Signal segmentation using maximum a posteriori probability estimator with application in EEG data analysis

Theodor D. Popescu
National Instituite for Research and Development in Informatics
8-10 Averescu Avenue, 011455 Bucharest, Romania

Received: March 21, 2021. Revised: August 22, 2021. Accepted: September 4, 2021. Published: September 6, 2021.

Abstract- The "corrected" EEG data, after artifact removing, may be the subject of further investigations, for example segmentation, resulting new information to be used for feature extraction, of great help for medical diagnosis. The paper has as object a generally approach for segmentation, making use of Maximum A posteriori Probability (MAP) estimator. The proposed procedure has been used in the analysis of a sample lowpass EEG signals recorded with 13 scalp and 1 EOG electrodes, event-related potential (ERP) data.

Keywords- Signal processing, segmentation, MAP estimator, feature extraction, EEG analysis.

I. Introduction

THE "corrected" EEG signals, after artifact source removing, may be the subject of future investigations, to detect a possible change in individual or simultaneous EEG recordings. The analysis of the behavior of such signals reveals that the most of the changes that occur are either changes in the mean level, or changes in spectral characteristics. In this framework, the problem of segmentation between "homogenous" part of the signals (or detection of changes in signals) arises more or or less explicitly.

Several methods for change detection and segmentation have been suggested earlier, with application in different fields. The general framework for change detection/segmentation make the object of [1] and [2]. Some authors typically employ explicit management of multiple model AFMM (adaptive forgetting by multiple models), formulate the segmentation problem as a least-squares problem with sum-of-norm regularization over the state parameter jumps [3], use a sequential algorithm [4], time-frequency decomposition and statistical modeling [5], or segmentation fragments optimization in empirical wavelet transform [6], among others.

Also, the segmentation problem is present in many applications for EEG signal analysis. So, [7] performs adaptive segmentation of EEG signals, [8] proposes a an efficient event-driven segmentation and de-noising technique, founded on the principles of level crossing and activity selection, [9] deals with a nonparametric method for the EEG signal segmentation, [10] makes use of a time-frequency approach for signal segmentation combining the empirical mode decomposition (EMD) and Hilbert transform (HT), [11] uses a genetic algorithm to choose appropriate values for parameters in any signal segmentation application, [12] performs multi-channel EEG signal segmentation and feature extraction, while [13] presents electrophysiological signals segmentation for EEG frequency bands and heart rate variability analysis.

The signal segmentation in EEG signal analysis is very important because the quasi-stationary segments resulted can be used for feature extraction, an important step in the analysis of EEG signals, using statistical methods [14], spectral analysis [15], cepstral analysis [16], and coherence analysis [17], among others.

The outline of this paper is as follows. Section 2 presents the change detection and segmentation problem in EEG signals analysis and gives the conceptual description of a segmentation algorithm, based on Maximum A posteriori Probability (MAP) estimator. Section 3 presents a case study having as subject, artifact removing and segmentation of "corrected" EEG recordings, collected from 13 scalp and 1 EOG electrodes, applied to a sample lowpass event-related potential (ERP) data for 2 epochs.

II. Segmentation of "corrected" EEG signals

A. *Preliminary*

The problem of change detection and segmentation is a key point which frequently occurs in many application areas, where analysis and modeling of non-stationary signals arises.

The change detection problem refers to detection of the change in signal dynamics, providing information, in some case, for feature extraction. The performance criterion of a change detection algorithm consists of its ability to correctly detect the changes, with minimum delay and minimum probability of false decisions. So, it must respond to the small changes (sensitivity to changes), and should not be affected by disturbances, noise or modeling errors (robustness of the algorithm).

Two basic approaches in change detection are reported as based on quantitative models and qualitative models, which can be conveniently combined to improve the robustness of the quantitative residuals generation [18]. In the case of analytical exact models absence, learning models, such as fuzzy and neural models, can be used. Moreover, the neural networks can be used for classification of the residuals, while fuzzy logic is useful in decision making. Some heuristics results, obtained from the previous experience, can be used by specialist doctor for diagnosing the origins of the change in the EEG signals, based on the change characteristics and his experience.

Almost all change detection solutions assume that the monitored signals can be described with sufficient precision by a finite-dimensional linear model. In practice, if the signal is more complex than the dynamics, described by a finite-dimensional model, the parameter estimates will still converge, but their values can be strongly dependent on the experiment conditions. The algorithms will not be able to separate the changes determined by the external conditions from those produced by the internal dynamics of the signal making the object of the analysis, so the classical tests will fail. The problems mentioned above point out the requirement of the robust change detection algorithms, able to separate the changes determined by the external conditions from the changes of the internal dynamics of the signal. The problem is of great interest, especially when the artifact removing has not been completed.

The first generation of change detection algorithms is based on strong hypotheses, or strong assumptions, which are difficult to verify in practice. So, a second generation of solutions was required, insensitive to the uncertainty of the signal dynamics, to the environment, and to large noise, statistically unknown. The central problems to be addressed in the change detection area refer to robustness, sensitivity and versatility. The lack of robustness of the classical algorithms is linked to the failure of the detection, if one or more of the hypotheses assumed during the design are not verified in practice. The sensitivity is linked to the ability of the algorithm to detect the change, even if there are small scale incipient changes. Finally, the versatility is linked to the ability of the methods and techniques to solve more change detection problems, using the same set of algorithms.

The change detection problem can be solved by change point estimation (mean change) [19], change detection using one and two model approach, with different distance measures and stoping rules [20], multiple change detection [21], detection and diagnosis of model parameter and noise variance changes [22], for mono- and multivariable signals. Some algorithms, making the object of [1] and [2] in change detection, represented the starting points in developing of these algorithms.

In change detection, the problem of segmentation between "homogenous" parts of a signal arises more or less explicitly. It is often the case that a mono- or multivariate signal can be represented as a sequence of discrete segments of finite length. There are two general approaches to this problem. The first involves looking for change points in the signal: for example, one may assign a segment boundary whenever there is a large jump in the average value of the signal. The second approach involves assuming that each segment in the signal is generated by a system with distinct parameters, and then inferring the most probable segment locations and the system parameters that describe them. While the first approach tends to only look for changes in a short window of time, the second approach generally takes into account the entire signal.

The goal in segmentation is to find the time instants for changes in the properties or dynamics of a signal. The problem is closely related to change detection, where the objectives are to detect a change as fast as possible, to isolate the change time and to diagnose the cause of the change. In segmentation however, only the change times are primarily of interest. One way to segment a signal using a change detection method, is to process the data sequentially and when a change is detected the detector is restarted. This is the natural method for online purposes and it is thoroughly surveyed in [2].

The proposed problem formulation in this paper assumes off-line or batch-wise data processing, although the solution is sequential in data and a recursive approximation is suggested as well. The segmentation model is the simplest possible extension of linear regression models to signals with abruptly changing properties, or piecewise linearizations of non-linear models.

B. *The linear regression model with piecewise constant parameters*

We introduce now the linear regression model with piecewise constant parameters to be used in EEG signals segmentation. As we mentioned above, in segmentation the goal is to find a sequence of time indices $k^n = k_1, k_2, \ldots, k_n$, where both the number n and the locations k_i are unknown, such that a linear regression model with piecewise constant parameters,

$$y_t = \phi_t^T \theta(i) + e_t, \quad E(e_t^2) = \lambda(i) R_t \tag{1}$$

when $k_{i-1} < t \leq k_i$ is a good description of the observed signal y_t. Here $\theta(i)$ is the d-dimensional parameter vector in segment i, ϕ_t is the regressor and k_i denotes the change times. The noise e_t is assumed to be Gaussian with variance $\lambda(i)R_t$, where $\lambda(i)$ is a possibly segment dependent scaling of the noise and R_t is the nominal covariance matrix of the noise. We can think of λ either as a scaling of the noise variance or variance itself (R_t = 1). Neither $\theta(i)$ or $\lambda(i)$ are known. The Gaussian assumption on the noise is a standard one, partly because it gives analytical expressions and partly because it has proven to work well in practice. We will assume R_t to be known and the scaling as a possibly unknown parameter. The model (1) is referred to as changing regression, because it changes between regression models. Its important feature is that the jumps divide the measurements into a number of independent segments, since the parameter vectors in different segments are independent. Some important cases of the model (1) are the changing mean model, the autoregressive (AR) model, the autoregressive model with exogenous variable (ARX) and finite impulse response (FIR) model, etc, where ϕ_t has different expressions.

The assumption on the regression models in (1) is not too restrictive since many stationary processes encountered in practice can be closely approximated by such models. The identification and parameters estimation methods represent only tools to perform change detection and segmentation. Good and precise models offers high performance in these schemes, but also biased parametric models can be used for change detection and segmentation. This bias decreases, but does not annihilate the performance of the detection and segmentation procedures.

C. *Algorithm description*

We present here the conceptual description of the segmentation algorithm using the Maximum A posteriori Probability (MAP) estimator [21].

The segmentation problem is solved, searching all segmentation k^n values, that minimizes an optimality criteria of the form:

$$\widehat{k^n} = \arg \min_{n\geq 1, 0<k_1<\ldots<k_n=N} V(k^n) \tag{2}$$

In each segment is determined a linear regression model. For the measurements belonging to i-th segment, $y_{k_{i-1}+1}, \ldots y_{k_i} = y_{k_{i-1}+1}^{k_i}$, are determined the least square estimate of model parameters and its covariance matrix:

$$\hat{\theta}(i) = P(i) \sum_{t=k_{i-1}+1}^{k_i} \phi_t R_t^{-1} y_t, \tag{3}$$

$$P(i) = \left(\sum_{t=k_{i-1}+1}^{k_i} \phi_t R_t^{-1} \phi_t^T \right)^{-1}. \tag{4}$$

In optimal segmentation algorithm, the following quantities are used:

$$V(i) = \sum_{t=k_{i-1}+1}^{k_i} (y_t - \phi_t^T \hat{\theta}(i))^T R_t^{-1} (y_t - \phi_t^T \hat{\theta}(i)) \tag{5}$$

$$D(i) = -\log \det P(i) \tag{6}$$

$$N(i) = k_i - k_{i-1} \tag{7}$$

The values used in k^n segmentation, having $n-1$ degrees of freedom are presented in Table 1, for better understanding of the conceptual description of the algorithm.

It can be noted the number of segmentations k^n is 2^N and the dimensionality problem is a difficult one. To solve the optimal segmentation in [2] are proposed different optimality criteria, one of these being the MAP estimator, for which the conceptual description is given in the following; the information from Table 1 is used, as well as three assumptions on noise scaling, $\lambda(i)$ given below:

Table 1: Values used in MAP estimator.

Data	$y_1, y_2, \ldots, y_{k_1}$	$\ldots$	$y_{k_{n-1}+1}, \ldots, y_{k_n}$
Segment	Segment 1	$\ldots$	Segment n
LS est.	$\hat{\theta}(1), P(1)$	$\ldots$	$\hat{\theta}(n), P(n)$
Statistics	$V(1), D(1), N(1)$	$\ldots$	$V(n), D(n), N(n)$

Data: Signal y_t, $t = 1 \ldots N$

Step 1: Analyze every segmentation, with the number of jumps n and jump times k^n, for each case.

Step 2: For each segmentation resulted, the best models for each segment are computed under the form of the least square estimates $\hat{\theta}(i)$ and covariance matrices $P(i)$.

Step 3: For each segment compute:

$$\begin{aligned} V(i) &= \textstyle\sum_{t=k_{i-1}+1}^{k_i} (y_t - \phi_t^T \hat{\theta}(i))^T R_t^{-1} (y_t - \phi_t^T \hat{\theta}(i)) \\ D(i) &= -\log \det P(i) \\ N(i) &= k_i - k_{i-1} \end{aligned}$$

Step 4: Use MAP estimator to determine $\widehat{k^n}$, for three assumptions on noise scaling, $\lambda(i)$, with q $(0 < q < 1)$ the change probability at each time instants.

(i) known $\lambda(i) = \lambda_0$,
$$\widehat{k^n} = \arg\min_{k^n,n} \textstyle\sum_{i=1}^{n} (D(i) + V(i)) + 2n \log \frac{1-q}{q}$$

(ii) unknown but constant $\lambda(i) = \lambda$,
$$\widehat{k^n} = \arg\min_{k^n,n} \textstyle\sum_{i=1}^{n} D(i) + (Np - nd - 2) \times \times \log \sum_{i=1}^{n} \frac{V(i)}{Np-nd-4} + 2n \log \frac{1-q}{q}$$

(iii) unknown and changing $\lambda(i)$,
$$\widehat{k^n} = \arg\min_{k^n,n} \textstyle\sum_{i=1}^{n} (D(i) + (N(i)p - d - 2) \times \times \log \frac{V(i)}{N(i)p-d-4}) + 2n \log \frac{1-q}{q}$$

Results : Number n and locations k_i , $k^n = k_1, k_2, \ldots, k_n$

According to the assumption on noise scaling, only one of the equations **Step 4** is used to estimate $\widehat{k^n}$. For the exact likelihood estimation can be used recursive local search techniques, as well as numerical searches based on dynamic programming or Markov Chain Monte Carlo (MCMC), [2].

Based on the results of optimal segmentation and the data analysis, for each segment, it is possible to locate and diagnose a possible anomaly occurred in EEG signals making the object of investigation.

D. Computational aspects

The real challenge in segmentation is to cope with the problem of the dimensionality. It can be noted that the number of segmentations k^n is 2^N, because it can be a change or no change at each time instant. Several strategies have been proposed for MAP segmentation, mainly, implementing numerical searches based on dynamic programming or MCMC (Markov Chain Monte Carlo) techniques, and recursive local search techniques [2].

An overview of the literature on computational complexity of the Metropolis-Hastings based MCMC methods for sampling probability measures in high dimensions is given in [23]. The computational complexity of MCMC-based estimators in large samples is discussed in [24], where the implications of the statistical large sample theory for the computational complexity of Bayesian and quasi-Bayesian estimation carried out using Metropolis random walks are examined.

The progress in MCMC has been impressive and seems to be accelerating. Problems that appeared impossible have been solved. For combinatorial counting problems, recent advances have been remarkable. Despite of this fact many problems in this field are still open, and a solid theory for these approaches is still almost nonexistent [25].

The optimal segmentation algorithm, MAP, uses as input, the data vector, the model structure: mean model, regression model, AR(na) model, ARX(na,nb,nk), etc., treating mode of the measurement covariance ((i) known noise scaling, (ii) unknown but constant noise scaling, and (iii) unknown and changing noise scaling), different options for the penalty terms occuring in model order selection: AIC, BIC/MDL, etc), the probability that the system jumps at each sample, q, and some design parameter for the search scheme: the number of filters used (M), the minimum lifelength of a jump sequence guaranteed (ll) and the minimum allowed segment size ($mseg$). In practice, if we have no other information, it is recommended to use the following values, [21]: the number of filter, M, is recommended to be chosen $\dim(\theta) + 8$, the choice of minimum lifelength, ll, is related to the identifiability of the model, and should be chosen larger than $\dim(\theta) + 2$, and the minimum allowed segment size, $mseg$, can be chosen 0, when a change or no change can be produced at each time instant, [21]. Finally, the jump probability, q, is used to tune the number of segments. Based on some information from the correct scientific knowledge of the physical process or from previous empirical evidence, an appropriate value for q can be chosen. For $q = 0.5$, the MAP estimator becomes Maximum Likelihood (ML) estimator, [21].

III. Experimental results

A. EEG data

The data used in this case study represent an EEG time series collected from 13 scalp and 1 EOG electrodes. For artifact removing from data, has been used Independent Component Analysis (ICA) with high-order statistics [26], applied to sample lowpass ERP data for 2 epochs and 312 frames per data epoch, at 312.5 Hz sampling rate. The data are used in many case studies (see [27], among others).

The "corected" EEG data for 2 conditions, after artifact removing, are shown in Fig. 1 and Fig. 2 for the channels 1-7 (Fz, Cz, Pz, Oz, F3, F4, C3) and for the channels 8-14 (C4, T3, T4, P3, P4, Fpz, EOG), respectively.

B. Segmentation of artifact "corrected" EEG signals

The EEG signals, after artifact removing, made the object of segmentation using MAP algorithm, presented in Section 2. Visual inspection for the artifact "corrected" EEG signals, for all components shows a clearly visible change in energy and frequency content. A piecewise constant model (1), could lead to a acceptable trade-off concerning the complexity and efficiency of the described algorithm for the estimation of the change time in EEG signals, making the object of investigation. The segmentation algorithm has been applied for "unknown and constant noise scaling" hypothesis, using MCMC algorithm with the following parameters; $q = 0.3$, $M = 9$, $ll = 5$ and $mseg = 10$. The segmentation results for an AR(1) model (see equation (1), for $n = 1$) are given in Fig. 3 and Fig. 4, for channels 1-7 and channels 8-14, respectively.

The "corrected" EEG signals segmentation evaluation can be performed using time-frequency analysis. For this, the reduced interference distribution (RID), [28], has been evaluated (see Fig. 5 and Fig. 6, for channels 1-7 and channels 8-14, respectively). The RID belongs to Cohen's class, and reprezents an extension of Wigner-Wille distribution [29]. It overcomes some problems in time-frequency analysis [29], and it appears to be a good choice for EEG signal analysis. The RID has been computed using a kernel based on a Hanning window, a number of frequency bins identical with the time instants, N = 624, a time smoothing window of length Lg = 204, a frequency smoothing window of length Lh = 512, and a threshold of 2%.

The time-frequency analysis results, presented above, can be used for energy concentration of the EEG signals and feature extraction. It consists of energy concentration analysis at specific time instant or frequency band, or in some specific time and frequency areas. A such analysis is able to provide more information about the EEG signals making the object of evaluation. The problem of energy concentration, making use of time-frequency analysis is discussed in [30]. Other works deal with Rényi entropy, as a measure in evaluation of information amount encoded in time-frequency distribution [31]. A generally method for energy distribution evaluation using measures of Rényi entropy in EEG signals is presented in [32].

IV. Conclusions

The results presented highlight the effectiveness of the approach used, making use of MAP estimator, for

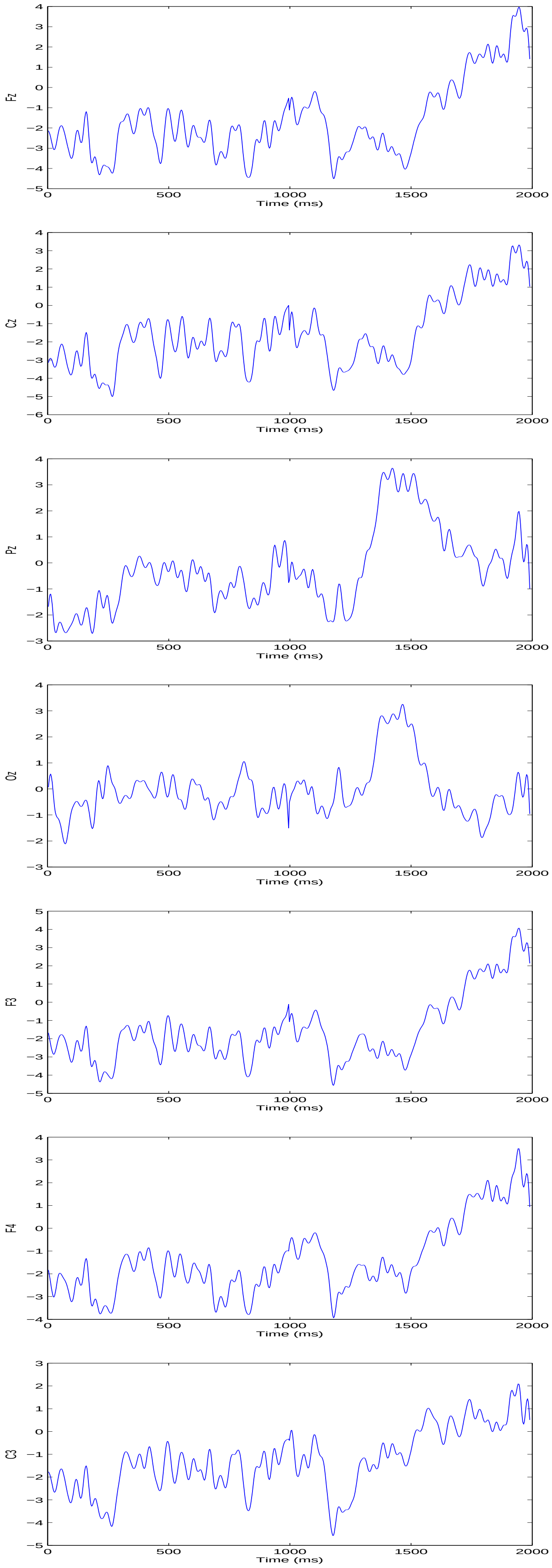

Fig. 1: ERP filtered data after artifact removing for 2 epochs, channels 1-7

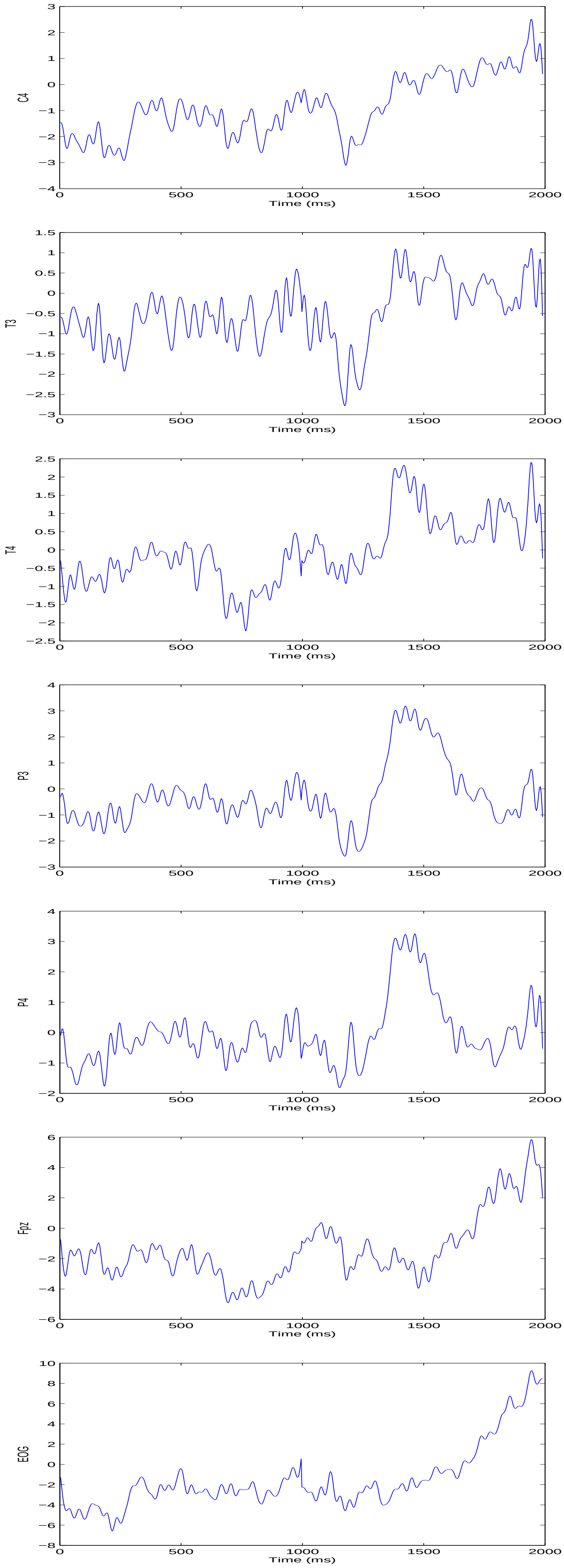

Fig. 2: ERP filtered data after artifact removing for 2 epochs, channels 8-14

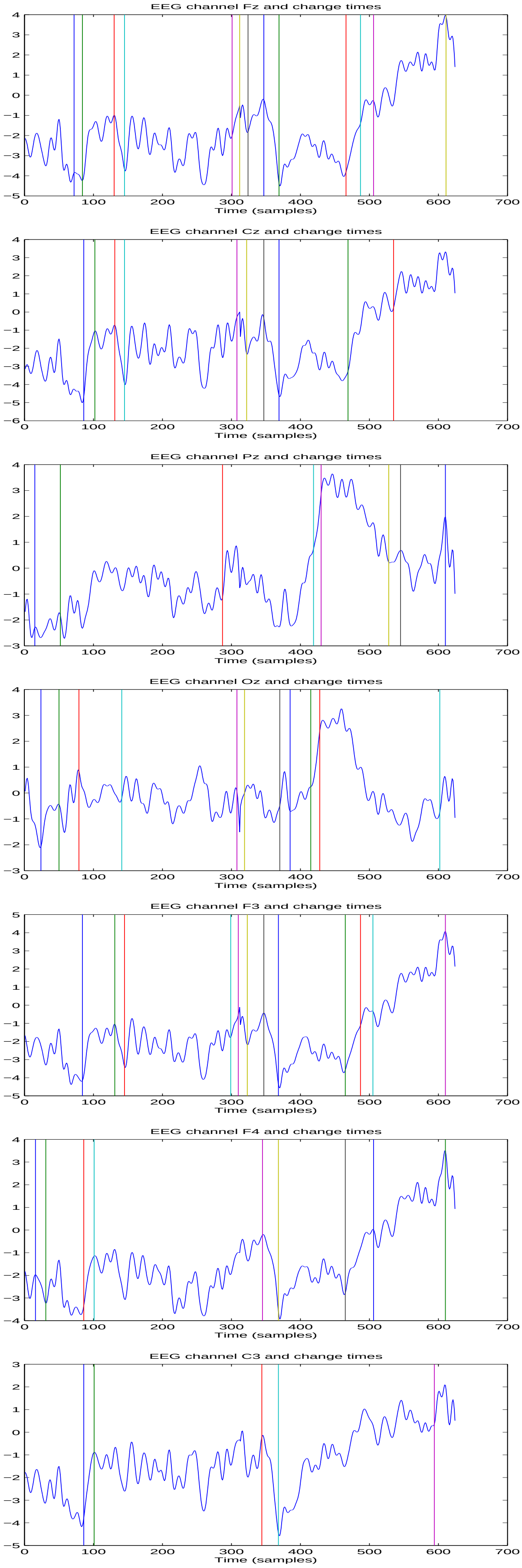

Fig. 3: ERP filtered data segmentation for channels 1-7 (2 epochs) after artifact removing

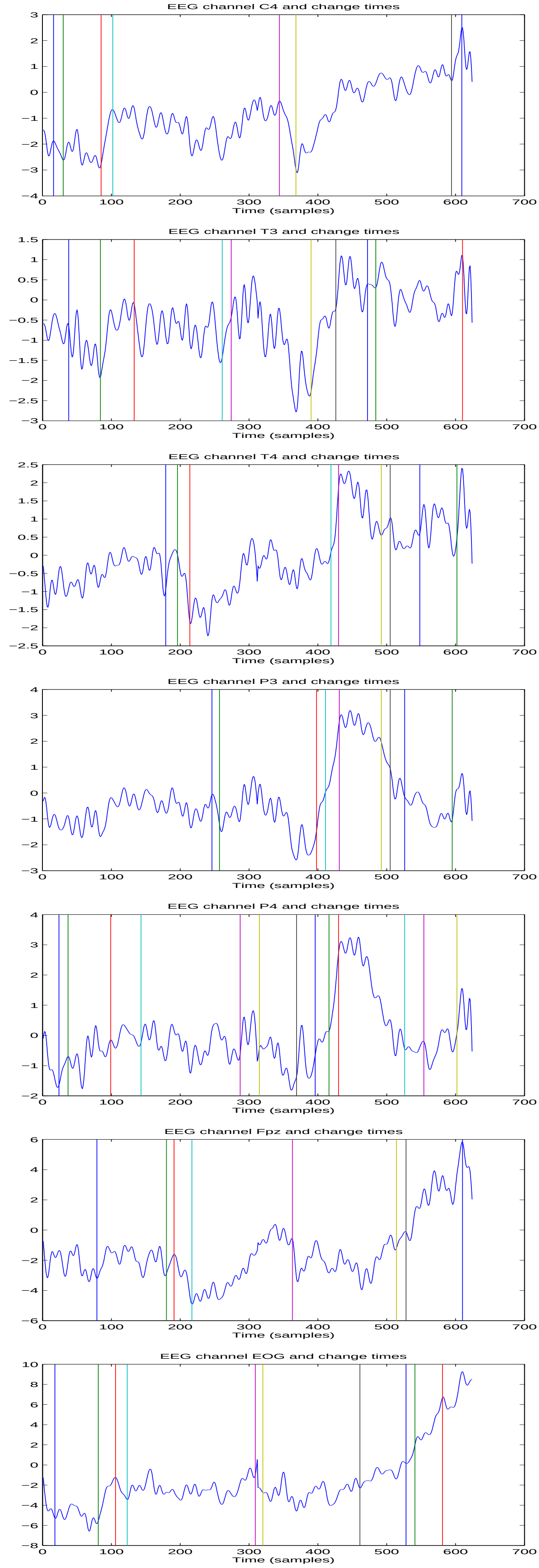

Fig. 4: ERP filtered data segmentation for channels 8-14 (2 epochs) after artifact removing

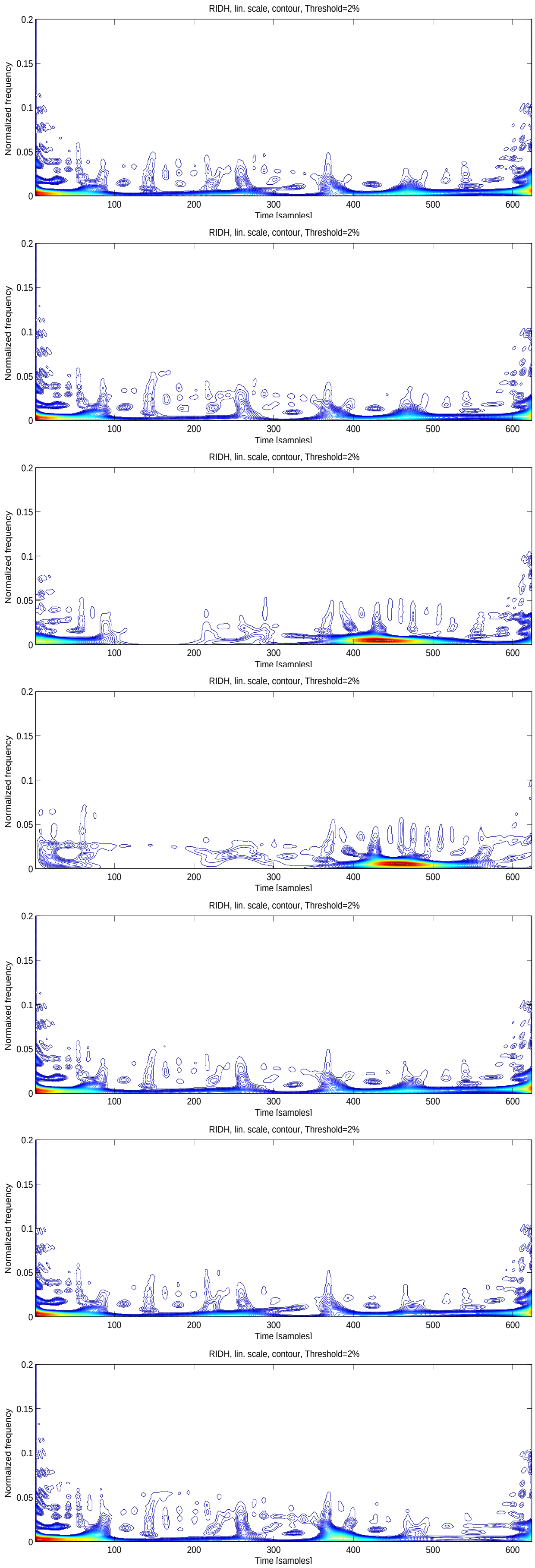

Fig. 5: Reduced interference distribution for ERP filtered data without artifacts: channels 1-7 (2 epochs)

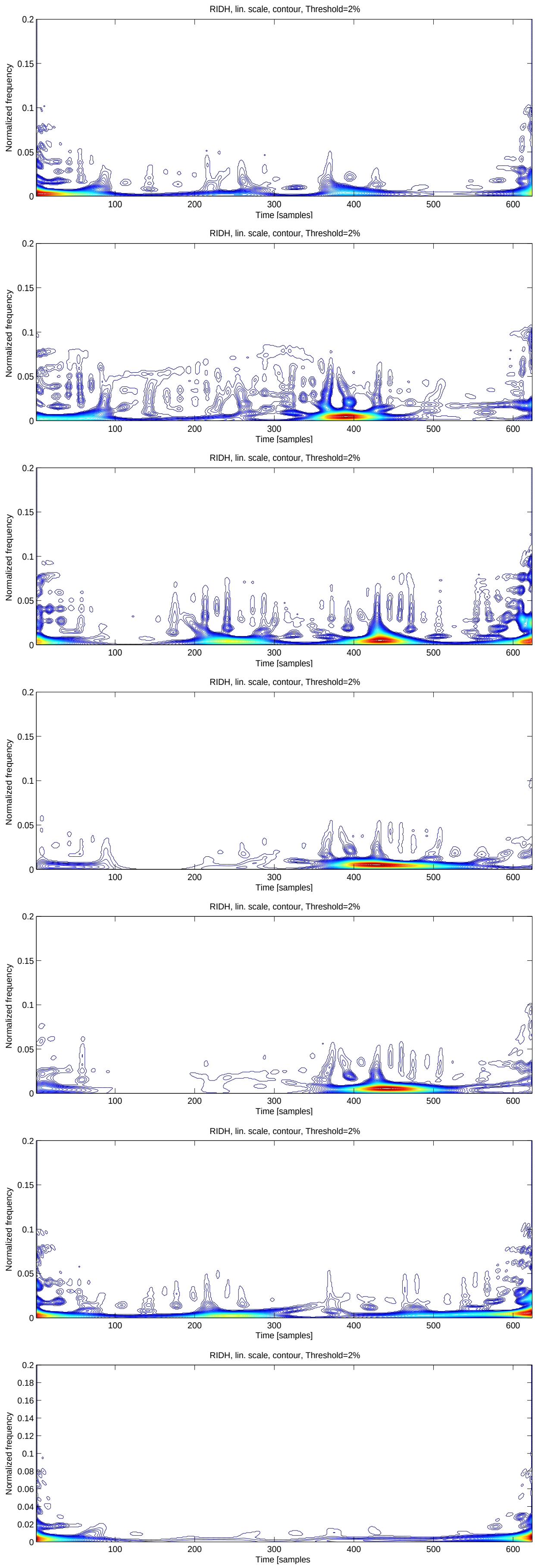

Fig. 6: Reduced interference distribution for ERP filtered data without artifacts: channels 8-14 (2 epochs)

optimal segmentation of "corrected" EEG signals. It offers a simpler analysis and interpretation of the scalar, or multivariable EEG signals, having as final goal feature extraction and providing facilities for its realization. The procedure does not raise problems in choosing the input parameters and is robust in choosing the q parameter, change probability at each time instants [21]. The model used is a simple one, offers a convenient framework in tracking the EEG signal dynamics and assures a good segmentation, for scalar and multivariable signals.

Acknowledgment

The author thanks the Executive Agency for Higher Education, Research, Development and Innovation Funding (UEFISCDI) and Ministry of Research and Innovation, for the support under Contract 2019-2022 Core Program, Project PN 301, RO-SmartAgeing.

References

[1] M. Basseville, I. Nikiforov, Detection of Abrupt Changes - Theory and Applications, Prentice Hall, N.J., 1993.

[2] F. Gustafsson, Adaptive Filtering and Change Detection, Wiley, 2001.

[3] H. Ohlson, L. Ljung, S. Boyd, "Segmentation ARX-models using sum-of-norms regularization", Automatica IFAC, vol. 46, pp. 1107-1111, 2010.

[4] P. Hubert, L. Padovese, J. M. Stern, "A sequential algorithm for signal segmentation", Entropy MDPI, pp. 1-20, 2018. 1-20.

[5] R. Zimroz , M. Madziarz, G. Zak , A. Wylomanska, J. Obuchowski, Seismic signal segmentation procedure using time-frequency decomposition and statistical modeling", Journal of Vibroengineering, vol. 17, pp. 3111-3121, 2015.

[6] D. Wang, K.-L. Tsui, Y. Qin, "Optimization of segmentation fragments in empirical wavelet transform and its applications to extracting industrial bearing fault features", Measurement, vol. 133, pp. 328-340, 2019.

[7] R. Aufrichtigl, S. B. Pedersen, P. Jennum, "Adaptive segmentation of EEG signals". Proc. of the Annual International Conference of the IEEE Engineering in Medicine and Biology Society, pp. 453-454, 1991.

[8] S. M. Qaisar, "A computationally efficient EEG signals segmentation and de-noising based on an adaptive rate acquisition and processing", Proc. of the IEEE 3rd International Conference on Signal and Image Processing (ICSIP), Shenzhen, China, pp. 182-186, 2018.

[9] B. E. Brodsky, B. S. Darkhovsky, A. Y. Kaplan, S. L. Shishkin, "A nonparametric method for the segmentation of the EEG", Computer Methods and Programs in Biomedicine, vol. 60, pp. 93-106, 1999.

[10] M. Azarbad, H. Azami, S. Sanei, A. Ebrahimzadeh, "A time-frequency approach for EEG signal segmentation", Journal of AI and Data Mining, vol. 2, pp. 63-71, 2014.

[11] H. Azamin, K. Mohammadi, H. Hassanpour, "An improved signal segmentation method using genetic algorithm", International Journal of Computer Applications, vol. 29, pp. 5-9, 2011.

[12] A. Prochazka, M. Mudrova, O. Vysata, R. Hava and C. P. Suarez Araujo, "Multi-channel EEG signal segmentation and feature extraction", The IEEE 14th International Conference on Intelligent Engineering Systems, Las Palmas, Spain, pp. 317-320, 2010.

[13] H. Abdullah, D. Cvetkovic, "Electrophysiological signals segmentation for EEG frequency bands and heart rate variability analysis", Proc. of The 15th International Conference on Biomedical Engineering, Springer, pp. 695-698, 2014.

[14] G. J. Miao, M. A. Clements, Digital signal processing and statistical classification. Artech House signal processing library. Artech House, 2002.

[15] P. J. Durka. Time-frequency analyses of EEG. PhD thesis, Warsaw University, 1996.

[16] A. Aarabi, R. Grebe, F. Wallois. "A multistage knowledge-based system for EEG seizure detection in newborn infants", Clin Neurophysiol, vol. 118, pp. 2781-2797, 2007.

[17] K. Akrofi, M. C. Baker, M. W. O'Boyle, R. B. Schiffer. "Clustering and modeling of EEG coherence features of Alzheimer's and mild cognitive impairment patients", Proc IEEE Eng Med Biol Soc Conf., pp. 1092-1095, 2008.

[18] M. Basseville, "Detecting changes in signals and systems - A survey", Automatica, vol. 24, pp. 309-326, 1988.

[19] S. Aminikhanghahi, D. J Cook, "A survey of methods for time series change point detection", Knowl Inf Syst., pp. 339-367, 2017.

[20] Th. D. Popescu, "Blind separation of vibration signals and source change detection - Application to machine monitoring", Applied Mathematical Modelling, pp. 3408-3421, 2010.

[21] Th. D. Popescu, "Signal segmentation using changing regression models with application in seismic engineering", Digital Signal Processing, pp. 14-26, 2014.

[22] Th. D. Popescu, "Detection and diagnosis of model parameter and noise variance changes with application in seismic signal processing", Mechanical Systems and Signal Processing, pp. 1598-1616, 2011.

[23] A. Beskos, A. Stuart, "Computational complexity of Metropolis-Hastings methods in high dimension", Monte Carlo and Quasi-Monte Carlo methods, Springer, pp. 61-71, 2009.

[24] A. Belloni, V. Chernozhukov, "Computational complexity of MCMC-based estimators in large samples", Ann. Statist., vol. 37 pp. 2011-2055, 2009.

[25] I. Beichl, F. Sullivan, "The Metropolis Algorithm", Computing in Science and Engineering, vol. 2, pp. 65-69, 2000.

[26] T. D. Popescu, "Artifact removing from EEG recordings using independent component analysis

with high-order statistics", International Journal of Mathematical Models and Methods in Applied Sciences, vol. 15, pp. 76-85, 2021.

[27] A. Delorme, S. Makeig, "EEGLAB: an open source toolbox for analysis of single-trial EEG dynamics including independent component analysis", Journal of Neuroscience Methods, vol. 134 pp. 9-21, 2004.

[28] J. Jeong, W. J. Williams, "Kernel design for reduced interference distributions", IEEE Transactions on Signal Processing, vol. 40, pp. 402-4012, 1992.

[29] L. Cohen, Time-Frequency Distribution, Prentice Hall, New York, 1995.

[30] L. Stankovic, "A measure of some time-frequency distributions concentration", Signal Processing, pp. 621-631, 2001.

[31] S. Aviyente, "Information processing on the time-frequency plane", Proc. IEEE International Conference Acoustics, Speech, and Signal Processing (ICASSP '04), pp. 617-620, 2004.

[32] T. D. Popescu, "Energy distribution of EEG signals using Rényi entropy measures", submitted to Digital Signal Processing, 2021, Elsevier.

Sources of funding for research presented in a scientific article or scientific article itself

Executive Agency for Higher Education, Research, Development and Innovation Funding (UEFISCDI) and Ministry of Research and Innovation, for the support under Contract 2019-2022 Core Program, Project PN 301, RO-SmartAgeing

Theodor D. Popescu was born in Rosiori de Vede, Romania, on July 4, 1949. He received his M.Sc. degree and Eng.Sc.D. (PhD) degree, both in Automatic Con- trol, from ”Politehnica” University of Bucharest in 1972 and 1983, respectively. He has been a senior research scientist with the National Institute for Research and Development in Informatics, ICI Bucharest, since 1972. His research activities are focused on the fields of sys- tem identification and parameter estimation, time series analysis, signal processing, change detection and diagno- sis, independent component analysis, blind source sepa- ration, time-frequency analysis, biomedical signal analy- sis. He is author or co-author of more than 150 papers and of five books. He was awarded with ”Tudor Tanas- escu” Prize of the Romanian Academy in 1995, for his works in the field of detection of changes in dynamics of signals and systems and General Association of Engi- neers in Romania Prize in 2000 in the field of Informa- tion Technology. He was granted by The Royal Society (1992), DAAD (1992, 1995, 1999), JSPS (1996, 1999), Australian Research Council(2002), CERGE-EI (2002). He held conferences and seminars in the Institute of Sys- tems Science, Beijing, Institute of Technology, Beijing, University of Lancaster, Wuppertal University, Kyushu Institute of Technology, Oita University, Saga Univer- sity, Tottori University, Kyoto Institute of Technology, CERGE-EI Prague and Spatial Co. Kuwait. He has been a Senior Member of IEEE since 2002.