

# Forecasting time series for power consumption data in different buildings using the fractional Brownian motion

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**Abstract**—In the present paper will be discussed the problem related to the individual household electric power consumption of objects in different areas – industry, farmers, banks, hospitals, theaters, hostels, supermarkets, universities. The main goal of the directed research is to estimate the active  $P$  and full  $S$  power consumptions for all studied buildings. The defined goal is achieved by solving of the following three problems. The first problem studies which buildings increase their power consumption. The second one finds which objects have the greatest increase of power consumption. And the third problem regards if it is possible to make a short-term forecast, based on the solutions of previous two problems. The present research and solving of the aforementioned problems is conducted using fractional Brownian motion theory. The applicability of this approach is illustrated on the example with 20 real objects in different areas. The paper ends with conclusion notes about possibilities to make short-term forecasts about power consumption of the considered buildings.

**Index Terms**—stochastic model, fractional Brownian motion (fBm), parameters estimation, forecasting power consumption

## I. INTRODUCTION

THE act of predicting the variation of some index characterizing the behavior of a particular object for a future period of time is named forecast. It uses data for the considered index for previous moments, which form the time series. Typically, the forecasts are based upon specific assumptions, such as targeted prospects or a preliminarily defined strategy. There exist different mathematical

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approaches for forecasting: stochastic, intervals, based on artificial intelligent: cluster analysis, neural networks etc. To make a more adequate forecast, on one side it is necessary to have initial data as huge as possible. On the other side, it makes longer the processing time and it requires more powerful computers.

These forecasts could be very useful for different applications such as optimizing the active and the full power energy distribution as a function of the consumption.

There are two types of forecasts: short-time and long-time ones. It depends on the type of the considered object, respectively the associated forecasting parameter. In particular the reliability of the forecast depends on the type of the considered time series and on the amount of the primary data for the analyzed index.

The paper is organized as follows. In the next section are defined three basic problems for analysis of real data about the consumed active power  $P$  and the full power  $S$  for 20 different objects. The basis of fractional Brownian motion (fBm) theory is presented in Section 3. In Section 4 is discussed the algorithm for analyzing the considered time series. The real examples for the consumed active power  $P$  and the full power  $S$  are discussed in Section 5. The paper ends with concluding remarks about power consumption and its future forecasting for the considered objects.

## II. PROBLEM STATEMENT

Let us consider the power consumption (active power  $P$  and full power  $S$ ) in different administrative buildings where the consumption has different daily and nightly behavior. The analyzed objects correspond to different life’s areas such as: ‘Bank’, ‘Automobile Industry’, ‘BPO Industry’, ‘Cement Industry’, ‘Farmers 1’, ‘Farmers 2’, ‘Health Care Resources’, ‘Textile Industry’, ‘Poultry Industry’, ‘Residential (individual)’, ‘Residential (apartments)’, ‘Food Industry’, ‘Chemical Industry’, ‘Handlooms’, ‘Fertilizer Industry’, ‘Hostel’, ‘Hospital’, ‘Supermarket’, ‘Theatre’ and ‘University’.

The main aim of the research is to estimate the actual power consumption ( $P$  and  $S$ ) of each analyzed object and to make forecast about their future consumptions. This aim is realized by solving the following problems:

Problem 1: Which buildings increase their consumption ( $P$  and  $S$ )?

Problem 2: Which buildings have the greatest increase of their consumption ( $P$  and  $S$ )?

Problem 3: If it is possible to make a short-terms forecast, based on the results from the solutions of Problems 1 and 2?

### III. BASIC CONCEPTIONS IN FRACTIONAL BROWNIAN MOTION (FBM) THEORY

Consider an observed trajectory  $x(t), t \in [0; T]$ , which is describing the stochastic evolution of some dynamic object. The mathematical model of this trajectory is defined as a random process  $\xi(t)$ , where:

$$x(t) = X(t),$$

where  $X(\bullet)$  is realization of the process  $\xi$ .

As a rule, it is chosen as a model random process with known characteristics. Direct use of this definition requires a broad class of these processes. On the other hand, this class includes Gaussian and Markov processes. Let us introduce another definition of continuous mathematical models for the observed trajectory  $x(\bullet) \in C[0; T]$  using nonlinear conversion.

Definition. Mathematical model of observed trajectory  $x(t)$  is a pair  $(\Phi, \xi)$ , where  $x(t) = \Phi(X(\bullet))(t)$ ,  $\xi(t)$  is a random process with known characteristics,  $\Phi$  is a reversible conversion in  $C(0; t)$ .

Let's assume  $t = t_k, k = 1, \dots, n, t_k = t_1 + (k-1) \frac{T-0}{n}, t_1 = 0,$

$x_k = \Phi(X(\bullet))(t_k)$  is a model of the observed time series  $\{x_1, x_2, \dots, x_n\}$ .

Let us call  $\xi$  as a basic process of the model.

One of the most popular Markov models of time series is Gaussian random process, and fractional Brownian motion (fBm). The large use of this process is caused by its "convenient" properties, which are described below.

Fractional Brownian motion is defined as a Gaussian random process with characteristics:

$$B_H(t), E\{B_H(t)\} = 0, B_H(0) = 0$$

$$E\{B_H(t) \cdot B_H(s)\} = \frac{1}{2} (t^{2H} + s^{2H} - |t-s|^{2H})$$

Note that with  $H = 0.5$  we get a standard Wiener process, where  $H$  indicates the Hurst exponent.

The smoothness of the trajectories of the process  $B_H(t)$  is defined by the parameter  $H$ : almost all the trajectories satisfy the Holder condition for regularity:

$$|X(t) - X(s)| \leq c |t - s|^\alpha, \alpha < H. \quad (1)$$

which generalizes known Levy's result for the Wiener process [2, 3, 4].

The increments of fBm

$$B_H(t_2) - B_H(t_1), B_H(t_4) - B_H(t_3), t_1 < t_2 < t_3 < t_4$$

form a Gaussian random vector with a correlation between the coordinates:

$$\frac{1}{2} \left( (t_4 - t_1)^{2H} + (t_3 - t_2)^{2H} - (t_4 - t_2)^{2H} - (t_3 - t_1)^{2H} \right)$$

For discrete time:

$$\xi_k = B_H\left(\frac{k}{n}\right) - B_H\left(\frac{k-1}{n}\right),$$

we obtain the correlation coefficient:

$$\rho(\xi_j, \xi_k) = \frac{1}{2} (|k-j+1|^{2H} + |k-j-1|^{2H} - 2|k-j|^{2H}), \quad (2)$$

It means that increments form a stationary (in the narrow sense) sequence.

Let us mention some properties of fBm:

1. Changing time scale is equivalent to changing the "amplitude" of the process:

$$Law(B_H(at)) = Law(a^H B_H(t)).$$

This equality denotes the coincidence of one-dimensional distributions of the processes:

$$B_H(at) \text{ and } a^H B_H(t).$$

This property is called self-similarity and it is useful for the analysis of time series.

2. Let us put in the formulae (2)  $j = k + n$ . Then the correlation coefficient:

$$\rho(\xi_k, \xi_{k+n}) = \frac{1}{2} \left[ (n+1)^{2H} + (n-1)^{2H} - 2n^{2H} \right] \quad (3)$$

When  $n \rightarrow \infty$  it follows:

$$\rho_n = \rho(\xi_k, \xi_{k+n}) \sim H(2H-1)n^{2H-2} \quad (4)$$

So, the increments memory decrease exhibits a power law; the increments are independent for  $H = 0.5$ . With  $H < 0.5$  the increments form the sequence with short memory, and for  $H > 0.5$  a sequence with a long memory. The sign of the correlation coefficient  $\rho_n$ , defined by formula (4), depends on the value of  $H$ :  $\rho_n < 0, H < 0.5$ ;  $\rho_n > 0, H > 0.5$ .

For  $H < 0.5$  the sequence  $\xi_n$  of increments fBm is called "pink noise" and negativity of the variations indicates fast variability values. The process of fBm,  $H < 0.5$  is known as "antipersistent".

For  $H > 0.5$  the sequence  $y_n$  of increments fBm is called "black noise" and the fBm process is known as "persistent".

The original approach using stochastic and statistical analysis is based on real time series modeling assuming a hypothesis that their increments can be modeled by fBm. If the estimated Hurst exponent is bigger than 0.5 the process is called persistent and a forecast can be made.

### IV. THE SELECTION OF THE CONVERSION OF TIME SERIES

It requires initial analysis of the increments  $\{y_1, y_2, \dots, y_n\}$  to determine the conversion  $\Phi$ .

In particular, is it necessary to estimate the one-dimensional distribution of the samples and the correlation. These actions are possible only with a large sample size (  $n > 5000$  ). We propose new empirical method of increments transformation  $\{y_1, y_2, \dots, y_n\}$  into  $\{z_1, z_2, \dots, z_n\}$  for a small sample, in order to satisfy required statistical properties (e.g. gaussianity).

Let us consider the increments  $y_k = x_{k+1} - x_k, k = 1, \dots, (n-1)$  and construct the statistics (kurtosis)

$$d_n = \frac{\left( \frac{1}{n-1} \cdot \sum_{k=2}^n |y_k| \right)^2}{\frac{1}{n-1} \cdot \sum_{k=2}^n y_k^2},$$

where  $d_n$  is equal to  $2/\pi$  or Gaussian model.

If  $d_n$  is significantly different from  $2/\pi$ , it is necessary to replace the time series  $\{y_1, y_2, \dots, y_{n-1}\}$  by the new sequence  $\{z_1, z_2, \dots, z_{n-1}\}$ .

The general idea of approximation is a one-dimensional functional transformation  $g$  of each increment  $y_k$ , where  $g$  is an increasing odd function,  $z_k = g(y_k)$ .

Let us assume

$$\lim_{n \rightarrow \infty} \frac{\left( \frac{1}{(n-1)} \cdot \sum_{k=1}^{n-1} |g(y_k)| \right)^2}{\frac{1}{(n-1)} \cdot \sum_{k=1}^{n-1} (g(y_k))^2} = \frac{2}{\pi},$$

where  $z_k = g(y_k)$  is assumed as a Gaussian random value. Let us demonstrate proposed algorithm with  $g$  as a power function.

Assume

$$z_k = \text{sgn } y_k |y_k|^\lambda = \Phi^{-1}(y_k), \tag{5}$$

$$y_k = \text{sgn } y_k |z_k|^{1/\lambda} = \Phi(z_k), \lambda > 0, \tag{5a}$$

then

$$d_n = \frac{\left( \frac{1}{n-1} \cdot \sum_{k=1}^{n-1} |z_k|^\lambda \right)^2}{\frac{1}{n-1} \cdot \sum_{k=1}^{n-1} |z_k|^{2\lambda}} \tag{6}$$

$d = \lim_{n \rightarrow \infty} d_n$  is equal to the ratio of the corresponding mathematical expectations.

$$\text{If } \xi \sim \Xi(0, \sigma^2) \Rightarrow E|\xi|^\alpha = \frac{2^{\frac{\alpha}{2}}}{\sqrt{\pi}} \cdot \sigma^\alpha \cdot \Gamma\left(\frac{\alpha+1}{2}\right),$$

$$\text{then } d = \frac{1}{\sqrt{\pi}} \cdot \frac{\Gamma^2\left(\frac{\lambda+1}{2}\right)}{\Gamma\left(\lambda + \frac{1}{2}\right)}, \quad \sqrt{\phantom{x}}$$

where the parameter  $\lambda$  is defined from the equations (5, 5a). Thus, the proposed approximation leads to the following model of original time series:

$$x_k = \sum_{j=1}^k \text{sgn}\left(y_j \cdot |z_j|^\lambda\right).$$

If we assume that the values of the sequence  $\{z_1, z_2, \dots, z_n\}$  are increments of fBm, the Hurst exponent can be calculated by the following algorithm, as it can be seen from the proposed method:

1) Construct the statistic:

$$R_1 = \frac{1}{n} \sum_{k=1}^n |z_k| = |\bar{z}|$$

2) Calculate the matrix  $S_H^{-1}$ , where  $S_H$  is a correlation matrix of increments fBm :

$$s_{jk} = E\left[ B_H\left(\frac{k}{n}\right) - B_H\left(\frac{k-1}{n}\right) \right] \cdot \left[ B_H\left(\frac{j}{n}\right) - B_H\left(\frac{j-1}{n}\right) \right] = \rho(\xi_j, \xi_k)$$

$$3) \quad Q = \frac{0,8}{R_1} \sqrt{\frac{(S_H^{-1} z, z)}{n-1}},$$

The statistics  $Q$  is calculated for different Hurst exponent values with step (0.05 – 0.1) and the corresponding  $\hat{H}$  is calculated, such as:

$$|Q(H) - 1| \rightarrow \min, \hat{H} = \arg \min |Q(H) - 1|. \tag{6}$$

4) Verify the following hypothesis  $T$ : *The statistics  $\{z_1, z_2, \dots, z_n\}$  which have been obtained by transformation (5) of real data can be simulated by fractional Brownian motion increments.* The algorithm with known  $H$  is the following. Denote

$$c = \frac{1}{n} \sum_{k=1}^n z_k^2,$$

and assume that hypothesis  $T$  is verified

$$z_k = \sqrt{c} \cdot \xi_k = \sqrt{c} \cdot n^H \left( B \left( \frac{k+1}{n} \right) - B \left( \frac{k}{n} \right) \right).$$

Assume  $v_k = \sum_{j=1}^{k-1} z_j$  and construct the statistics [5, 6]

$$A_n = \frac{1}{n} \sum_{k=1}^n v_k \cdot z_k^3, \quad H \in (0; 0.5)$$

$$B_n = \frac{1}{n^{1+H}} \sum_{k=1}^n v_k^2 \cdot z_k^3, \quad H \in (0; 0.5)$$

$$D_n = \frac{1}{n^{2H}} \sum_{k=1}^n v_k \cdot z_k^3, \quad H \in (0.5; 1)$$

If hypothesis  $T$  is true, there is convergence [1]:

$$A_n \rightarrow -\frac{3}{2}c^2; \quad B_n \rightarrow 3c^{2.5}\eta; \quad D_n \rightarrow \frac{3}{2}c^2B^2.$$

The decision about the veracity of the hypothesis  $T$  is accepted by comparing the real values of the statistics with their theoretical limit values. Therefore, let us determine the deviation from the limit values for the statistic  $A_n$

$$\delta = \left| \frac{A_n - A}{A} \right|.$$

The limit distribution functions for the statistics  $B_n, D_n$ :

$$F_1(x) = P \left\{ 3c^{2.5}\eta < x \right\} = \Phi \left( \frac{x}{3\sigma} c^{-2.5} \right);$$

$$F_2(x) = 2\Phi \left( \frac{1}{c} \sqrt{\frac{2}{3}} x \right) - 1, \quad x > 0,$$

where  $\Phi$  is the Laplace function,  $\sigma = (2H + 2)^{-0.5}$ .

The hypothesis  $T$  is accepted, if:

$$\delta < \beta_0, |B_n| < \beta_1, \quad H < 0.5$$

or

$$0 < D_n < \beta_2, \quad H > 0.5$$

where  $\beta_1, \beta_2$  are quantile distributions from  $F_1, F_2$  which correspond to the selected significance level  $\alpha = 0.1$ .

$$\beta_1 = \frac{4,95c^{2,5}}{\sqrt{2H+2}}, \quad \beta_2 = 4,08c^2, \quad \text{where } c = \bar{z}^2.$$

## V. REAL EXAMPLES

Let us consider the consumed active power  $P$  and full power  $S$  for the following 20 objects. The data for individual household electric power consumption are saved each hour [7]. The considered buildings are: 'Bank', 'Automobile Industry', 'BPO Industry', 'Cement Industry', 'Farmers 1',

'Farmers 2', 'Health Care Resources', 'Textile Industry', 'Poultry Industry', 'Residential (individual)', 'Residential (apartments)', 'Food Industry', 'Chemical Industry', 'Handlooms', 'Fertilizer Industry', 'Hostel', 'Hospital', 'Supermarket', 'Theatre' and 'University'.

The research aim is to estimate the power consumption ( $P$  and  $S$ ) of each analyzed object. This aim is realized by solving the following problems:

**Problem 1:** Which buildings increase their consumption ( $P$  and  $S$ )?

**Problem 2:** Which buildings have the greatest increase in their consumption ( $P$  and  $S$ )?

**Problem 3:** If it is possible to make a short forecasting, based on the results from the solutions of Problems 1 and 2?

Before starting the data processing all primary data have been normalized and the respective values of  $P$  and  $S$  take values in interval  $[0, 1]$ . Next, the primary data have been processed using linear approximation of the trend of the initial data and new sequences have been obtained in every example.

This is realized by the following steps:

1. Calculation of the increments

$$y_k = x_{k+1} - x_k, \quad k = 1, \dots, (n-1).$$

2. Construction of the new sequence

$\{z_1, z_2, \dots, z_{n-1}\}$  by (5) and (5a).

3. Hurst exponent estimation by (6).

4. Checking the quality of the approximation by  $A_n$  and  $B_n$  (7).

The processing results are shown in Tables IA and IB.

As it can be seen from the TABLE IA, for all the data we have antipersistent parameter  $\hat{H} < 0.5$ . For the examples (except Theater and Food Industry), the condition is clearly verified:  $|B_n| < \beta_1$ .

For all examples (except Theater and Textile Industry), the condition is clearly verified:  $|B_n| < \beta_1$  and the process has short-memory.

According to the results gotten in the previous section it can make the following conclusions about three problems defined in the beginning of the paper:

**Problem 1:** Which buildings increase their consumption ( $P$  and  $S$ )?

The approximation by fractional Brownian motion of the considered real data shows that Bank, Automobile Industry, BPO Industry, Textile Industry, Residential (individual), Chemical Industry, Hospital and University increase their consumption of the active power  $P$ , because for these buildings we've got the highest antipersistent Hurst parameter  $\hat{H} = 0.4$  (which means that the processing data change more smoothly).

TABLE IA  
ACCESSING RATES OF THE CONSIDERED PARAMETERS  
FOR CONSUMED ACTIVE POWER  $P$

Analyzed object	$\hat{H}$	$A_n$	$B_n$	$A$	$\beta_1$	$\beta_2$
Bank	0.4	-1.58	0.22	-1.57	2.98	
University	0.4	-1.49	0.55	-1.62	2.93	
Cement Industry	0.1	-1.41	1.61	-1.51	3.34	—
Automobile Industry	0.4	-1.54	0.04	-1.5	2.99	
BPO Industry	0.4	-1.72	-1.61	-1.51	5.28	—
Farmers 1	0,3	-1.42	1.68	-1.5	2.59	
Farmers 2	0.1	-3.44	-6.38	-3.57	9.883	—
Health resources	0.1	-1.5	-1.24	-1.5	3.53	
Textile Industry	0.4	398.8	-2.125	-1.5	2.95	—
Poultry Industry	0.1	-1.47	-1.58	-1.5	3.82	
Residential (individual)	0.4	-1.72	-1.61	-1.51	5.28	—
Residential (Apartments)	0.1	-1.57	-2.25	-1.5	3.11	
Food Industry	0,1	-1.47	-2.98	-1.5	2.83	—
Chemical Industry	0.4	-1.6	0.09	-1.5	2.35	
Handlooms	0.4	-1.78	0.46	-1.5	2.99	—
Fertilizer Industry	0.1	-1.56	-0.28	-1.5	2.87	
Hostel	0.1	-1.54	-0.97	-1.5	2.81	—
Hospital	0.4	-1.47	0.16	-1.5	2.91	
Supermarket	0.25	-1.51	0.64	-1.5	2.16	—
Theater	0.1	-1.59	3.07	-1.5	2.05	—

TABLE IB  
ACCESSING RATES OF THE CONSIDERED PARAMETERS  
FOR CONSUMED FULL POWER  $S$

Analyzed object	$\hat{H}$	$A_n$	$B_n$	$A$	$\beta_1$	$\beta_2$
Bank	0.4	-1.34	0.31	-1.47	2.77	
University	0.36	-1.25	0.88	-1.52	2.10	
Cement Industry	0.24	-1.38	1.92	-1,29	3.01	—
Automobile Industry	0.39	-1.59	0.2	-1.68	2.57	
BPO Industry	0.46	-1,87	-1,23	-1.08	4,01	—
Farmers 1	0.28	-1.80	1.32	-1.6	2.0006	
Farmers 2	0,17	-3.34	-5.36	-3.03	10.01	—
Health resources	0.28	-1.34	-0.99	-1.45	2.44	
Textile Industry	0.46	201.2	-118	-1.8	4.5	—
Poultry Industry	0.18	-1.59	-2.21	-1.48	5.77	
Residential (individual)	0.39	-1.22	-1.85	-1.50	6.54	—
Residential (Apartments)	0.2	-1.33	-3.77	-1.2	5.5	
Food Industry	0.17	-1.99	-2.42	-1.66	4.5	—
Chemical Industry	0.36	-1.21	0.5	-1.01	4.66	
Handlooms	0.38	-1.56	0.43	-1.39	2.2	—
Fertilizer Industry	0.21	-1.88	-0.44	-1.5	2.56	
Hostel	0.11	-1.2	-0.5	-1.9	2.4	—
Hospital	0.29	-1.73	0.12	-1.5	2.37	
Supermarket	0.29	-1.58	0.23	-1.5	2.89	—
Theater	0.11	-1.43	3.55	-1.5	2.14	—

*Problem 2:* Which buildings have the greatest increase in their consumption ( $P$  and  $S$ )?

The approximation by fractional Brownian motion of the considered real data shows that Bank, BPO Industry and Textile Industry increase their full power  $S$  consumption because for these buildings we've got the highest antipersistent Hurst parameters:  $\hat{H} = 0.4$ ,  $\hat{H} = 0.46$  and  $\hat{H} = 0.46$ , respectively (which means that the processing data change more smoothly).

*Problem 3:* Is it possible to make a short forecasting, based on the results from the solutions of *Problems 1* and *2*?

All of the approximations have antipersistent character ( $\hat{H} < 0.5$ ) and it is adequate, if the conditions (5) are satisfied. Therefore, it is impossible to construct a short-term forecast. Then other methods will be used for approximation of the analyzed real data to make more reliable prognoses. But it will be the focus of the future investigations, where some newer techniques such as cluster analysis will be applied.

Numerical experiment has shown that for a lot of data it is possible to implement approximation by fractional Brownian motion, but without forecast and with short memory.

For the examples Food Industry, Textile Industry and Theater conditions (5) are not satisfied, so it means that they can not be assumed as Gaussian processes and it is possible to approximate them by other distributions.

## VI. CONCLUSION

Fractional Brownian motion is used for approximation of real object data to estimate a capability of short and long-term prognosis about an active power  $P$  and full power  $S$  consumption. Different buildings consumptions have been analyzed, and the relevance of the chosen modeling by fBm increments has been evaluated using limit theorems. The used methodology is appropriated for short-term prognosis for all considered objects except of the Theater and Textile Industry.

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