for a function $f : \mathbb{R} \to \mathbb{R}$.

Note that the error term in (14) is dominated by

$$O\left(K^{D-1}x^{\frac{1}{2}}\right) + O\left(K^{-D-1}x^{2D+\frac{1}{2}}d^{-2D}\right).$$

Putting $K = x^{\alpha}$, $d = x^{\beta}$, and requiring that

$$\alpha D - \alpha + \frac{1}{2} = \frac{3}{4},$$
$$-\alpha D - \alpha + 2D + \frac{1}{2} - 2D\beta = \frac{3}{4},$$

we find that $\alpha = \frac{1}{4(D-1)}$, $\beta = \frac{4D-5}{4D-4}$, i.e., that $K = x^{\frac{1}{4(D-1)}}$, $d = x^{\frac{4D-5}{4D-4}}$.

Thus, the error term $O\left(x^{\frac{3}{4}}\right)$ in (13) follows as required.

In [13] and [5], the authors applied the method developed in [14] and [15] for compact Riemann surfaces.

The method described in this paper is very well applied in [12], [1], [9], and [7] in the case of real hyperbolic manifolds with cusps, compact, odd-dimensional, real hyperbolic spaces, and compact, even-dimensional locally symmetric Riemannian manifolds of strictly negative curvature, respectively.

In order to derive their results, the authors usually apply approximate formulas for the logarithmic derivative of the corresponding zeta function, such as the Riemann or the Selberg, or the Ruelle zeta function (see, [16], [11], [8], [2]).

References

[1] M. Avdispahić and Dž. Gušić, "On the error term in the prime geodesic theorem," *Bull. Korean Math. Soc.*, vol. 49, pp. 367–372, 2012.

[2] M. Avdispahić and Dž. Gušić, "On the logarithmic derivative of zeta functions for compact even-dimensional locally symmetric spaces of real rank one," *Mathematica Slovaca*, vol. 69, pp. 311–320, 2019.

[3] A. Borel, *Introduction aux groupes arithmetiques.* Paris: Publications de I'Institut de Mathematique de I'Universite de Strasbourg XV. Actualites Scientifiques et Industrielles, No. 1341., 1969.

[4] H. Chandra, "Harmonic analysis on real reductive groups. I. The theory of the constant term," *J. Functional Analysis*, vol. 19, pp. 104–204, 1975.

[5] A. Deitmar and M. Pavey, "A prime geodesic theorem for SL_4," *Ann. Anal. Glob. Geom.*, vol. 33, pp. 161–205, 2008.

[6] D. Fried, "The zeta functions of Ruelle and Selberg. I," *Ann. Sci. Ec. Norm. Sup.*, vol. 19, pp. 491–517, 1986.

[7] Dž. Gušić, "Prime geodesic theorem for compact even-dimensional locally symmetric Riemannian manifolds of strictly negative sectional curvature," *WSEAS Trans. on Math.*, vol. 17, pp. 188–196, 2018.

[8] Dž. Gušić, "On the Logarithmic Derivative of Zeta Functions for Compact, Odd-dimensional Hyperbolic Spaces," *WSEAS Trans. on Math.*, vol. 18, pp. 176–184, 2019.

[9] Dž. Gušić, "On the Length Spectrum for Compact, Odd-dimensional, Real Hyperbolic Spaces," *WSEAS Trans. on Math.*, vol. 18, pp. 211–222, 2019.

[10] Dž. Gušić, "Approximate Formulas for Zeta Fucntions of Selberg's Type in Quotients of SL_4," submitted.

[11] D. Hejhal, *The Selberg trace formula for $PSL\,(2, \mathbb{R})$. Vol. I. Lecture Notes in Mathematics 548.* Berlin–Heidelberg: Springer–Verlag, 1976.

[12] J. Park, "Ruelle zeta function and prime geodesic theorem for hyperbolic manifolds with cusps," in *Casimir force, Casimir operators and Riemann hypothesis*, Berlin 2010, pp. 89–104.

[13] M. Pavey, "Class Numbers of Orders in Wuartic Fields," Ph.D. dissertation, University of Tubingen, 2006.

[14] B. Randol, "On the asymptotic distributon of closed geodesics on compact Riemann surfaces," *Trans. Amer. Math. Soc.*, vol. 233, pp. 241–247, 1977.

[15] B. Randol, "The Riemann hypothesis for Selberg's zeta-function and the asymptotic behaviour of eigenvalues of the Laplace operator," *Trans. Amer. Soc.*, vol. 236, pp. 209–233, 1978.

[16] E. C. Titchmarsh, *The Theory of the Riemann Zeta-function.* Oxford: Oxford Science Publications, Clarendon Press, 1986.

[17] J. A. Wolf, "Discrete groups, symmetric spaces, and global holonomy," *Amer. J. Math.*, vol. 84, pp. 527–542, 1962.