shows the recognition rate of NFL, RNFLE, SFLS, NCE and SDC algorithms. The recognition rate of SFLS, NCE and SDC algorithms is same, except 0.70 proportion, SDC is better than others, 0.65 proportion, SFLS is better than others and 0.60 proportion, NCE is better than others.

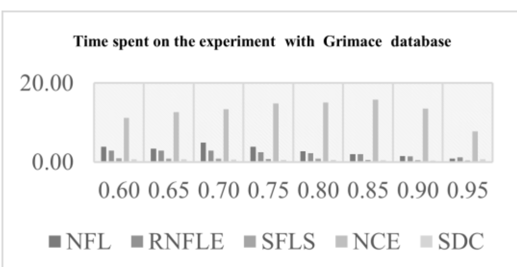

Fig. 7 shows time spent on the experiment with various algorithms for Grimace database

Table 4. Time spent on the experiment with various algorithms for Faces94 (female) database

Faces94 (female)	**Time spent on the experiment (second)**				
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**	**SDC**
0.95	0.89	0.95	0.28	10.41	0.26
0.90	1.65	1.52	0.45	12.76	0.38
0.85	2.31	2.16	0.60	16.66	0.47
0.80	3.21	3.33	0.76	17.69	0.56
0.75	3.24	3.09	0.87	16.71	0.60
0.70	3.65	3.15	0.97	16.01	0.69
0.65	4.27	3.20	1.03	14.32	0.92
0.60	4.29	3.26	1.05	12.79	0.80

Table 4 shows time spent on the experiment of NFL, RNFLE, SFLS, NCE and SDC algorithms. The minimum time spent on the experiment is SDC algorithm and the maximum time spent on the experiment is NCE algorithm

Table 5. The recognition rate of Faces94 (male staff) database with various algorithms

Faces94 (male staff)	**the recognition rate**				
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**	**SDC**
0.95	20.00	95.00	100.00	100.00	95.00
0.90	15.00	97.50	97.50	97.50	97.50
0.85	10.00	93.33	98.33	96.67	98.33
0.80	10.00	95.00	97.50	98.75	98.75
0.75	12.00	98.00	95.00	97.00	98.00
0.70	7.50	94.17	98.33	97.50	98.33
0.65	9.29	95.00	97.86	95.71	97.14
0.60	6.25	96.88	97.50	97.50	96.88

Table 5 shows the recognition rate of NFL, RNFLE, SFLS, NCE and SDC algorithms. The recognition rate of SFLS is better than other algorithms.

Table 6. Time spent on the experiment with various algorithms for Faces94 (male staff) database

Faces94 (male staff)	**Time spent on the experiment (second)**				
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**	**SDC**
0.95	1.20	1.01	0.32	9.31	0.27
0.90	2.23	2.35	0.51	16.59	0.42
0.85	2.65	2.97	0.68	16.54	0.51
0.80	3.27	3.02	0.81	21.44	0.59
0.75	3.84	3.76	0.91	21.82	0.73
0.70	4.23	4.39	1.05	18.71	0.74
0.65	4.53	3.44	1.10	17.31	0.78
0.60	4.78	3.38	1.10	14.35	0.83

Table 6 shows time spent on the experiment of NFL, RNFLE, SFLS, NCE and SDC algorithms. The minimum time spent on the experiment is SDC algorithm and the maximum time spent on the experiment is NCE algorithm.

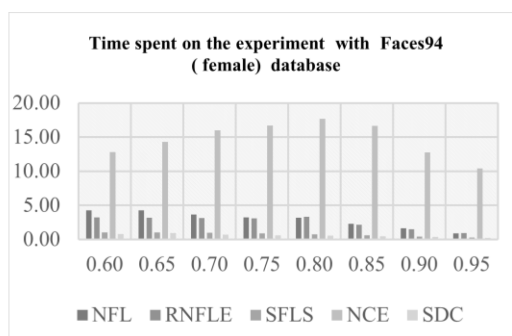

Fig. 8 shows time spent on the experiment with various algorithms for Faces94 (female) database

Table 7 shows percentage relative error of Time spent on the experiment of SDC algorithm is clearly less than NFL, RNFLE, and NCE. Percentage Relative Error of SDC is less than NFL average 74%, RNFLE average 72%, NCE average 95% and SFLS average 16%.

Table 7. Percentage Relative Error of Time spent with Grimace database

Grimace	Percentage Relative Error of Time spent for SDC and Other algorithms			
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**
0.95	20.72	42.72	-43.11	91.49
0.90	76.07	74.97	10.49	97.29
0.85	77.88	77.38	16.40	97.17
0.80	81.6	77.58	41.14	96.67
0.75	85.14	76.93	25.64	96.13
0.70	87.45	78.44	26.15	95.38
0.65	80.57	76.94	24.72	94.76
0.60	82.52	76.53	25.38	93.95

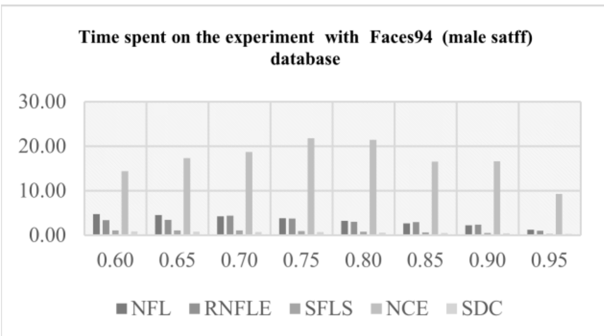

Fig. 9 shows time spent on the experiment with various algorithms for Faces94 (male staff) database

Table 8. Percentage Relative Error of Time spent with Faces94 (female) database

Faces94 (female)	Percentage Relative Error of Time spent for SDC and Other algorithms			
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**
0.95	25.58	30.60	-56.26	93.65
0.90	77.86	75.98	11.97	97.14
0.85	80.62	79.30	23.11	97.31
0.80	84.41	84.98	34.22	97.17
0.75	82.33	81.50	34.43	96.57
0.70	83.05	80.40	36.41	96.14
0.65	84.49	79.28	35.59	95.38
0.60	84.19	79.20	35.45	94.70

Table 8 shows percentage relative error of Time spent on the experiment of SDC algorithm which is clearly less than NFL, RNFLE, and NCE. Percentage Relative Error of SDC is less than NFL average 73%, RNFLE average 74%, NCE average 96% and SFLS average 19%.

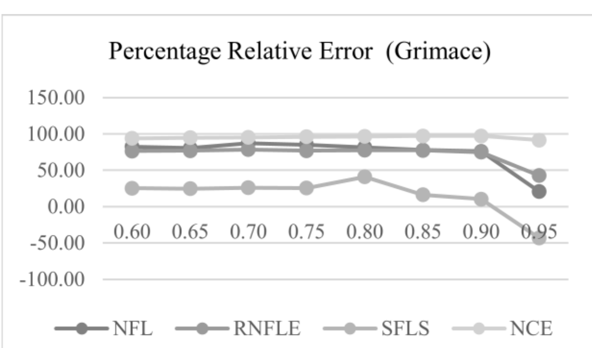

Fig. 10 shows Percentage Relative Error of Time spent with Grimace database

Table 9. Percentage Relative Error of Time spent with Faces94 (male staff) database

Faces94 (male staff)	Percentage Relative Error of Time spent for SDC and Other algorithms			
Proportion	**NFL**	**RNFLE**	**SFLS**	**NCE**
0.95	44.72	34.22	-49.99	92.89
0.9	83.68	84.51	21.95	97.80
0.85	83.13	84.92	34.62	97.29
0.8	84.68	83.45	38.49	97.67
0.75	85.08	84.80	37.13	97.38
0.7	85.40	85.92	41.37	96.70
0.65	85.38	80.75	39.66	96.17
0.6	85.83	79.92	38.52	95.28

Table 9 shows percentage relative error of Time spent on the experiment of SDC algorithm which is clearly less than NFL, RNFLE, and NCE. Percentage Relative Error of SDC is less than NFL average 79%, RNFLE average 77%, NCE average 96% and SFLS average 25%.

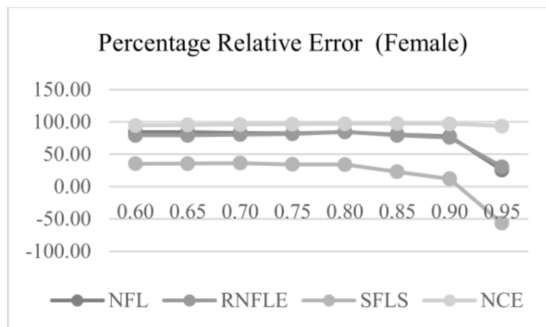

Fig. 11 shows Percentage Relative Error of Time spent with Faces94 (female) database

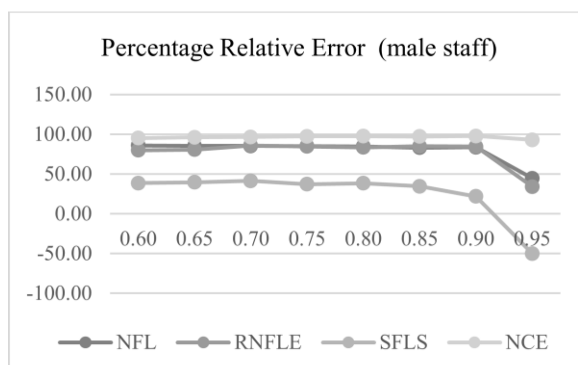

Fig. 12 shows Percentage Relative Error of Time spent with Faces94 (Male staff) database

V. CONCLUSIONS

In this paper, we introduced an algorithm for face recognition system based on PCA by using the centroid of a triangle which is generalized by three points of the same class, then the tested image is in class C_{SDC} when the distance between the tested image and the centroid of the triangle of class C_{SDC} is minimum distance.

We perform 2 comparisons consisting of the recognition rate and time spent on experiment. Mostly, the recognition rate of NCE is better than the others, followed by SDC and SFLS, respectively. However, we consider time spent on experiment of NFL, RNFLE, SFLS, NCE and SDC. We can see that time spent on experiment of SDC is lower than the other algorithms, followed by SFLS, RNFLE and NFL, respectively. Moreover, we compare time spent on experiment by percentage relative error to see the results of the time easier, that is the time spent of SDC algorithm is lower than NCE about 95.92%, SFLS about 20.15%, RNFLE about 74.63% and NFL about 76.35%.

REFERENCES

[1] M. Turk and A. Pentland, "Eigenfaces for recognition," *J. Cognitive Neurosci*., vol. 3, no.1, pp. 71–86, 1991.

[2] P. Belhumeur, J. Hespanha and D. Kriegman, "Eigenfaces vs. fisherfaces: recognition using class specific linear projection", *IEEE Trans. Pattern. Anal*., vol. 19, no. 7, pp. 711–720, 1997.

[3] M.S. Bartlett, J.R. Movellan and T.J. Sejnowski, "Face recognition by independent component analysis", *IEEE Trans Neural Netw*., vol. 13, no. 6, pp.1450–1464, 2002.

[4] H.B. Kekre and K. Shah, "Performance comparison of kekre's transform with PCA and other conventional orthogonal transforms for face recognition", *J. Inf. Hiding and Multimedia Signal Proc*., vol. 3, no.3, pp. 240-247, 2012.

[5] C. Y. Chang, C. W. Chang and C. Y. Hsieh, "Applications of block linear discriminant analysis for face recognition", *J. Inf. Hiding and Multimedia Signal Proc.*, vol. 2, no. 3, pp. 259-269, 2011.

[6] D. Y. Huang, C. J. Lin and W. C. Hu, "Learning-based face detection by adaptive switching of skin color models and AdaBoost under varying illumination," *J. Inf. Hiding and Multimedia Signal Proc*., vol. 2, no. 3, pp. 204-216, 2011.

[7] X. He, S. Yan,Y. Hu, P. Niyogi and H.J. Zhang, "Face recognition using laplacianfaces," *IEEE T. Pattern. Anal*., vol. 27, no.3, pp. 1–13, 2005.

[8] J.T. Chien and C.C. Wu, "Discriminant waveletfaces and nearest feature classifiers for face recognition," *IEEE Trans. Pattern Anal. Machine Intell*., vol. 24, no. 12, pp. 1644-1649, Dec 2002.

[9] S. Ullman and R. Basri, "Recognition by linear combinations of models," *IEEE Trans. Pattern Anal. Machine Intell*., vol. 13, pp. 992-1,006. 1991.

[10] T. Vetter and T. Poggio, "Linear Object Classes and Image Synthesis from a Single Example Image," *IEEE Trans. Pattern Anal. Machine Intell*., vol. 19, no. 7, pp. 733-742, July 1997.

[11] M. Bichsel and A.P. Pentland, "Human face recognition and the face image set's topology," *CVGIP: Image Understanding*, vol. 59, pp. 254-261, 1994.

[12] S. Ullman and R. Basri, "Recognition by linear combinations of models," *IEEE Trans. Pattern Anal. Machine Intell*., vol. 13, pp. 992-1006, 1991.

[13] L. Sirovich and M. Kirby, "Low-dimensional procedure for the characterization of human faces," *J. Opt. Soc. Amer. A*, vol. 4, no. 3, pp. 519-524, Mar. 1987.

[14] T. Cove and, P. Hart, "Nearest neighbor pattern classification," *IEEE Trans. Inf. Theory*., vol. 13, pp. 21–27, Jan 1967.

[15] S.Z. Li and J. Lu, "Face recognition using the nearest future line method," *IEEE Trans Neural Netw.*, vol. 10, no. 2, pp. 439-43, Mar 1999.

[16] Q. Feng, J.S. Pan, and L. Yan, "Nearest feature centre classifier for face recognition*," J. Electr. Lett*., vol. 48, no. 18, pp. 1120-1120, 2012.

[17] Y. Zhou, C. Zhang and J. Wang, "Extended nearest feature line classifier," in *Proc. 8th Pacific Rim Int. Conf. Artif. Intell. Trends Artif. Intell*. Berlin, Heidelberg, 2004, pp. 183–190.

[18] D.Q. Han, C.Z. Han and Y. Yang, "A novel classifier based on shortest feature line segment," *Pattern Recognit. Lett*., vol. 32, pp. 485–493, 2011.

[19] Q. Feng, J. Pan and L. Yan, " Restricted nearest feature line with ellipse for face," *J. Inf. Hiding and Multimedia Signal Proc.*, vol. 3, pp. 297–305, 2012.

[20] H. Tameem, "Face recognition by using nearest feature midpoint algorithm," *J. of AL-Qadisiyah for comp. Sci. Math.*, vol. 9, no.1 pp. 144-152, 2017.

[21] W. Ieosanurak and W. Klongdee, "A Novel Classification based on Ellipse for Face Recognition," *J. Inf. Hiding and Multimedia Signal Proc.*, vol. 9, no. 3, pp. 620-628, 2018.

[22] X. Zhou, Z. Nie and Y. Li, Statistical analysis of human facial expressions, *J. Inf. Hiding and Multimedia Signal Proc.*, vol. 1, no. 3, pp. 241-260, 2010.

[23] I. J. Cox, J. Ghosn and P. N. Yianilos, "Feature-based face recognition using mixture-distance," *Proc. IEEE Comput. Soc. Conf. Comput. Vision Patt. Recognit.*, pp. 209-216, 1996.

[24] W. Zheng, L. Zhao and C. Zou, "Locally nearest neighbor classifiers for pattern classification," *Patt. Recognit.,* vol.37, no.6, pp.1307-1309. 2004.

[25] L. Spacek, Description of the collection of facial images, (2008). Available: http://cswww.essex.ac.uk/mv/allfaces/.