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Analytical model for separated frequency signature of outer race bearing fault from static eccentricity

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Abstract—**The present paper addresses a precise and an accurate mathematical model for three-phase squirrel cage induction motors, based on winding function theory. Through an analytical development, a comparative way is presented to separate the signature between the existence of the outer race bearing fault and the static eccentricity concerning the asymmetry of the air gap between the stator and the rotor. This analytical model proposes an effective signature of outer race defect separately from other signatures of static eccentricity. Simulation and experimental results are presented to validate the proposed analytical model.**

***Keywords*—analytical model, induction motors, outer race bearing fault, static eccentricity.**

I. INTRODUCTION

Industrial processes have picked up a great deal of attention in the area of fault tolerance of electrical machines. A defects prognosis and diagnosis has become indispensable. Requirement of a mathematical model of three-phase induction machine, appropriate for the simulation of machines behavior under fault circumstances, has received considerable interest. From all induction motors used in electric drives we chose the squirrel cage induction motor because of its simplicity construction, robustness, and reliability. Despite these advantages, various effects can reduce the performance of these induction motors. Among various failures we can distinguish stator fault (unbalanced supply, short circuit, phase break down, ...) which is known as the most existing fault. Rotor fault can appear due to irregular fabrication of the machine or both thermal and overloading effects (bar breakage, end ring breakage, …). The eccentricity fault which is defined by the mismatch between axis of rotation and the axis of symmetry (static, dynamic and mixed eccentricity). Bearing defect is known the damage one of rolling elements like the outer race bearing fault, the inner race bearing fault or the cage of rolling elements. In order to avoid an unexpected stop of the industrial process, several methods exist to maintain the machines performances. The most important is an early diagnosis of collapse which is a big challenge for researchers in this domain. A series of models has been carried out to estimate the sensitivity and reliability of the stator and rotor feature [1]-[10], [22], [23]. Strong diagnosis methods stand on the understanding the behavior of the machines in different states (healthy, faulty). It is so important to know correctly electrical, mechanical, and magnetic performances of the machine. For that an analytical model of diagnosis becomes important to describe a technical behavior of machines under different states of performance [5]-[9], [13]. In [20], Schoen proposed and developed an analytic model to assume the pattern of defect appearance. This model was also used in works as [16]. In [18], Blodt elaborate more on the previous approach saying that the contact between the ball and the defect produces a small vibration caused by the movement of the rotor center. In his model, he uses the series of Dirac function to generalize the impact of the defect. Eccentricity in induction machines also was studied before, like in [1], [9], [11], [14], [15]. The main content of these works is to extract the fault harmonics that appear in the stator current spectrum. These harmonic frequencies appear around sides of the rotor slot harmonics (RSH). This group of significant harmonics is related directly to the presence of static or dynamic eccentricity and it's described with the following relation:

$$f_{ecc} = \left(\frac{\lambda n_b}{p} (1 \pm g) \pm k \right) f_s \qquad (1)$$

where: n_b is rotor bars number,
p: pole pairs number,
g: slip coefficient,
k=1,3,5… is the stator harmonics order.

Many indices concerning this fault don't take into account the existence of the faulty angle. In [10], Hadjami corrects the unreliability of the approach by taking into account in his analytical development the angle of the defect. Basely, rotor eccentricity has two forms: static and dynamic. Static eccentricity (SE) is defined as a fixed position of the minimal radial air-gap length in space [1], [9]-[15]. After outer race bearing fault situation, the rolling elements (the part of machine that allows one part to rotate or move) rotate irregularity. This phenomenon produces a vibration considered as temporary static eccentricity. The main objective of this article is to offer a correct and general mathematical formula of physical phenomenon caused by regularity and irregularity in the air gap length of an induction motor. Because the inductance's calculation is highly time consuming, we use the decomposition into Fourier series to calculate all inductances without any kind of reference frame transformation. To demonstrate the effectiveness of the proposed model, certain frequency harmonics that can be predicted theoretically, are shown. Simulation and experimental results are given to justify the proposed analytical model.

II. AIR GAP VARIATION

In healthy condition the air gap function for the induction machine is fixed along the interface between the stator and rotor frame (g0). For defected machines with static eccentricity or bearing fault, the air gap function becomes [10]–[13], [16], [17]:
For static eccentricity:

$$g(\theta) = g_0(1+\delta_s \cos(\theta)). \tag{2}$$

The approximation to the first harmonic gives:

$$g^{-1}(\theta) = \frac{1}{g_0}(1+\delta_s \cos(\theta)). \tag{3}$$

δ_s : The level of static eccentricity
The outer race bearing is known that a part of the machine that allows another part to turn or move with as little friction as possible. If any irregularity manifests itself in the form of the outer stroke, it will generate vibrations with a periodic frequency. The Bale Pass Frequency (BPFO) of the outer ring is created when rolling elements pass through the irregularity (see fig.1).

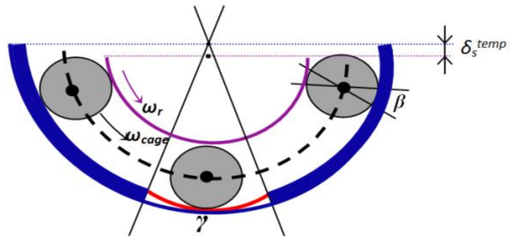

Fig. 1. Outer race bearing fault.

The defect frequency which for outer race is given by:

$$f_{out} = \frac{N_b}{2} F_r\left(1-\frac{D_b\cos(\beta)}{D_c}\right). \tag{4}$$

A new way to represent this contact between the rolling elements and the defect in the outer race was presented in [11], by the signal:

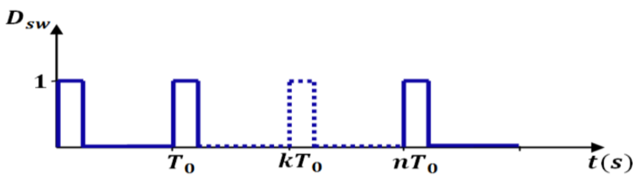

Fig. 2. Appearance signal for outer race bearing fault.

$$D_{sw} = \frac{\gamma}{2\pi} + \frac{2}{\pi}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)\cos\left(\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right). \tag{5}$$

The formula to the temporary level of static eccentricity is:

$$\delta_{st}^{temp} = \frac{\gamma}{2\pi}\delta_{st}\cos(\theta) + \frac{\delta_{st}}{\pi}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)\cdot\cos\left(\theta\pm\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right) \tag{6}$$

By Compiling (1) and (5) we get respectively, the new formula for small variation of the air gap and the permeance functions:

$$g_{out}(\theta) = g_0\left(1-\frac{\gamma}{2\pi}\delta_{st}\cos(\theta) - \frac{\delta_{st}}{\pi}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)\cos\left(\theta\pm\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right)\right) \tag{7}$$

$$g_{out}^{-1}(\theta) = \frac{1}{g_0}\left(1-\frac{\gamma}{2\pi}\delta_{st}\cos(\theta) - \frac{\delta_{st}}{\pi}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)\cos\left(\theta\pm\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right)\right) \tag{8}$$

The cases for fault conditions we have:
$\gamma = 0$: Healthy state.
$\gamma \neq 0, 2\pi$: Temporary static eccentricity (outer race fault).
$\gamma = 2\pi$: Permanent static eccentricity.

III. INDUCTANCES CALCULATION

Referring to winding function theory, the development of mutual inductances between two windings in electrical machines [5]-[7], [10] is:

$$L_{AB} = \mu_0 r l\int_0^{2\pi} n_A(\theta)\, M_B(\theta)\, g^{-1}(\theta)\, d\theta \;. \tag{9}$$

$$M_B(\theta_s,\theta) = n(\theta_s,\theta) - \frac{1}{2\pi * \langle g^{-1}(\theta_s,\theta)\rangle} * \int_0^{2\pi} n(\theta_s,\theta)\, g^{-1}(\theta_s,\theta)\, d\theta_s \;. \tag{10}$$

$$\langle g^{-1}(\theta_s,\theta)\rangle = \frac{1}{2\pi}\int_0^{2\pi} g(\theta_s,\theta)\, d\theta_s \;. \tag{11}$$

$n_A(\theta)$ The turn function, $M_B(\theta)$ The winding function, $\langle g^{-1}(\theta_s,\theta)\rangle$ The mean value of the inverse of the air gap function.

A. Winding stator Functions

In healthy state the air gap is uniform, so:

$$n_{sq}(\theta_s) = C_0 + \frac{2N_t}{p\pi}\sum_{h=1}^{\infty}\frac{k_{bo}}{h}\cos\left[hp\left(\theta_s-\theta_0-(q-1)\frac{2\pi}{3p}\right)\right] \tag{12}$$

$$M^{healthy}{}_{sq}(\theta_s) = \frac{2N_t}{p\pi}\sum_{h=1}^{\infty}\frac{k_{bo}}{h}\cos\left[hp\left(\theta_s-\theta_0-(q-1)\frac{2\pi}{3p}\right)\right] \tag{13}$$

With

$$\begin{cases} N_t = N_c N_e P \\ K_{bo} = K_{racc} K_{dis} \\ K_{dis} = \dfrac{\sin\left(hp\,\pi\dfrac{N_e}{N_s}\right)}{N_e\sin\left(hp\dfrac{\pi}{N_s}\right)}, \quad K_{racc} = \sin\left(hp\,\pi\dfrac{Q}{N_s}\right) \\ \theta_0 = (N_e-1+Q)\dfrac{\pi}{N_s} \end{cases} \tag{14}$$

K_{bo} Winding factor of the h[th] harmonic, K_{dis} Pitch factor of the h[th] harmonic, K_{racc} Distribution factor of the h[th] harmonic, N_t Number of stators turns in series, N_c Number of conductors per stator slot, N_e Number of slots per pole and per phase, N_s Number of stator slots, θ_0 Angle between stator and rotor reference axes at t=0.

In Static Eccentricity state the air gap is non-uniform, so:
If $p \neq 1$.

$$M^{mod}{}_{sq}\left(\theta_s\right) = M^{healthy}{}_{sq}\left(\theta_s\right) \tag{15}$$

If $p = 1$.

$$\begin{cases} M^{mod}{}_{sq}\left(\theta_s\right) = M^{healthy}{}_{sq}\left(\theta_s\right) - \delta_s \dfrac{N_t}{\pi} K_{bo-st} \times \\ \times \cos\left(\theta_0 - (q-1)\dfrac{2\pi}{3p}\right) \end{cases} \tag{16}$$

$$M^{mod}{}_{sq}\left(\theta_s\right) = M^{healthy}{}_{sq}\left(\theta_s\right) + M^{sta-ecc}{}_{sq}\left(\theta_s\right) \tag{17}$$

With:

$$M^{sta-ecc}{}_{sq}\left(\theta_s\right) = -\delta_s \frac{N_t}{\pi} K_{bo-st} \cos\left(\theta_0 - (q-1)\frac{2\pi}{3p}\right) \tag{18}$$

$M^{mod}{}_{sq}\left(\theta_s\right)$: modified winding functions

$M^{sta-ecc}{}_{sq}\left(\theta_s\right)$: additional winding function for static eccentricity.

$$\begin{cases} N_t = N_c N_e P \\ K_{bo-st} = K_{racc-st} K_{dis-st} \\ K_{dis-st} = \dfrac{\sin\left(p\,\pi \dfrac{N_e}{N_s}\right)}{N_e \sin\left(p \dfrac{\pi}{N_s}\right)}, \quad K_{racc-st} = \sin\left(p\,\pi \dfrac{Q}{N_s}\right) \\ \theta_0 = (N_e - 1 + Q)\dfrac{\pi}{N_s} \end{cases} \tag{19}$$

In outer race bearing fault state:

If $p \neq 1$.

$$M^{mod}{}_{sq}\left(\theta_s\right) = M^{healthy}{}_{sq}\left(\theta_s\right) \tag{20}$$

If $p = 1$.

$$\begin{aligned} M^{mod}{}_{sq}\left(\theta_s\right) &= M^{healthy}{}_{sq}\left(\theta_s\right) + \frac{\gamma}{2\pi} M^{sta-ecc}{}_{sq}\left(\theta_s\right) \\ &+ \frac{2}{\pi} M^{sta-ecc}{}_{sq}\left(\theta_s\right) \sum_{\lambda=1}^{\infty} \frac{1}{\lambda} \sin\left(\lambda \frac{\gamma}{2}\right) \cos\left(\theta \pm \lambda\left(\theta_{out} - \frac{\gamma}{2}\right)\right) \end{aligned} \tag{21}$$

B. *Winding rotor Functions*

In healthy conditions $\delta_s = 0$ (healthy state):

$$n_{rk}\left(\phi\right) = \frac{\alpha_r}{2\pi} + \frac{2}{\pi} \sum_{h=1}^{\infty} \frac{1}{h} \sin\left(h \frac{\alpha_r}{2}\right) \cos\left[h\left(\phi - \left(k - \frac{1}{2}\right)\alpha_r\right)\right] \tag{22}$$

$$M^{healthy}{}_{rk}\left(\phi\right) = \frac{2}{\pi} \sum_{h=1}^{\infty} \frac{1}{h} \sin\left(h \frac{\alpha_r}{2}\right) \cos\left[h\left(\phi - \left(k - \frac{1}{2}\right)\alpha_r\right)\right] \tag{23}$$

$\alpha_r = \dfrac{2\pi}{n_b}$ Rotor loop opening.

In Static Eccentricity state:

$$M^{mod}{}_{rk}\left(\phi\right) = M^{healthy}{}_{rk}\left(\phi\right) - \frac{\delta_s}{\pi} \sin\left(\frac{\alpha_r}{2}\right) \cos\left(\theta_r + \left(k - \frac{1}{2}\right)\alpha_r\right) \tag{24}$$

$$M^{mod}{}_{rk}\left(\phi\right) = M^{healthy}{}_{rk}\left(\phi\right) + M^{sta-ecc}{}_{rk}\left(\theta_r\right) \tag{25}$$

$$M^{sta-ecc}{}_{rk}\left(\theta_r\right) = -\frac{\delta_s}{\pi} \sin\left(\frac{\alpha_r}{2}\right) \cos\left(\theta_r + \left(k - \frac{1}{2}\right)\alpha_r\right) \tag{26}$$

In outer race bearing fault state:

$$\begin{aligned} M^{mod}{}_{rk}\left(\phi\right) &= \frac{2}{\pi} \sum_{h=1}^{\infty} \frac{1}{h} \sin\left(h \frac{\alpha_r}{2}\right) \cos\left[h\left(\phi - \left(k - \frac{1}{2}\right)\alpha_r\right)\right] \\ &- \frac{\gamma}{2\pi} \frac{\delta_s}{\pi} \sin\left(\frac{\alpha_r}{2}\right) \cos\left(\theta_r + \left(k - \frac{1}{2}\right)\alpha_r\right) \\ &- \frac{2}{\pi} \frac{\delta_s}{\pi} \sin\left(\frac{\alpha_r}{2}\right) \cos\left(\theta_r + \left(k - \frac{1}{2}\right)\alpha_r\right) \\ &* \sum_{\lambda=1}^{\infty} \frac{1}{\lambda} \sin\left(\lambda \frac{\gamma}{2}\right) * \cos\left(\lambda\left(\theta_{out} - \frac{\gamma}{2}\right)\right) \end{aligned} \tag{27}$$

$$\begin{aligned} M^{mod}{}_{rk}\left(\phi\right) &= \frac{2}{\pi} \sum_{h=1}^{\infty} \frac{1}{h} \sin\left(h \frac{\alpha_r}{2}\right) \cos\left[h\left(\phi - \left(k - \frac{1}{2}\right)\alpha_r\right)\right] \\ &- \frac{\gamma}{2\pi} M^{sta-ecc}{}_{rk}\left(\theta_r\right) - \frac{2}{\pi} M^{sta-ecc}{}_{rk}\left(\theta_r\right) \\ &* \sum_{\lambda=1}^{\infty} \frac{1}{\lambda} \sin\left(\lambda \frac{\gamma}{2}\right) * \cos\left(\lambda\left(\theta_{out} - \frac{\gamma}{2}\right)\right) \end{aligned} \tag{28}$$

By using the formula of the winding function theory, the different inductions (stator, rotor, mutual stator-rotor) depend on the value of "p", (number of poles pairs). We use in our case the condition $p \neq 1$. We are adding $\pm\theta_r$.

Static Eccentricity state：（$p \neq 1$）

The self-magnetizing inductance is:

$$L^{mod}{}_{sqm} = L^{healthy}{}_{sqm}\left(\theta_s\right) \tag{29}$$

$$L^{healthy}{}_{sqm}\left(\theta_s\right) = \frac{4\mu_0 r l}{g_0} \frac{N_t^2}{p^2\pi} \sum_{h=1}^{\infty} \frac{k_{bo}^2}{h^2} \tag{30}$$

The mutual inductance between any two stator phases is:

$$L^{mod}{}_{sq1sq2} = L^{healthy}{}_{sq1sq2} \tag{31}$$

$$L^{healthy}{}_{sq1sq2}\left(\theta_s\right) = \frac{4\mu_0 r l}{g_0} \frac{N_t^2}{p^2\pi} \sum_{h=1}^{\infty} \frac{k_{bo}^2}{h^2} \cos\left(h\left(q1 - q2\right)\frac{2\pi}{3}\right) \tag{32}$$

The inductance of a rotor loop k is defined by:

$$L^{mod}{}_{mrj} = L^{healthy}{}_{mrj} + L^{sta-ecc}{}_{mrj} \tag{33}$$

$$L^{mod}_{mrj} = \left(\frac{\mu_0 rl}{g_0}\frac{4}{\pi}\sum_{h=1}^{\infty}\frac{1}{h^2}\sin^2\left(h\frac{\alpha_r}{2}\right)\right) - \frac{\mu_0 rl}{g_0}\frac{2\delta_s^2}{\pi}\sin^2\left(\frac{\alpha_r}{2}\right)\cos^2\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) \quad (34)$$

$$L^{sta-ecc}_{mrj} = -\frac{\mu_0 rl}{g_0}\frac{2\delta_s^2}{\pi}\sin^2\left(\frac{\alpha_r}{2}\right)\cos^2\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) \quad (35)$$

The mutual inductance between loop "j" and no adjacent loop "k", can be obtained by:

$$L^{mod}_{rjrk} = L^{healthy}_{rjrk} + L^{sta-ecc}_{rjrk} \quad (36)$$

$$L^{mod}_{rjrk} = \left(\frac{\mu_0 rl}{g_0}\frac{4}{\pi}\sum_{h=1}^{\infty}\frac{1}{h^2}\sin^2\left(h\frac{\alpha_r}{2}\right)\cos\left(h\left(k-j\right)\alpha_r\right)\right) - \frac{\mu_0 rl}{g_0}\frac{2\delta_s^2}{\pi}\sin^2\left(\frac{\alpha_r}{2}\right)\cos\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) * \cos\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) \quad (37)$$

$$L^{sta-ecc}_{rjrk} = -\frac{\mu_0 rl}{g_0}\frac{2\delta_s^2}{\pi}\sin^2\left(\frac{\alpha_r}{2}\right) * \cos\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) * \cos\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right). \quad (38)$$

The expression of the mutual inductance M_{sr} between a stator winding and a rotor loop is:

$$M^{mod}_{sr} = M^{healthy}_{sr} + M^{sta-ecc}_{sr} \quad (39)$$

$$M^{mod}_{sr} = M^{healthy}_{sr} + \frac{\mu_0 rl}{g_0}\sum_{h=1}^{\infty}\frac{1}{h\left(hp\pm 1\right)}\frac{2N_t\delta_s K_{bo}}{p\pi}\sin\left(\left(hp\pm 1\right)\frac{\alpha_r}{2}\right)\cos\left[\left(hp\pm 1\right)\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) - hp\left(\theta_0 + \left(q-1\right)\frac{2\pi}{3p}\right)\right] \quad (40)$$

$$M^{sta-ecc}_{sr} = \frac{\mu_0 rl}{g_0}\sum_{h=1}^{\infty}\frac{1}{h\left(hp\pm 1\right)}\frac{2N_t\delta_s K_{bo}}{p\pi}\sin\left(\left(hp\pm 1\right)\frac{\alpha_r}{2}\right) * \cos\left[\left(hp\pm 1\right)\left(\theta_r + \left(k-\frac{1}{2}\right)\alpha_r\right) - hp\left(\theta_0 + \left(q-1\right)\frac{2\pi}{3p}\right)\right] \quad (41)$$

Outer Race bearing Fault：（$p \neq 1$)

With the same development the different inductances for Outer race bearing fault is:

$$L^{mod}_{sqm} = L^{healthy}_{sqm} \quad (42)$$

$$L^{mod}_{sq1sq2} = L^{healthy}_{sq1sq2} \quad (43)$$

$$L^{mod}_{mrj} = L^{healthy}_{mrj} + \left(\frac{\gamma}{2\pi}\right)^2 L^{sta-ecc}_{mrj} + 2\left(\frac{2}{\pi}\right)\left(\frac{\gamma}{2\pi}\right)L^{sta-ecc}_{mrj}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)*\cos\left(\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right) + \left(\frac{2}{\pi}\right)^2 L^{sta-ecc}_{mrj}\left(\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)*\cos\left(\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right)\right)^2 \quad (44)$$

$$L^{mod}_{rjrk} = L^{healthy}_{rjrk} + \left(\frac{\gamma}{2\pi}\right)^2 L^{sta-ecc}_{rjrk} + 2\left(\frac{2}{\pi}\right)\left(\frac{\gamma}{2\pi}\right)L^{sta-ecc}_{rjrk}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)*\cos\left(\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right) + \left(\frac{2}{\pi}\right)^2 L^{sta-ecc}_{rjrk}\left(\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)*\cos\left(\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right)\right)^2 \quad (45)$$

$$M^{mod}_{sr} = M^{healthy}_{sr} + \left(\frac{\gamma}{2\pi}\right)M^{sta-ecc}_{sr\,(p\neq 1)} + \left(\frac{2}{\pi}\right)M^{sta-ecc}_{sr\,(p\neq 1)}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda}\sin\left(\lambda\frac{\gamma}{2}\right)*\cos\left(\lambda\left(\theta_{out}-\frac{\gamma}{2}\right)\right) \quad (46)$$

IV. Simulation And Experimental Results

We introduce the different inductance formulas in the system equations of the induction machine as in [12], [13], [15]. Those equations can be solved by using fourth-order Runge-Kutta method. We take a simulated machine with 2 pairs poles ($p \neq 1$), 3 phases, 50Hz and 22 rotor bars.

In experimental test bench, we use a 1.1kW three-phase induction machine FIMET manufacturer with 24 stator and 22 rotor slots, 2 pairs poles. The stator current was recorded using the LeCroy oscilloscope (WR60500). After acquisition, we use MATLAB for frequency-domain analysis.

Fig. 3 shows the influence of the variation level for static eccentricity fault on the mutual inductances Msr. We can notice that the deformation of this mutual inductance is proportionally with the level of static eccentricity (δ_s). The deformation of the mutual inductances seems important when the static eccentricity level (δ_s) increases. We also notice that the deformation is clearly greater with the inclination of the rotor bars. Fig. 4 shows the impact of outer race bearing fault in the mutual inductance. The influence of outer bearing fault is considering like a temporary static eccentricity that is marked by a defect angle (γ) and a level of static eccentricity ($\delta_s = 0.4$). The two outer rings bear default parameters that have a direct relationship between the fault duration and the angle (γ), and between the defect importance and the level of eccentricity (δ_s). Fig. 5 illustrate FFT spectrum of stator line current in healthy state (showed in black), static eccentricity (in blue) and outer race bearing fault (in red). In the healthy state, the appearance of rotor slot harmonics (RSH) components in the stator current of induction machines appear like those existing in the literature [1], [10], [11]. In healthy condition, these harmonics orders define by $h = \lambda n_b/2 \pm 1$ appear with specified slip coefficient. This case is confirmed when the order of harmonics belongs to the following harmonics group:

$$G = \left\{\left(h = \lambda n_b/2 \pm 1\right) \cap h = 6v \pm 1\right\}$$

In this figure (in blue color), additional components appear caused by the existence of static eccentricity (SE). These additional components are the result of the retreat of the order harmonics $h=\frac{\lambda n_b}{2}\pm 1$ to the order $h=\frac{\lambda n_b \pm 1}{2}\pm 1$ or to the order $h=\frac{\lambda n_b \pm 2}{2}\pm 1$. This phenomenon is caused by the increase of slip coefficient when eccentricity defect occurs. So, the static eccentricity harmonics order group is:

$$C=\left\{G\cup\left(h=\frac{\lambda n_b \pm 1}{2}\pm 1 \cup h=\frac{\lambda n_b \pm 2}{2}\pm 1\right)\right\}$$

In red figure, we show the FFT stator current spectrum under the outer race bearing fault. We remark additional harmonic that appears related to RSH harmonics of SE.
The increasing of slip coefficient provokes the retreat of harmonics from the order $h=\frac{\lambda n_b}{2}\pm 1$ to the order $h=\frac{\lambda n_b \pm 1}{2}\pm 1$ or to the order $h=\frac{\lambda n_b \pm 2}{2}\pm 1$.
The outer race bearing fault harmonics order group is:

$$C=\left\{G\cup\left(h=\frac{\lambda n_b \pm 1}{2}\pm 1 \cup h=\frac{\lambda n_b \pm 2}{2}\pm 1\right)\right\}$$

The tables I, II, III describe the related frequencies for each simulated stator line current with healthy and faulty conditions.

Table I, describe the order frequencies components that appears in stator current spectrum with healthy condition. Table II, describe the frequencies components that appears in stator current spectrum with (SE) additionally to those in healthy state.

The Table III describes the frequency components that appear on both sides of each frequency under the (SE) state.

In Fig. 6, the representation of the experimental result for normalized FFT spectrum of stator line current of both healthy and outer race bearing machines shows. Some specific harmonics exist in both healthy state and outer race bearing fault. These harmonics exist in all induction motors due to a little eccentricity of rotor slot during manufacture. Experimental result justifies the appearance of these harmonics in Table IV. On the other hand, the specific harmonics shows for only outer race bearing fault and represent an accurate signature for this default (show the Table V).

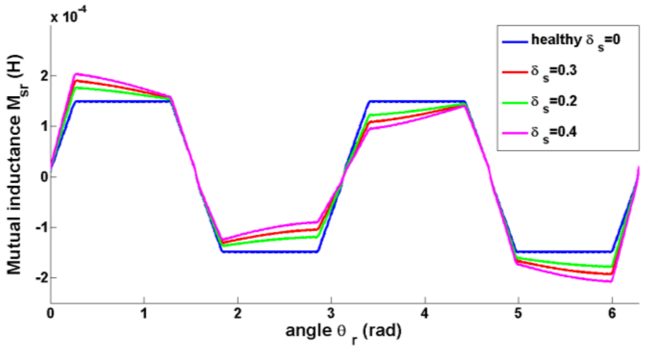

Fig. 3 Mutual inductance *Msr* for different values of δ_σ

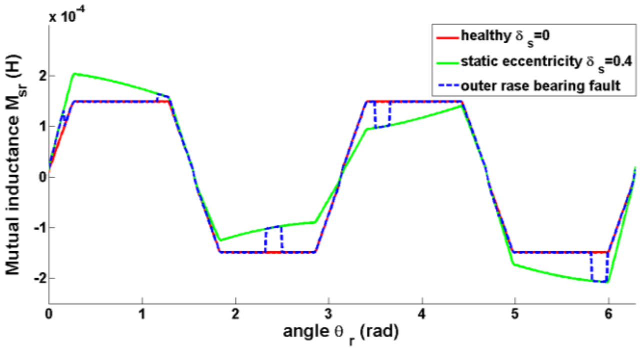

Fig. 4 Mutual inductance *Msr* for Healthy, static eccentricity and outer race bearing fault (δ_σ=0.4)

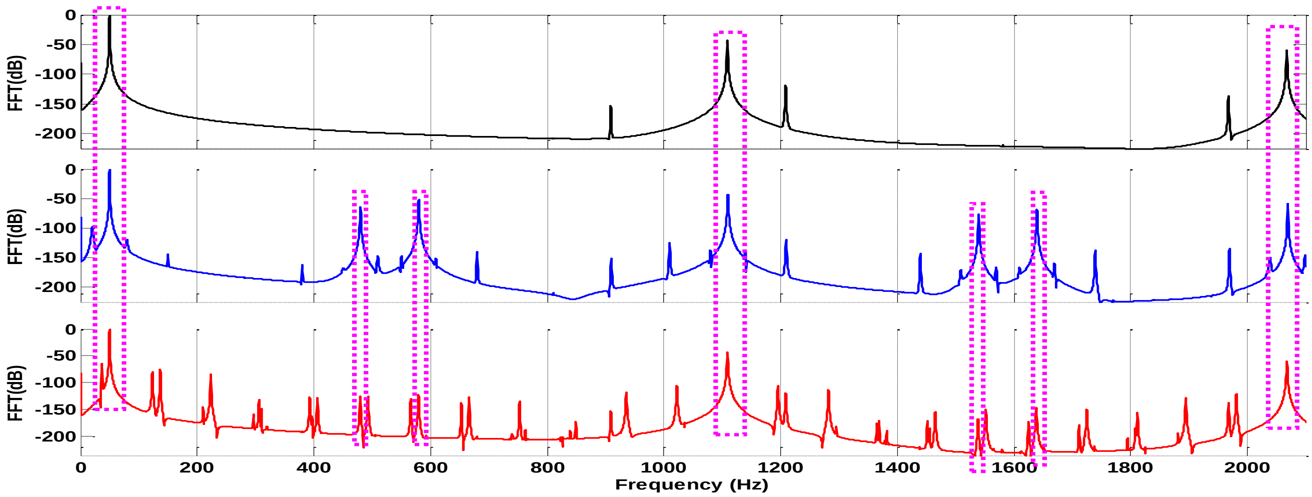

Fig. 3. Simulated, normalized FFT spectrum of stator line current of healthy induction machine ab-bc model (black), static eccentricity (blue) and outer race bearing fault (red).

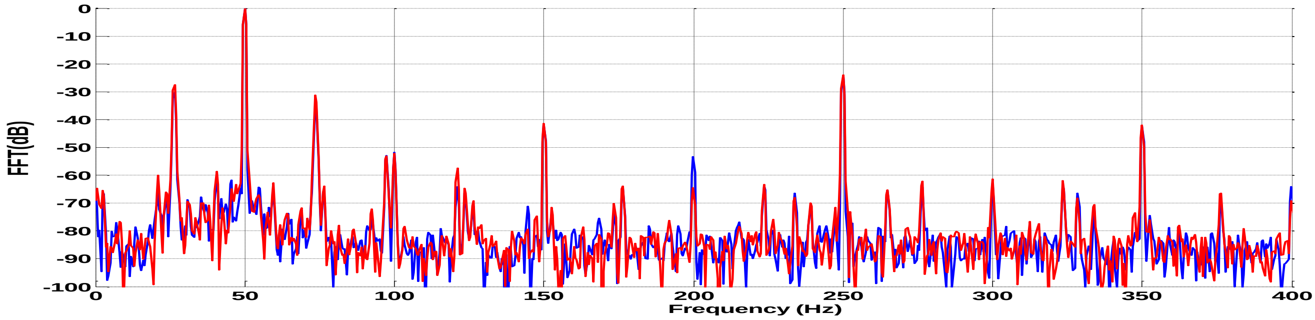

Fig. 4. Experimental, normalized FFT spectrum of stator line current, of healthy induction machine (blue), outer race bearing fault (red).

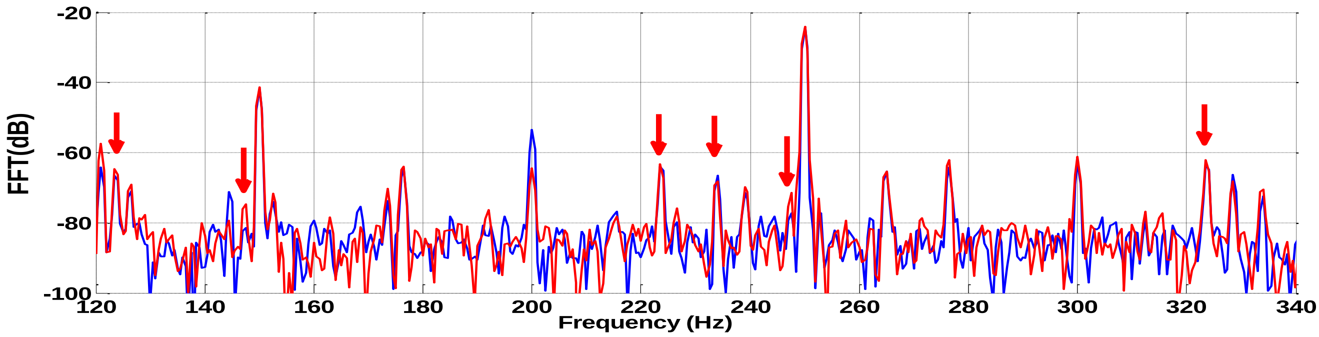

Fig. 5. Zoom of experimental, normalized FFT spectrum of stator line current, of healthy induction machine (blue), outer race bearing fault.

TABLE I: STATOR CURRENT FREQUENCY COMPONENTS IN HEALTHY CONDITION AND THEIR ORDERS

Frequency f (λ)	**Order (h)**
$\left(\frac{\lambda n_b}{2}(1-s)+1\right) f_s$	$h = \lambda n_b/2 + 1$
$\left(\frac{\lambda n_b}{2}(1-s)-1\right) f_s$	$h = \lambda n_b/2 - 1$

TABLE II: STATOR CURRENT FREQUENCY COMPONENTS IN SE CONDITION AND THEIR ORDERS

Frequency f (λ)	**Order (h)**
$\left(\frac{\lambda n_b}{2}(1-s)+1\right) f_s$	$h = \lambda n_b/2 + 1$ $h = \lambda n_b \pm 1/2 + 1$ $h = \lambda n_b \pm 2/2 + 1$
$\left(\frac{\lambda n_b}{2}(1-s)-1\right) f_s$	$h = \lambda n_b/2 - 1$ $h = \lambda n_b \pm 1/2 - 1$ $h = \lambda n_b \pm 2/2 - 1$

TABLE III: STATOR CURRENT FREQUENCY COMPONENTS IN OUTER RACE BEARING FAULT CONDITION AND THEIR ORDERS

Frequency f (λ)	**Order (h)**
$\left(\frac{\lambda n_b}{2}(1-s)+1\right) f_s \pm k\ f_{out}$	$h = \lambda n_b/2 + 1$ $h = \lambda n_b \pm 1/2 + 1$ $h = \lambda n_b \pm 2/2 + 1$
$\left(\frac{\lambda n_b}{2}(1-s)-1\right) f_s \pm k\ f_{out}$	$h = \lambda n_b/2 - 1$ $h = \lambda n_b \pm 1/2 - 1$ $h = \lambda n_b \pm 2/2 - 1$

TABLE IV: SPECIFIC HARMONICS APPEARS IN BOTH HEALTHY STATE AND OUTER RACE BEARING FAULT

$f_{ecc-out} = \left(\frac{\lambda n_b}{2}(1-s)\pm 1\right) f_s \pm k\ f_{out}$		
	Healthy state	**Outer race bearing fault**
$\lambda = 0$, k=2	123.72 223.72	123.72 223.72
$\lambda = 2$, k=12	232.98 332.98	232.98 332.98

TABLE V: SPECIFIC HARMONICS CONCERNING ONLY OUTER RACE BEARING FAULT

$f_{ecc-out} = \left(\frac{\lambda n_b}{2}(1-s)\pm 1\right) f_s \pm k\ f_{out}$	
$\lambda = 2$ k=13	$(28(1-s)+1) f_s - 13\ f_{out} = 146{,}12$ Hz $(28(1-s)-1) f_s - 13\ f_{out} = 246{,}12$ Hz

V. Conclusion

In this paper, analytical study of spectral content of the stator current and a development of calculation for different inductances of squirrel cage induction machine has been presented. This analytical development, based on winding function approach, takes into account the presence of static eccentricity and outer race bearing fault. Through the proposed analytic model, we showed that it is possible to studies the effects of static eccentricity, and outer race bearing fault separately. This analytical development, permit the administration of one specific harmonic signature to outer race bearing faults. The principal main result of this analytical model is to explain by simulation and experimental results, the specific signatures for each studied defect and to separate different frequency signatures. This will allow us to predict the type of occurring fault. Experimental results represent the stator current spectra of the induction machine with outer race bearing fault. Expression of the characteristic frequency of this fault allows us to definitively separate it with the static eccentricity fault. This paper shows clearly the efficacy of the proposed analytical model that can be used to identify the presence of the outer race bearing fault and to definitively differentiate it from the static eccentricity.

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