Analytical Approach to Transient Solution of Single-Line and Double-Line Faults in Three-Phase Circuits

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***Abstract*—**In this work a general methodology is introduced to obtain the analytical solution of transients due to single-line and double-line faults in three-phase circuits. The proposed approach is based on the Clarke transformation, and it is shown that the constraints introduced by the considered faults result in the coupling, through an ideal transformer, between the modal circuits α and 0 of the transformation. The exact analytical solution provided by the proposed approach is validated numerically by implementing a reference three-phase circuit in Matlab/Simulink. Thus, it is shown that single-line and double-line fault transients in simple three-phase circuits do not need a rough numerical approach, but an analytical methodology with educational value can be effectively exploited.

***Keywords*—**Clarke transformation, Fault analysis, Power system analysis, Three-phase circuits, Time-domain analysis, Transients.

I. INTRODUCTION

TRANSIENT analysis of three-phase power systems is of paramount importance in modern electrical power systems. In fact, increasing system complexity and widespread use of sophisticated electrical/electronic apparatus result in high components susceptibility to transient overcurrents and overvoltages originating from fault conditions [1]. Nevertheless, analytical tools for transient analysis have not received extensive attention in the technical literature, whereas numerical tools (e.g., the software EMTP) have been thoroughly investigated [2]. The main reason can be identified in the asymmetrical conditions introduced by single-line or double-line faults. In fact, conventional analytical tools for three-phase circuit analysis (e.g., the well-known symmetrical component transformation) are based on the assumption of equal phases, i.e., each phase must be characterized by the same topology and electrical parameters [3]. Since asymmetrical faults introduce system asymmetry, the conventional analytical approaches cannot be applied in a straightforward way [4]. Some papers introduced approximate analytical methods [5]-[8], but a general and theoretically consistent methodology leading to non-approximate results would be preferable.

The general analytical approach proposed in this paper is based on the well-known Clarke transformation since it operates in the time domain [9]-[12]. Asymmetry introduced by a single-line and double-line faults is taken into account in a rigorous way by deriving a circuit representation of coupling between the α, β, and 0 modal circuits of the Clarke transformation. The methodology presented in this paper is the generalized version of [13]-[14] since: 1) the proposed model for the single-line-to-ground fault is enriched by taken into account the fault resistance; 2) the double-line-to-ground and the isolated double-line faults are modeled.

The analytical models derived in the paper are numerically validated by implementing a reference three-phase circuit in Matlab/Simulink.

The paper is organized as follows. The Clarke transformation is briefly recalled in Section II. The analytical derivation of the single-line-to-ground fault model is treated in Section III. The double-line-to-ground and the isolated double-line faults are modeled in Section IV. Numerical simulations are presented in Section V in order to validate the analytical results derived in the paper. Finally, concluding remarks are discussed in Section VI.

II. CLARKE TRANSFORMATION

Time-domain analysis of three-phase circuits can be effectively performed by means of the Clarke transformation [11]. In fact, under the assumption of three equal phases (with equal coupling parameters, if any), Clarke transformation allows the derivation of three uncoupled circuits (i.e., the α, β, and 0 modal circuits) whose solution is straightforward when compared with the original coupled three-phase circuit.

The Clarke transformation applies to each triplet of phase time-domain variables *a, b, c* (voltages or currents) in a given three-phase circuit. For example, by considering a triplet of phase currents:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \mathbf{T} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}. \quad (1)$$

Notice that in (1) the rational form of the Clarke transformation was used (i.e., the factor $\sqrt{2/3}$ was introduced). This choice corresponds to orthogonality of the transformation

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matrix **T**, leading to the conservation of power through the transformation.

In previous papers it was shown that the Clarke transformation (similarly to the well-known Symmetrical Component Transformation) results also in topological properties of the modal circuits [15]. In particular, it was shown that: 1) the star centers in the three-phase circuit correspond, in the α and β modal circuits, to shorted terminals with respect to a common reference point; 2) a single-phase circuit, possibly connected to the three-phase star centers, results in ideal transformers (with ratio $\sqrt{3}$) connecting such single-phase circuit to the 0 modal circuit.

The Clarke transformation, as a linear transformation, can be used also to operate in the phasor and in the Laplace domains. This property will be exploited in the transient analysis derived in the next Section.

III. Single-Line-to-Ground Fault with Fault Resistance

The problem under analysis can be investigated by considering the simplified three-phase system depicted in Fig. 1. The fault network is represented by a triplet of switches where only the switch *a* will be operated in order to simulate a single-line-to-ground fault. Notice that the investigation proposed in this paper is more general than in [13]-[14] where the fault resistance R_F was assumed equal to zero (i.e., a shorted line to ground). Here the fault resistance is included in the fault network, and its effects on the transients will be evaluated.

As mentioned in Section II, the Clarke transformation can only be applied in case of phase symmetry. A single-line-to-ground fault, however, results in a loss of phase symmetry since one phase is connected to ground whereas the other two phase remain open circuited. As a consequence, the Clarke transformation cannot be applied directly to the circuit in Fig. 1.

The problem, however, can be overtaken by resorting to the well-known Thevenin theorem. In fact, by treating the fault network (including the fault resistance) as a load, by following the Thevenin theorem such fault network can be detached and the Thevenin equivalent of the remaining three-phase circuit can be derived. Notice that, once the load is removed, the remaining three-phase circuit shows the phase symmetry property. Thus, the Thevenin equivalent (i.e., the open-circuit voltage and the equivalent impedance) can be evaluated by resorting to the Clarke transformation.

Fig. 2 shows the no-load three-phase circuit for Thevenin theorem evaluation. Such circuit can be analyzed by using the Clarke transformation to derive the modal circuits α, β, and 0. The modal circuit α is depicted in Fig. 3 where, according to the rules mentioned in Section II, the star centers have been shorted. The modal circuit β (not reported in the figures) has the same topology as α, whereas voltages and currents of course assume different values. Finally, the modal circuit 0 is shown in Fig. 4. In this case, according to the ideal transformers with ratio $\sqrt{3}$ mentioned in Section II, the ground resistances $R_{n1,2}$ must be reported to the three-phase network through the factor 3.

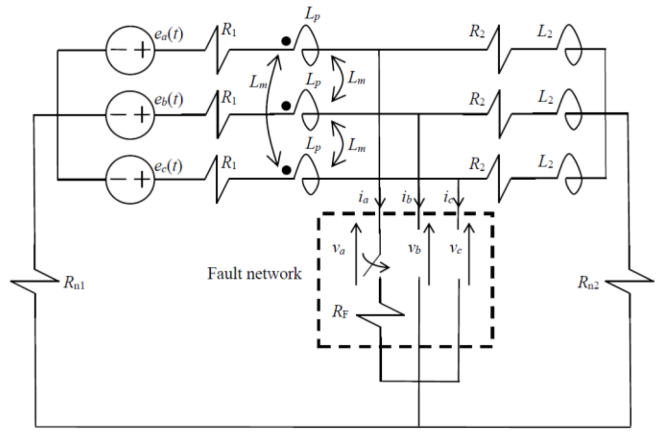

Fig. 1. Three-phase circuit used as a reference system to illustrate the proposed methodology. The fault network includes a fault resistance R_F for more general modeling.

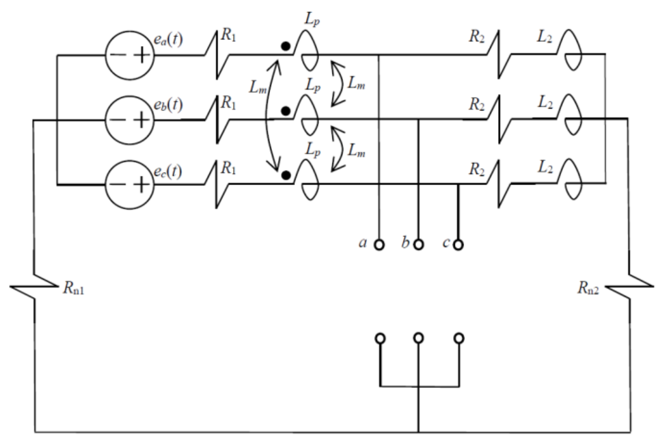

Fig. 2. Reference three-phase system where the asymmetrical fault network in Fig. 1 has been removed. The remaining circuit is a symmetrical three-phase circuit where the conventional Clarke transformation can be applied.

The time-domain modal circuits α, β, and 0 can be readily analyzed by means of the Laplace transform to derive the three corresponding Thevenin equivalents. At this point, the Thevenin theorem foresees the connection of the load previously detached. In the present analysis the load consists in the fault network identified in Fig. 1. Such network is characterized by the following relationships:

$$v_a = R_F i_a, \quad i_b = 0, \quad i_c = 0. \tag{2}$$

By using the Clarke transformation applied to the load voltages and currents we obtain:

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} R_F i_a \\ v_b \\ v_c \end{bmatrix} =$$

$$= \sqrt{\frac{2}{3}} \begin{bmatrix} R_F i_a - 1/2\,(v_b + v_c) \\ \sqrt{3}/2\,(v_b - v_c) \\ 1/\sqrt{2}\,(R_F i_a + v_b + v_c) \end{bmatrix}, \tag{3a}$$

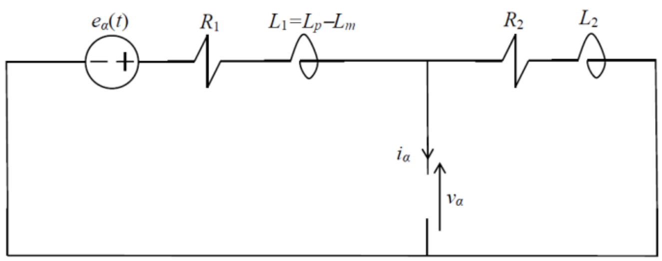

Fig. 3. Modal circuit α corresponding to the three-phase circuit in Fig. 2, according to the Clarke transformation. The star centers, once transformed, are all shorted to a common reference point. The modal circuit β, not reported here, has the same topology as the α circuit.

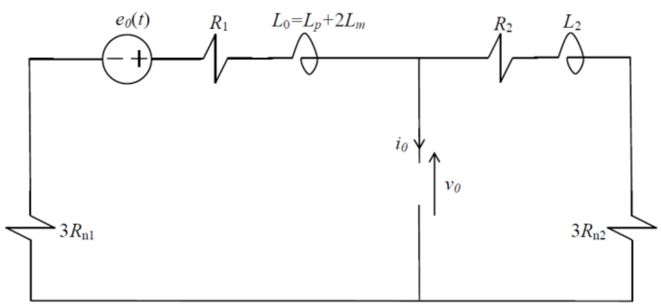

Fig. 4. Modal circuit 0 corresponding to the three-phase circuit in Fig. 2, according to the Clarke transformation. The single-phase network (i.e., the resistances R_{n1} and R_{n2}), once transformed, are reported through the star centers by the multiplicative factor 3.

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_a \\ 0 \\ 0 \end{bmatrix} =$$

$$= \sqrt{\frac{2}{3}} \begin{bmatrix} i_a \\ 0 \\ i_a/\sqrt{2} \end{bmatrix}. \tag{3b}$$

From (3b) we obtain:

$$i_a = \sqrt{\frac{3}{2}} i_\alpha = \sqrt{3} i_0. \tag{4}$$

By substituting (4) into (3a), and eliminating $(v_b + v_c)$ between the first and third equations we obtain:

$$v_\alpha = R_F i_\alpha - \frac{1}{\sqrt{2}}(v_0 - R_F i_0). \tag{5}$$

From (4) and (5) we readily obtain that the two modal circuits α and 0 are coupled through an ideal transformer with ratio

$$k = -\frac{1}{\sqrt{2}} \tag{6}$$

as shown in Fig. 5. From the second equation in (3b) the modal circuit β is open, and therefore it is uncoupled with the other two modal circuits.

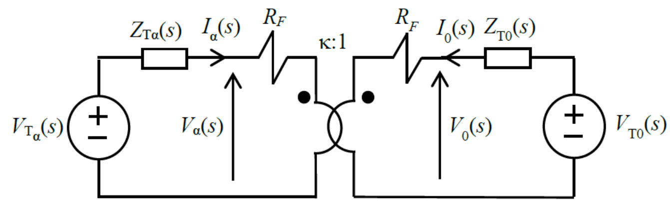

Fig. 5. The fault network constraints result in the coupling of Thevenin equivalents (Laplace domain) of the α and 0 modal circuits through an ideal transformer with ratio $k = -1/\sqrt{2}$, including the fault resistance R_F on both sides.

By solving the simple circuit in Fig. 5 (Laplace domain):

$$I_\alpha = \frac{V_{T\alpha} - kV_{T0}}{Z_{T\alpha} + R_F + k^2(Z_{T0} + R_F)} \tag{7}$$

and by taking the inverse Laplace transform, from (4) we obtain the time-domain fault current i_a.

Finally, the modal voltages:

$$V_\alpha = V_{T\alpha} - Z_{T\alpha} I_\alpha \tag{8a}$$

$$V_\beta = V_{T\beta} \tag{8b}$$

$$V_0 = V_{T0} - Z_{T0} I_0 \tag{8c}$$

allow the evaluation of the phase voltages v_a, v_b and v_c through inverse Clarke transformation of the inverse Laplace transforms:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 1/\sqrt{2} \\ -1/2 & \sqrt{3}/2 & 1/\sqrt{2} \\ -1/2 & -\sqrt{3}/2 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix}. \tag{9}$$

Notice that v_a is simply given by $R_F i_a$, whereas (9) is needed in order to evaluate the other two voltages v_b and v_c.

IV. Double-Line Faults

A. Double-Line-to-Ground Faults

The proposed methodology for the transient analysis of double-line-to-ground faults will be introduced in this Section with reference to the simplified three-phase circuit represented in Fig. 6 (same as Fig. 1 apart from the fault network). The fault network is represented by three switches. Under normal operating conditions the three switches are open. Notice that, for the sake of simplicity, in the double-line cases the fault resistances are assumed equal to zero. Inclusion of the fault resistances in the model can be readily done by following the same procedure outlined in Section III.

Since the objective of the analysis presented in this Subsection is the double-line-to-ground fault, it is assumed that at $t = 0$ the two switches b and c are closing. As a consequence, for $t \geq 0$ the two phases b and c are shorted and grounded.

The crucial point, as in Section III, is that the fault network in Fig. 6 is an asymmetrical component since one switch is open

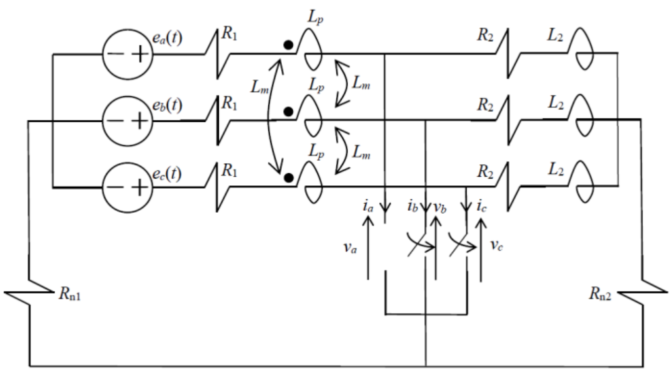

Fig. 6. Three-phase circuit used to illustrate the proposed methodology for the double-line faults.

and two switches are closed. Therefore, the Clarke transformation cannot be used in a straightforward way since the basic assumption is the phase symmetry as mentioned in Section II.

The methodology already outlined in Section III can be applied also in this case. In fact, by resorting to the Thevenin theorem, detaching the load (i.e., the fault network) leads to the symmetrical three-phase circuit shown in Fig. 2. By applying the Clarke transformation, the corresponding modal circuits are represented in Fig. 3 (α and β circuits) and in Fig. 4 (0 circuit).

The time-domain circuits in Figs. 3-4 can be transformed into the Laplace domain and the corresponding Thevenin equivalents can be readily derived.

Finally, the load consisting in the asymmetrical fault network must be taken into account. The constraints corresponding to the double-line-to-ground fault can be written as:

$$i_a = 0, \quad v_b = v_c = 0 \tag{10}$$

By putting (10) into the corresponding Clarke transformation defined in (1) we obtain for currents and voltages:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} -1/2\,(i_b + i_c) \\ \sqrt{3}/2\,(i_b - i_c) \\ 1/\sqrt{2}\,(i_b + i_c) \end{bmatrix} \tag{11a}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} v_a \\ 0 \\ 0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} v_a \\ 0 \\ v_a/\sqrt{2} \end{bmatrix} \tag{11b}$$

From (11a) we obtain

$$i_0 = -\sqrt{2} i_\alpha, \tag{12}$$

whereas from (11b) we obtain

$$v_\alpha = \sqrt{2} v_0. \tag{13}$$

Thus, the double-line-to-ground fault network results in the coupling of the Thevenin modal circuits α and 0, and such coupling can be represented as an ideal transformer with ratio $\sqrt{2}$. In the Laplace domain, this result can be represented as in Fig. 7a.

Moreover, from (11b) we obtain $v_\beta = 0$. Therefore, the Thevenin modal circuit β is shorted and uncoupled with the other modal circuits (see Fig. 7b).

The modal currents can be readily obtained by direct solution of the circuits in Fig. 7:

$$I_\alpha(s) = \frac{V_{T\alpha}(s) - \sqrt{2} V_{T0}(s)}{Z_{T\alpha}(s) + 2 Z_{T0}(s)} \tag{14a}$$

$$I_0(s) = -\sqrt{2} I_\alpha(s) \tag{14b}$$

$$I_\beta(s) = \frac{V_{T\beta}(s)}{Z_{T\beta}(s)} \tag{14c}$$

Finally, from the inverse Clarke transformation the fault currents can be obtained:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 1/\sqrt{2} \\ -1/2 & \sqrt{3}/2 & 1/\sqrt{2} \\ -1/2 & -\sqrt{3}/2 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \\ I_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 0 \\ -3/2\, I_\alpha + \sqrt{3}/2 I_\beta \\ -3/2\, I_\alpha - \sqrt{3}/2 I_\beta \end{bmatrix} \tag{15}$$

Of course from (15) $I_a = 0$, whereas:

$$I_b(s) = -\sqrt{\frac{3}{2}} I_\alpha(s) + \frac{1}{\sqrt{2}} I_\beta(s) \tag{16a}$$

$$I_c(s) = -\sqrt{\frac{3}{2}} I_\alpha(s) - \frac{1}{\sqrt{2}} I_\beta(s) \tag{16b}$$

Notice that the ground current:

$$I_g(s) = I_b(s) + I_c(s) = -\sqrt{6} I_\alpha(s) \tag{17}$$

is independent of I_β.

As far as the voltage $V_a(s)$ is concerned, from the inverse Clarke transformation we obtain:

$$V_a(s) = \sqrt{\frac{2}{3}} V_\alpha(s) + \frac{1}{\sqrt{3}} V_0(s) = \frac{1}{\sqrt{3}} \left(\sqrt{2} + \frac{1}{\sqrt{2}} \right) V_\alpha(s) \tag{18}$$

where from Fig. 7a:

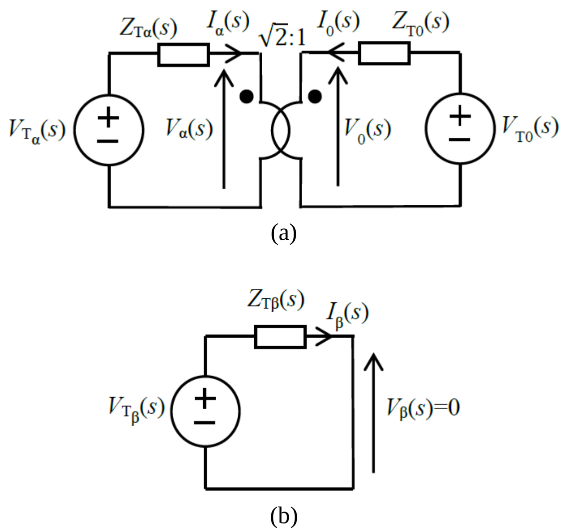

Fig. 7. Thevenin modal circuits in the Laplace domain. The double-line-to-ground fault results in coupled α and 0 circuits through an ideal transformer (a), and an uncoupled and shorted circuit β (b).

$$V_\alpha(s) = V_{T\alpha}(s) - Z_{T\alpha}(s) I_\alpha(s) \tag{19}$$

and $I_\alpha(s)$ is given by (14a).

B. Isolated Double-Line Faults

The isolated double-line fault is a special case of the fault investigated in the previous Subsection. In fact, if the ground connection of the fault network in Fig. 6 is missing, the current i_0 is forced to zero (indeed, from (1) i_0 is proportional to $i_a + i_b + i_c$ which is zero in case of no ground connection).

Therefore, the equivalent circuit in Fig. 7a should be modified by including an open switch on the 0 modal circuit (i.e., the right-hand side). As a consequence, the following results can be readily obtained:

$$I_\alpha = I_0 = 0 \tag{20a}$$

$$I_\beta(s) = \frac{V_{T\beta}(s)}{Z_{T\beta}(s)} \tag{20b}$$

where (20b) is the same as (14c). The corresponding fault currents, according to (15), are given by:

$$I_b(s) = -I_c(s) = \frac{1}{\sqrt{2}} I_\beta(s) \tag{21}$$

As far as the voltage V_a is considered, the result in (18) is still valid where, however, V_α is given by (19) with $I_\alpha = 0$. Thus, we obtain:

$$V_a(s) = \frac{1}{\sqrt{3}}\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) V_{T\alpha}(s) \tag{22}$$

V. Numerical Validation

A. Single-line-to-ground fault

The analytical results derived in Section III were validated through numerical simulation in Matlab/Simulink of the three-phase circuit depicted in Fig. 1.

The voltage source consists in three 1 kV (rms) symmetrical star-connected positive-sequence sources. The phase φ of the voltage source a is assumed as a parameter in the following simulations. The three-phase source is grounded through $R_{n1} = 0.1\ \Omega$.

The transmission line on the left side of the fault network is simulated by means of a three-phase coupled inductor with self-inductance $L_p = 3\ mH$, mutual inductance $L_m = 1\ mH$, and resistance $R_1 = 0.1\ \Omega$. The load on the right side is modeled as three uncoupled inductors $L_2 = 5\ mH$ and resistors $R_2 = 0.1\ \Omega$. The load star-center is grounded through the resistance $R_{n2} = 0.1\ \Omega$.

The fault network consists in three open switches (only the switch a will be operated) and a fault resistance R_F in series with switch a. The resistance R_F will be treated as a parameter in the following simulations. The special case $R_F = 0$ corresponds to the ideal short circuit already considered in [13]-[14].

Fig. 8 shows the time behavior of the fault current i_a when the switch a is closed at $t = 0$. Solid lines refer to the analytical results derived in Section III, whereas dashed lines refer to Matlab/Simulink results. The phase φ of the voltage source a is assumed equal to -90°. Three values for the fault resistance R_F were selected: $R_F = 0, R_n, 2R_n$, where $R_n = R_{n1} = R_{n2} = 0.1\ \Omega$. The comparison between solid and dashed curves in Fig. 6 validates the analytical results.

Fig. 9 is similar to Fig. 8, apart from two parameters. First, the phase of the voltage source a is assumed as $\varphi = 30^\circ$ instead of -90°. Second, the case of high fault resistance R_F is assumed as $R_F = 10R_n$ instead of $2R_n$. This point explains why the red curve is much lower than the other two curves. Comparison between Figs. 8 and 9 shows the strong dependence of the current peak on the transient initial time.

Finally, in Fig. 10 all the inductance values were doubled with respect to Fig. 8. This choice results in larger time constant (slower transients) and lower steady-state values of the short-circuit current.

B. Double-line faults

The analytical results concerning double-line faults derived in Section IV were validated by means of numerical simulation (Matlab/Simulink) of the three-phase circuit shown in Fig. 6. The circuit parameters are the same as in the previous Subsection. According to Section IV, two kinds of simulation were performed.

In the first case, the double-line-to-ground fault was considered. In particular, the results in (16) were validated against simulation. To this aim, the transient behavior of the short-circuit currents i_b (Fig. 11) and i_c (Fig. 12) were evaluated for three different values of the phase φ of the voltage source a. Solid curves refer to the analytical model (16), whereas dashed curves refer to Matlab/Simulink results. The two figures put into evidence the strong dependence of the peak overcurrent on the initial time instant of the transient. Fig. 13 shows the behavior of the short-circuit current in the ground connection, i.e., the total current $i_b + i_c$.

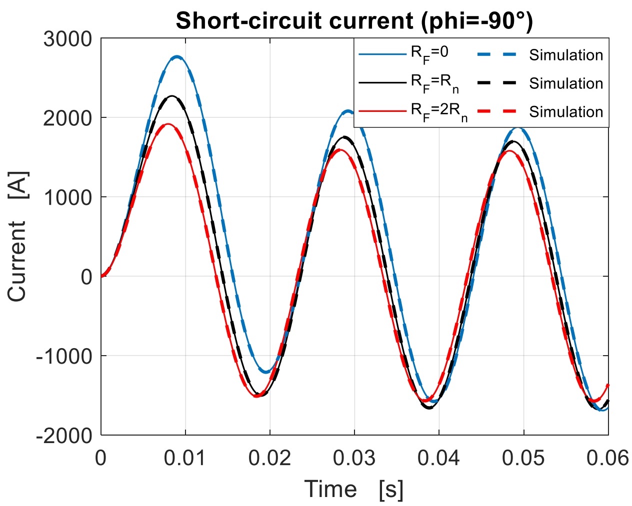

Fig. 8. Time behavior of the short-circuit current i_a for three different values of the fault resistance R_F. The phase of the voltage source a is assumed as $\varphi = -90^{\circ}$. Analytical results (solid lines) are compared with simulation results (dashed lines) obtained by Matlab/Simulink.

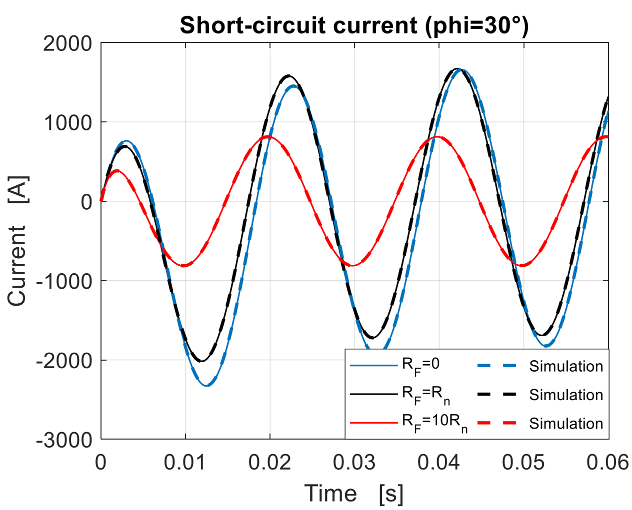

Fig. 9. Time behavior of the short-circuit current i_a for three different values of the fault resistance R_F. Unlike in Fig. 8, the phase of the voltage source a is assumed as $\varphi = 30^{\circ}$, and the high R_F case is assumed as $10R_n$. Analytical results (solid lines) are compared with simulation results (dashed lines) obtained by Matlab/Simulink.

Finally, Fig. 14 validates the case of isolated double-line fault. In particular, the analytical result (21) is validated against simulations for the three phase angles φ mentioned above. It is worth noticing that, in this case, the peak overcurrent in the line-to-line short circuit can change by a factor of 2 depending on the initial time instant (compare peaks in blue and red curves).

VI. Conclusions

The main result derived in the paper is the circuit representation, through an ideal transformer, of the coupling between the α and 0 Clarke circuits of the Thevenin equivalents at the fault location in case of single-line and double-line faults. Thus, the analytical solution of the time-domain short-circuit currents was readily evaluated and successfully compared with numerical simulations. It was shown that the peak overcurrent

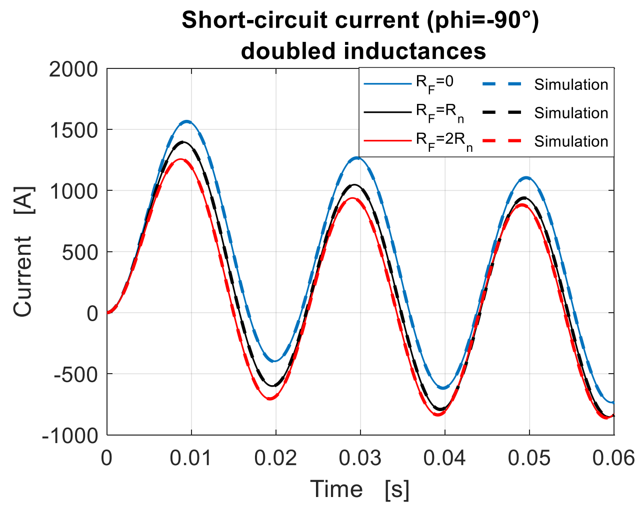

Fig. 10. Time behavior of the short-circuit current i_a for three different values of the fault resistance R_F. All the inductance values were doubled with respect to Fig. 8.

is strongly dependent on the fault initial time and the fault resistance.

The proposed transient analysis shows that, in case of in simple three-phase circuits, the rough numerical approach is not needed. By a proper use of the Clarke transformation the exact analytical solution can be readily derived, and a methodology with educational value can be suitably used.

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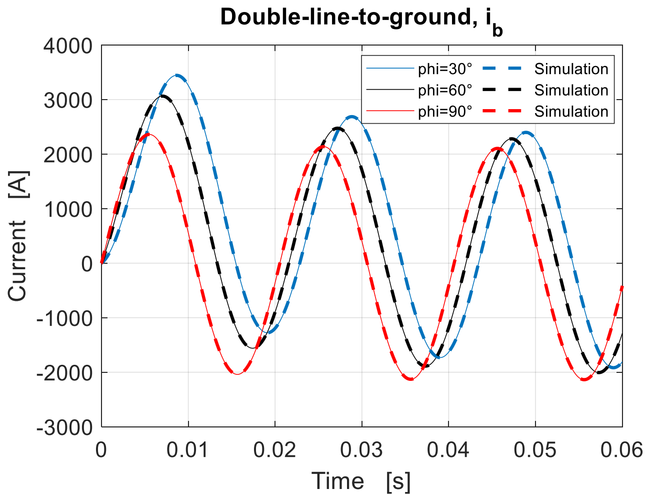

Fig. 11. Time behavior of the short-circuit current i_b in double-line-to-ground fault for different values of the phase φ of the voltage source *a*. Comparison between analytical (solid lines) and numerical (dashed lines) results.

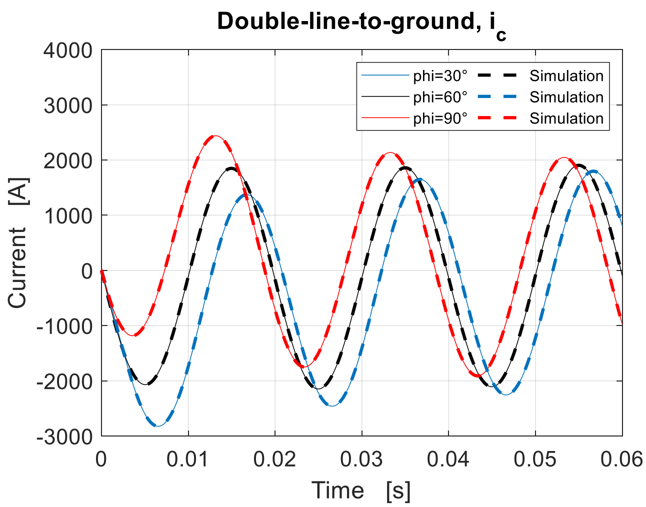

Fig. 12. Time behavior of the short-circuit current i_c in double-line-to-ground fault for different values of the phase φ of the voltage source *a*. Comparison between analytical (solid lines) and numerical (dashed lines) results.

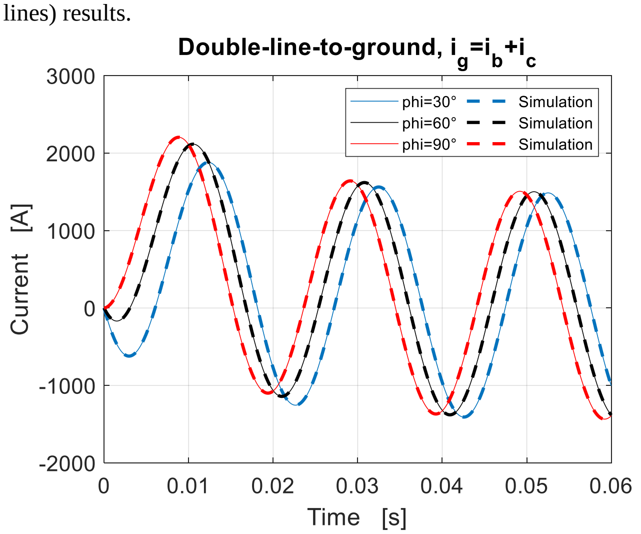

Fig. 13. Time behavior of the short-circuit ground current $i_b + i_c$ in double-line-to-ground fault for different values of the phase φ of the voltage source *a*.

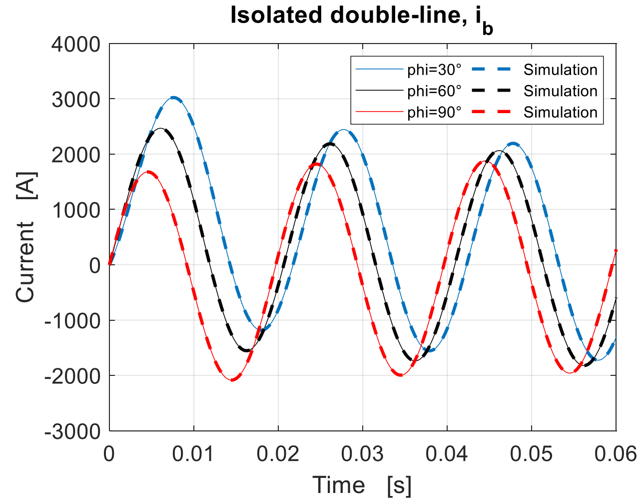

Fig. 14. Time behavior of the short-circuit current i_b in isolated double-line fault for different values of the phase φ of the voltage source *a*. Comparison between analytical (solid lines) and numerical (dashed lines) results.

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