Co-evolution of construction waste recycling industrial chain based on Lotka–Volterra model

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Abstract—Under the green building policy, the recycling of construction waste has become an important issue. However, many obstructions in the industrial chain of construction waste recycling, slow integration of production elements in the chain, and inefficient resource allocation hinder the development of the construction waste recycling industry. In this study, a co-evolution model of the industrial chain of construction waste recycling is constructed based on the Lotka–Volterra model, and the Jacobian matrix is used for stability analysis. Finally, a numerical simulation is performed. The simulation results indicate that: (1) There is a high product correlation between upstream and downstream enterprises in the industrial chain of construction waste recycling; (2) The conversion coefficient of supply and demand of upstream and downstream enterprises has a higher impact on and is more sensitive to the evolution of this industrial chain; (3) The co-evolution of upstream and downstream enterprises promotes the added value of products and maximizes the overall benefits of the industrial chain, which provides reference value and theoretical basis for the development of the industrial chain of construction waste recycling.

Keywords—Construction waste, Lotka–Volterra, co-evolution, recycling, industrial chain.

I. INTRODUCTION

CONSTRUCTION waste recycling has been the focus of experts and government departments around the world. However, continuous deepening of urbanization construction in China has also resulted in a large amount of construction waste, a part of which is simply landfilled or inadequately disposed, resulting in waste of resources and land, and seriously affecting the surrounding environment of the city [1]. According to statistics, the amount of construction waste in China accounts for 30%-40% of the total urban waste [2]. Although China has progressed in reducing construction waste, the efficiency of construction waste recycling is relatively low, and the utilization rate of resources is less than 5%. In contrast, in accordance with the European Revised Waste Framework Directive 2008/98/EC, European countries should recover at least 70% of nonhazardous construction and demolition waste by 2020 [3] - [5]. Thus, China exhibits a discrepancy compared to developed countries regarding the recycling of construction waste.

The industrial chain of construction waste recycling refers to an integration including value, enterprise, supply, and space chains, which are mainly composed of

construction contractor enterprises, construction waste recycling enterprises, and recycled product dealers, with construction waste and recycled products as series products, guided by market demand [6]. Considering the industrial chain of construction waste recycling in China, the industrial chain ecosystem is shown in Fig. 1.
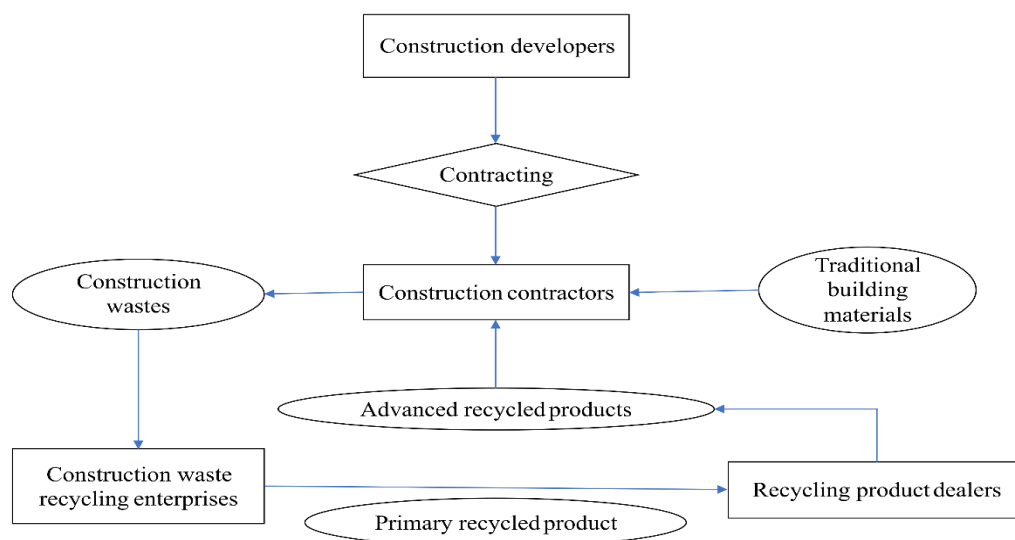

Fig. 1 Ecosystem of the construction waste recycling industrial chain

The construction developers contract the project to the construction contractors, who produce construction wastes during the implementation of the project. The construction contractors transport construction wastes to the recycling enterprises, which transform the wastes into construction material products (primary recycled products) through classification, processing, and other processes. The construction waste recycling enterprises then transport the recycled products to the recycled product dealers to produce advanced recycled products that will be sold to the construction contractors for the development of a project. This industrial chain reflects the process of sharing, interaction, integration, and distribution of production factors such as capital, knowledge, technology, information, and resources, regardless of the type of enterprise, whether it is an internal enterprise such as construction waste recycling enterprises, developers, building material dealers, and construction contractors, or the interaction between upstream and downstream enterprises in the industrial chain (including circulation and information transmission between upstream and downstream enterprises, such as construction waste and recycled products) [7] [8]. However, there are many obstructions in China's construction waste recycling industry chain, such as slow integration of production elements in the chain and inefficient resource allocation, which hinder the development of the construction waste recycling industry [9]. This is mainly due to: (1) Lack of systematic resource management thinking, which results in the diversity and complexity of participants, dynamics and competitiveness of the market environment, inadequate macro-control efforts of the government, and untimely management of industrial chain problems in the process of construction waste recycling management. An example is the "12·20" particularly serious landslide accident at a residue acceptance site of the Guangming New Area in Shenzhen, which was mainly caused by the upstream industry in the industrial chain of urban construction waste [10]. In this case, Shenzhen Municipal Government could only compensate the loss after the incident by attaching considerable importance to the recycling of construction waste, promulgating corresponding support and subsidy

policies, promoting the classification of the sources, and improving the recycling efficiency of construction waste; (2) Lack of regular understanding of the coordinated development of the industrial chain. The development and perfection of the industrial chain of construction waste recycling is regular: The government should support the construction waste recycling industry differently at each stage and under diverse economic development conditions, as the main body of the ecosystem of the industrial chain reacts differently. However, the laws of internal development and evolution have not been understood by local governments, and the economic incentive policies formulated during the growth of industrial chains have little effect, combined with insufficient support and insufficient power to support industrial development, which hinders the development and improvement of the industrial chain of construction waste recycling. Based on the aforementioned information, in this study, the relationship between the participants in the industrial chain of construction waste recycling is studied, the co-evolution law of the industrial chain of construction waste is discussed, and strategies and suggestions for promoting the development of this chain are proposed.

II. LITERATURE REVIEW

A mature industrial chain of construction waste recycling and its smooth operation and development have significant importance to solve the problem of construction waste disposal. In recent years, researchers have focused on the industrial chain of construction waste recycling [11]. Based on the diamond model, Hu *et al*. analyzed the effects of production factors, demand conditions, enterprise strategy, related industries, development opportunities, and government actions on the development of the construction waste recycling industry [12]. Gao and Xie based on the development of an industrial chain structure model of construction waste recycling, cited as implementation strategies the promotion of the development of recycling enterprises, maintenance of a stable supply of raw materials in the industrial chain, and improvement of the market competitiveness of recycling products [13]. Liu *et al*. constructed a green building

industry chain model from the perspective of recycling economy and analyzed the interaction between the government and builders in this chain [14]. They suggested the strengthening of the management of recycling parts in the chain, acceleration of the complex development of the green building industry chain, establishment of a green building industry alliance, and formulation and improvement of relevant incentive policies by the government. Kucukvar *et al*. analyzed the optimal strategy selection of recycling, landfilling, and incineration strategies in the entire economic supply chain under different evaluation indices by constructing a mixed life cycle assessment model based on economic input-output [15]. Li *et al*. discussed the logistics network design scheme of the construction waste reduction system under the condition of site treatment capacity availability, and constructed a logistics network model of the reduction system [16]. Wu *et al*. believed that the cross-regional flow of construction waste can affect the quantity of waste in the regional waste management system and produce economic, social, and environmental impacts to the system [17].

Lotka and Volterra established the predator-prey system and the competitive system models, respectively and Odum extended these models to the reciprocal system, which is known as the Lotka–Volterra model [18] - [20]. Nowadays, this model is widely applied by many researchers in the fields of inter-industry coupling relationships, energy industry competition, futures index competition, relations between majority and minority shareholders in a shareholder population, economic growth, and energy consumption relationships. For example, Zhou and Wang used the Lotka–Volterra model to measure the symbiotic relationship and symbiotic coordination degree of industrial economy and industrial ecology in 30 provinces and regions of China, respectively, and classified the coordination types of industrial eco-economic systems according to symbiotic relationships and development levels, analyzing influencing factors [21]. Yang and Hu used the Lotka–Volterra model to measure the degree of coordination of the eco-economic system in ethnic minority areas from 2011 to 2015, and applied a neural network model to analyze the main factors affecting

the degree of coordination [22]. Szolnoki *et al*. utilized the Lotka–Volterra model to study the coordination and competition among biological populations, and used the RP model and Ginzburg–Landau theory to build a large ecological system [23]. Lauro *et al*. used the Lotka–Volterra model to construct the coordination of microorganisms in a lake, highlighting the coordination relationship among populations, and contributing to the study of the ecological environment system in the lake [24]. Bao *et al*. used the Lotka–Volterra model to study the degree of coordination of population dynamics [25]. In the current research on industrial chains, the Lotka–Volterra model is a more advanced and effective method for dynamic research on synergy, competition, and symbiosis between upstream and downstream of the industrial chain. Thus, the Lotka–Volterra model is first introduced to study the relationship between upstream and downstream of the construction waste recycling industry chain. Then, the evolution process of the upstream and downstream enterprises in the construction waste recycling industrial chain is analyzed from an ecological perspective, a co-evolution model of this chain is constructed, and strategies and suggestions to promote the development of the industrial chain of construction waste recycling are drawn using a numerical simulation.

III. METHODOLOGY

Interdependence and cooperation between organisms and between organisms and the environment are the basis for survival, and co-evolution occurs every moment in the biological field. During the operation of the industrial chain of construction waste recycling, the synergy between the upstream and downstream of the industrial chain reflects the process in which these parts correlate, benefit, and restrict each other. The similarities between organisms and enterprises suggest a co-evolution process between upstream and downstream enterprises in the development and perfection of the industrial chain of construction waste recycling, as they interact and develop together in the system and constantly adjust in the process of co-evolution to maximize the value of the industrial chain. Therefore, in

this study, a co-evolution model on the industrial chain of construction waste recycling is constructed by referring to the collaborative evolution theory in ecology and the Lotka–Volterra model. First, the basic hypotheses of the Lotka–Volterra model are discussed. In sequence, the co-evolution model of the construction waste industry chain is constructed, and numerical simulation and analysis are performed. Finally, case simulation and modeling are performed to draw conclusions and suggestions.

A. Basic hypotheses

The following reasonable hypotheses are made according to the actual situation and many studies before building a co-evolution model for the industrial chain of construction waste recycling [26] - [32]:

1) To simplify, the impact of minor enterprises, such as subcontractors, is ignored. The recycling industry chain analyzed consists of construction contractors, construction waste recycling enterprises, and recycled product dealers.

2) The delay in the production and transportation of construction waste recycling enterprises is not considered, and the supply and demand of construction waste and recycled products are assumed to be uniform.

3) Regardless of the impact of the type of project, it is assumed that the total amount of building materials (including recycled products and traditional building materials) equals the amount of construction waste generated + the amount used in the building.

B. Definition of parameters and related decision variables

Based on the basic hypotheses of the model, the parameters and definitions of the co-evolution model are shown in Table I.

Table I. Parameters and definitions of the co-evolution model

Parameters	Definitions	Value range
β_1	Natural growth rate of construction contractors	$\beta_1 \in [-1,1]$
β_2	Natural growth rate of construction waste recycling enterprises	$\beta_2 \in [-1,1]$
β_3	Natural growth rate of dealers of recycled construction waste products	$\beta_3 \in [-1,1]$
ε_1	Conversion coefficient of demand for construction waste of the downstream construction waste recycling enterprises (considering the upstream and downstream industrial chain between construction contractors and construction waste recycling enterprises)	$\varepsilon_1 > 0$
ε_2	Conversion coefficient of demand for recycled products of the downstream recycled products dealers (considering the upstream and downstream industrial chain between construction waste recycling enterprises and recycled products dealers)	$\varepsilon_2 > 0$
τ_1	Conversion coefficient of supply of construction waste from upstream construction contractors (considering the upstream and downstream industrial chain between construction contractors and construction waste recycling enterprises)	$\tau_1 > 0$
τ_2	Conversion coefficient of supply of recycled products of the upstream construction waste recycling enterprises (considering the upstream and downstream industrial chain between construction waste recycling enterprises and recycled products dealers)	$\tau_2 > 0$
D	Maximum amount of construction waste (including waste from recycled products and traditional building materials, as well as waste such as residues) generated in the upstream and downstream industrial chain between the construction contractors and the construction waste recycling enterprises, with sufficient resources for the upstream construction contractors	$D > 0$
K	Market demand for recycled products of downstream construction contractors (considering the upstream and downstream industrial chain between the construction contractors and recycled products dealers)	$D \geq K \geq 0$
λ	Proportion of the demand for recycled products of construction contractors in the total demand for building materials, including recycled products and traditional building materials	$1 \geq \lambda \geq 0$

The assumptions and definitions of the decision variables are as follows:

x_1: Output of construction wastes disposed to recycling by construction contractors;

x_2: Output of recycled products of a construction waste recycling enterprise;

x_3: Volume of sales of recycled products by the recycled products dealers.

Note: Initial variables of x_1, x_2 and x_3 are x_{10}, x_{20} and x_{30}.

C. Construction of the co-evolution model based on Lotka–Volterra

According to Zhou, the co-evolution between upstream and downstream enterprises in the residential industry chain is studied, and an ecological Lotka–Volterra model is used as a reference to build a co-evolution model between upstream (design units, product suppliers) and downstream enterprises (developers and properties) [32]. Based on the studies of Green [27], Gao and Xie [13], the industrial chain of construction waste recycling, composed of construction contractors, construction waste recycling enterprises, and recycled products dealers, is considered as the research object, and a co-evolution model is constructed under the condition of abundant supply of building materials (including recycled products and traditional building materials), as shown in (1) [13] [27]:

$$\begin{cases} \dfrac{dx_1}{dt} = \beta_1 x_1 \left(D - x_1\right)\left(\varepsilon_1 x_2 - x_1\right) \\[2mm] \dfrac{dx_2}{dt} = \beta_2 x_2 \left(\tau_1 x_1 - x_2\right)\left(\varepsilon_2 x_3 - x_2\right) \\[2mm] \dfrac{dx_3}{dt} = \beta_3 z \left(\tau_2 x_2 - x_3\right)\left(K - x_3\right) \\[2mm] K = \lambda D \end{cases} \qquad (1)$$

Where,

$\dfrac{dx_1}{dt}$ = rate of change in the output of construction waste recycled by a construction contractor (reflecting the growth rate of the construction contractor);

$\dfrac{dx_2}{dt}$ = rate of change in the output of recycled products of a construction waste recycling enterprise (reflecting the growth rate of the construction waste recycling enterprise);

$\dfrac{dx_3}{dt}$ = rate of change in sales of recycled products by a recycled products dealer (reflecting the growth rate of the recycled products dealer);

$\left(\varepsilon_1 x_2 - x_1\right)$ = increased demand for construction waste of the downstream construction waste recycling enterprises (considering the upstream and downstream industrial chain between construction contractors and construction waste recycling enterprises); The condition $\left(\varepsilon_1 x_2 - x_1\right) \geq 0$ should always be met; Otherwise, the output of the construction waste from construction contractors will tend to 0;

$\left(\tau_1 x_1 - x_2\right)$ = increased supply of construction waste from upstream construction contractors (considering the upstream and downstream industrial chain between construction contractors and construction waste recycling enterprises); The condition $\left(\tau_1 x_1 - x_2\right) \geq 0$ should always be met; Otherwise, the output of recycled products of the construction waste recycling enterprises will tend to 0;

$\left(\varepsilon_2 x_3 - x_2\right)$ = increased market demand for recycled products of downstream recycled products dealers (considering the upstream and downstream industrial chain between the construction waste recycling enterprises and recycled product dealers); The condition $\left(\varepsilon_2 x_3 - x\right)_2 \geq 0$ should always be met; Otherwise, the output of recycled products of the construction waste recycling enterprises will tend to 0;

$\left(\tau_2 x_2 - x_3\right)$ = increased supply of recycled products of the upstream construction waste recycling enterprises (considering the upstream and downstream industrial chain between construction waste recycling enterprises and recycled products dealers); The condition $\left(\tau_2 x_2 - x_3\right) \geq 0$ should always be met; Otherwise, the sales volume of recycled products of the recycled products dealers will tend to 0.

Let $\dfrac{dx_1}{dt}, \dfrac{dx_2}{dt}, \dfrac{dx_3}{dt}$ be 0. Then, the steady-state solutions of $\left(x_1, x_2, x_3\right)$ in the model are $A_1\left(0,0,0\right)$, $A_2\left(0,0,\lambda D\right)$, $A_3\left(D,0,0\right)$, $A_4\left(D,0,\lambda D\right)$, $A_5\left(D,\tau_1 D,0\right)$, $A_6\left(D,\dfrac{D}{\varepsilon_1},0\right)$, $A_7\left(0,\varepsilon_2 \lambda D,\lambda D\right)$,

$A_8\left(0,\dfrac{\lambda D}{\tau_2},\lambda D\right)$, $A_9\left(D,\tau_1 D,\lambda D\right)$, $A_{10}\left(D,\varepsilon_2\lambda D,\lambda D\right)$,

$A_{11}\left(\varepsilon_1\varepsilon_2\lambda D,\varepsilon_2\lambda D,\lambda D\right)$, and $A_{12}\left(D,\tau_1 D,\tau_1\tau_2 D\right)$.

Among them, the isoclinic of the model is shown in (2):

$$\begin{cases} \beta_1 x_1\left(D-x_1\right)\left(\varepsilon_1 x_2 - x_1\right) = 0 \\ \beta_2 x_2\left(\tau_1 x_1 - x_2\right)\left(\varepsilon_2 x_3 - x_2\right) = 0 \\ \beta_3 x_3\left(\tau_2 x_2 - x_3\right)\left(K - x_3\right) = 0 \end{cases}$$

$$(2)$$

D. Judgment of stability of steady-state solutions

According to the Jacobian matrix proposed by Friedman (1991), the stability judgment is made on the local equilibrium points $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$ of the above 10 steady-state solutions of the co-evolution model for the industrial chain of construction waste recycling, and the partial derivatives of the $\dfrac{dx_1}{dt}, \dfrac{dx_2}{dt}, \dfrac{dx_3}{dt}$ pairs $\left(x_1, x_2, x_3\right)$ in Equation (1) are obtained by partial differentiation, respectively, from which the corresponding Jacobian matrix is obtained ((3) - (11)):

$$\dfrac{\partial\left(\dfrac{dx_1}{dt}\right)}{\partial x_1} = \beta_1\left(D-x_1\right)\left(\varepsilon_1 x_2 - x_1\right) - \beta_1 y\left(\varepsilon_1 x_2 - x_1\right) - \beta_1 x_1\left(D-x_1\right)$$

$$(3)$$

$$\dfrac{\partial\left(\dfrac{dx_1}{dt}\right)}{\partial x_2} = \beta_1 x_1\left(D-x_1\right)\varepsilon_1$$

$$(4)$$

$$\dfrac{\partial\left(\dfrac{dx_1}{dt}\right)}{\partial x_3} = 0$$

$$(5)$$

$$\dfrac{\partial\left(\dfrac{dx_2}{dt}\right)}{\partial x_1} = \beta_2 x_2 \tau_1\left(\varepsilon_2 x_3 - x_2\right)$$

$$(6)$$

$$\dfrac{\partial\left(\dfrac{dx_2}{dt}\right)}{\partial x_2} = \beta_2\left(\tau_1 x_1 - x_2\right)\left(\varepsilon_2 x_3 - x_2\right) - \beta_2 x\left(\varepsilon_2 x_3 - x_2\right) - \beta_2 x_2\left(\tau_1 x_1 - x_2\right)$$

$$(7)$$

$$\dfrac{\partial\left(\dfrac{dx_2}{dt}\right)}{\partial x_3} = \beta_2 x_2\left(\tau_1 x_1 - x_2\right)\varepsilon_2$$

$$(8)$$

$$\dfrac{\partial\left(\dfrac{dx_3}{dt}\right)}{\partial x_1} = 0$$

$$(9)$$

$$\dfrac{\partial\left(\dfrac{dx_3}{dt}\right)}{\partial x_2} = \beta_3 x_3 \tau_2\left(K - x_3\right)$$

$$(10)$$

$$\dfrac{\partial\left(\dfrac{dx_3}{dt}\right)}{\partial x_3} = \beta_3\left(\tau_2 x_2 - x_3\right)\left(K - x_3\right) - \beta_3 x_3\left(K - x_3\right) - \beta_3 x_3\left(\tau_2 x_2 - x_3\right)$$

$$(11)$$

To summarize, the Jacobian matrix is

$$J = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}.$$

After organizing:

$$J = \begin{vmatrix} \dfrac{\partial\left(\dfrac{dx_1}{dt}\right)}{\partial x_1} & \dfrac{\partial\left(\dfrac{dx_1}{dt}\right)}{\partial x_2} & \dfrac{\partial\left(\dfrac{dx_1}{dt}\right)}{\partial x_3} \\ \dfrac{\partial\left(\dfrac{dx_2}{dt}\right)}{\partial x_1} & \dfrac{\partial\left(\dfrac{dx_2}{dt}\right)}{\partial x_2} & \dfrac{\partial\left(\dfrac{dx_2}{dt}\right)}{\partial x_3} \\ \dfrac{\partial\left(\dfrac{dx_3}{dt}\right)}{\partial x_1} & \dfrac{\partial\left(\dfrac{dx_3}{dt}\right)}{\partial x_2} & \dfrac{\partial\left(\dfrac{dx_3}{dt}\right)}{\partial x_3} \end{vmatrix};$$

Thus, the Jacobian matrix can be used to judge whether the steady-state solutions are stable. The Jacobian matrix indicates that when $\det(J) > 0$ and $tr(J) < 0$, the steady-state solutions will approach the stable state. Therefore, the stability of the local equilibrium points $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$ of 10 steady-state solutions of the co-evolution model for the construction waste recycling industrial chain is determined as shown in Table II.

Table II. Stability judgment of local equilibrium points in the co-evolution model for the construction waste recycling industrial chain

Local equilibrium points	$\det(J)$	$tr(J)$	State of stability
$A_1(0,0,0)$	0	0	Saddle point
$A_2(0,0,\lambda D)$	0	$\beta_3\lambda^2 D^2$	Unstable point
$A_3(D,0,0)$	0	$\beta_1 D^2 > 0$	Unstable point
$A_4(D,0,\lambda D)$	$\beta_1\beta_2\beta_3\tau_1\varepsilon_2\lambda^3 D^6$	$\beta_2\tau_1\varepsilon_2\lambda D^2 + \beta_1 D^2 + \beta_3\lambda^2 D^2$	Unstable point
$A_5(D,\tau_1 D,0)$	$\beta_1\beta_2\beta_3\tau_1^{3}\tau_2\lambda D^6 (1-\varepsilon_1\tau_1)$	$\beta_2\tau_1^{2}D^2 - \beta_1 D(\varepsilon_1\tau_1 D - D) + \beta_3\tau_1\tau_2\lambda D^2$	Unstable point
$A_6\left(D,\dfrac{D}{\varepsilon_1},0\right)$	0	$\beta_2\dfrac{D^2}{\varepsilon_1^{2}} + 2\beta_2\dfrac{D^2}{\varepsilon_1}\left(\dfrac{1}{\varepsilon_1}-\tau_1\right)$	Unstable point
$A_7(0,\varepsilon_2\lambda D,\lambda D)$	$\beta_1\beta_2\beta_3\varepsilon_1\varepsilon_2^{3}\lambda^5 D^6 (1-\tau_2\varepsilon_2)$	$\beta_2\varepsilon_2^{2}\lambda^2 D^2 + \beta_1\varepsilon_1\varepsilon_2\lambda D^2 + \beta_3\lambda^2 D^2(1-\varepsilon_2\tau_2)$	Unstable point
$A_8\left(0,\dfrac{\lambda D}{\tau_2},\lambda D\right)$	0	$2\beta_2\dfrac{\lambda^2 D^2}{\tau_2}\left(\dfrac{1}{\tau_2}-\varepsilon_2\right)+\beta_2 \;\; \dfrac{\lambda^2 D^2}{\tau_2^{2}}+\beta_1\varepsilon_1 D^2\dfrac{\lambda}{\tau_2}$	Unstable point
$A_9(D,\tau_1 D,\lambda D)$	$\beta_1\beta_2\beta_3\lambda\tau_1 D^6 (\varepsilon_2\lambda-\tau_1)(1-\varepsilon_1\tau_1)(\tau_1\tau_2-\lambda)$	$-\beta_2\tau_1 D^2(\varepsilon_2\lambda-\tau_1)-\beta_1 D^2(\varepsilon_1\tau_1-1)-\beta_3\lambda D^2(\tau_1\tau_2-\lambda)$	—
$A_{10}(D,\varepsilon_2\lambda D,\lambda D)$	$\beta_1\beta_2\beta_3\varepsilon_2\lambda^3 D^6 (\tau_1-\varepsilon_2\lambda)(1-\varepsilon_2\tau_2)(\varepsilon_1\varepsilon_2\lambda-1)$	$-\beta_2\varepsilon_2\lambda D^2(\tau_1-\varepsilon_2\lambda)-D^2(\varepsilon_1\varepsilon_2\lambda-1)-\lambda^2 D^2(\varepsilon_2\tau_2-1)$	—
$A_{11}\left(\begin{array}{l}\varepsilon_1\varepsilon_2\lambda D,\varepsilon_2\lambda D,\\ \lambda D\end{array}\right)$	$-\beta_1\beta_2\beta_3\varepsilon_1\varepsilon_2^{3}\lambda^5 D^6 (\tau_1\varepsilon_1-1)(1-\varepsilon_1\varepsilon_2\lambda)(\tau_2\varepsilon_2-1)$	$-\beta_2\varepsilon_2^{2}\lambda^2 D^2(\tau_1\varepsilon_1-1)-\beta_1\varepsilon_1\varepsilon_2\lambda D^2(1-\varepsilon_1\varepsilon_2\lambda)-\beta_3\lambda^2 D^2(\tau_2\varepsilon_2-1)$	—
$A_{12}(D,\tau_1 D,\tau_1\tau_2 D)$	$-\beta_1\beta_2\beta_3\tau_1^{3}\tau_2 D^6 (\varepsilon_2\tau_2-1)(\varepsilon_1\tau_1-1)(\lambda-\tau_1\tau_2)$	$-\beta_2\tau_1^{2}D^2(\tau_2\varepsilon_2-1)-\beta_1 D^2(\varepsilon_1\tau_1-1)-\beta_3\tau_1\tau_2 D^2(\lambda-\tau_1\tau_2)$	—

(Note: $\det(J)=m_{11}m_{22}m_{33}-m_{12}m_{23}m_{31}-m_{13}m_{21}m_{32}$, $tr(J)=m_{11}+m_{22}+m_{33}$, $K=\lambda D$, "-" indicates uncertain stability).

Table II shows that the stability of $A_9, A_{10}, A_{11}, A_{12}$ cannot be judged. Thus, the stability judgment of $A_9(D, \tau_1 D, \lambda D)$, $A_{10}(D, \varepsilon_2 \lambda D, \lambda D)$, $A_{11}(\varepsilon_1 \varepsilon_2 \lambda D, \varepsilon_2 \lambda D, \lambda D)$, and $A_{12}(D, \tau_1 D, \tau_1 \tau_2 D)$ can be analyzed as follows:

(1) Stability judgment and analysis of $A_9(D, \tau_1 D, \lambda D)$

Case 1: When
$$\begin{cases} \beta_1 \beta_2 \beta_3 (\varepsilon_2 \lambda - \tau_1)(1 - \varepsilon_1 \tau_1)(\tau_1 \tau_2 - \lambda) > 0 \\ \beta_2 \tau_1 (\varepsilon_2 \lambda - \tau_1) + \beta_1 (\varepsilon_1 \tau_1 - 1) + \beta_3 \lambda (\tau_1 \tau_2 - \lambda) > 0 \end{cases}$$, the local equilibrium point A_9 is a stable point.

Case 2: When
$$\begin{cases} \beta_1 \beta_2 \beta_3 (\varepsilon_2 \lambda - \tau_1)(1 - \varepsilon_1 \tau_1)(\tau_1 \tau_2 - \lambda) = 0 \\ \beta_2 \tau_1 (\varepsilon_2 \lambda - \tau_1) + \beta_1 (\varepsilon_1 \tau_1 - 1) + \beta_3 \lambda (\tau_1 \tau_2 - \lambda) = 0 \end{cases}$$, the local equilibrium point A_9 is a saddle point.

Case 3: If neither Case 1 nor Case 2 is met, the local equilibrium point A_9 is an unstable point.

(2) Stability judgment and analysis of $A_{10}(D, \varepsilon_2 \lambda D, \lambda D)$

Case 4: When
$$\begin{cases} \beta_1 \beta_2 \beta_3 (\varepsilon_2 \lambda - \tau_1)(1 - \varepsilon_2 \tau_2)(1 - \varepsilon_1 \varepsilon_2 \lambda) > 0 \\ \beta_2 \varepsilon_2 \lambda (\varepsilon_2 \lambda - \tau_1) + \beta_1 (1 - \varepsilon_1 \varepsilon_2 \lambda) + \beta_3 \lambda^2 (1 - \varepsilon_2 \tau_2) < 0 \end{cases}$$, the local equilibrium point A_{10} is a stable point.

Case 5: When
$$\begin{cases} \beta_1 \beta_2 \beta_3 (\varepsilon_2 \lambda - \tau_1)(1 - \varepsilon_2 \tau_2)(1 - \varepsilon_1 \varepsilon_2 \lambda) = 0 \\ \beta_2 \varepsilon_2 \lambda (\varepsilon_2 \lambda - \tau_1) + \beta_1 (1 - \varepsilon_1 \varepsilon_2 \lambda) + \beta_3 \lambda^2 (1 - \varepsilon_2 \tau_2) = 0 \end{cases}$$, the local equilibrium point A_{10} is a saddle point.

Case 6: If neither Case 4 nor Case 5 is met, the local equilibrium point A_{10} is an unstable point.

(3) Stability judgment and analysis of $A_{11}(\varepsilon_1 \varepsilon_2 \lambda D, \varepsilon_2 \lambda D, \lambda D)$

Case 7: When
$$\begin{cases} \beta_1 \beta_2 \beta_3 \varepsilon_1 \varepsilon_2^3 \lambda^5 (\tau_1 \varepsilon_1 - 1)(1 - \varepsilon_1 \varepsilon_2 \lambda)(\tau_2 \varepsilon_2 - 1) < 0 \\ -\beta_2 \varepsilon_2^2 \lambda^2 (\tau_1 \varepsilon_1 - 1) - \beta_1 \varepsilon_1 \varepsilon_2 \lambda (1 - \varepsilon_1 \varepsilon_2 \lambda) - \beta_3 \lambda^2 (\tau_2 \varepsilon_2 - 1) < 0 \end{cases}$$

, the local equilibrium point A_{11} is a stable point.

Case 8: When
$$\begin{cases} \beta_1 \beta_2 \beta_3 \varepsilon_1 \varepsilon_2^3 \lambda^5 (\tau_1 \varepsilon_1 - 1)(1 - \varepsilon_1 \varepsilon_2 \lambda)(\tau_2 \varepsilon_2 - 1) = 0 \\ -\beta_2 \varepsilon_2^2 \lambda^2 (\tau_1 \varepsilon_1 - 1) - \beta_1 \varepsilon_1 \varepsilon_2 \lambda (1 - \varepsilon_1 \varepsilon_2 \lambda) - \beta_3 \lambda^2 (\tau_2 \varepsilon_2 - 1) = 0 \end{cases}$$

, the local equilibrium point A_{11} is a saddle point.

Case 9: If neither Case 7 nor Case 8 is met, the local equilibrium point A_{11} is an unstable point.

(4) Stability judgment and analysis of $A_{12}(D, \tau_1 D, \tau_1 \tau_2 D)$

Case 10: When
$$\begin{cases} -\beta_1 \beta_2 \beta_3 \tau_1^3 \tau_2 (\tau_2 \varepsilon_2 - 1)(\varepsilon_1 \tau_1 - 1)(\lambda - \tau_1 \tau_2) > 0 \\ -\beta_2 \tau_1^2 (\tau_2 \varepsilon_2 - 1) - \beta_1 (\varepsilon_1 \tau_1 - 1) - \beta_3 \tau_1 \tau_2 (\lambda - \tau_1 \tau_2) < 0 \end{cases}$$, the local equilibrium point A_{12} is a stable point.

Case 11: When
$$\begin{cases} -\beta_1 \beta_2 \beta_3 \tau_1^3 \tau_2 (\tau_2 \varepsilon_2 - 1)(\varepsilon_1 \tau_1 - 1)(\lambda - \tau_1 \tau_2) = 0 \\ -\beta_2 \tau_1^2 (\tau_2 \varepsilon_2 - 1) - \beta_1 (\varepsilon_1 \tau_1 - 1) - \beta_3 \tau_1 \tau_2 (\lambda - \tau_1 \tau_2) = 0 \end{cases}$$, the local equilibrium point A_{12} is a saddle point.

Case 12: If neither Case 10 nor Case 11 is met, the local equilibrium point A_{12} is an unstable point.

F. Analysis of the synergy effect

Discussions on Cases 1, 4, 7, and 10:

a) When $\tau_1 > \varepsilon_2 \lambda$, $x = \varepsilon_2 \lambda D, z = \lambda D$.

Under conditions where the parameter meets Case 1 or 4 or 7 or 10, if $\varepsilon_1 \varepsilon_2 \lambda < 1$, then $y = \varepsilon_1 \varepsilon_2 \lambda D$, that is, $A_{11}(\varepsilon_1 \varepsilon_2 \lambda D, \varepsilon_2 \lambda D, \lambda D)$ is a stable local equilibrium point; If $\varepsilon_1 \varepsilon_2 \lambda > 1$, then $y = D$, that is, $A_{10}(D, \varepsilon_2 \lambda D, \lambda D)$ is a stable local equilibrium point; If $\varepsilon_1 \varepsilon_2 \lambda = 1$, then $A_{11}(\varepsilon_1 \varepsilon_2 \lambda D, \varepsilon_2 \lambda D, \lambda D)$ or $A_{10}(D, \varepsilon_2 \lambda D, \lambda D)$ is a stable local equilibrium point.

b) When $\tau_1 = \varepsilon_2 \lambda$, under conditions where the parameter meets Case 1 or 4 or 7 or 10, if $\varepsilon_1 \varepsilon_2 \lambda > 1$ and $\tau_1 \tau_2 > \lambda$, then $A_9(D, \tau_1 D, \lambda D)$ or $A_{10}(D, \varepsilon_2 \lambda D, \lambda D)$ is a stable local equilibrium point; If $\varepsilon_1 \varepsilon_2 \lambda < 1$ and

$\tau_1\tau_2 > \lambda$, then $A_{11}\left(\varepsilon_1\varepsilon_2\lambda D, \varepsilon_2\lambda D, \lambda D\right)$ is a stable local equilibrium point; If $\varepsilon_1\varepsilon_2\lambda > 1$ and $\tau_1\tau_2 < \lambda$, then $A_{12}\left(D,\tau_1 D,\tau_1\tau_2 D\right)$ is a stable local equilibrium point; If $\varepsilon_1\varepsilon_2\lambda = 1$ and $\tau_1\tau_2 = \lambda$, then $A_9\left(D,\tau_1 D,\lambda D\right)$, or $A_{10}\left(D,\varepsilon_2\lambda D,\lambda D\right)$, or $A_{11}\left(\varepsilon_1\varepsilon_2\lambda D,\varepsilon_2\lambda D,\lambda D\right)$, or $A_{12}\left(D,\tau_1 D,\tau_1\tau_2 D\right)$ is a stable local equilibrium point.

IV. NUMERICAL SIMULATIONS

Based on the above analysis of (a), (b), and (c), MATLAB (R2015a) was used to simulate the co-evolution of the construction waste recycling industrial chain in Cases 1, 4, 7, and 10, aiming to verify the correctness of the above local equilibrium point stability analysis and study the influence of various factors on the co-evolution process of the industrial chain, including simulation analysis of tendencies and sensitivity.

A. Co-evolution tendency and sensitivity analyses under Case 1 conditions

(1) Analysis of the tendency of co-evolution of construction waste recycling industrial chain

The industrial chain of construction waste recycling is considered as the research object. The initial output of enterprises (including construction waste recycling enterprises, recycled products dealers, and construction contractors), market demand K of construction contractors on recycled products, and maximum production D of construction waste (including waste from recycled products and traditional building materials as well as waste such as residues) of construction contractors are assigned in the co-evolution model of

c) When $\tau_1 < \varepsilon_2\lambda$, then $x = \tau_1 D, y = D$.

Under conditions where the parameter meets Case 1 or 4 or 7 or 10, if $\tau_1\tau_2 < \lambda$, then $z = \tau_1\tau_2 D$, that is, $A_{12}\left(D,\tau_1 D,\tau_1\tau_2 D\right)$ is a stable local equilibrium point; If $\tau_1\tau_2 > \lambda$, then $z = \lambda D$, that is, $A_9\left(D,\tau_1 D,\lambda D\right)$ is a stable local equilibrium point; If $\tau_1\tau_2 = \lambda$, then $A_9\left(D,\tau_1 D,\lambda D\right)$ or $A_{12}\left(D,\tau_1 D,\tau_1\tau_2 D\right)$ is a stable local equilibrium point.

$x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $\lambda = 0.64$ (i.e.,)

$K = 160$ $D = 250$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, $\beta_3 = 0.2$, $\varepsilon_1 = 1.7$, $\varepsilon_2 = 0.8$, $\tau_1 = 0.6$, and $\tau_2 = 1.4$.

Based on the above data, the stability analysis of the local equalization points $A_9\left(\tau_1 D, D, \lambda D\right)$, $A_{10}\left(\varepsilon_2\lambda D, D, \lambda D\right)$, $A_{11}\left(\varepsilon_2\lambda D, \varepsilon_1\varepsilon_2\lambda D, \lambda D\right)$, and $A_{12}\left(\tau_1 D, D, \tau_1\tau_2 D\right)$ can be performed. The results show that if the conditions in Case 1 are met

$$\begin{cases} \beta_1\beta_2\beta_3\left(\varepsilon_2\lambda - \tau_1\right)\left(1 - \varepsilon_1\tau_1\right)\left(\tau_1\tau_2 - \lambda\right) > 0 \\ \beta_2\tau_1\left(\varepsilon_2\lambda - \tau_1\right) + \beta_1\left(\varepsilon_1\tau_1 - 1\right) + \beta_3\lambda\left(\tau_1\tau_2 - \lambda\right) > 0 \end{cases}, \text{ then}$$

$\tau_1 > \varepsilon_2\lambda$ and $\varepsilon_1\varepsilon_2\lambda < 1$. Therefore, $A_{11}\left(128, 217.6, 160\right)$ is a stability point in the co-evolution process of the construction waste recycling industrial chain.

The simulation validation was performed using 0.005 as the simulation cycle. The simulation results of the co-evolution of the industrial chain of construction waste recycling meeting the conditions of Case 1 are shown in Fig. 2.
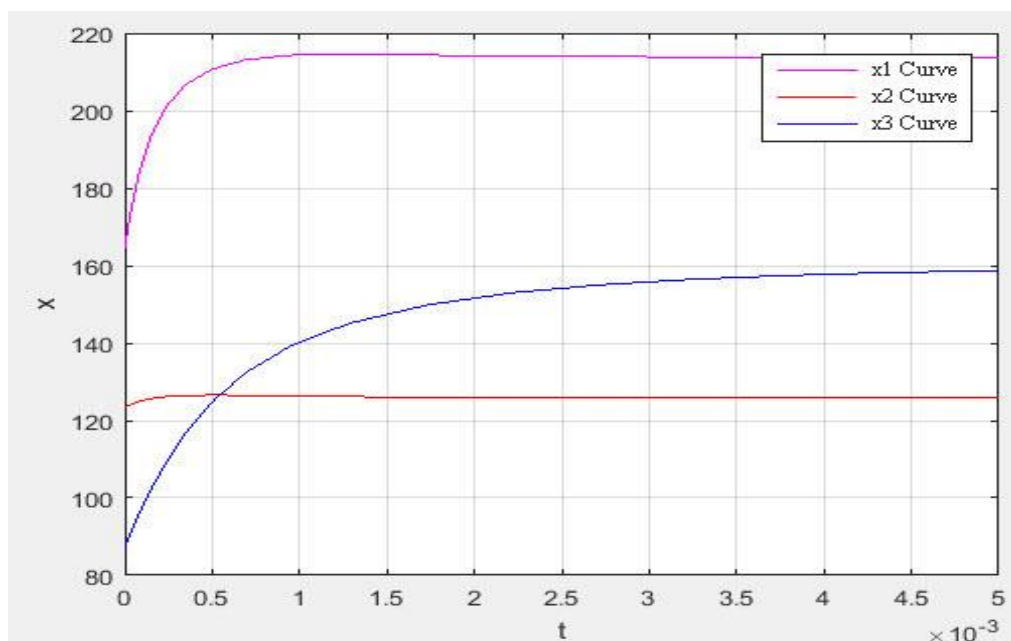
Fig. 2 Simulation of the co-evolution of the industrial chain of construction waste recycling meeting the conditions of Case 1 (Note: x1 curve represents y, i.e., the output of construction waste from construction contractors; x2 curve represents x, i.e., the output of recycled products of construction waste recycling enterprises; x3 curve represents z, i.e. the sales volume of the recycled products by the recycled products dealers. The same applies to Figs. 5 and 8).

Analysis of Fig. 2 shows that the output of recycled products of construction waste recycling enterprises gradually increases from the initial output ranging from 123.45 to 124.80 with time, reaching a stable state at 0.3×10^{-3}, and the growth rate of the output gradually decreases and tends to be stable at 0; The output of construction wastes of construction contractors increases gradually from an initial output ranging from 164.6 to 213.9 with time, reaching a stable state at 1×10^{-3}, and the growth rate of construction waste output gradually decreases and becomes stable at 0; The sales volume of recycled products of the recycled products dealers gradually increases from an initial output of 87.5 to 158.7 over time, reaching a stable state at 5×10^{-3}, and the growth rate of construction waste output gradually decreases and becomes stable at 0. Thus, it is clear that the simulation results are in accordance with the analysis results.

(2) Sensitivity analysis of co-evolution of the construction waste recycling industrial chain

Assuming that other conditions remain unchanged, that is $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $K = 160$, $D = 250$, $\varepsilon_1 = 1.7$, $\varepsilon_2 = 0.8$, $\tau_1 = 0.6$, and $\tau_2 = 1.4$, and increasing and reducing by 30% the natural growth rates $\beta_1 = 0.5$, $\beta_2 = 0.15$, and $\beta_3 = 0.2$, the simulation results of the co-evolution sensitivity of the natural growth rate of the construction waste recycling industrial chain are shown in Fig. 3.
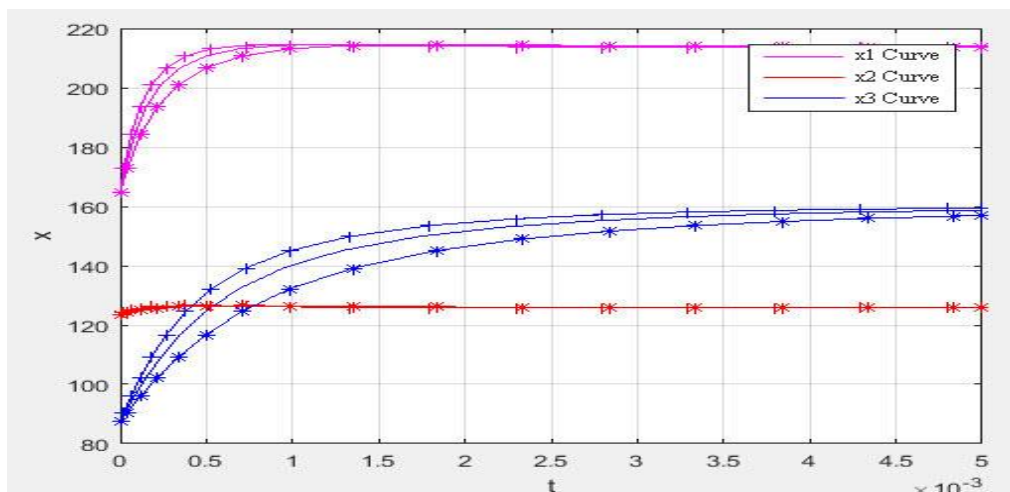
Fig. 3 Sensitivity analysis of the natural growth rate of the construction waste recycling industrial chain meeting the conditions of Case 1 (Note: "+solid line" means an increase of 30%; "* solid line" means a reduction of 30%; "Solid line" indicates the original value; Pink indicates y, i.e., the output of construction contractors; Red indicates x, i.e., the output of construction waste recycling enterprises; And blue indicates z, i.e., the output of recycled products dealers. The same applies to Figs 4, 6, 7, 9, and 10)

Analysis of Fig. 3 shows that when the natural growth rates are increased by 30%, the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors, and the sales of recycled products by the recycled products dealers are 128, 217.6 and 160, respectively, which are the same as the original stable values; The evolution rates of these three parameters are accelerated at the same time, and the evolution rate is evident for the construction wastes from construction contractors and sales of recycled products by the recycled products dealers, but not for the recycled product output of the construction waste recycling enterprises. When the natural growth rates are reduced by 30%, the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors and the sales of recycled products by the recycled products dealers are 128,

217.6, and 160, respectively, which are the same as the original stable values. The evolution rates of the three parameters are decelerated at the same time, and the evolution rate is evident for the construction wastes from construction contractors and sales of recycled products by the recycled products dealers, but not for the recycled product output of the construction waste recycling enterprises.

Assuming that other conditions remain unchanged, that is $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $K = 160$, $D = 250$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, and $\beta_3 = 0.2$, and increasing and reducing by 30% the conversion coefficients $\varepsilon_1 = 1.7$, $\varepsilon_2 = 0.8$, $\tau_1 = 0.6$ and $\tau_2 = 1.4$, the simulation results of the co-evolution sensitivity for the conversion coefficient of the construction waste recycling industrial chain are shown in Fig. 4.
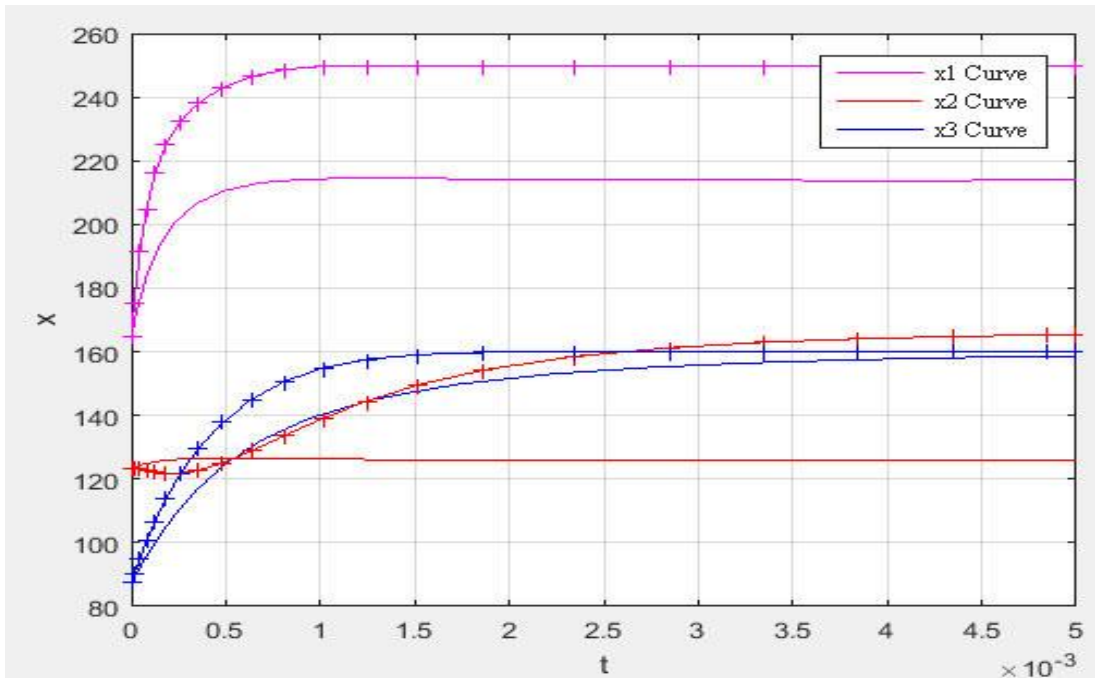
Fig. 4 Sensitivity analysis of the conversion coefficient of the construction waste recycling industrial chain meeting the conditions of Case 1

When the conversion coefficients are increased by 30%, the conditions of Case 4 are met, and the analysis of Fig. 4 shows that the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors, and the sales of recycled products by the recycled product dealers are 166.4, 250, and 160, respectively, which are consistent with the stable equilibrium point A_{10} (166.4, 250, 160) in Case 4. In addition, the changes in evolution rates of the three parameters are evident: The evolution rate of the recycled product output of the construction waste recycling enterprises increases first and then decreases, and finally tends to be stable at 0, whereas that of construction wastes from construction contractors and sales of recycled products by the recycled products dealers is accelerated. During this evolution process, it should be highlighted that the output of recycled products of construction waste recycling enterprises first decreases, then increases, and tends to be stable at 166.4 with strong fluctuation. When the conversion coefficients are reduced by 30%, the parameters do not meet the condition for judging the stability of local equilibrium points.

B. Co-evolution tendency and sensitivity analyses under conditions of Cases 4 and 10

1) The tendency of co-evolution of the construction waste recycling industrial chain

The industrial chain of construction waste recycling is considered as the research object, and the initial output of enterprises, market demand K of construction contractors on recycled products, and maximum production D of construction waste of construction contractors are assigned in the co-evolution model of construction waste recycling industrial chain, $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $\lambda = 0.64$ (i.e.,) $K = 160$ $D = 250$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, $\beta_3 = 0.2$, $\varepsilon_1 = 1.8$, $\varepsilon_2 = 1.1$, $\tau_1 = 1.6$, and $\tau_2 = 1.5$.

Based on the above data, the stability analysis of the local equalization points $A_9\left(\tau_1 D, D, \lambda D\right)$, $A_{10}\left(\varepsilon_2 \lambda D, D, \lambda D\right)$, $A_{11}\left(\varepsilon_2 \lambda D, \varepsilon_1 \varepsilon_2 \lambda D, \lambda D\right)$, and $A_{12}\left(\tau_1 D, D, \tau_1 \tau_2 D\right)$ can be performed. The results show

that if the conditions in Cases 4 and 10 are met,

$$\begin{cases} \beta_1\beta_2\beta_3(\varepsilon_2\lambda-\tau_1)(1-\varepsilon_2\tau_2)(1-\varepsilon_1\varepsilon_2\lambda) > 0 \\ \beta_2\varepsilon_2\lambda(\varepsilon_2\lambda-\tau_1)+\beta_1(1-\varepsilon_1\varepsilon_2\lambda)+\beta_3\lambda^2(1-\varepsilon_2\tau_2) < 0 \end{cases}$$ and

$$\begin{cases} -\beta_1\beta_2\beta_3\tau_1{}^3\tau_2(\tau_2\varepsilon_2-1)(\varepsilon_1\tau_1-1)(\lambda-\tau_1\tau_2) > 0 \\ -\beta_2\tau_1{}^2(\tau_2\varepsilon_2-1)-\beta_1(\varepsilon_1\tau_1-1)-\beta_3\tau_1\tau_2(\lambda-\tau_1\tau_2) < 0 \end{cases},$$

then $\tau_1 > \varepsilon_2\lambda$ and $\varepsilon_1\varepsilon_2\lambda > 1$. Therefore,

$A_{10}(176,250,160)$ is a stability point in the co-evolution process of the construction waste recycling industrial chain.

The simulation validation was performed in 0.005 as the simulation cycle. The simulation results of the co-evolution of the industrial chain of construction waste recycling are shown in Fig. 5.
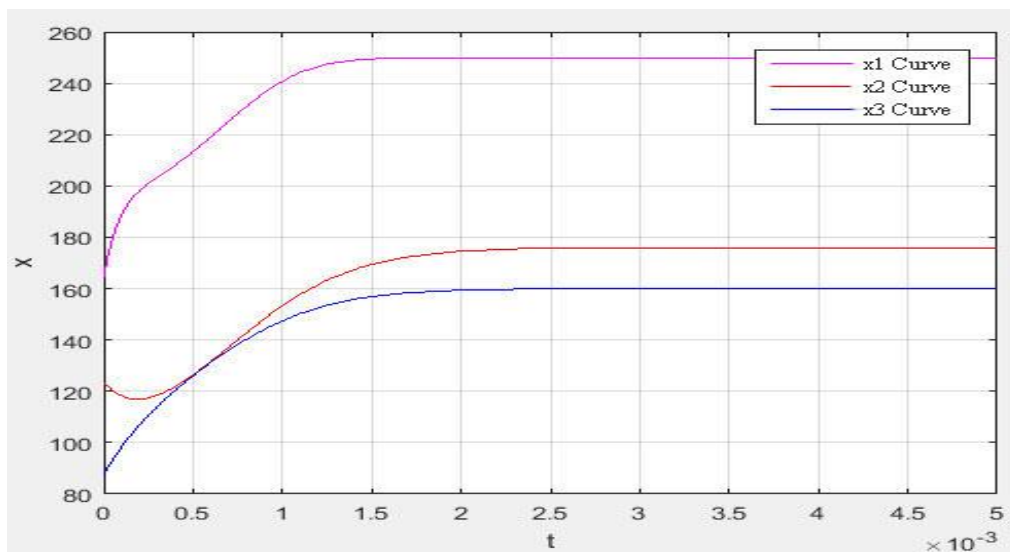
Fig. 5 Simulation of the co-evolution of the industrial chain of the construction waste recycling meeting the conditions of Case 4

The analysis of Fig. 5 shows that the output of recycled products of construction waste recycling enterprises initially decreases gradually from an initial output of 123.45, and then increases to 176 with time. It reaches a stable state at 2.3×10^{-3}. The growth rate of the output of recycled products first increases and then decreases and tends to be stable at 0; The output of construction wastes of construction contractors gradually increases from an initial output of 164.6 to 250, reaches a stable state at 1.5×10^{-3}, and then the growth rate of construction waste output gradually decreases and becomes stable at 0; The sales volume of recycled products of the recycled products dealers gradually increases from an initial output of 87.5 to 160, and reaches a stable state at 2×10^{-3}. The growth rate of construction waste output

gradually decreases and becomes stable at 0. Thus, it is clear that the simulation results are in accordance with the analysis results.

2) Sensitivity analysis of the co-evolution of construction waste recycling industrial chain

Assuming that other conditions remain unchanged, that is $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $K = 160$, $D = 250$, $\varepsilon_1 = 1.8$, $\varepsilon_2 = 1.1$, $\tau_1 = 1.6$, and $\tau_2 = 1.5$, and increasing and reducing by 30% the natural growth rates $\beta_1 = 0.5$, $\beta_2 = 0.15$, and $\beta_3 = 0.2$, the simulation results of the co-evolution sensitivity of the natural growth rate of the construction waste recycling industrial chain are shown in Fig. 6.
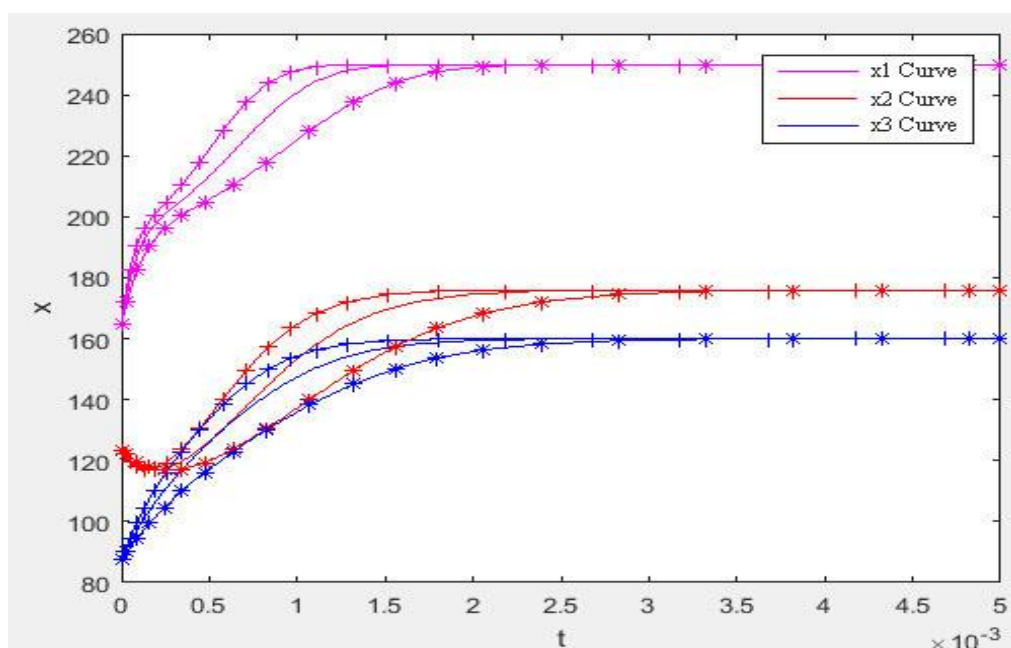
Fig. 6 Sensitivity analysis of the natural growth rate of the construction waste recycling industrial chain meeting the conditions of Case 4

Analysis of Fig. 6 shows that when the natural growth rates are increased by 30%, the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors and the sales of recycled products by the recycled products dealers are 176, 250 and 160, respectively, which are the same as the original stable values; The evolution rates of these three parameters are accelerated at the same time. When the natural growth rates are reduced by 30%, the stable values are 176, 250, and 160, respectively, which are the same as the original stable values; The evolution rates of the three parameters are decelerated at the same time. Thus, it is clear that whether the natural growth rates are increased or decreased by 30%, the output curves are basically similar to the original values.

Assuming that other conditions remain unchanged, that is $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $K = 160$, $D = 250$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, and $\beta_3 = 0.2$, and increasing and reducing by 30% the conversion coefficients $\varepsilon_1 = 1.8$, $\varepsilon_2 = 1.1$, $\tau_1 = 1.6$, and $\tau_2 = 1.5$, the simulation results of the co-evolution sensitivity of the conversion coefficient of the construction waste recycling industrial chain are shown in Fig. 7.
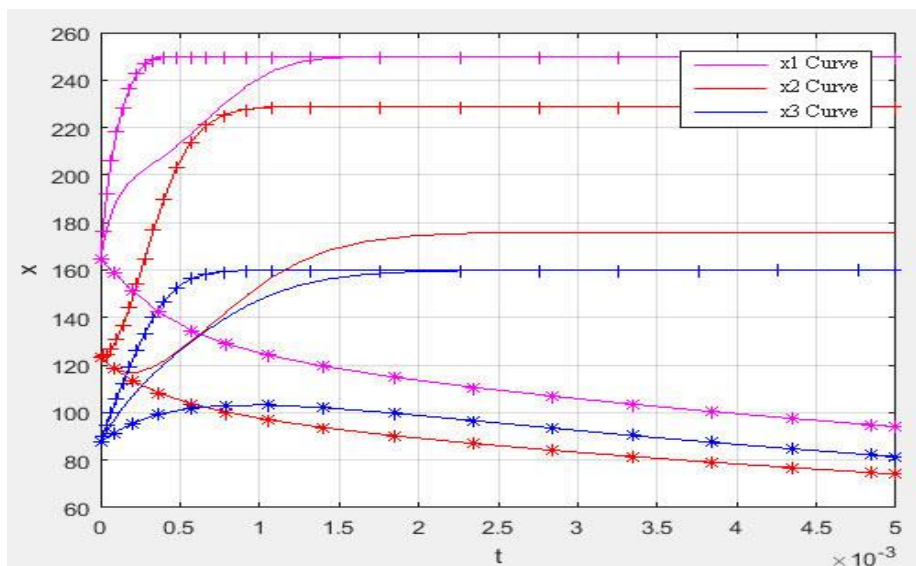
Fig. 7 Sensitivity analysis of the conversion coefficient of the construction waste recycling industrial chain meeting the conditions of Case 4

When the conversion coefficients are increased by 30%, the conditions of Case 4 are met, and the analysis of Fig. 7 shows that the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors, and the sales of recycled products by the recycled products dealers are 228.8, 250, and 160, respectively, which are identical to the stable equilibrium point A_{10} (228.8, 250, 160) in Case 4. In addition, the changes in the evolution rates of the three parameters are evident, accelerating and then reaching stability at 1×10^{-3}, 0.4×10^{-3}, and 0.8×10^{-3}, respectively, which are better than the original values of 2.3×10^{-3}, 1.5×10^{-3}, and 2×10^{-3}. When the conversion coefficients are reduced by 30%, the parameters do not meet the requirements of the local equilibrium stability judgment.

C. Co-evolution tendency and sensitivity analyses under conditions of Case 7

1) Analysis of the tendency of co-evolution of construction waste recycling industrial chain

The industrial chain of construction waste recycling is considered as the research object, and the initial output of enterprises, market demand K of construction contractors on recycled products, and maximum production D of construction waste of construction contractors are assigned in the co-evolution model of the construction waste recycling industrial chain, $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $\lambda = 0.64$ (i.e.,) $K = 160$ $D = 250$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, $\beta_3 = 0.2$, $\varepsilon_1 = 1.5$, $\varepsilon_2 = 1.1$, $\tau_1 = 0.7$, and $\tau_2 = 1.3$.

Based on the above data, the stability analysis of the local equalization points $A_9\left(\tau_1 D, D, \lambda D\right)$, $A_{10}\left(\varepsilon_2 \lambda D, D, \lambda D\right)$, $A_{11}\left(\varepsilon_2 \lambda D, \varepsilon_1 \varepsilon_2 \lambda D, \lambda D\right)$, and $A_{12}\left(\tau_1 D, D, \tau_1 \tau_2 D\right)$ can be performed. The results show that if the conditions in Case 7 are met

$$\begin{cases} \beta_1 \beta_2 \beta_3 \varepsilon_1 \varepsilon_2{}^3 \lambda^5 \left(\tau_1 \varepsilon_1 - 1\right)\left(1 - \varepsilon_1 \varepsilon_2 \lambda\right)\left(\tau_2 \varepsilon_2 - 1\right) < 0 \\ -\beta_2 \varepsilon_2{}^2 \lambda^2 \left(\tau_1 \varepsilon_1 - 1\right) - \beta_1 \varepsilon_1 \varepsilon_2 \lambda \left(1 - \varepsilon_1 \varepsilon_2 \lambda\right) - \beta_3 \lambda^2 \left(\tau_2 \varepsilon_2 - 1\right) < 0 \end{cases}$$

, then $\tau_1 < \varepsilon_2 \lambda$ and $\tau_1 \tau_2 > \lambda$. Therefore, $A_9\left(175, 250, 160\right)$ is a stability point in the co-evolution process of the construction waste recycling industrial chain.

The simulation validation was performed in 0.100 as the simulation cycle. The simulation results of the co-evolution of the industrial chain of construction waste recycling meeting the conditions of Case 7 are shown in Fig. 8.
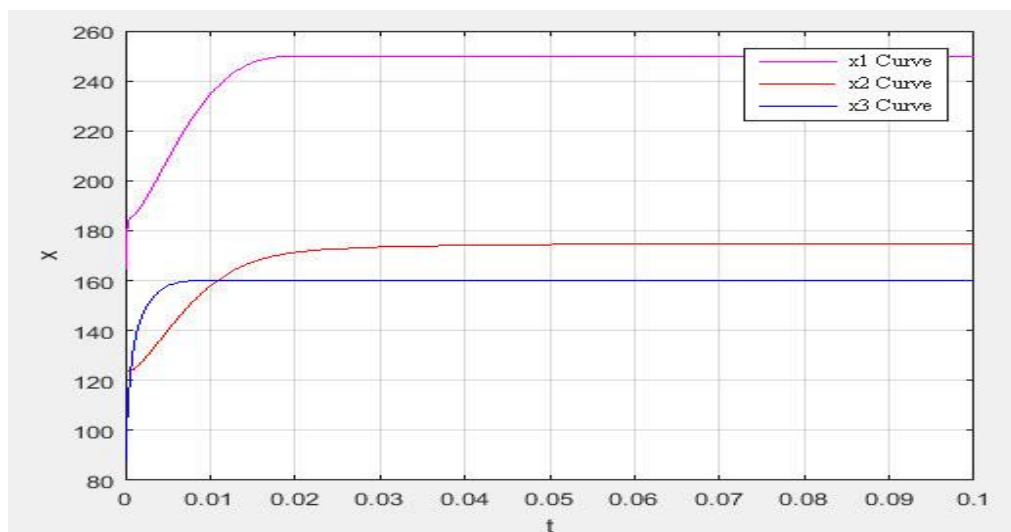
Fig. 8 Simulation of the co-evolution of the industrial chain of the construction waste recycling meeting the conditions of Case 7

The analysis of Fig. 8 shows that the output of recycled products of construction waste recycling enterprises gradually increases from the initial output of 123.45 to 174.9, reaching a stable state at 0.05, and the growth rate of this output gradually decreases and tends to be stable at 0; The output of construction wastes of construction contractors gradually increases from an initial output of 164.6 to 250, reaching a stable state at 0.02, and the growth rate of this output gradually decreases and becomes stable at 0; The sales volume of recycled products of the recycled products dealers gradually increases from an initial output of 87.5 to 160, reaching a stable state at 0.01, and the growth rate of this output gradually decreases and becomes stable at 0. Thus, it is clear that the simulation results are in accordance with the analysis results.

2) Sensitivity analysis of the co-evolution of the construction waste recycling industrial chain

Assuming that other conditions remain unchanged, that is $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $K = 160$, $D = 250$, $\varepsilon_1 = 1.5$, $\varepsilon_2 = 1.1$, $\tau_1 = 0.7$, and $\tau_2 = 1.3$, and increasing and reducing by 30% the natural growth rates $\beta_1 = 0.5$, $\beta_2 = 0.15$, and $\beta_3 = 0.2$, the simulation results of the co-evolution sensitivity of the natural growth rate of the construction waste recycling industrial chain are shown in Fig. 9.
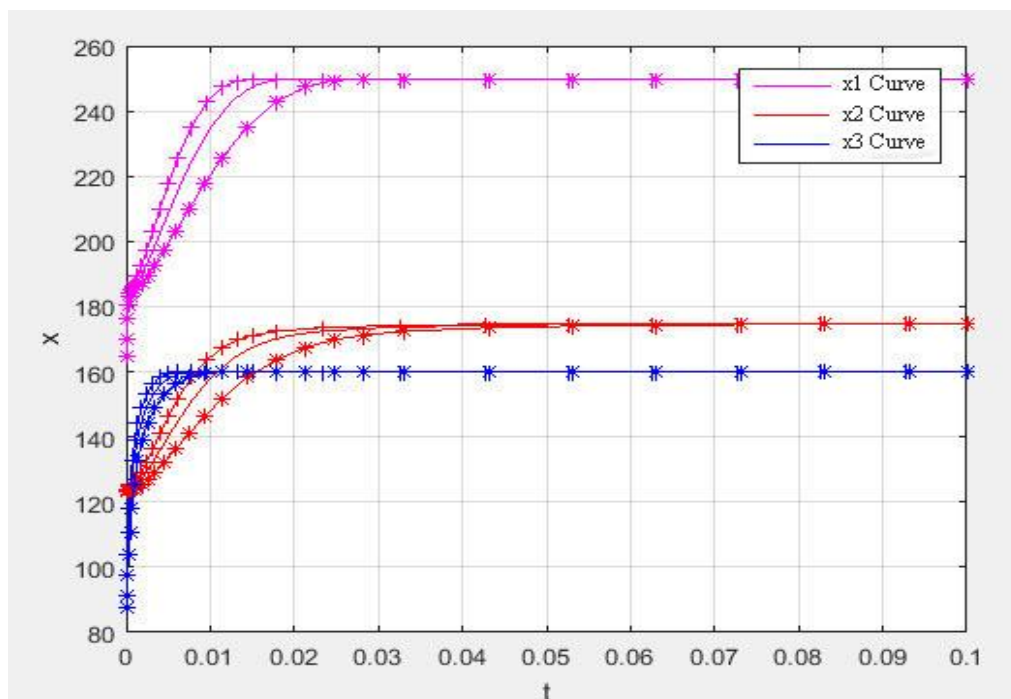
Fig. 9 Sensitivity analysis of the natural growth rate of the construction waste recycling industrial chain meeting the conditions of Case 7

Analysis of Fig. 9 shows that when the natural growth rates are increased by 30%, the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors and the sales of recycled products by the recycled products dealers are 174.9, 250 and 160, respectively, which are the same as the original stable values; The evolution rates of these three parameters are accelerated at the same time. When the natural growth rates are reduced by 30%, the stable values of these outputs are 174.9, 250, and 160, respectively, which are the same as the original stable values. The evolution rates of the three parameters are decelerated at the same time. Thus, it is clear that whether the natural growth rates are increased or decreased by 30%, the output curves are basically similar to the original values.

Assuming that other conditions remain unchanged, that is $x_{10} = 164.6$, $x_{20} = 123.45$, $x_{30} = 87.5$, $K = 160$, $D = 250$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, and $\beta_3 = 0.2$, and increasing and reducing by 30% the conversion coefficients $\varepsilon_1 = 1.5$, $\varepsilon_2 = 1.1$, $\tau_1 = 0.7$, and $\tau_2 = 1.3$, the simulation results of the co-evolution sensitivity of the conversion coefficient of the construction waste recycling industrial chain are shown in Fig. 10.
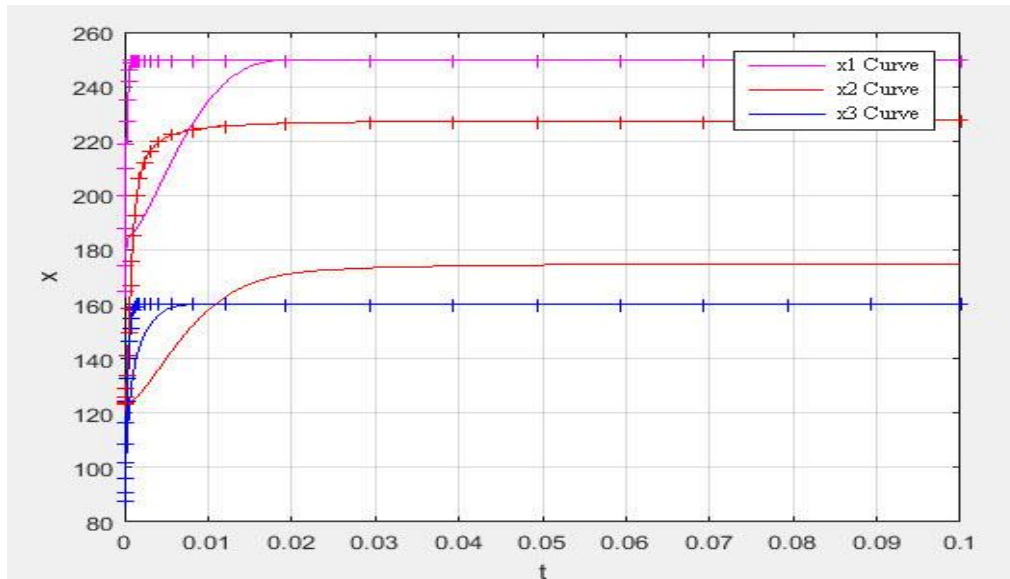
Fig. 10 Sensitivity analysis of the conversion coefficient of the construction waste recycling industrial chain meeting the conditions of Case 7

When the conversion coefficients are increased by 30%, the conditions of Case 7 are met, and the analysis of Fig. 10 shows that the stable values of the output of recycled products from the construction waste recycling enterprises, the output of construction waste from the construction contractors, and the sales of recycled products by the recycled products dealers are 174.9, 250, and 160, respectively, which are basically consistent with the stable equilibrium point A_{10} (175, 250, 160) in Case 7. In addition, the changes in the evolution rates of the three parameters are evident, accelerating and then reaching stability at 0.03, 0.001, and 0.001, respectively, which are better than the original values of 0.05, 0.02 and 0.01. When the

conversion coefficients are reduced by 30%, the parameters do not meet the requirements of the local equilibrium stability judgment.

D. Analysis and discussion

According to the above analysis in Figs. 2, 5, and 8, the comparison between the validation results of the simulation of the co-evolution of the construction waste recycling industrial chain and the results of the stability analysis by the Jacobian matrix under the conditions of parameters meeting Cases 1, 4, 7, and 10 is shown in Table III.

Table III. Error analysis of stability analysis results and simulation verification results

Conditions of parameters	Stability analysis results	Simulation verification results	Error rate
Meeting Case 1	(128, 217.6, 160)	(124.8, 213.9, 158.7)	(2.5%, 1.7%, 0.8%)
Meeting Cases 4 and 10	(176, 250, 160)	(176, 250, 160)	(0%, 0%, 0%)
Meeting Case 7	(175, 250, 160)	(174.9, 250, 160)	(0.05%, 0%, 0%)

Table III shows that when the parameters meet the conditions of Case 1, the error rates of output of recycled products of construction waste recycling enterprises, output of construction wastes from construction

contractors, and sales of recycled products of recycled products dealers are 2.5%, 1.7%, and 0.8%, respectively, by simulation verification results and stability analysis results, which meet the error range. When the parameters

meet the conditions of Cases 4 and 10, the error rates of simulation verification results and stability analysis results are 0%, 0%, and 0%, respectively, which meet the error range. When the parameters meet the conditions of Case 7, the error rates of the simulation verification results and stability analysis results are 0.05%, 0%, and 0%, respectively, which meet the error range.

According to the above analysis of Figs. 3, 4, 6, 7, 9, and 10, it is clear that when parameters meet the conditions

V. CASE SIMULATION

After confirming the validity of the model, the co-evolution of the construction waste recycling industrial chain in Guangzhou was simulated and forecasted. Based on the studies of Kloppers and Greeff the initial output of enterprises (construction waste recycling enterprises, recycled product dealers, and construction contractors), market demand K of construction contractors on recycled products, and maximum production D of construction waste (including waste from recycled products and traditional building materials as well as waste

of Cases 1, 4, 7, and 10, the influence of the conversion coefficient is more sensitive than that of the natural growth rate on the industrial chain, which is in accordance with the research conclusions of Zhou [32].

Based on the above analysis, the simulation results of the co-evolution model of construction waste recycling industrial chain are in accordance with the stability analysis results, which verifies the reliability of the model and analysis [33] - [35].

such as residues) of construction contractors are reasonably predicted from 2020 to 2030: $x_{20} = 1157.43$, $x_{10} = 1543.24$, $x_{30} = 820$, $K = 2360.6$, $D = 4551.6$ (in 10 000 tons per year) [36].

The natural growth rate and other parameters between 2020 and 2030 were collected through a questionnaire survey. The parameters are listed in Table IV. The change in the simulation forecast curve and the numerical forecast of nodes in each year (year-end) about the co-evolution of construction waste recycling industrial chain in Guangzhou are shown in Fig. 11 and Table IV.

Table IV. Parameter values

Parameters	β_1	β_2	β_3	ε_1	ε_2	τ_1	τ_2
Values	0.55	0.17	0.28	0.64	1.45	1.72	0.83

Table V Node value forecast for each year (in 10 000 tons/year)

Year/variable	x_1	x_2	x_3
2020	1543.24	1157.43	820
2021	1842.74	1159.62	938.43
2022	1981.91	1155.1	1184.84
2023	1985.63	1155.21	1528.03
2024	2042.47	1189.31	1694.7
2025	2447.99	1426.19	2015.53
2026	3004.23	1750.33	2328.35
2027	3328.12	1935.64	2360.6
2028	3370.02	1959.3	2360.6
2029	3369.99	1959.3	2360.6
2030	3369.99	1959.3	2360.6

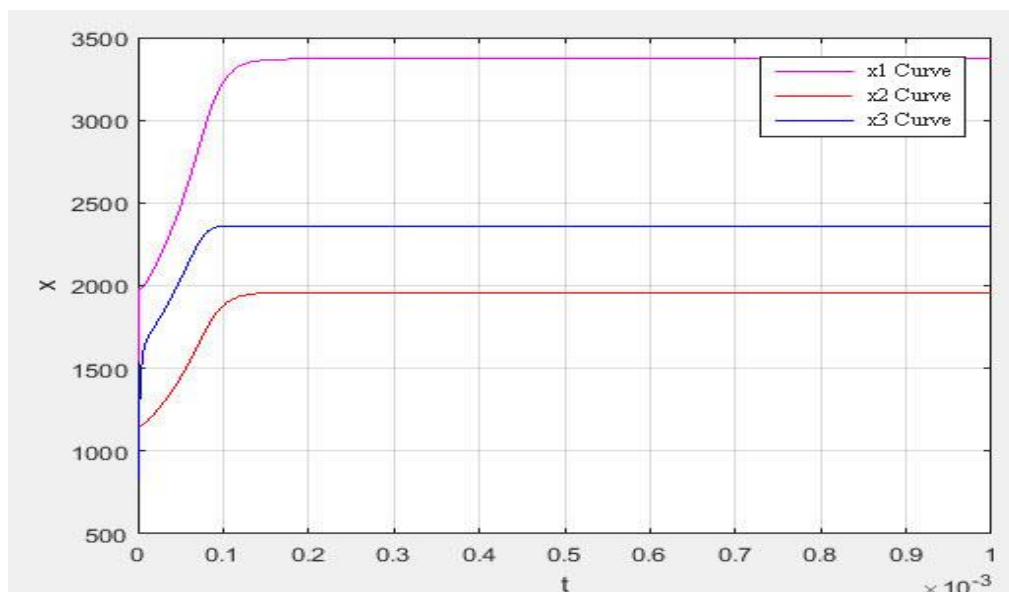Data from SBGP and GBS [37] [38]

Fig. 11 Forecast of the co-evolution of the construction waste recycling industrial chain in Guangzhou City

The simulation results in Fig. 11 and Table V show that the output of recycled products of construction waste recycling enterprises reaches a stable state in 2028, with an annual output of 19.593 million tons. The output of construction waste recycled by construction contractors reaches a stable state in 2028, with an annual handling capacity of 33.7 million tons, and the sales of recycled products of the recycled products dealers reaches a stable state in 2027, with annual sales of 23.606 million tons, indicating that the construction waste recycling industrial chain in Guangzhou will finally reach a stable state with a very high construction waste recycling rate, but lower than 100%.

VI. CONCLUSIONS

Aiming at the low utilization rate of construction waste resources in China, a Lotka–Volterra model of co-evolution of construction waste recycling industry chain was built from the perspective of the construction waste recycling industry chain. The stability of the model was analyzed using the Jacobian matrix. The analysis results were verified by MATLAB numerical simulation. The simulation results indicated the following:

1) Regardless of the parameters, the error rate of the verification results and the stability analysis results from the simulation of output of recycled products of construction waste recycling enterprises, output of construction waste recycled by the construction contractors, and sales of recycled products of the recycled products dealers is less than 5%, which proves that the co-evolution model is reliable;

2) The output of recycled products of construction waste recycling enterprises, output of construction waste recycled by the construction contractors, and sales of recycled products of the recycled products dealers will reach their maximum. This indicates that the upstream and downstream enterprises in the entire construction waste recycling industrial chain will flourish, develop and progress together to realize industrial value added, and that they will constitute a stable product supply chain and maximize the total output value of the industrial chain;

3) The evolution rate of upstream and downstream enterprises in the construction waste recycling industrial chain is ultimately 0, and the change rate is basically the same in the process of evolution, which indicates that there is a high product correlation between upstream and downstream enterprises in the construction waste recycling industrial chain;

4) Compared with the natural growth coefficient, the conversion coefficient has a higher influence and sensitivity on the evolution of construction waste recycling industrial chains. In the process of co-evolution of the construction waste recycling industrial chain, the 30% increase in the natural growth rate and in the conversion

factor of the industrial chain leads to a maximum increase of 0.2 and 19 times the rate of evolution of the industrial chain, respectively, resulting in a substantial increase in the synergistic efficiency.

In the process of co-evolution of the construction waste recycling industrial chain, only when the upstream and downstream enterprises of industrial chain work closely together to improve the conversion coefficient of enterprise output by means of information sharing, use of high-efficiency technology, and diversified processing of products to accelerate the co-evolution of the industrial chain and achieve a stable state, can the synergy effect of industrial chain finally be highlighted. As the propeller of the construction waste recycling industrial chain, the government should provide guidance, regulation, and incentive in the development and perfection, such as the formulation of standards for use of recycled products, government subsidy policies, government penalties, etc., and increase the market publicity to promote an increase in the market demand for recycled products.

In terms of practical significance, this study can guide the effective treatment of construction wastes on construction sites, help to improve the utilization rate of construction wastes as resources, help the government formulate incentive policies, and then improve the efficiency of policies to maximize the overall value of the industrial chain. In terms of theory, the related theories of construction waste management in China have been deepened in this study, as the basis for the government to formulate economic compensation policies and relevant laws and regulations in the process of resource management of construction waste, which can be used by the government to guide construction waste recycling and promote the development of the construction waste recycling industrial chain through market-oriented control and economic policies, to achieve the strategic goal of sustainable development.

There are many deficiencies in this study. For example, the construction waste recycling industry chain involves more stakeholders, and the related network structure is more complex, but only the main stakeholders are reflected in this study. Furthermore, the process of construction waste recycling is studied macroscopically from the perspective of industrial chains. In future studies, the impact of microindividual interactions in the construction waste recycling industrial chain can be explored from a micro perspective, in combination with self-organization and system complexity theories, to find its internal mechanism and evolution path.

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