(Fig. 6, right panel), function $h^*_{\Lambda_t}$ gets values above 1 which in the discrete domain make no sense.

Therefore for the discrete Weibull distribution, we prefer usage of the accumulated hazard function instead of the cumulative hazard function.

VI. Conclusion

The analysis presented here shows that a priority in the discrete domain should be given to the accumulated hazard function, as H_t function has more similar properties to the cumulative hazard function as defined in the continuous domain.

By applying H_t function, the same memoryless effect is obtained in discrete domain (geometric distribution) as in continuous domain (exponential distribution). The similarity criterion also favors the accumulated hazard function in case of the discrete Weibull distribution. In the case of small p and non-increasing hazard function ($0 < \beta < 1$) of the Weibull distribution, the two functions provide almost identical information, except at $t = 0$. In other cases the accumulated hazard function seems to be more credible. If the results for the discrete Weibull distribution can be assumed representative, this shows that defining the cumulative hazard function based on the analogy between mathematical operations (switching integration with summation) is likely correct.

Finally, these examples show a potential to resolve the Leemis dilemma. We are also confident that the alternative definitions of discrete hazard functions, such as pseudo-hazard rate, are not needed for this purpose.

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