$$y = a_i \cos\theta + b_i \sin\theta ,$$

so that

$$-(a_i^2 + b_i^2)^{1/2} \le y \le (a_i^2 + b_i^2)^{1/2} ,$$

and

$$\min_{|z|=1} |\lambda(z\mathrm{I}-A)| \le [1-(a_i + b_i)^{1/2}] ,$$

take the minimum i on both sides of the equation, then we have

$$\min_i \min_{|z|=1} |\lambda(z\mathrm{I}-A)| \le \min_{|z|=1} \min_i [1-(a_i + b_i)^{1/2}] = 1 - r(A) ,$$

from the theorem condition above, yielding

$$\xi_n(x) \le \lambda_i(x) ,$$

so that

$$\xi_n(z\mathrm{I}\text{-}A) \le \min_i |\lambda_i(z\mathrm{I}-A)| .$$

Take the minimum values of the two sides of the upper formula in the unit circle, yielding

$$\min_{|z|=1} \xi_n(z\mathrm{I}\text{-}A) \le \min_{|z|=1} \min_i |\lambda_i(z\mathrm{I}-A)| .$$

Because $\min_{|z|=1} \min_i |\lambda_i(z\mathrm{I}-A)| = 1 - r(A)$, by synthesizing the upper two expressions, one has

$$\min_{|z|=1} \xi_n(z\mathrm{I}\text{-}A) \le 1 - r(A) .$$

By definition above, there has

$$m_d = \min_{|z=1|} \xi_n(z\mathrm{I}\text{-}A) ,$$

namely

$$m_d \le 1 - r(A) .$$

This completes the proof.

V. Conclusion

Based on the hypothesis of networked control system, this paper analyses the networked control system model with packet dropout through multi-packet transmission, and studies the causes of packet dropout in networked control system. The system performance of networked control system with multi-packet loss under different circumstances is analyzed from the point of view of asynchronous dynamic system, and the influence of multi-packet dropout on system performance is illustrated by simulation. The stability conditions and stability boundaries of NCS with data packet loss are obtained.

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