New Model of a Separately Excited D.C. Motor

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***Abstract*—**In the paper a new model of a separately excited D.C. motor is presented. It is assumed that the mechanical torque do not cause the motion of the motor. This condition implies the model is nonlinear. The non-linearity of the model is reshaped to the linear model with time delay. The value of time delay depends on the value of mechanical torque which loads the motor. The parametric optimization problem for a new model of a separately existed D.C. motor with a P-controller is considered. The general quadratic performance index is used. The value of the quadratic performance index is equal to the value of the Lyapunov functional at initial function of time delay system. In the paper Repin's method is used to determine the Lyapunov functional coefficients. The parametric optimization results for the separately excited D.C. motor Siemens 1GH6 size 225, the catalog number: 1GH6 226-ONA40-1VV3 are presented.

***Index Terms*—**Lyapunov functional, parametric optimization, separately excited D.C. motor, time delay systems

I. Introduction

In the paper a new model of a separately existed D.C. motor is presented. It is assumed that the mechanical torque do not cause the motion of the motor. This condition implies the model is nonlinear. The novelty of this paper consists in reshaping of the nonlinear model to the linear model with time delay. The value of time delay depends on the value of mechanical torque which loads the motor. In the paper the method how to reshape the nonlinear model to the linear model with time delay is presented. For a control system which consists of linear time delay model of motor and P controller we survey the parametric optimization process. In parametric optimization the general quadratic index of quality is used. The value of the quality index is equal to the value of the Lyapunov functional at initial function of time delay system. There are two methods of determination of the Lyapunov functional. The first method was proposed by Repin [14] and was developed in [1], [8]. In the paper the Repin's method is used to obtain the value of the performance index. The author does not know the papers of other authors developing the Repin's method. In the second method the Lyapunov functional is determined by means of the Lyapunov matrix. The second method is most popular and is presented in [9]-[13], [15]. The Lyapunov matrices approach is used in the parametric optimization problems described in [2]-[7]. The paper is organized as follows. In Section II the method of reshaping of the nonlinear model into the linear model with time delay is presented. In Section III the parametric optimization problem for a new model of a separately excited D.C. motor with a P-controller is considered. In Section IV optimization results are given. Conclusions end the paper.

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II. Mathematical model of a Separately Excited D.C. Motor

It is assumed that in the field coil of excitation the electric current is fixed and constant. The motor input voltage is the control signal, $u(t)$. The motor is loaded by the mechanical torque which is treated as a disturbance, $z(t)$. The angular speed $\omega(t)$ and a motor current $i(t)$ will be controlled. It is also assumed that the mechanical torque do not cause the motion of the motor. We will analyze the motor in two periods. In the first period the load of the motor is greater than motor electrical torque. The motor current increases but motor stops, the rotational speed is equal to zero. In the second period motor starts. It appears the angular speed and equations describing the dynamics change. The time instant when motor starts is denoted by r and will be treated as time delay.

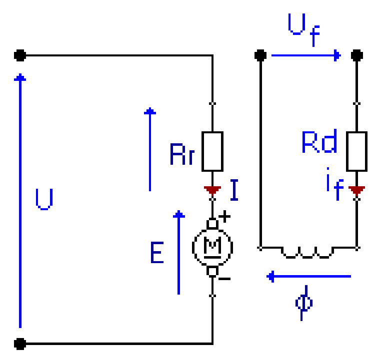

Fig. 1 The separately excited D.C. motor

The first period for $0 \le t \le r$
The angular velocity

$$\omega(t) = 0 \tag{1}$$

Voltage balance equation

$$Ri(t) + L\frac{di(t)}{dt} = u(t) \tag{2}$$

where R is a motor resistance, L is a motor inductance
Electrical torque equation

$$m(t) = k_m i(t) \tag{3}$$

where k_m is a torque equation constant

$$z(t) = z_s \tag{4}$$

$$u(t) = u_0 = const \tag{5}$$

The time delay r is computed from relation

$$m(r) = k_m i(r) = z_s \tag{6}$$

The current $i(t)$ for $t \in [0,r]$ is obtained by solving the differential equation (2) with initial condition $i(0) = i_0$ for $u(t) = u_0$.

$$i(t) = \frac{u_0}{R}\left(1 - e^{-\frac{t}{T}}\right) + i_0 e^{-\frac{t}{T}} \tag{7}$$

where $T = \frac{L}{R}$ is a time constant.
Taking into account equation (7) we can write relation (6) in the form

$$m_z\left(1 - e^{-\frac{r}{T}}\right) + k_m i_0 e^{-\frac{r}{T}} = z_s \tag{8}$$

where $m_z = \frac{u_0}{R}k_m$ is called a short-circuit moment.
From equation (8) we can calculate the time delay r

$$r = T \ln \frac{m_z - k_m i_0}{m_z - z_s} \tag{9}$$

The second period for $t \geq r$
Voltage balance equation

$$Ri(t-r) + L\frac{di(t-r)}{dt} + k_e \omega(t) = u(t-r) \tag{10}$$

where k_e is a speed equation constant
Electrical torque equation

$$m(t-r) = k_m i(t-r) \tag{11}$$

Mechanical torque equation

$$z(t-r) = z_s + J_m \frac{d\omega(t)}{dt} \tag{12}$$

where J_m is a moment of inertia.
There holds a relation for each time instant

$$m(t-r) = z(t-r) \tag{13}$$

Relation (13) implies

$$\frac{d\omega(t)}{dt} = \frac{k_m}{J_m} i(t-r) - \frac{z_s}{J_m} \tag{14}$$

Equation (10) can be reshaped to the form

$$\frac{di(t-r)}{dt} = -\frac{R}{L} i(t-r) - \frac{k_e}{L}\omega(t) + \frac{1}{L}u(t-r) \tag{15}$$

We will consider a control system with a P controller. Control signal $u(t)$ has the form

$$u(t) = u_0 - p\omega(t) \tag{16}$$

where p is a gain of the P controller. We substitute the term (16) into equation (15).

$$\frac{di(t-r)}{dt} = -\frac{R}{L} i(t-r) - \frac{k_e}{L}\omega(t) + \frac{1}{L}u_0 - \frac{p}{L}\omega(t-r) \tag{17}$$

We introduce the new variables and the new coefficients

$$y_1(t) = \omega(t) \tag{18}$$

$$y_2(t) = i(t-r) \tag{19}$$

$$\frac{k_0}{T} = \frac{k_m}{J_m} \tag{20}$$

$$z_0 = -\frac{z_s}{J_m} \tag{21}$$

$$k_1 = \frac{k_e}{L} \tag{22}$$

$$w_0 = \frac{u_0}{L} \tag{23}$$

$$k = \frac{p}{L} \tag{24}$$

Taking relations (18)-(24) into account we can write the equations (14) and (17) in the form

$$\begin{cases} \frac{dy_1(t)}{dt} = \frac{k_0}{T} y_2(t) + z_0 \\ \frac{dy_2(t)}{dt} = -k_1 y_1(t) - \frac{1}{T} y_2(t) - k y_1(t-r) + w_0 \end{cases} \tag{25}$$

for $t \geq r$.
Initial functions for equation (25) are obtained from (1), (7) and (19)

$$\begin{cases} y_1(\xi) = 0 \\ y_2(\xi) = \frac{u_0}{R}\left(1 - e^{-\frac{\xi - r}{T}}\right) + i_0 e^{-\frac{\xi - r}{T}} \end{cases} \tag{26}$$

for $\xi \in [0,r]$.
We shift functions (26) to the interval $[-r,0]$ by substitution $\xi = r + \theta$ for $\theta \in [-r,0]$

$$\begin{cases} y_1(r+\theta) = 0 \\ y_2(r+\theta) = \frac{u_0}{R}\left(1 - e^{-\frac{\theta}{T}}\right) + i_0 e^{-\frac{\theta}{T}} \end{cases} \tag{27}$$

System (25) has non zero equilibrium point $\begin{bmatrix} \frac{w_0 + \frac{z_0}{k_0}}{k_1 + k} \\ -\frac{T z_0}{k_0} \end{bmatrix}$

We introduce new variables to obtain a system with zero equilibrium point

$$\begin{cases} x_1(t) = y_1(t) - \frac{w_0 + \frac{z_0}{k_0}}{k_1 + k} \\ x_2(t) = y_2(t) + \frac{T z_0}{k_0} \end{cases} \tag{28}$$

In such a manner we obtained a system which dynamics is described by functional-differential equation

$$\begin{cases} \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = A \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + B \begin{bmatrix} x_1(t-r) \\ x_2(t-r) \end{bmatrix} \\ \begin{bmatrix} x_1(r+\theta) \\ x_2(r+\theta) \end{bmatrix} = \begin{bmatrix} -\frac{w_0 + \frac{z_0}{k_0}}{k_1 + k} \\ \frac{u_0}{R}\left(1 - e^{-\frac{\theta}{T}}\right) + i_0 e^{-\frac{\theta}{T}} + \frac{T z_0}{k_0} \end{bmatrix} \end{cases} \tag{29}$$

for $t \geq r,\ \theta \in [-r,0]$, where

$$A=\begin{bmatrix} 0 & \frac{k_0}{T} \\ -k_1 & -\frac{1}{T} \end{bmatrix} \tag{30}$$

$$B=\begin{bmatrix} 0 & 0 \\ -k & 0 \end{bmatrix} \tag{31}$$

III. Parametric optimization

In parametric optimization we will consider the following performance index

$$J=\int_r^\infty \left\{ x^T(t)x(t)+\int_{-r}^0 x_t^T(\theta)x_t(\theta)d\theta \right\} dt \tag{32}$$

The value of the performance index is equal to the value of the functional v for initial function φ of system (29), see [1]

$$J=v(\varphi) \tag{33}$$

where

$$v(x_t)=x_t^T(0)\alpha x_t(0)+$$

$$+\int_{-r}^0 x_t^T(0)\beta(\theta)x_t(\theta)d\theta+\int_{-r}^0 x_t^T(\theta)\gamma(\theta)x_t(\theta)d\theta+$$

$$+\int_{-r}^0\int_{\theta}^0 x_t^T(\theta)\delta(\theta,\sigma)x_t(\sigma)d\sigma d\theta \tag{34}$$

and

$$\varphi(\theta)=\begin{bmatrix} -\frac{w_0+\frac{z_0}{k_0}}{k_1+k} \\ \frac{u_0}{R}\left(1-e^{-\frac{\theta}{T}}\right)+i_0e^{-\frac{\theta}{T}}+\frac{Tz_0}{k_0} \end{bmatrix} \tag{35}$$

for $\theta\in[-r,0]$.

Now will be presented formulas for matrices α, $\beta(\theta)$, $\gamma(\theta)$ and $\delta(\theta,\sigma)$ without introduction. Introduction of that formulas is given in [1].

Matrix α has a form

$$\alpha=\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12} & \alpha_{22} \end{bmatrix} \tag{36}$$

where

$$\alpha_{11}=\frac{1+r}{W}\{\frac{T^2s_1s_2(s_1^2-s_2^2)}{k_0(k_1+k)}(\frac{k_0}{T}+k_1-k+$$

$$+\frac{1}{T^2(k_1-k)})\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+$$

$$+(k_1s_1+ks_1-\frac{k_0(k_1-k)^2}{Ts_1}-\frac{2k_0^2k_1}{T^2s_1}-\frac{k_0s_2^2}{Ts_1}+$$

$$+\frac{s_1}{T^2(k_1+k)}-\frac{s_1s_2^2}{k_1+k})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+$$

$$+(-k_1s_2-ks_2+\frac{k_0(k_1-k)^2}{Ts_2}+$$

$$+\frac{2k_0^2k_1}{T^2s_2}+\frac{k_0s_1^2}{Ts_2}-\frac{s_2}{T^2(k_1+k)}+$$

$$+\frac{s_1^2s_2}{k_1+k})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+T(s_1^2-s_2^2)(\frac{k_0}{T}+$$

$$+k_1+k+\frac{1}{T^2(k_1+k)})\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}\} \tag{37}$$

where

$$s_1=\frac{1}{T}\sqrt{\frac{1-2Tk_0k_1+P}{2}} \tag{38}$$

$$s_2=\frac{1}{T}\sqrt{\frac{1-2Tk_0k_1-P}{2}} \tag{39}$$

$$P^2=4T^2k_0^2k^2-4Tk_0k_1+1 \tag{40}$$

$$W=2k_0(s_1^2-s_2^2)\{\frac{k_0(k_1-k)}{Ts_1s_2}\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+$$

$$+(s_1+\frac{k_0(k_1-k)}{Ts_1})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+$$

$$+(s_2+\frac{k_0(k_1-k)}{Ts_2})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+$$

$$+\frac{1}{T}\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}\} \tag{41}$$

$$\alpha_{12}=\frac{1+r}{W}\{\frac{k_0^2(s_1^2-s_2^2)}{T^2s_1s_2}\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+$$

$$+(-\frac{k_0^2}{T^3s_1}+2k_0ks_1+\frac{k_0s_1}{T^2(k_1+k)}+$$

$$+\frac{2kTs_1s_2^2}{k_1+k})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+(\frac{k_0^2}{T^3s_2}-2k_0ks_2+$$

$$-\frac{k_0s_2}{T^2(k_1+k)}-\frac{2kTs_1^2s_2}{k_1+k})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+$$

$$+\frac{k_0(s_1^2-s_2^2)}{T(k_1+k)}\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}\} \tag{42}$$

$$\alpha_{22}=\frac{1+r}{W}\{(\frac{k_0^3}{T^2s_1s_2}+\frac{Ts_1s_2}{k_1+k})(s_1^2-s_2^2)\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+$$

$$+(-\frac{k_0^3}{T^3s_1}+Tk_0s_1^3+\frac{k_0^2k_1(s_1^2-s_2^2)}{s_1}+\frac{k_0^2k(s_1^2+s_2^2)}{s_1}+$$

$$+\frac{k_0^2s_1}{T^2(k_1+k)}-\frac{Tk_0s_1s_2^2(k_1-k)}{k_1+k})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+$$

$$+(\frac{k_0^3}{T^3 s_2} - Tk_0 s_2^3 + \frac{k_0^2 k_1 (s_1^2 - s_2^2)}{s_2} - \frac{k_0^2 k (s_1^2 + s_2^2)}{s_2} +$$

$$-\frac{k_0^2 s_2}{T^2 (k_1 + k)} + \frac{Tk_0 s_1^2 s_2 (k_1 - k)}{k_1 + k}) \sinh \frac{s_2 r}{2} \cosh \frac{s_1 r}{2} +$$

$$+(k_0 + \frac{k_0^2}{T(k_1 + k)})(s_1^2 - s_2^2) \cosh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2}\} \quad (43)$$

Matrix $\beta(\theta)$ has the form

$$\beta(\theta) = \begin{bmatrix} \beta_{11}(\theta) & 0 \\ \beta_{21}(\theta) & 0 \end{bmatrix} \quad (44)$$

for $\theta \in [-r, 0]$ where

$$\beta_{11}(\theta) = \frac{2+2r}{W}\{[(-\frac{k_0^2 k s_2}{T^2 s_1} + ks_1 s_2) \sinh \frac{s_1 r}{2} \sinh \frac{s_2 r}{2} +$$

$$+(Tks_1 s_2^2 - \frac{k_0^2 k s_2^2}{Ts_1} - \frac{k_0 k s_1 s_2^2}{k_1 - k} +$$

$$+\frac{T^2 k s_1^3 s_2^2}{k_0 (k_1 - k)}) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2} +$$

$$+(-\frac{T^2 k s_1^2 s_2^3}{k_0 (k_1 + k)} + \frac{k_0 k s_1 s_2^3}{k_1 + k} - \frac{Tks_1^2 s_2 (k_1 - k)}{T(k_1 + k)} +$$

$$+\frac{k_0^2 k s_2 (k_1 - k)}{T(k_1 + k)}) \sinh \frac{s_2 r}{2} \cosh \frac{s_1 r}{2} +$$

$$+(\frac{k_0 k s_2^2}{T(k_1 + k)} - \frac{Tks_1^2 s_2^2}{k_0 (k_1 + k)}) \cosh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2}] \cosh s_1 \theta +$$

$$+[(-\frac{T^2 k s_1^2 s_2^3}{k_0 (k_1 + k)} + \frac{k_0 k s_2^3}{k_1 + k} - \frac{Tks_1^2 s_2 (k_1 - k)}{T(k_1 + k)} +$$

$$+\frac{k_0^2 k s_2 (k_1 - k)}{T(k_1 + k)}) \sinh \frac{s_1 r}{2} \sinh \frac{s_2 r}{2} +$$

$$+(\frac{k_0 k s_2^2}{T(k_1 + k)} - \frac{Tks_1^2 s_2^2}{k_0 (k_1 + k)}) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2} +$$

$$+(-\frac{k_0^2 k s_2}{T^2 s_1} + ks_1 s_2) \sinh \frac{s_2 r}{2} \cosh \frac{s_1 r}{2} +$$

$$+(Tks_1 s_2^2 - \frac{k_0^2 k s_2^2}{Ts_1} - \frac{k_0 k s_1 s_2^2}{k_1 - k} +$$

$$+\frac{T^2 k s_1^3 s_2^2}{k_0 (k_1 - k)}) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2}] \sinh s_1 \theta +$$

$$+[(\frac{k_0^2 k s_1}{T^2 s_2} - ks_1 s_2) \sinh \frac{s_1 r}{2} \sinh \frac{s_2 r}{2} +$$

$$+(\frac{T^2 k s_1^3 s_2^2}{k_0 (k_1 + k)} - \frac{k_0 k s_1^3}{k_1 + k} + \frac{Tks_1 s_2^2 (k_1 - k)}{T(k_1 + k)} +$$

$$-\frac{k_0^2 k s_1 (k_1 - k)}{T(k_1 + k)}) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2} +$$

$$+(-Tks_1^2 s_2 + \frac{k_0^2 k s_1^2}{Ts_2} + \frac{k_0 k s_1^2 s_2}{k_1 - k} +$$

$$-\frac{T^2 k s_1^2 s_2^3}{k_0 (k_1 - k)}) \sinh \frac{s_2 r}{2} \cosh \frac{s_1 r}{2} +$$

$$+(-\frac{k_0 k s_1^2}{T(k_1 + k)} + \frac{Tks_1^2 s_2^2}{k_0 (k_1 + k)}) \cosh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2}] \cosh s_2 \theta +$$

$$+[(\frac{T^2 k s_1^3 s_2^2}{k_0 (k_1 + k)} - \frac{k_0 k s_1^3}{k_1 + k} + \frac{Tks_1 s_2^2 (k_1 - k)}{T(k_1 + k)} +$$

$$-\frac{k_0^2 k s_1 (k_1 - k)}{T(k_1 + k)}) \sinh \frac{s_1 r}{2} \sinh \frac{s_2 r}{2} +$$

$$+(\frac{k_0^2 k s_1}{T^2 s_2} - ks_1 s_2) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2} +$$

$$+(-\frac{k_0 k s_1^2}{T(k_1 + k)} + \frac{Tks_1^2 s_2^2}{k_0 (k_1 + k)}) \sinh \frac{s_2 r}{2} \cosh \frac{s_1 r}{2} +$$

$$+(-Tks_1^2 s_2 + \frac{k_0^2 k s_1^2}{Ts_2} + \frac{k_0 k s_1^2 s_2}{k_1 - k} +$$

$$-\frac{T^2 k s_1^2 s_2^3}{k_0 (k_1 - k)}) \cosh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2}] \sinh s_2 \theta\} \quad (45)$$

$$\beta_{21}(\theta) = \frac{2+2r}{W}\{[(k_0 k s_1 s_2 - \frac{k_0^3 k s_2}{T^2 s_1} + \frac{Tks_1^3 s_2}{k_1 + k} +$$

$$-\frac{k_0^2 k s_1 s_2}{T(k_1 + k)}) \sinh \frac{s_1 r}{2} \sinh \frac{s_2 r}{2} +$$

$$+(-\frac{k_0^3 k}{T^3 s_1} + \frac{k_0 k s_1}{T}) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2} +$$

$$+(\frac{k_0^2 k s_2}{T^2 (k_1 + k)} - \frac{k s_1^2 s_2}{k_1 + k}) \sinh \frac{s_2 r}{2} \cosh \frac{s_1 r}{2} +$$

$$+(-k_0 k s_1^2 + \frac{k_0^3 k}{T^2} - \frac{Tks_1^2 s_2^2}{k_1 + k} +$$

$$+\frac{k_0^2 k s_2^2}{T(k_1 + k)}) \cosh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2}] \cosh s_1 \theta +$$

$$+[(\frac{k_0^2 k s_2}{T^2 (k_1 + k)} - \frac{k s_1^2 s_2}{k_1 + k}) \sinh \frac{s_1 r}{2} \sinh \frac{s_2 r}{2} +$$

$$+(-k_0 k s_1^2 + \frac{k_0^3 k}{T^2} - \frac{Tks_1^2 s_2^2}{k_1 + k} +$$

$$+\frac{k_0^2 k s_2^2}{T(k_1 + k)}) \sinh \frac{s_1 r}{2} \cosh \frac{s_2 r}{2} +$$

$$+(k_0 k s_1 s_2 - \frac{k_0^3 k s_2}{T^2 s_1} + \frac{Tks_1^3 s_2}{k_1 + k} +$$

$$-\frac{k_0^2 k s_1 s_2}{T(k_1+k)})\sinh\frac{s_2 r}{2}\cosh\frac{s_1 r}{2}+$$

$$+(-\frac{k_0^3 k}{T^3 s_1}+\frac{k_0 k s_1}{T})\cosh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}]\sinh s_1\theta+$$

$$+[(-k_0 k s_1 s_2+\frac{k_0^3 k s_1}{T^2 s_2}-\frac{T k s_1 s_2^3}{k_1+k}+$$

$$+\frac{k_0^2 k s_1 s_2}{T(k_1+k)})\sinh\frac{s_1 r}{2}\sinh\frac{s_2 r}{2}+$$

$$+(-\frac{k_0^2 k s_1}{T^2(k_1+k)}+\frac{k s_1 s_2^2}{k_1+k})\sinh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}+$$

$$+(\frac{k_0^3 k}{T^3 s_2}-\frac{k_0 k s_2}{T})\sinh\frac{s_2 r}{2}\cosh\frac{s_1 r}{2}+$$

$$+(k_0 k s_2^2-\frac{k_0^3 k}{T^2}+\frac{T k s_1^2 s_2^2}{k_1+k}+$$

$$-\frac{k_0^2 k s_1^2}{T(k_1+k)})\cosh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}]\cosh s_2\theta+$$

$$+[(-\frac{k_0^2 k s_1}{T^2(k_1+k)}+\frac{k s_1 s_2^2}{k_1+k})\sinh\frac{s_1 r}{2}\sinh\frac{s_2 r}{2}+$$

$$+(-k_0 k s_1 s_2+\frac{k_0^3 k s_1}{T^2 s_2}-\frac{T k s_1 s_2^3}{k_1+k}+$$

$$+\frac{k_0^2 k s_1 s_2}{T(k_1+k)})\sinh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}+$$

$$+(k_0 k s_2^2-\frac{k_0^3 k}{T^2}+\frac{T k s_1^2 s_2^2}{k_1+k}+$$

$$-\frac{k_0^2 k s_1^2}{T(k_1+k)})\sinh\frac{s_2 r}{2}\cosh\frac{s_1 r}{2}+$$

$$+(\frac{k_0^3 k}{T^3 s_2}-\frac{k_0 k s_1}{T})\cosh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}]\sinh s_2\theta \qquad (46)$$

Matrix $\gamma(\theta)$ is given by formula

$$\gamma(\theta)=\begin{bmatrix}\theta+r & 0\\ 0 & \theta+r\end{bmatrix} \qquad (47)$$

for $\theta\in[-r,0]$

Matrix $\delta(\theta,\sigma)$ has the form

$$\delta(\theta,\sigma)=\begin{bmatrix}\delta_{11}(\theta,\sigma) & 0\\ 0 & 0\end{bmatrix} \qquad (48)$$

for $\theta\in[-r,0]$ and $\sigma\in[\theta,0]$ where

$$\delta_{11}(\theta,\sigma)=\frac{2+2r}{W}\{[(k_0 k^2 s_1 s_2-\frac{k_0^3 k^2 s_2}{T^2 s_1}+$$

$$+\frac{T k^2 s_1^3 s_2}{k_1+k}-\frac{k_0^2 k^2 s_1 s_2}{T(k_1+k)})\sinh\frac{s_1 r}{2}\sinh\frac{s_2 r}{2}+$$

$$+(-\frac{k_0^3 k^2}{T^3 s_1}+\frac{k_0 k^2 s_1}{T})\sinh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}+$$

$$+(-\frac{k_0^2 k^2 s_2}{T^2(k_1+k)}+\frac{k^2 s_1^2 s_2}{k_1+k})\sinh\frac{s_2 r}{2}\cosh\frac{s_1 r}{2}+$$

$$+(k_0 k^2 s_1^2-\frac{k_0^3 k^2}{T^2}+\frac{T k^2 s_1^2 s_2^2}{k_1+k}+$$

$$-\frac{k_0^2 k^2 s_2^2}{T(k_1+k)})\cosh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}]\cosh s_1(\theta-\sigma)+$$

$$+[(-\frac{k_0^2 k^2 s_2}{T^2(k_1+k)}+\frac{k^2 s_1^2 s_2}{k_1+k})\sinh\frac{s_1 r}{2}\sinh\frac{s_2 r}{2}+$$

$$+(k_0 k^2 s_1^2-\frac{k_0^3 k^2}{T^2}+\frac{T k^2 s_1^2 s_2^2}{k_1+k}+$$

$$-\frac{k_0^2 k^2 s_2^2}{T(k_1+k)})\sinh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}+$$

$$+(k_0 k^2 s_1 s_2-\frac{k_0^3 k^2 s_2}{T^2 s_1}+\frac{T k^2 s_1^3 s_2}{k_1+k}+$$

$$-\frac{k_0^2 k^2 s_1 s_2}{T(k_1+k)})\sinh\frac{s_2 r}{2}\cosh\frac{s_1 r}{2}+$$

$$+(-\frac{k_0^3 k^2}{T^3 s_1}+\frac{k_0 k^2 s_1}{T})\cosh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}]\sinh s_1(\theta-\sigma)+$$

$$+[(-k_0 k^2 s_1 s_2+\frac{k_0^3 k^2 s_1}{T^2 s_2}-\frac{T k^2 s_1 s_2^3}{k_1+k}+$$

$$+\frac{k_0^2 k^2 s_1 s_2}{T(k_1+k)})\sinh\frac{s_1 r}{2}\sinh\frac{s_2 r}{2}+$$

$$+(\frac{k_0^2 k^2 s_1}{T^2(k_1+k)}-\frac{k^2 s_1 s_2^2}{k_1+k})\sinh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}+$$

$$+(\frac{k_0^3 k^2}{T^3 s_2}-\frac{k_0 k^2 s_2}{T})\sinh\frac{s_2 r}{2}\cosh\frac{s_1 r}{2}+$$

$$+(-k_0 k^2 s_2^2+\frac{k_0^3 k^2}{T^2}-\frac{T k^2 s_1^2 s_2^2}{k_1+k}+$$

$$+\frac{k_0^2 k^2 s_1^2}{T(k_1+k)})\cosh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}]\cosh s_2(\theta-\sigma)+$$

$$+[(\frac{k_0^2 k^2 s_1}{T^2(k_1+k)}-\frac{k^2 s_1 s_2^2}{k_1+k})\sinh\frac{s_1 r}{2}\sinh\frac{s_2 r}{2}+$$

$$+(-k_0 k^2 s_1 s_2+\frac{k_0^3 k^2 s_1}{T^2 s_2}-\frac{T k^2 s_1 s_2^3}{k_1+k}+$$

$$+\frac{k_0^2 k^2 s_1 s_2}{T(k_1+k)})\sinh\frac{s_1 r}{2}\cosh\frac{s_2 r}{2}+$$

$$+(-k_0 k^2 s_2^2+\frac{k_0^3 k^2}{T^2}-\frac{T k^2 s_1^2 s_2^2}{k_1+k}+$$

$$+\frac{k_0^2k^2s_1^2}{T(k_1+k)})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+$$

$$+(\frac{k_0^3k^2}{T^3s_2}-\frac{k_0k^2s_2}{T})\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}]\sinh s_2(\theta-\sigma) \tag{49}$$

Having matrices α, $\beta(\theta)$, $\gamma(\theta)$ and $\delta(\theta,\sigma)$ we can calculate the value of the Lyapunov functional (34) for arbitrary initial function φ.

The value of the performance index (32) is equal to the value of the Lyapunov functional (34) at initial function (35). After computation we obtain

$$J=\left(\frac{w_0+\frac{z_0}{k_0}}{k_1+k}\right)^2\left[\alpha_{11}+\int_{-r}^{0}\beta_{11}(\theta)d\theta+\int_{-r}^{0}\int_{\theta}^{0}\delta_{11}(\theta,\sigma)d\sigma d\theta\right]+$$

$$-\left(i_0+\frac{Tz_0}{k_0}\right)\frac{w_0+\frac{z_0}{k_0}}{k_1+k}\left[2\alpha_{12}+\int_{-r}^{0}\beta_{21}(\theta)d\theta\right]+$$

$$+\left(i_0+\frac{Tz_0}{k_0}\right)^2\alpha_{22}+\left(\frac{w_0+\frac{z_0}{k_0}}{k_1+k}\right)^2\int_{-r}^{0}(\theta+r)d\theta+$$

$$+\int_{-r}^{0}(\theta+r)\left(\frac{u_0}{R}\left(1-e^{-\frac{\theta}{T}}\right)+i_0e^{-\frac{\theta}{T}}+\frac{Tz_0}{k_0}\right)^2d\theta \tag{50}$$

The last term of formula (50) does not depend on parameter k and therefore it can be omitted because it has not an effect on the optimal solution only on the value of the performance index.

After calculations we obtain

$$J=\frac{1+r}{W}\left(\frac{w_0+\frac{z_0}{k_0}}{k_1+k}\right)^2\{(s_1^2-s_2^2)(\frac{k_0}{T^2s_1s_2}+$$

$$+\frac{k_0(k_1+k)^2}{s_1s_2}+\frac{k_0^2(k_1-k)}{Ts_1s_2}-\frac{4k_0^3k_1k}{T^4s_1^3s_2^3}+$$

$$+\frac{4k_0^2k_1k}{Ts_1s_2(k_1-k)}+\frac{2k_0^2k^2r}{T^2s_1s_2(k_1+k)})\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+$$

$$+(k_1s_1+ks_1-\frac{k_0s_2^2}{Ts_1}-\frac{2k_0^2k_1}{T^2s_1}-\frac{k_0(k_1+k)^2}{Ts_1}+\frac{4k_0^3k_1k}{T^3s_1^3}+$$

$$+\frac{2k_0k_1s_1}{T(k_1+k)}+\frac{s_1^3}{k_1+k}+\frac{2k_0^2k^2r(s_1^2-s_2^2)}{Ts_1(k_1+k)}+$$

$$+\frac{2k_0^3k^2r(s_1^2-s_2^2)}{T^2s_1s_2^2})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+$$

$$+(-k_1s_2-ks_2+\frac{k_0s_1^2}{Ts_2}+\frac{2k_0^2k_1}{T^2s_2}+\frac{k_0(k_1+k)^2}{Ts_2}+$$

$$-\frac{4k_0^3k_1k}{T^3s_2^3}-\frac{2k_0k_1s_2}{T(k_1+k)}-\frac{s_2^3}{k_1+k}+\frac{2k_0^2k^2r(s_1^2-s_2^2)}{Ts_2(k_1+k)}+$$

$$+\frac{2k_0^3k^2r(s_1^2-s_2^2)}{T^2s_1^2s_2})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+T(s_1^2-s_2^2)(\frac{k_0}{T}+$$

$$+k_1+k+\frac{1}{T^2(k_1+k)}+\frac{2k_0^3k^2r}{T^4s_1^2s_2^2})\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}\}+$$

$$-\frac{2+2r}{W}\left(i_0+\frac{Tz_0}{k_0}\right)\frac{w_0+\frac{z_0}{k_0}}{k_1+k}\{\frac{k_0^2(k_1-k)(s_1^2-s_2^2)}{T^2s_1s_2(k_1+k)}\times$$

$$\times\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+(-\frac{k_0^2}{T^3s_1}+\frac{2k_0^3k}{T^2s_1}+\frac{k_0s_1}{T^2(k_1+k)}+$$

$$+\frac{2k_0^2ks_2^2}{Ts_1(k_1+k)})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+(\frac{k_0^2}{T^3s_2}-\frac{2k_0^3k}{T^2s_2}+$$

$$-\frac{k_0s_2}{T^2(k_1+k)}-\frac{2k_0^2ks_1^2}{Ts_2(k_1+k)})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+$$

$$+\frac{k_0(s_1^2-s_2^2)}{T(k_1+k)}\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}\}+$$

$$+\frac{1+r}{W}\left(i_0+\frac{Tz_0}{k_0}\right)^2\{(s_1^2-s_2^2)(\frac{k_0^3}{T^2s_1s_2}+$$

$$+\frac{Ts_1s_2}{k_1+k})\sinh\frac{s_1r}{2}\sinh\frac{s_2r}{2}+(-\frac{k_0^3}{T^3s_1}+$$

$$+Tk_0s_1^3+\frac{k_0^2k_1(s_1^2-s_2^2)}{s_1}+\frac{k_0^2k(1-2Tk_0k_1)}{T^2s_1}+$$

$$+\frac{k_0^2s_1}{T^2(k_1+k)}-\frac{Tk_0s_1s_2^2(k_1-k)}{k_1+k})\sinh\frac{s_1r}{2}\cosh\frac{s_2r}{2}+$$

$$+(\frac{k_0^3}{T^3s_2}-Tk_0s_2^3+\frac{k_0^2k_1(s_1^2-s_2^2)}{s_2}-\frac{k_0^2k(1-2Tk_0k_1)}{T^2s_2}+$$

$$-\frac{k_0^2s_2}{T^2(k_1+k)}+\frac{Tk_0s_1^2s_2(k_1-k)}{k_1+k})\sinh\frac{s_2r}{2}\cosh\frac{s_1r}{2}+$$

$$+(s_1^2-s_2^2)(k_0+\frac{k_0^2}{T(k_1+k)})\cosh\frac{s_1r}{2}\cosh\frac{s_2r}{2}\}+$$

$$+\left(\frac{w_0+\frac{z_0}{k_0}}{k_1+k}\right)^2\frac{r^2}{2} \tag{51}$$

Formula (24) implies

$$p=Lk \tag{52}$$

We will search for optimal gain p_{opt} which minimizes the performance index (51).

IV. Optimization results

We will make the parametric optimization for the separately excited D.C. motor Siemens 1GH6 size 225, the catalog number: 1GH6 226-ONA40-1VV3, which has the following properties:

Rated speed $n_N = 745\, rpm$
Rated angular velocity $\omega_N = 77,98\, rad/s$
Rated armature voltage $u_0 = 420\, V$
Rated output $P_N = 96\, kW$
Rated torque $m_N = 1230\, Nm$
Maximum field weakening speed $n_{F_{max}} = 2020\, rpm$
Rated current $i_N = 264\, A$
Efficiency $\eta = 85\%$
Armature circuit Resistance $R = 180\, m\Omega$
Armature circuit Inductance $L = 4,71\, mH$
Rated field voltage 310 V
Field power $P_{field} = 2,9\, kW$
Moment of inertia of the rotor $J_r = 2,2\, kgm^2$
Mechanical limit speed $n_{mech} = 3000\, rpm$
Now we can compute the value of parameters:
$k_m = 4,659,\ k_e = 4,777,\ m_z = 10871\, Nm,\ w_0 = 89172,\ k_1 = 1014,\ T = 0,0262,\ k = 212,31p$
We suppose that $i_0 = 0$
The terms (9), (20) and (21) imply $r = 0,0262 \ln \frac{10871}{10871 - z_s}$, $k_0 = \frac{0,1220658}{J_m}$ and $z_0 = -\frac{z_s}{J_m}$
Moment of inertia J_m must be greater than moment of inertia of the rotor J_r. The relation $J_m > J_r = 2,2\, kgm^2$ holds.
We will search for optimal gain p_{opt} for varies z_s and J_m. Optimization results are given in Table I to Table III.

Table I: Optimization results for $z_s = 2000\, Nm$ and r =0,0053 s

J_m $[kgm^2]$	p_{opt}	current [A]	angular velocity [rad/s]
3	7,9336	433,9	27,0403
4	11,4522	438,1	21,0067
5	14,9698	434,5	17,1196
6	18,4862	427,1	14,5021

Table II: Optimization results for $z_s = 2500\, Nm$ and r =0,0068 s

J_m $[kgm^2]$	p_{opt}	current [A]	angular velocity [rad/s]
3	5,4370	536,7	31,6613
4	8,1505	536,5	25,0115
5	10,8613	536,1	20,6776
6	13,5706	535,8	17,6289

Table III: Optimization results for $z_s = 3000\, Nm$ and r =0,0084 s

J_m $[kgm^2]$	p_{opt}	current [A]	angular velocity [rad/s]
3	3,7338	643,9	35,7305
4	5,9088	643,9	28,4575
5	8,0799	643,9	23,6523
6	10,2488	643,8	20,2389

The values of the optimal gains of the P-controller were obtained by means of Matlab function *fminsearch.* Current and angular velocity values given in Table I to Table III denote steady state values.
Fig. 2 to Fig. 7 show the current and the angular velocity for optimal value of the gain of the P-controller and for varies value of the mechanical torque z_s and for fixed value of moment of inertia $J_m = 3\, kgm^2$.

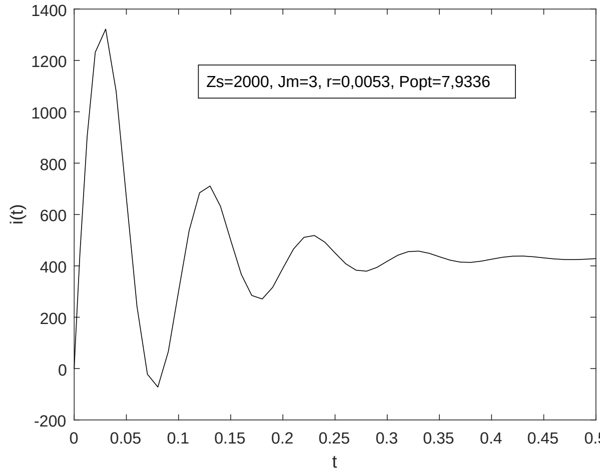

Fig. 2 The current for the optimal gain and Zs=2000 Nm

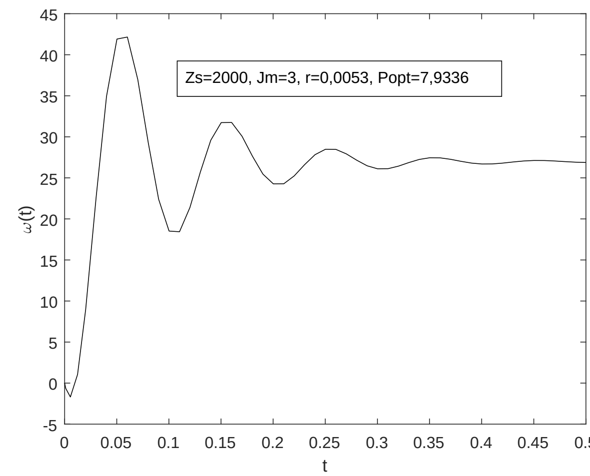

Fig. 3 The angular velocity for the optimal gain and Zs=2000 Nm

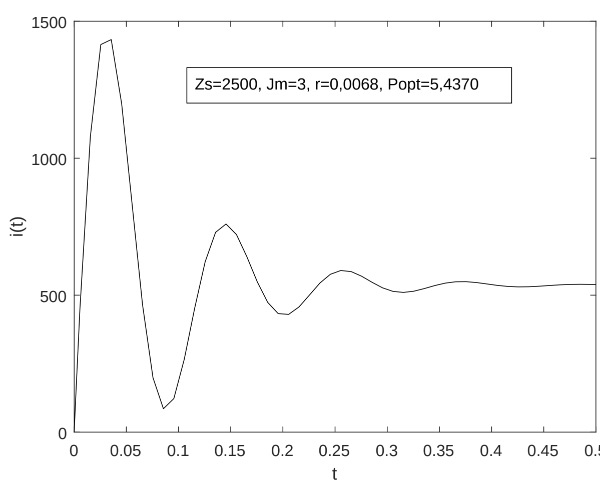

Fig. 4 The current for optimal gain and Zs=2500 Nm

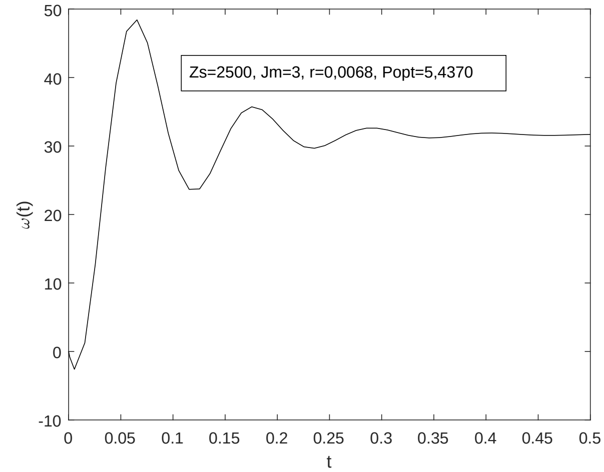

Fig. 5 The angular velocity for optimal gain and Zs=2500 Nm

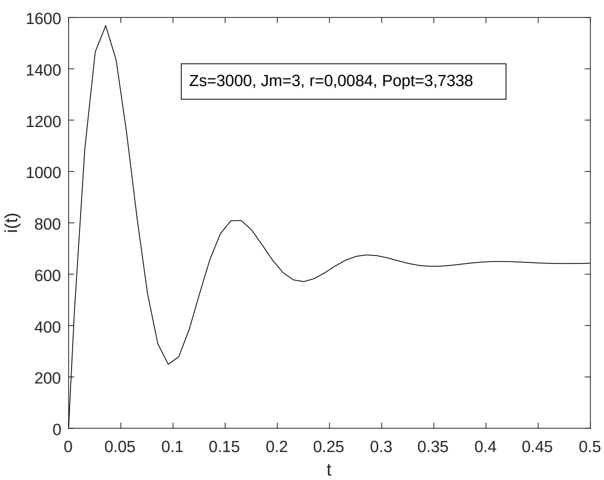

Fig. 6 The current for optimal gain and Zs=3000 Nm

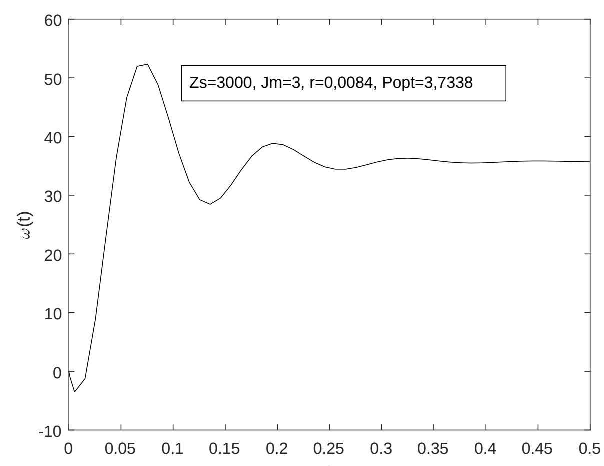

Fig. 7 The angular velocity for optimal gain and Zs=3000 Nm

Analyzing graphs on figures we can see oscillations with big overshoot. The overshoot is less for greater mechanical torque. Oscillations are typical for performance index of type integral of squared error.

V. CONCLUSIONS

In the paper a new model of a separately excited D.C. motor was presented. The parametric optimization problem for a new model of a separately excited D.C. motor with a P-controller was considered. The general quadratic performance index was used. The value of the quadratic performance index was determined by means of the Lyapunov functional using Repin's method. The optimization results for the separately excited D.C. motor Siemens 1GH6 size 225 were given. Author intends to develop results of that paper for the control system with PI-controller. It is important to find the performance index which enables less overshoot of oscillations.

ACKNOWLEDGMENT

I thank the associate editor and the reviewers for their suggestions, which have improved the quality of the paper.

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