Novel digital integrators and differentiators using fractional delay−A Biomedical Application

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Abstract—This paper deals with application of fractional delay to IIR type digital integrators. A study of various digital integrators is presented. Often applications in controls, wave-shaping, oscillators and communications require a constant -900 phase for integrators. When the design neglects the phase, a phase equalizer is often needed to compensate for the phase error or a phase lock loop should be added. In this paper an attempt is made to study about the variation of both magnitude and phase angle of digital integrators with the application of fractional delay. The differentiators have performed well at low frequencies. The designed differentiators are compared by taking QRS detection as an application.

Keywords—Digital integrator, Fractional delay, Interpolation, differentiator, finite impulse response, Allpass system, QRS Complex, Pan-Tompkins method.

I. Introduction

Digital integrators are used to find the time-integral of the incoming signal. These devices are used in almost all fields of engineering like instrumentation, control systems, digital signal and Image processing, bio-medical engineering, radar and other allied fields. The methods of designing digital integrators can be of either FIR or IIR type. In FIR approach the filter coefficients are obtained by using maximum flatness conditions. In IIR approach the filter coefficients are obtained directly from the well known Rectangular, Trapezoidal and Simpson methods of Integration.

FD filters have been widely used before in areas as diverse as arbitrary sampling rate conversion, synchronization of digital modems and speech coding etc.In this paper an attempt is made to apply fractional delay to digital integrators to achieve constant phase angle.

QRS detection is a widely used technique to identify the function of the heart[10]. Out of many of the existing procedures Pan-Tompkins method is widely used one.Digital differentiator forms an essential component in the Pan-Tompkins method. In this paper an attempt is made to replace the proposed differentiators with conventional one and the QRS complex detection is carried out.

The paper is organized as follows. Section2 deals with digital integrators. IIR and FIR approximations of fractional delay are studied in Section3.Design of fractional delay based digital integrators is presented in section 4. QRS detection is discussed in Section 5.Finally Results and con-clusions are presented in Section 6.

II. Digital Integrators

An ideal integrator is defined by the following transfer function [1-4],

$$H(j\omega) = \frac{1}{j\omega} \tag{1}$$

The following are the transfer functions of digital integrators. Back ward integrator [1],

$$H_1(z) = \frac{zT}{z-1} \tag{2}$$

Bilinear Integrator [1],

$$H_2(z) = \frac{T(z+1)}{2(z-1)} \tag{3}$$

In 1993, by the interpolation of Backward and Bilinear integrators Al-Alaoui has produced a new non-minimum phase integrator given by[2-6],

$$H_{AL}(z) = \frac{T(z+7)}{8(z-1)} \tag{4}$$

Simpson's Integrator is defined as,

$$H_3(z) = \frac{T(z^2+4z+1)}{3(z^2-1)} \tag{5}$$

Tick Integrator is defined as,

$$H_4(z) = \frac{T(0.3585z^2+1.2832z+0.3584)}{(z^2-1)} \tag{6}$$

Al-Alaoui second order digital integrator is defined as,

$$H_{AL2}(z) = \frac{0.4T(z^2+2.5z+1)}{(z^2-1)} \tag{7}$$

Magnitude comparison of all the above mentioned digital integrators is shown in Fig.1. It can be observed that the ideal integrator magnitude response lies between that of the bilinear and backward integrators, bilinear and Simpson integrator etc.

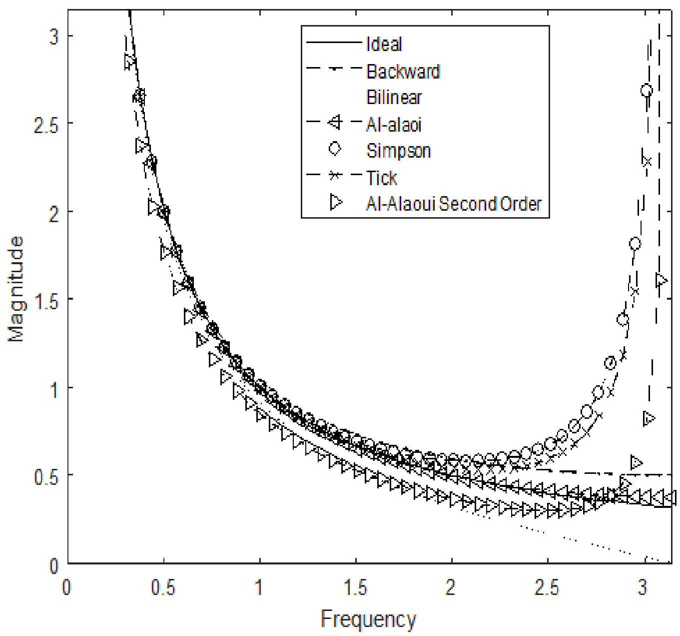

Fig. 1. Comparison of Magnitude Response various digital integrators

III. Fractional Delay

The delayed version of a discrete-time signal x(n) may be represented as[7-8],

$$y[n] = x[n - D] \tag{8}$$

Where D is a positive integer that denotes the amount by which the signal is delayed. In traditional digital signal processing theory, D can only take integer values.In many applications it is desirable that the delay D accurately represent the fractional delay, rather than the integer delay. If the Z-transform of Eq.8 is taken, the transfer function of an ideal delay element may be written as,

$$H(z) = \frac{Y(z)}{X(z)} = z^{-D} \tag{9}$$

For the sake of discussion, assume that D is a positive real number, def ned as the sum of its integer part, N, and the fractional part, d.

$$D = N + d \tag{10}$$

In the frequency domain, the ideal fractional-delay f lter can be described as,

$$H(e^{j\omega}) = e^{-j\omega D} \tag{11}$$

i.e., the magnitude response for an ideal delay element is unity for all frequencies, while the phase response is linear with a slope of -D. This can be called an *all pass system* with linear phase response.

$$\left|H(e^{j\omega})\right| = 1$$

$$\angle H(e^{j\omega}) = -D\omega \tag{12}$$

A. Thiran All pass filter

With this approximation the total delay D is approximated by[7-8],

$$z^{-D} \approx \frac{a_N + a_{N-1}z^{-1} + - - z^{-N}}{1 + a_1 z^{-1} + - - a_N z^{-N}} \tag{13}$$

$$a_k = (-1)^k C_k^N \prod_{n=0}^{N} \frac{N - n - D}{N - k - n - D} \tag{14}$$

Where $C_k^N = \frac{N!}{k!(N-k)!}$ and D=N+d.

B. Lagrange interpolation FIR Delay filter

With this approximation, a total delay D is approximated by [7-8],

$$z^{-D} \approx \sum_{n=0}^{L} h[n] z^{-n} \tag{15}$$

The f lter-order L is chosen such that (L - 1)/2 $<$ D $<$ (L +1)/2.Moreover, the total delay D is expressed as D = N + d, where N is the integer part of the delay and d the fractional part. In general for a fractional delay of d, using Taylor series expansion $z^{-d} = d + dz^{-1}$ (Neglecting higher order terms).Considering half sample delay the expressions for FIR and IIR cases will be,

$$z^{-\frac{1}{2}} = \frac{1}{2} + \frac{1}{2} z^{-1} \tag{16}$$

$$z^{-\frac{1}{2}} = \frac{1 + 3z^{-1}}{3 + z^{-1}} \tag{17}$$

The transfer functions obtained for different values of d are tabulated in Table.1.

TABLE I
Approximations of Fractional delay,d

Delay,d	FIRApproximation	IIRApproximation
0.1	$\frac{1+z^{-1}}{10}$	$\frac{9+11z^{-1}}{11+9z^{-1}}$
0.2	$\frac{1+z^{-1}}{5}$	$\frac{8+12z^{-1}}{12+8z^{-1}}$
0.3	$\frac{1+z^{-1}}{10/3}$	$\frac{7+13z^{-1}}{13+7z^{-1}}$
0.4	$\frac{1+z^{-1}}{2.5}$	$\frac{6+14z^{-1}}{14+6z^{-1}}$
0.5	$\frac{1+z^{-1}}{2}$	$\frac{5+15z^{-1}}{15+5z^{-1}}$
0.6	$\frac{1+z^{-1}}{5/3}$	$\frac{4+16z^{-1}}{16+4z^{-1}}$
0.7	$\frac{1+z^{-1}}{10/7}$	$\frac{3+17z^{-1}}{17+3z^{-1}}$
0.8	$\frac{1+z^{-1}}{5/4}$	$\frac{2+18z^{-1}}{18+2z^{-1}}$
0.9	$\frac{1+z^{-1}}{10/9}$	$\frac{1+19z^{-1}}{19+z^{-1}}$

IV. Design of Digital Differentiators and Integrators

In this section, initially a novel class of integrators are derived by applying fractional delay to the conventional integrators. The digital differentiators are designed by following the procedure as mentioned below,

1) Consider the transfer function of an integrator from which the desired differentiator is to be designed.
2) Invert the transfer function of the integrator proposed in step (1) and stabilize it.
3) Compensate the change in magnitude.

A. design of digital integrators

1) Al-Alaoui First order Integrator: When the sampling rate is decreased by half, the expression for the digital integrator will be,

$$H(z) = 7\frac{T/2}{8}\frac{1+\frac{1}{7}z^{-\frac{1}{2}}}{1-z^{-\frac{1}{2}}} = \frac{7T}{16}\frac{1+\frac{1}{7}z^{-1}+\frac{8}{7}z^{-\frac{1}{2}}}{1-z^{-1}} \tag{18}$$

Substituting the values of $z^{-\frac{1}{2}}$ the expressions will be, for IIR case

$$H(z) = \frac{T}{16}\frac{29+34z^{-1}+z^{-2}}{3-2z^{-1}-z^{-2}} \tag{19}$$

for FIR case

$$H(z) = \frac{T}{16}\frac{11+5z^{-1}}{1-z^{-1}} \tag{20}$$

2) Simpson Integrator : For a Simpson Integrator the equation reduces for half sample delay as,

$$H(z) = \frac{T}{6}\frac{1+4z^{-\frac{1}{2}}+z^{-1}}{1-z^{-1}}$$

$$= \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \qquad FIR \tag{21}$$

$$= \frac{T}{6}\frac{7+16z^{-1}+z^{-2}}{3-2z^{-1}-z^{-2}} \qquad IIR$$

3) Tick Integrator: The Transfer functions of half sampled Tick Integrator for FIR and IIR cases will be, For FIR case it reduces to Bilinear Integrator.

$$H(z) = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \tag{22}$$

Whereas for an IIR Case,

$$H(z) = T\frac{2.3587+5.2833z^{-1}+0.3584z^{-2}}{3-2z^{-1}-z^{-2}} \tag{23}$$

4) Al-Alaoui second order Integrator: For FIR Case

$$H(z) = \frac{0.45}{T}\frac{1+z^{-1}}{1-z^{-1}} \tag{24}$$

Whereas for an IIR Case

$$H(z) = \frac{T}{5}\frac{5.5+11.5z^{-1}+z^{-2}}{3-2z^{-1}-z^{-2}} \tag{25}$$

B. design of digital differentiators

By following the above mentioned procedure, the transfer functions of the differentiators will be,

1) Al-Alaoui First order case: The expressions for IIR case

$$G(z) = \frac{14}{T}\frac{3-2z^{-1}-z^{-2}}{1+0.9056z^{-1}+0.02644z^{-2}} \tag{26}$$

for FIR case

$$G(z) = \frac{16}{T}\frac{1-z^{-1}}{11+5z^{-1}} \tag{27}$$

2) Simpson Integrator case: for FIR case,

$$G(z) = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{28}$$

for IIR case,

$$G(z) = \frac{6}{2.2215T}\frac{3-2z^{-1}-z^{-2}}{1+2.2921z^{-1}+0.1571z^{-2}} \tag{29}$$

3) Tick Integrator case: For FIR case

$$G(z) = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{30}$$

Whereas for an IIR Case,

$$G(z) = \frac{2.17}{T}\frac{3-2z^{-1}-z^{-2}}{1+0.530025z^{-1}+0.03277z^{-2}} \tag{31}$$

4) Al-Alaoui second order Integrator case: For FIR Case

$$G(z) = \frac{0.45}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{32}$$

Whereas for an IIR Case

$$G(z) = \frac{5}{T}\frac{3-2z^{-1}-z^{-2}}{1+(13/22)z^{-1}+(1/22)z^{-2}} \tag{33}$$

The magnitude and phase responses of the fractionally delayed digital integrators presented in the previous sections are shown in Fig.2 and 3 respectively.

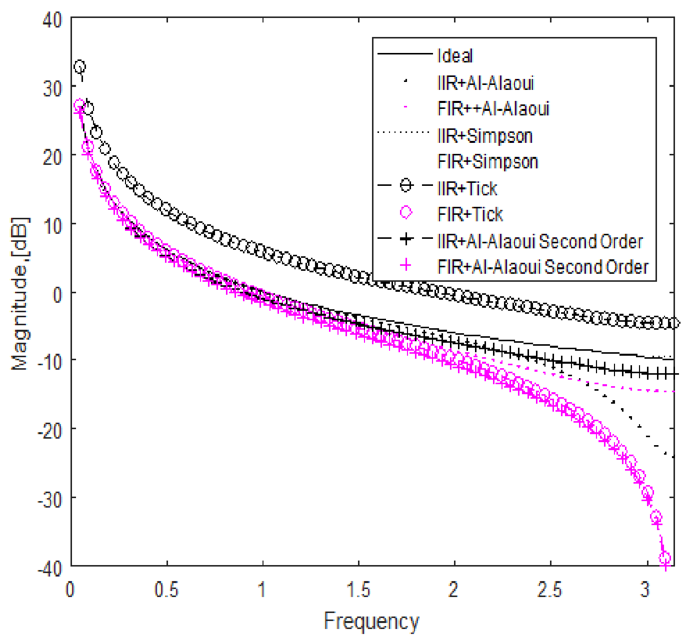

Fig. 2. Comparison of magnitude responses of fractionally delayed digital integrators

The magnitude and phase responses of the fractionally delayed digital differentiators presented in the previous sections are shown in Fig.4 and 5 respectively.

The simulations are carried out at T=1sec.

V. QRS COMPLEX DETECTION

The diagram of the conventional Pan-Tompkins method is as shown in Fig.6. The differentiator block is replaced with the proposed differentiators. The data is taken from the MITBIH data base.The signal under test is as shown in Fig.7. The outputs obtained are as shown in Figs.8-11.

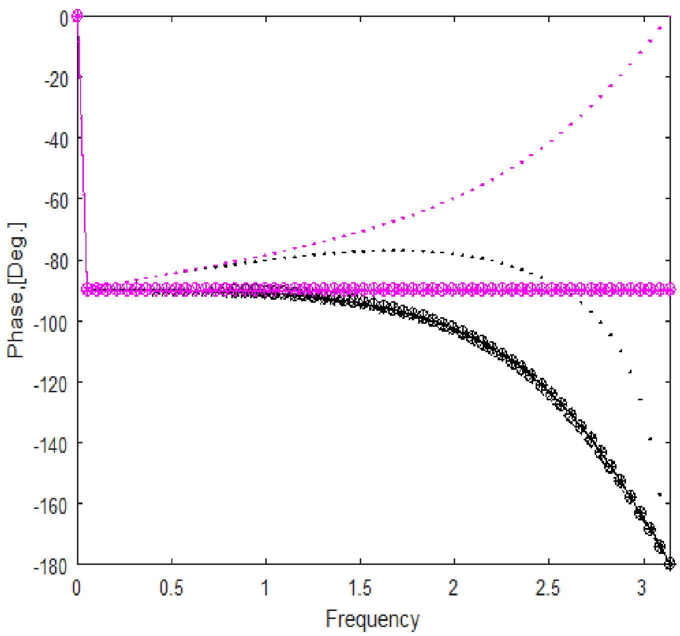

Fig. 3. Comparison of Phase responses of fractionally delayed digital integrators

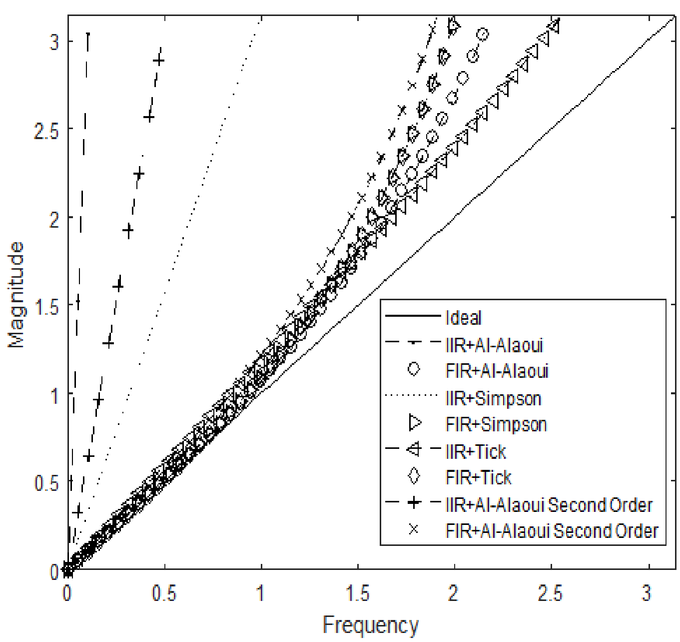

Fig. 4. Comparison of magnitude responses of fractionally delayed digital Differentiators

VI. Results and Conclusions

From Fig.2 it is evident that Tick integrator designed by using IIR type delay is having a large deviation from the ideal characteristics. From Fig.3 it is clear that FIR Al-Alaoui second order, FIR Simpson and FIR tick integrators are producing constant phase angle of -90 degrees. It is also evident that IIR Simpson, IIR Tick, IIR Al-Alaoui Second order integrators are exhibiting the characteristic of linear phase digital differentiator. They are behaving as leading elements rather than behaving as lagging elements by the application of fractional delay. The application of different

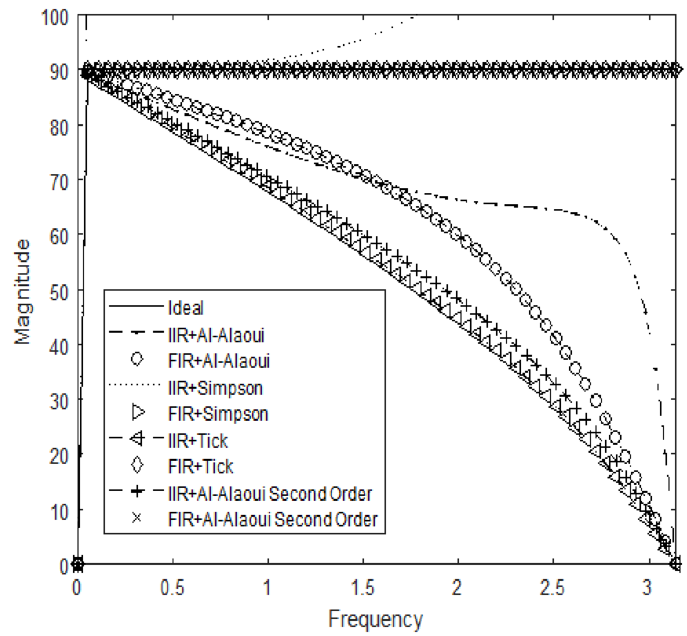

Fig. 5. Comparison of Phase responses of fractionally delayed digital Differentiators

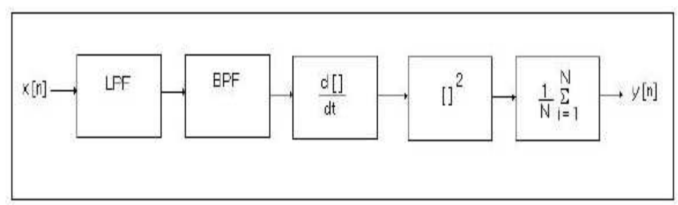

Fig. 6. Pan-Tompkins algorithm

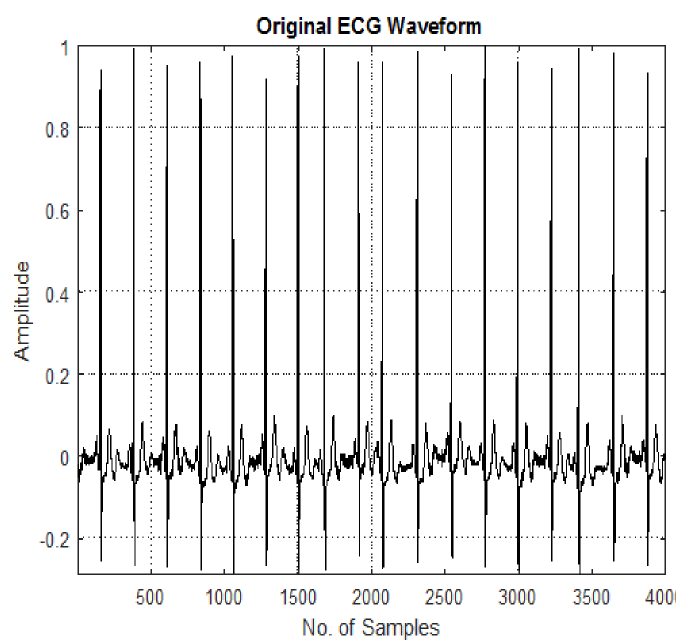

Fig. 7. Original ECG Signal

values of fractional delay will def nitely produce more useful results for achieving constant phase angle.From f gures 4 and 5 it is observed that the differentiators from the FIR simpson

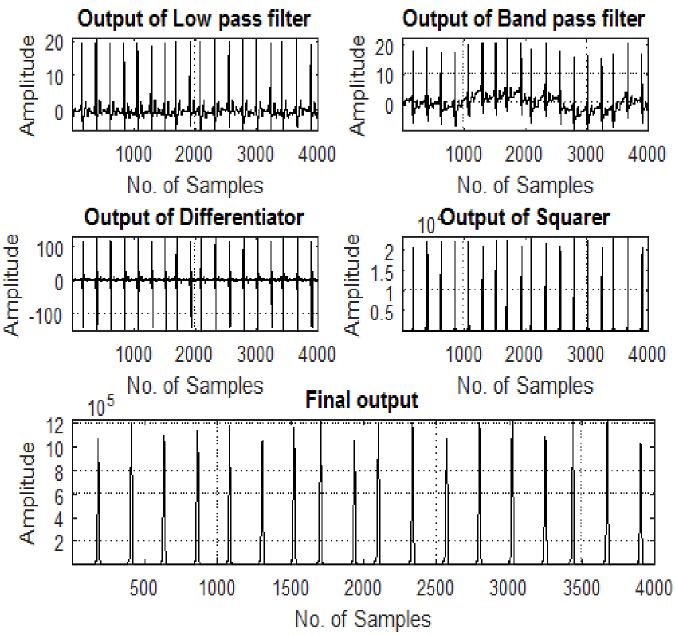

Fig. 8. Output using Al-Alaoui Differentiator of IIR type

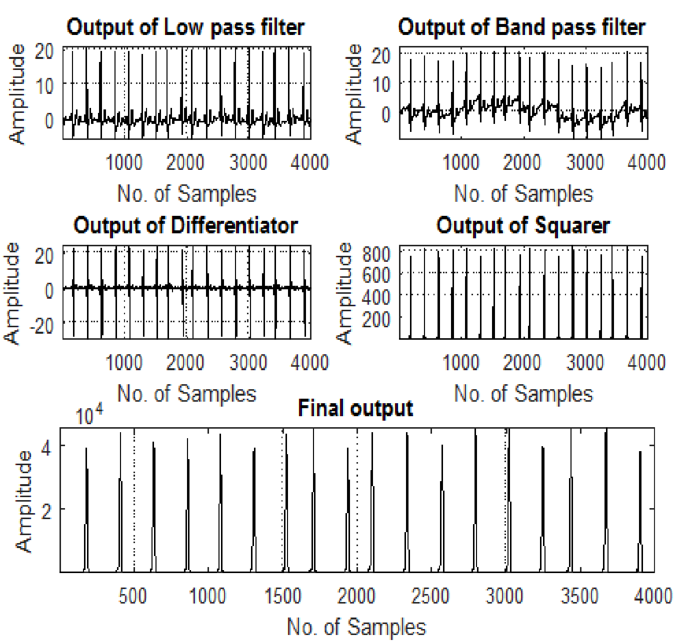

Fig. 9. Output using Tick Differentiator of IIR type

is similar to wideband differentiator with some change in amplitude. The differentiators perform well at low frequencies. Many of the differentiators are exhibiting the linear phase characteristics. These can be used for real time applications like edge detection, QRS complex etc.

From Figs.8-11 it is evident that the proposed differentiators can be used in the place of the conventional differentiators. It can be observed that there some change in the amplitude of the output with the proposed f lters compared to conventional one.

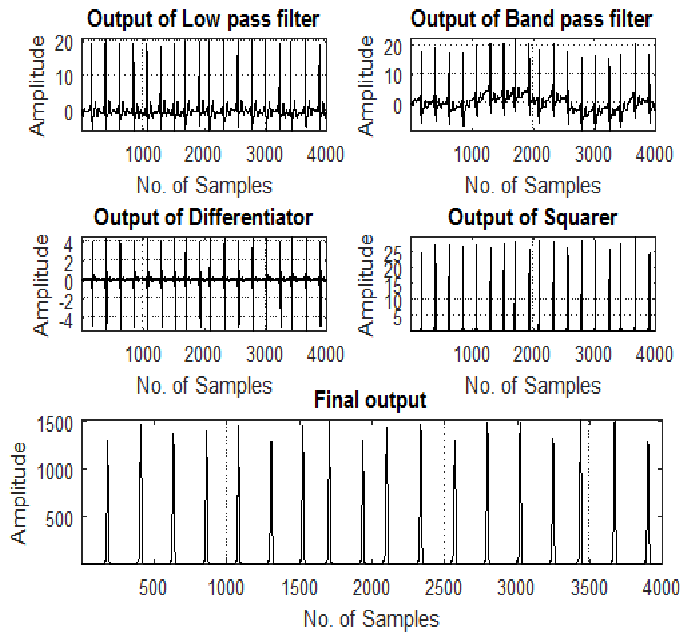

Fig. 10. Output using Simpson Differentiator of FIR type

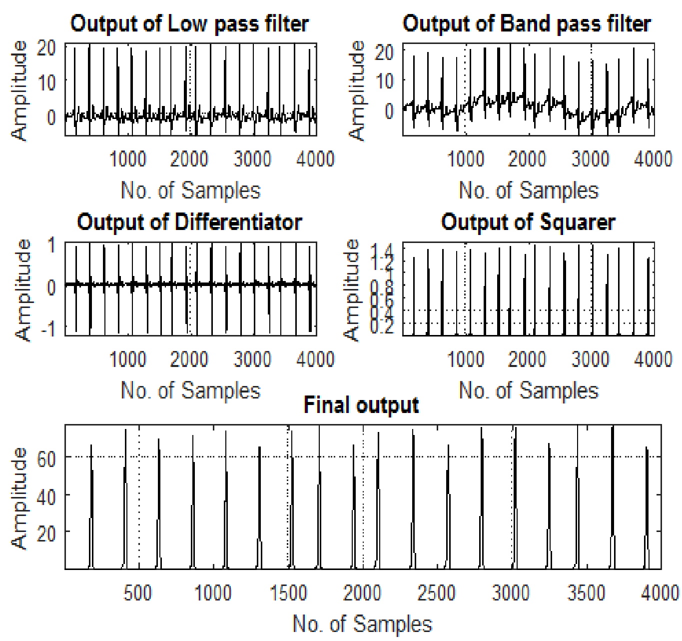

Fig. 11. Output using Second order Al-Alaoui Differentiator of FIR type

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