Possible Approach to Control of Multi-variable Control Loop by Using Tools for Determining Optimal Control Pairs

Pavel Navratil and Libor Pekar

***Abstract*—**The paper describes one of possible approaches to control of multi-variable control loops. In the proposed approach to control is used the so called RGA (Relative Gain Array) tool and also RNGA (Relative Normalized Gain Array) tool, further the correction members, the auxiliary controllers and simple approach to a design of the primary controllers. The RGA tool and also the RNGA tool serve to determine the optimal input-output variable pairings in a multi-variable controlled plant. Correction members are generally considered for ensuring invariance of control loop. Auxiliary controllers are considered to ensure at least partial decoupling control loop. Further, it is considered that the primary controllers are determined by arbitrary single-variable synthesis method for optimal input-output variable pairings. Simulation verifications of the mentioned way of control are carried out for three-variable controlled plant of a steam turbine.

***Keywords*—**Decoupling control loop, Invariance of control loop, MIMO control loop, RGA, RNGA, Simulation.

I. Introduction

CONTROLLED plants with only one output variable (controlled variable) which are controlled by a one input variable (manipulated variable, disturbance variable) are called as SISO (Single-Input Single-Output or single-variable) controlled plants. But, there are not a little cases where it is more than one output variable controlled simultaneously by means of more than one input variable, e.g. aircraft autopilots, air condition plants, chemical reactors, helicopter, tank processes, steam boilers, etc. [1], [2]. In these cases, it means that there is larger numbers of dependent SISO control loops. These control loops are complex and have multiple dependencies and multiple interactions between different input variables (manipulated variables and disturbance variables) and output variables (controlled variables). These control loops are known as MIMO (Multi-Input Multi-Output or multi-variable) control loop and represent a complex of mutually influencing single-variable control loops [1]. Special case of the MIMO control loop is SISO control loop [3].

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Multi-variable control methods have received increased industrial interest [4]. It is often no easy to tell when these control methods are necessary for improved performance in practice and when usage of simpler control structures are sufficient. Therefore, it is useful to know functional limits and structure of the whole control loop, i.e. a controlled plant, a controller and separate signals in the control loop. [2]

Control methods of MIMO controlled plants can be verified not only by using simulation tools, but also on the laboratory models. Some MIMO laboratory models have been described in the literature, e.g. heating plant [5], helicopter model [6], [7], tank model [2], [8], etc.

One of possible examples of a MIMO controlled plant is also steam turbine [9] - [12]. In one of the other part of the paper is considered three-variable controlled plant of steam turbine [9]. The proposed approach to control of the MIMO controlled plant uses the RGA tool and also the RNGA tool to determination optimal pairs in the MIMO controlled plant. Further this method of control used the primary controllers, the correction members and the auxiliary controllers. Parameters of the primary controllers can be determined via arbitrary SISO synthesis methods, e.g. [1], [3], [13] - [16]. Correction members ensure an elimination of influence of disturbance variables on a MIMO control loop and they are determined from parameters of a MIMO controlled plant. Auxiliary controllers ensure decoupling control loop. Mentioned method of control of a MIMO control loop is considered for a MIMO controlled plant with same number input signals and output signals.

Simulation experiments were performed, for the chosen MIMO controlled plant, in MATLAB/ SIMULINK software [17], [18]. The MATLAB software serves for programming and technical computing in many areas. The SIMULINK software is part of the MATLAB environment and serves to analyzing, modelling and simulation of dynamics systems. It is possible to use the MATLAB/SIMULINK software for education and also for research [5], [19], [20].

II. Analysis and Control Design of Multi-Variable Control Loop

A MIMO controlled plant generally consists of m input variables and n output variables. It is generally a non-square controlled plant type $n \times m$. It means, there are three possible cases, i.e. $m = n$, $m > n$, $m < n$. In the next part of the paper, it is mostly considered that controlled plant have a same number of input variables and output variables, i.e. $m = n$ (square controlled plant type $n \times n$). There are several possible configurations of a MIMO control loop. The number of possible configurations of a MIMO control loop for $n \times n$ controlled plant is $n!$ (n factorial).

One of possible approach to analysis and control design of a MIMO control loop, for a controlled plant in steady state, is using the so called RGA (Relative Gain Array) tool [21], [22]. The RGA is useful for MIMO controlled plants that can be decoupled. The other approaches to analysis and control design of MIMO control loop can be found e.g. in [23] - [25].

A. Description of the Multi-variable Control Loop

It will be generally considered a multi-variable control loop with measurement of the disturbance variables. The modified scheme of the control loop is shown in the Fig. 1.

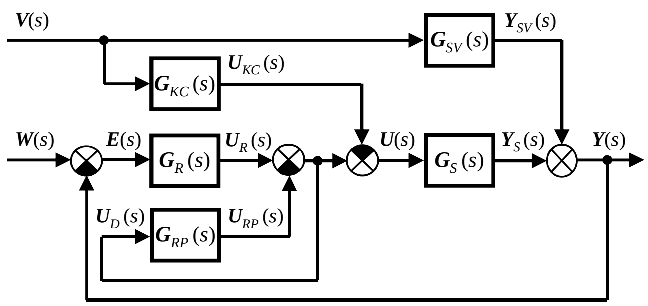

Fig. 1 Modified scheme of multi-variable control loop with measurement of disturbance variables

The description of the parameters in the figure is following, i.e. matrices $\boldsymbol{G}_S(s)$, $\boldsymbol{G}_{SV}(s)$, $\boldsymbol{G}_R(s)$, $\boldsymbol{G}_{RP}(s)$ and $\boldsymbol{G}_{KC}(s)$ denote the transfer function matrices of the controlled plant, the measurable disturbance variables, the primary controllers, the auxiliary controllers and the correction members. Signal $\boldsymbol{Y}(s)$ $[n \times 1]$ denotes the Laplace transform of the vector of controlled variables, $\boldsymbol{U}(s)$ $[n \times 1]$ is the Laplace transform of the vector of manipulated variables, $\boldsymbol{V}(s)$ $[m \times 1]$ is the Laplace transform of the vector of measurable disturbance variables, $\boldsymbol{W}(s)$ $[n \times 1]$ is the Laplace transform of the vector of setpoints and $\boldsymbol{E}(s)$ $[n \times 1]$ is the Laplace transform of the vector of control error, where $\boldsymbol{E}(s) = \boldsymbol{W}(s) - \boldsymbol{Y}(s)$.

It is considered that the transfer function matrices of the controlled plant $\boldsymbol{G}_S(s)$ and the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ are in the following forms, i.e.

$$\boldsymbol{G}_S(s) = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix}, \quad S_{ij} = \frac{Y_{S,i}(s)}{U_j(s)} \tag{1}$$

where $i, j = <1, ..., n>$ and

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix}, \quad S_{Vij} = \frac{Y_{SV,i}(s)}{V_j(s)} \tag{2}$$

where $i = <1, ..., n>$, $j = <1, ..., m>$, $m \le n$.

Transfer function matrices of the primary controllers $\boldsymbol{G}_R(s)$, the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ and the correction members $\boldsymbol{G}_{KC}(s)$ are considered in the following forms

$$\boldsymbol{G}_R(s) = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix}, \quad R_{ij} = \frac{U_{R,i}(s)}{E_j(s)} \tag{3}$$

where $i, j = <1, ..., n>$ and

$$\boldsymbol{G}_{RP}(s) = \begin{bmatrix} RP_{11} & RP_{12} & \cdots & RP_{1n} \\ RP_{21} & RP_{22} & \cdots & RP_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ RP_{n1} & RP_{n2} & \cdots & RP_{nn} \end{bmatrix}, \quad RP_{ij} = \frac{U_{RP,i}(s)}{U_{D,j}(s)} \tag{4}$$

where $i, j = <1, ..., n>$ and

$$\boldsymbol{G}_{KC}(s) = \begin{bmatrix} KC_{11} & KC_{12} & \cdots & KC_{1m} \\ KC_{21} & KC_{22} & \cdots & KC_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ KC_{n1} & KC_{n2} & \cdots & KC_{nm} \end{bmatrix}, \quad KC_{ij} = \frac{U_{KC,i}(s)}{V_j(s)} \tag{5}$$

where $i = <1, ..., n>$, $j = <1, ..., m>$, $m \le n$.

The relations (1) - (5) can be used to build other the transfer function matrices that occur in the MIMO control loop with measurement of the disturbance variables (see Fig. 1), i.e. a closed loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ and a disturbance transfer function matrix $\boldsymbol{G}_{V/Y}(s)$.

$$\boldsymbol{G}_{W/Y}(s) = \left[\boldsymbol{I} + \boldsymbol{G}_S(s)(\boldsymbol{I} + \boldsymbol{G}_{RP}(s))^{-1}\boldsymbol{G}_R(s)\right]^{-1} \boldsymbol{G}_S(s)(\boldsymbol{I} + \boldsymbol{G}_{RP}(s))^{-1}\boldsymbol{G}_R(s) \tag{6}$$

$$\boldsymbol{G}_{V/Y}(s) = \left[\boldsymbol{I} + \boldsymbol{G}_S(s)(\boldsymbol{I} + \boldsymbol{G}_{RP}(s))^{-1}\boldsymbol{G}_R(s)\right]^{-1} \left[\boldsymbol{G}_{SV}(s) - \boldsymbol{G}_S(s)\boldsymbol{G}_{KC}(s)\right] \tag{7}$$

These transfer function matrices can be use at control design and also for ensuring invariance of the MIMO control loop and for ensuring decoupling MIMO control loop.

B. Optimal Input-Output Variable Pairings

Transfer function matrix of the controlled plant can be written in the following form

$$\boldsymbol{Y}(s) = \boldsymbol{G}_S(s)\boldsymbol{U}(s) \ \vee \ \boldsymbol{U}(s) = \boldsymbol{G}_S^{-1}(s)\boldsymbol{Y}(s) \tag{8}$$

where $\boldsymbol{U}(s) = [U_1(s), \ldots, U_n(s)]^T$ and $\boldsymbol{Y}(s) = [Y_1(s), \ldots, Y_n(s)]^T$ are considered as Laplace transform of n-dimensional vectors ($n \times 1$) of inputs and outputs variables and $\boldsymbol{G}_S(s)$ is considered as $n \times n$ transfer function matrix of the controlled plant.

One of possible tools to the analysis of the interactions between input variables u_j and output variables y_i of a MIMO controlled plant is the so called RGA (Relative Gain Array) tool. The relative gain technique has not only become a valuable tool for the selection of input-output pairings, it has also been used to predict the behaviour of controlled responses [26]. The RGA is a standardized form of the gain matrix that describes the influence of each input variable on each output variable.

Each element in the RGA matrix $\boldsymbol{\Lambda}$ (9), (10) is defined as the open control loop gain divided by the gain between the same two variables when all other loops are under so called "perfect" control [27]. Element λ_{ij} in the RGA matrix $\boldsymbol{\Lambda}$ are depended on frequency. They are usually determined for frequency equal to zero, i.e. for steady state. From the point of view of control, for a controlled plant with same number input and output variables, it is ideal state when values of diagonal elements of the RGA matrix are approaching to the value of one and aside-from-diagonal elements of RGA matrix approaching to the value of zero.

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{n1} & \cdots & \lambda_{nn} \end{bmatrix} \tag{9}$$

where

$$\lambda_{ij} = \frac{(\partial Y_i / \partial U_j)_{U_k}}{(\partial Y_i / \partial U_j)_{Y_l}} \qquad \begin{array}{l} i, j = 1, .., n \\ U_k = 0,\ k = 1,..,n \ \wedge k \neq j \\ Y_l = 0,\ l = 1,..,n \ \wedge \ l \neq i \end{array}$$

$$(\partial Y_i / \partial U_j)_{U_k} = [\boldsymbol{G}_S(s)]_{ij},\ (\partial Y_i / \partial U_j)_{Y_l} = \frac{1}{[\boldsymbol{G}_S^{-1}(s)]_{ji}}$$

λ_{ij} is the relative gain for the corresponding variable pairings, i.e. element of RGA, $(\partial Y_i / \partial U_j)_{U_k}$ is the open control loop gain with all control loop open, $(\partial Y_i / \partial U_j)_{Y_l}$ is the corresponding open control loop gain with all other control loop closed.

From relations (8) and (9), it follows that the RGA for the controlled plant $\boldsymbol{G}_S(s)$ can be determined as

$$\begin{aligned} \boldsymbol{\Lambda}(\boldsymbol{G}_S(s)) &= \boldsymbol{G}_S(s) \otimes (\boldsymbol{G}_S^{-1}(s))^T \\ \boldsymbol{\Lambda}(\boldsymbol{G}_S(0)) &= \boldsymbol{G}_S(0) \otimes (\boldsymbol{G}_S^{-1}(0))^T = \boldsymbol{K} \otimes \boldsymbol{K}^{-T} \end{aligned} \tag{10}$$

where

$$\lambda_{ij} = [\boldsymbol{G}_S(0)]_{ij} \cdot [\boldsymbol{G}_S^{-1}(0)]_{ji} = [\boldsymbol{K}]_{ij} \cdot [\boldsymbol{K}^{-1}]_{ji}$$

$s = j\omega \rightarrow \omega = 0 \rightarrow$ steady state ($t \rightarrow \infty$), $\otimes$ operator implies an element by element multiplication, i.e. Hadamard or Schur product, $\boldsymbol{K}$ is the gain matrix of the controlled plant $\boldsymbol{G}_S(s)$, i.e. $\boldsymbol{K} = [k_{ij}]_{n \times n}$.

In the literature [28] is presented the procedure to calculate RGA for non-square transfer function matrix by using the pseudoinverse.

Utilisation of the above mentioned approach serving to the analysis of interactions between input variables and output variables of a MIMO controlled plant can be illustrated on an example of two-variables controlled plant (see Fig. 2a). It is considered linearized steady state model of the controlled plant in the following form

$$\begin{aligned} Y_1(s) &= S_{11}(s)U_1(s) + S_{12}(s)U_2(s) = ... = k_{11}U_1(s) + k_{12}U_2(s) \\ Y_2(s) &= S_{21}(s)U_1(s) + S_{22}(s)U_2(s) = ... = k_{21}U_1(s) + k_{22}U_2(s) \end{aligned} \tag{11}$$

where k_{ij} is a gain of separate elements of the transfer function matrix of the controlled plant (11), i.e. $k_{ij} = [\boldsymbol{G}_S(0)]_{ij} = S_{ij}(0)$, U_j is Laplace transform of input variable and Y_i is Laplace transform of output variable.

In this case, elements λ_{ij} in the RGA matrix $\boldsymbol{\Lambda}$ are determined by the following way, i.e. element λ_{11}

$$\lambda_{11} = \frac{(\partial Y_1/\partial U_1)_{U_2=0}}{(\partial Y_1/\partial U_1)_{Y_2=0}} = \frac{k_{11}}{\frac{k_{11}k_{22} - k_{12}k_{21}}{k_{22}}} = \frac{k_{11}k_{22}}{k_{11}k_{22} - k_{12}k_{21}} \tag{12}$$

where numerator and denominator of λ_{11} are determined from the following relations

$$U_2(s) = 0 : Y_1(s) = k_{11}U_1(s) \Rightarrow (\partial Y_1/\partial U_1)_{U_2=0}$$

$$Y_2(s) = 0 : U_2(s) = -\frac{k_{21}}{k_{22}}U_1(s)$$

$$\rightarrow Y_1(s) = \frac{k_{11}k_{22} - k_{12}k_{21}}{k_{22}}U_1(s) \Rightarrow (\partial Y_1/\partial U_1)_{Y_2=0}$$

hence

$$(\partial Y_1/\partial U_1)_{U_2=0} = k_{11}$$

$$(\partial Y_1/\partial U_1)_{Y_2=0} = \frac{k_{11}k_{22} - k_{12}k_{21}}{k_{22}}$$

Other elements λ_{12}, λ_{21} and λ_{22} are determined via the following relations

$$\begin{aligned} \lambda_{12} &= \frac{(\partial Y_1/\partial U_2)_{U_1=0}}{(\partial Y_1/\partial U_2)_{Y_2=0}} = \frac{k_{12}k_{21}}{k_{12}k_{21} - k_{11}k_{22}} \\ \lambda_{21} &= \frac{(\partial Y_2/\partial U_1)_{U_2=0}}{(\partial Y_2/\partial U_1)_{Y_1=0}} = \frac{k_{12}k_{21}}{k_{12}k_{21} - k_{11}k_{22}} \\ \lambda_{22} &= \frac{(\partial Y_2/\partial U_2)_{U_1=0}}{(\partial Y_2/\partial U_2)_{Y_1=0}} = \frac{k_{11}k_{22}}{k_{11}k_{22} - k_{12}k_{21}} \end{aligned} \tag{13}$$

Elements λ_{ij} in the RGA matrix $\boldsymbol{\Lambda}$ can be determined also experimentally, e.g. element λ_{11} is possible determined in this way:

- all control loops, i.e. y_1 - u_1, y_2 - u_2 are opened and $u_2 = 0$ (see Fig. 2a)
- only one control loop, i.e. y_1 - u_1 is opened, other control loop(s), i.e. in this case only y_2 - u_2 is closed and $y_2 = 0$ (see Fig. 2b - controller R_{22} ensures "perfect" control)

then

$$\lambda_{11} = \frac{(\partial y_1/\partial u_1)_{\text{all-control-loops-open}, u_2=0}}{(\partial y_1/\partial u_1)_{\text{only-one-control-loop-open } (u_1 - y_1),\ y_2=0}} \tag{14}$$

Other elements λ_{12}, λ_{21} and λ_{22} can be determined by using the following relation

$$\lambda_{ij} = \frac{(\partial y_i / \partial u_j)_{\text{all-control-loops-open}, u_k=0\, (k \neq j)}}{(\partial y_i / \partial u_j)_{\text{only-one-control-loop-open}\, (u_j - y_i),\, y_l=0\, (l \neq i)}} \tag{15}$$

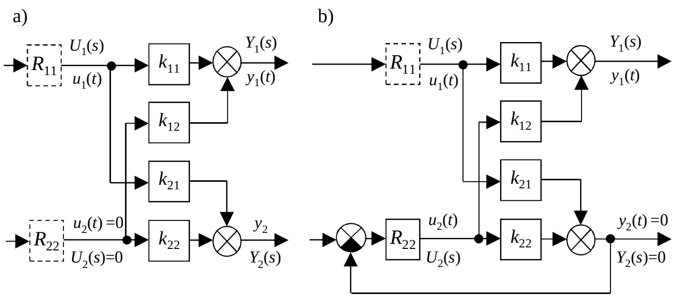

Fig. 2 Block schemes for a case of the experimental determination of the parameter λ_{11} for 2×2 controlled plant (11)
a) all control loops are opened, b) only one control loop is opened

There are some important properties and rules to understanding and analyzing the RGA. Determined values of the RGA mean following:

- Separate elements λ_{ij} are dimensionless and so independent of units.
- The sum of all the elements λ_{ij} of the RGA matrix (9) across any row, or any column will be equal to one

$$\sum_{i=1}^{n} \lambda_{ij} = \sum_{j=1}^{n} \lambda_{ij} = 1 \tag{16}$$

This relation can be verified e.g. for 2×2 controlled plant (11). It means, that it is possible to calculate for 2×2 controlled plant only one element λ_{ij}, e.g. only λ_{11} is calculated and then $\lambda_{12}=1-\lambda_{11}$, $\lambda_{21}=1-\lambda_{11}$, $\lambda_{22}=\lambda_{11}$.

So, the relation (16) simplifies calculation of elements λ_{ij}, e.g. in 2×2 case, only 1 element must be calculate to determine all elements, in 3×3 case, only 4 elements must be calculate to determine all elements, etc. The number of elements (*num*) necessary to calculate all elements of the RGA matrix in case *n*×*m* can be determined from the following relations

$$\begin{aligned} & m, n > 1 : num = (m-1)(n-1) \\ & m = 1 : \quad num = m-1; \;\; n = 1 : num = m-1 \end{aligned} \tag{17}$$

- Each row in the RGA represents one output variable y_i and each column in the RGA represents one input variable u_j. The interpretation of the determined elements λ_{ij} in the RGA can be classified as follows
  - λ_{ij} = 1: This implies that u_j influences y_i without any interaction from the other control loop. In this case, the control pair y_i - u_j can be ideal (however, this may not always be true - see sensitivity of a triangular matrix of the controlled plant in [29]).
  - λ_{ij} = 0: This means u_j has no effect on y_i. In this case, the control pair y_i - u_j is not recommended.
  - 0 < λ_{ij} < 1: This indicates that control pair u_j - y_i is influenced by the other control loops.
    If λ_{ij} < 0.5: Influence of the other control loops is a greater than influence of the control pair y_i - u_j.
    If λ_{ij} > 0.5: Influence of the control pair y_i - u_j is a greater than influence of the other control loops.
    In this case, it is recommended avoid pairing y_i with u_j whenever $\lambda_{ij} \leq 0.5$.
  - λ_{ij} > 1: The positive value of the RGA indicates that the control pair y_i - u_j represents dominant control loop. Others control loops have an influence on the control pair in the opposite direction. The higher the value of λ_{ij} means that the more correctional effects of the other control loops affect the control pair. In this case, it is recommended avoid pairing y_i with u_j whenever λ_{ij} has very high value, e.g. λ_{ij} > 10.
  - λ_{ij} < 0: This means that the control pair y_i - u_j causes instability of the control loop. In this case, it is recommended avoid pairing y_i with u_j.

Control pairs y_i - u_j whose input and output variables have positive RGA elements (λ_{ij}) and their values are close to one are considered as the optimal control pairs [23]. If the value of λ_{ij} fulfils above mentioned general rule the control of control loop for the control pair y_i - u_j is possible. For other values, the control can become difficult because the interaction rate is too high. According to above mentioned approach, it is possible to determine the optimal control pairs y_i - u_j by using RGA tool. Then it is possible to determine, for the optimal control pairs, the parameters of the so called primary controllers via classical SISO synthesis methods [30].

It is considered the RGA matrix for the case 3×3 controlled plant, i.e.

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0.36 & 0.76 & -0.12 \\ -1.11 & 0.22 & 1.89 \\ 1.75 & 0.02 & -0.77 \end{bmatrix} \tag{18}$$

the optimal control pairs are following: y_1-u_2, y_2-u_3, y_3-u_1.

The RGA pairing method has also some shortcoming, i.e. the RGA tool ignores process dynamics. If the transfer function has very large time delay or time constant relative to the others (see (22)), steady state the RGA analysis can provide an incorrect recommendation. In this case, it is then preferable to use e.g. the so called the RNGA (Relative Normalized Gain Array) pairing method (see (22)) for interaction measurement [31].

The RNGA matrix $\boldsymbol{\Lambda}_N$ is generally considered in the following form

$$\boldsymbol{\Lambda}_N = \begin{bmatrix} \lambda_{N,11} & \cdots & \lambda_{N,1n} \\ \vdots & \ddots & \vdots \\ \lambda_{N,n1} & \cdots & \lambda_{N,nn} \end{bmatrix} \tag{19}$$

The RNGA for the controlled plant $\boldsymbol{G}_S(s)$ can be determined as

$$\begin{aligned} \boldsymbol{\Lambda}_N(\boldsymbol{G}_S(s)) &= (\boldsymbol{G}_S(s) \odot \boldsymbol{T}_{ar}) \otimes (\boldsymbol{G}_S(s) \odot \boldsymbol{T}_{ar})^{-T} \\ \boldsymbol{\Lambda}_N(\boldsymbol{G}_S(0)) &= (\boldsymbol{G}_S(0) \odot \boldsymbol{T}_{ar}) \otimes (\boldsymbol{G}_S(0) \odot \boldsymbol{T}_{ar})^{-T} \\ &= (\boldsymbol{K} \odot \boldsymbol{T}_{ar}) \otimes (\boldsymbol{K} \odot \boldsymbol{T}_{ar})^{-T} = \boldsymbol{K}_N \otimes \boldsymbol{K}_N^{-T} \end{aligned} \quad (20)$$

where

$$\lambda_{N,ij} = [\boldsymbol{K}_N]_{ij} \cdot [\boldsymbol{K}_N^{-1}]_{ji}, \quad k_{N,ij} = \frac{k_{ij}}{\tau_{ar,ij}} = \frac{k_{ij}}{|T_{ij} + L_{ij}|}$$

whereas, separate elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$, i.e. $S_{ij}(s)$ are considered in the form

$$S_{ij}(s) = \frac{k_{ij}}{T_{ij}s + 1} e^{-L_{ij}s}$$

$s = j\omega \rightarrow \omega = 0 \rightarrow$ steady state ($t \rightarrow \infty$), $\otimes$ indicates element by element multiplication, $\odot$ indicates element by element division, $\boldsymbol{K}_N$ is the normalized gain matrix of the controlled plant, i.e. $\boldsymbol{K}_N = [k_{N,ij}]_{n\times n}$, k_{ij} is the gain of separate elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$, $\boldsymbol{T}_{ar}$ is the matrix of the average residence time, i.e. $\boldsymbol{T}_{ar} = [\tau_{ar,ij}]_{n\times n}$, T_{ij} is the time constant of separate elements of the transfer function matrix of the controlled plant, L_{ij} is the time delay of separate elements of the transfer function matrix of the controlled plant.

Control pairs y_i - u_j whose input and output variables have positive RNGA elements ($\lambda_{N,ij}$) and their values are close to one are considered as the optimal control pairs. Large values RNGA elements ($\lambda_{N,ij}$) are avoided. [31], [32]

The average residence time $\tau_{ar,ij}$ can be determined, e.g. for chosen types of individual elements $S_{ij}(s)$ of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$, by the following way, i.e.

$$\boldsymbol{Y}(s) = \boldsymbol{G}_S(s)\boldsymbol{U}(s), \quad S_{ij}(s) = [\boldsymbol{G}_S(s)]_{ij}$$

$$\begin{aligned} &{}^1S_{ij}(s) = \frac{5}{(2s+1)(5s+1)}, \quad {}^2S_{ij}(s) = \frac{3(6s+1)}{(9s+1)(5s+1)} e^{-4s} \\ &{}^1S_{ij}(s) \approx {}^1S_{ij,\mathrm{mod}}(s) = \frac{k_{ij}}{T_{ij}s+1} e^{-L_{ij}s} = \frac{5}{(7s+1)} \quad \rightarrow \tau_{ar,ij} = 7 \\ &{}^2S_{ij}(s) \approx {}^2S_{ij,\mathrm{mod}}(s) = \frac{k_{ij}}{T_{ij}s+1} e^{-L_{ij}s} = \frac{3}{(8s+1)} e^{-4s} \rightarrow \tau_{ar,ij} = 12 \end{aligned} \quad (21)$$

The average residence time $\tau_{ar,ij}$ can be also determined for other types of transfer functions $S_{ij}(s)$ (see [1], [16], [31], [33]).

Further, it is considered 2×2 controlled plant with transport delay, i.e.

$$\boldsymbol{Y}(s) = \boldsymbol{G}_S(s)\boldsymbol{U}(s)$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{20s+1} e^{-3s} & \frac{3}{s+1} e^{-5s} \\ \frac{2}{s+1} e^{-2s} & \frac{-4}{20s+1} e^{-4s} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad (22)$$

The RGA matrix $\boldsymbol{\Lambda}$ is in the form

$$\boldsymbol{K} = \boldsymbol{G}_S(0) = \begin{bmatrix} 5 & 3 \\ 2 & -4 \end{bmatrix} \rightarrow \boldsymbol{\Lambda} = \begin{bmatrix} 0.7692 & 0.2308 \\ 0.2308 & 0.7692 \end{bmatrix}$$

then, the optimal control pairs according to RGA tool are following: y_1-u_1, y_2-u_2. However, the aside from diagonal elements of the controlled plant indicates that y_1 responds about twenty times faster to u_2 than u_1 because of its time constant.

The RNGA matrix $\boldsymbol{\Lambda}_N$ is in the form

$$\boldsymbol{K}_N = \boldsymbol{K} \odot \boldsymbol{T}_{ar} = \begin{bmatrix} 0.2174 & 0.5 \\ 0.6667 & -0.1667 \end{bmatrix} \rightarrow \boldsymbol{\Lambda}_N = \begin{bmatrix} 0.0980 & 0.9020 \\ 0.9020 & 0.0980 \end{bmatrix}$$

where

$$\boldsymbol{K} = \boldsymbol{G}_S(0) = \begin{bmatrix} 5 & 3 \\ 2 & -4 \end{bmatrix} \quad \boldsymbol{T}_{ar} = \begin{bmatrix} 23 & 6 \\ 3 & 24 \end{bmatrix}$$

then, the optimal control pairs according to RNGA tool are following: y_1-u_2, y_2-u_1.

To determination of the stability of the resulting control structure, i.e. control loop that uses the optimal control pairs, can be used the Niederlinski index (NI). It is considered $n\times n$ controlled plant, according to (1), (8). Then, the NI value can be calculated by using following relation (it is considered the steady state ($t \rightarrow \infty$, $s = 0$)), i.e.

$$NI = \frac{\det(\boldsymbol{G}_S(0))}{\prod_{i=1}^{n} S_{ii}(0)} = \frac{\det(\boldsymbol{K})}{\prod_{i=1}^{n} k_{ii}} \quad (23)$$

A negative the NI value indicates instability in the proposed control loop, i.e. in the resulting control structure.

It is considered 3×3 controlled plant $\boldsymbol{G}_S(s)$ whose a gain matrix $\boldsymbol{K}$ is following

$$\boldsymbol{K} = \begin{bmatrix} 1.8 & 1 & 1 \\ 1 & 0.25 & 1 \\ 1 & 1 & 0.25 \end{bmatrix} \quad (24)$$

The RGA matrix $\boldsymbol{\Lambda}$ is determined in the form

$$\boldsymbol{\Lambda} = \begin{bmatrix} 9 & -4 & -4 \\ -4 & 0.73 & 4.27 \\ -4 & 4.27 & 0.73 \end{bmatrix}$$

the optimal control pairs according to the RGA tool are y_1-u_1, y_2-u_2 and y_3-u_3. According to the Niederlinsky rule

$$NI = \frac{\det(\boldsymbol{K})}{\prod_{i=1}^{3} k_{ii}} = \frac{-0.1875}{4.84} = -0.0387 < 0$$

i.e., chosen control pairs lead to unstable configuration.

In this case, it is possible to change the control pairs, i.e. y_1-u_1, y_2-u_3 and y_3-u_2, then

$$\boldsymbol{\Lambda} = \begin{bmatrix} 9 & -4 & -4 \\ -4 & 4.27 & 0.73 \\ -4 & 0.73 & 4.27 \end{bmatrix}$$

which corresponds to

$$\boldsymbol{K} = \begin{bmatrix} 1.8 & 1 & 1 \\ 1 & 1 & 0.25 \\ 1 & 0.25 & 1 \end{bmatrix}$$

and then value of the NI is following

$$NI = \frac{\det(\boldsymbol{K})}{\prod_{i=1}^{3} k_{ii}} = \frac{0.1875}{1.80} = 0.1042 > 0$$

which indicates that control pairs y_1-u_1, y_2-u_3 and y_3-u_2 should ensure the stability of the control loop.

It is a suitable to consider all the mentioned tools for the analysis of the optimal control pairs, i.e. the RGA tool, the RNGA tool and eventually also NI, when final decision is to be made. Further possible tool, which can be used not only for the analysis of the interactions in a MIMO controlled plant, is e.g. SVD (Singular Value Decomposition) tool [34].

C. Invariance of Multi-variable Control Loop and Decoupling Multi-variable Control Loop

It is often required at synthesis of a SISO or also a MIMO control loop, beside stability and quality of control, that influence of the measurable disturbance variables on controlled variables was eliminated. Such a MIMO control loop is called invariant. Control loop at which the influence of disturbance variables is eliminated only partially, e.g. only in steady state is called approximately invariant. Control loop at which the influence of disturbance variables on controlled variables is completely eliminated is called absolutely invariant. Other requirement at synthesis of a MIMO control loop can also be an elimination of effects of interactions of control variables in a MIMO control loop, i.e. one desired variable (setpoint) causes a change of only one corresponding controlled variable in a MIMO control loop. Such a MIMO control loop is called decoupled. [1]

In order to ensure invariance of a MIMO control loop decoupling MIMO control loop a disturbance transfer function matrix $\boldsymbol{G}_{V/Y}(s)$ (7) and a closed loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ (6) are used.

Absolute invariance of a MIMO control loop can be ensured if the transfer function matrix $\boldsymbol{G}_{V/Y}(s)$ (7) is zero. It can be possible if the following relation is valid

$$\boldsymbol{G}_{KC}(s) = \boldsymbol{G}_S^{-1}(s)\boldsymbol{G}_{SV}(s) \tag{25}$$

Separate elements of the transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$, i.e. KC can be determined as follows

$$KC_{ij} = \frac{1}{\det(\boldsymbol{G}_S)} \sum_{k=1}^{n} s_{ki} \cdot S_{V,kj} \qquad (26)$$
$$\det(\boldsymbol{G}_S) \neq 0,\ i = <1,\ldots,n>,\ j = <1,\ldots,m>,\ m \le n$$

where $\det(\boldsymbol{G}_S)$ is a determinant of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$, $S_{V,kj}$ are separate members of the transfer function matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ and s_{ki} are algebraic supplements of separate elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$.

In case that diagonal members of the transfer function matrix $\boldsymbol{G}_{SV}(s)$ and $\boldsymbol{G}_S(s)$ are considered as a dominant, the relation (26) can be simplified. In this case an influence of the other elements of the transfer function matrix $\boldsymbol{G}_{SV}(s)$ and $\boldsymbol{G}_S(s)$ is omitted at design of the correction members KC. So that invariance of the control loop is ensured only for diagonal elements. It is considered that corresponding number of SISO branched control loops with measurement of the disturbance variable is used to determine the correction members KC. Connection of all SISO branched control loops is the similar and they differ in separate transfer functions and variables (see Fig. 3). The influence of separate elements of the transfer function matrix $\boldsymbol{G}_{SV}(s)$ and $\boldsymbol{G}_S(s)$ can be verified by using one of possible tools to the analysis of the interactions between input and output variables in a MIMO controlled plant (see paragraph II.B, i.e. "Optimal Input-Output Variable Pairings"). Described approach ensures the so called approximate invariance of control loop. It means that influence of disturbance variables is generally eliminated only partially. [1]

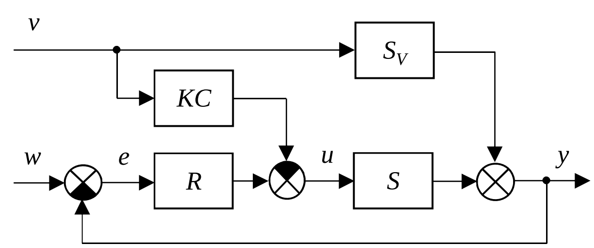

Fig. 3 Single-variable branched control loop with measurement of disturbance variable

The transfer functions of the correction members KC are determined by using the following relation

$$KC_{ii} = \frac{S_{V,ii}}{S_{ii}} \quad i = <1,\ldots,n>,\ S_{ii} \neq 0 \qquad (27)$$
$$KC_{ij} = 0 \qquad i,j = <1,\ldots,n>,\ i \neq j$$

where $S_{V,ii}$ are separate elements of transfer matrix $\boldsymbol{G}_{SV}(s)$, S_{ii} are separate elements of transfer function matrix $\boldsymbol{G}_S(s)$.

In case that diagonal elements of transfer function matrix $\boldsymbol{G}_{SV}(s)$ or $\boldsymbol{G}_S(s)$ have not dominant influence then corresponding the correction members KC may not ensure the desired behaviour of a control loop.

A more general relation that ensures approximate invariance of control loop via dominant diagonal elements transfer function matrix $\boldsymbol{G}_{SV}(s)$ or $\boldsymbol{G}_S(s)$ and also via dominant aside from diagonal elements transfer function matrix $\boldsymbol{G}_{SV}(s)$ or $\boldsymbol{G}_S(s)$ can be written in the following form

$$KC_{ij} = \frac{\tilde{S}_{V,kj}}{\tilde{S}_{ki}} \quad i,k = <1,\ldots,n>,\ j = <1,\ldots,m>,\ m \le n \qquad (28)$$
$$KC_{ij} = 0 \ \text{ for other correction members } KC$$

where $\tilde{S}_{V,kj}$ are separate dominant elements of the transfer matrix $\boldsymbol{G}_{SV}(s)$ in the k-th row, $\tilde{S}_{ki}$ are separate dominant elements of the transfer function matrix $\boldsymbol{G}_S(s)$ in the k-th row.

One of the above mentioned two approaches, which ensure invariance of a MIMO control loop, uses to determine the correction members KC analogy of SISO branched control loop with measurement of the disturbance variable(s). This approach can also be used for a reduction of interactions of separate non-dominant control loops, i.e. a reduction of influence of non-dominant elements of the transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ in the MIMO control loop. Such the control loop is then called decoupling control loop. In this case, it is considered that separate non-dominant elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ represent measurable disturbance variables.

For ensuring decoupling control loop the transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ (6) must be a diagonal. Because the sum and product of three diagonal matrices are diagonal matrices, and the inverse of diagonal matrix is also diagonal matrix, then the requirement can be ensured if transfer fiction matrix $\boldsymbol{G}_S(s)\cdot(\boldsymbol{I}+\boldsymbol{G}_{RP}(s))^{-1}\cdot\boldsymbol{G}_R(s)$ is diagonal. Further they are considered, for separate elements of the transfer function matrix of the primary controllers $\boldsymbol{G}_R(s)$ (3) and the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ (4), following conditions:

- Diagonal elements of the transfer function matrix of auxiliary controllers $\boldsymbol{G}_{RP}(s)$, i.e. RP_{ii}, are equal to 0, thus
  $RP_{ii} = 0,\ i=<1, \ldots, n>$
- Parameters of separate elements of the transfer function matrix of the primary controllers $\boldsymbol{G}_R(s)$, i.e. R_{ij} are determined for corresponding dominant elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$, i.e. S_{ji}, thus
  $R_{ij} \neq 0$ only for dominant elements S_{ji}
  $R_{ij} = 0$ for other (non-dominant) elements S_{ji}

Then, it is possible to determine elements of the transfer function matrix of the auxiliary controllers $\boldsymbol{G}_{RP}(s)$, i.e. RP_{ij}, in the following form

$$RP_{ij} = \frac{S_{kj}}{\tilde{S}_{ki}} \quad i,j,k = <1,\ldots,n>,\ i \neq j \tag{29}$$

where S_{kj} are separate non-dominant elements of the transfer matrix $\boldsymbol{G}_S(s)$ in the k-th row, $\tilde{S}_{ki}$ are separate dominant elements of the transfer function matrix $\boldsymbol{G}_S(s)$ in the k-th row.

D. Control Design of Multi-variable Control Loop

One of the possible approaches to control of MIMO control loops is described in the following part. This approach uses analysis of the interactions between input variables and output variables in a MIMO controlled plant (see paragraph II.B, i.e. "Optimal Input-Output Variable Pairings"). The chosen approach can be generally divided into several parts, i.e. determination of parameters of the primary controllers then ensuring invariance of control loop and also ensuring at least partial decoupling control loop, i.e. partial reduction of the influence of non-dominant elements (the non-optimal control pairs) of the transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ in the MIMO control loop (see paragraph II.C, i.e. "Invariance of Multi-variable Control loop and Decoupling Multi-variable Control Loop").

The so called primary controllers are designed by any synthesis methods of the SISO control loop. Parameters of the primary controllers are determined for the optimal control pairs in the MIMO controlled plant $\boldsymbol{G}_S(s)$. Optimal control pairs can be gained by using approaches described in the paragraph II.B, i.e. via the RGA tool (9), (10), the RNGA tool (19), (20), possible also via NI (23).

Invariance of the MIMO control loop is ensured by means of elements of the transfer function matrix of the correction members $\boldsymbol{G}_{KC}(s)$, i.e. KC (26) or (28) (see paragraph II.C). Relation (26) ensures absolute invariance. Relation (28) is simpler, but it can ensure only approximate invariance.

Decoupling MIMO control loop is ensured by means of the elements of the transfer function matrix of the auxiliary controllers $\boldsymbol{G}_{RP}(s)$, i.e. RP (29) (see paragraph II.C).

It is considered the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ and the transfer function matrix of the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ are in the following form

$$\boldsymbol{G}_S(s) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{s^2+6s+5}\begin{bmatrix} 4 & 7.5 \\ 6.3 & 2.2 \end{bmatrix} \tag{30}$$

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix} S_{V11} & S_{V12} \\ S_{V21} & S_{V22} \end{bmatrix} = \frac{1}{s^2+6s+5}\begin{bmatrix} 2 & 0.8 \\ 1.5 & 3 \end{bmatrix} \tag{31}$$

and

$$\boldsymbol{Y}(s) = \boldsymbol{G}_S(s)\boldsymbol{U}(s) + \boldsymbol{G}_{SV}(s)\boldsymbol{V}(s)$$

where $\boldsymbol{Y}(s)$ is the Laplace transform of the vector of controlled variables, $\boldsymbol{U}(s)$ is the Laplace transform of the vector of manipulated variables and $\boldsymbol{V}(s)$ $[m\times 1]$ is the Laplace transform of the vector of measurable disturbance variables.

The RGA matrix $\boldsymbol{\Lambda}$ for transfer function matrix (30) and (31) is in the following form

$$\boldsymbol{\Lambda}(\boldsymbol{G}_S(0)) = \begin{bmatrix} -0.223 & 1.223 \\ 1.223 & -0.223 \end{bmatrix}$$

thus the optimal pairs according to the RGA tool are following: y_1-u_2, y_2-u_1, i.e. S_{12} and S_{21} (dominant elements). Further

$$\boldsymbol{\Lambda}(\boldsymbol{G}_{SV}(0)) = \begin{bmatrix} 1.25 & -0.25 \\ -0.25 & 1.25 \end{bmatrix}$$

thus the optimal pairs according to the RGA tool are following: y_1-v_1, y_2-v_2, i.e. S_{V11} and S_{V22} (dominant elements).

Primary controllers R_{ij} are determined for corresponding dominant elements of the $\boldsymbol{G}_S(s)$, i.e. S_{12} and S_{21}. It means that parameters of the primary controllers R_{12} and R_{21} are designed by arbitrary SISO synthesis method for dominant elements of the $\boldsymbol{G}_S(s)$ S_{21} and S_{12} ($S_{21} \rightarrow R_{12}$ and $S_{12} \rightarrow R_{21}$).

The correction members KC are determined via (28), i.e.

$$KC_{21} = \frac{\tilde{S}_{V,11}}{\tilde{S}_{12}} = \frac{S_{V11}}{S_{12}} = 0.5, \quad KC_{12} = \frac{\tilde{S}_{V,22}}{\tilde{S}_{21}} = \frac{S_{V22}}{S_{21}} = 1.367$$

$$KC_{11} = 0,\ KC_{22} = 0$$

Auxiliary controllers RP are determined by using relation (29), i.e.

$$RP_{12} = \frac{S_{22}}{\tilde{S}_{21}} = \frac{S_{22}}{S_{21}} = 0.349,\ RP_{21} = \frac{S_{11}}{\tilde{S}_{12}} = \frac{S_{11}}{S_{12}} = 0.533$$

$$RP_{12} = 0,\ RP_{22} = 0$$

III. Simulation Verification of Described Approach to Control of Multi-variable Control Loop

A. Description of the Three-variable Controlled Plant of the Condensing Steam Turbine

One of typical examples of MIMO controlled plant is e.g. a condensing steam turbine [11], [12]. In this case it is considered the condensing steam turbine with two controlled withdrawals which drives electric generator supplying determined part of electric network, which means the turbine operates without phasing into power network [1], [9]. The scheme of three-variable controlled plant of the condensing steam turbine is shown in the Fig. 4 [9].

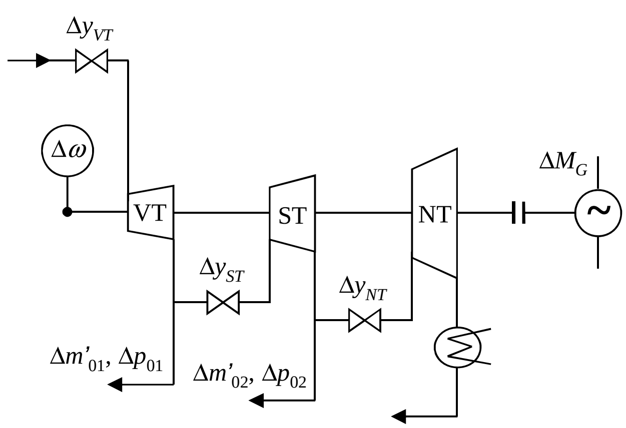

Fig. 4 Three-variable control plant of condensing steam turbine

Description of separate parameters in the Fig. 4 is following, i.e. Δy_{VT}, Δy_{ST}, Δy_{NT} are changes of opening position of control valves of high-pressure (VT), medium-pressure (ST) and low-pressure part of turbine (NT), ΔM_G is a change of electric load of turbo-generator and $\Delta m'_{01}$, $\Delta m'_{02}$ are changes of mass flows of withdrawn steam, $\Delta\omega$ is a change of angular speed of turbo-generator, Δp_{01}, Δp_{02} are changes of steam pressures in corresponding withdrawals.

Described parameters represent separate variables in the modified three-variable control loop with measurement of disturbance variables (see Fig. 1), i.e. manipulated variables (u_i) are parameters Δy_{VT}, Δy_{ST}, Δy_{NT} and disturbance variables (v_i) are parameters ΔM_G, $\Delta m'_{01}$, $\Delta m'_{02}$, controlled variables (y_i) are parameters $\Delta\omega$, Δp_{01}, Δp_{02}.

B. Mathematical Model of the Three-variable Controlled Plant of the Condensing Steam Turbine

Mathematical model of the controlled plant of the condensing steam turbine is given by three differential equations (32) - (34). These differential equations were gained already after deriving and using linearization from project OTROKOVICE, which was elaborated by the firm ALSTOM Power [9].

The first differential equation represents moment balance which is in the following form

$$\begin{aligned}518.4\Delta\dot{\omega} &= -63.3\Delta\omega + 656.9\Delta p_{01} + 4611.7\Delta p_{02} \\ &+ 1007.3\Delta y_{VT} + 200.6\Delta y_{ST} + 121.5\Delta y_{NT} - \Delta M_G\end{aligned} \quad (32)$$

second and third differential equations represent flow through flow spaces and they are in the forms

$$\begin{aligned}1.865\Delta\dot{p}_{01} &= -1.610\Delta p_{01} + 0.167\Delta p_{02} + 1.523\Delta y_{VT} \\ &- 0.361\Delta y_{ST} - \Delta m'_{01}\end{aligned} \quad (33)$$

$$\begin{aligned}13.45\Delta\dot{p}_{02} &= 1.563\Delta p_{01} - 10.517\Delta p_{02} + 0.361\Delta y_{ST} \\ &- 0.222\Delta y_{NT} - \Delta m'_{02}.\end{aligned} \quad (34)$$

The above mentioned equations (32) - (34) can be rewritten into better form (35) - (37) by introducing relative values, i.e. with regard to starting stable state-operational (the calculated point), at which relation of values can be generally written in the form

$$\varphi_X = \frac{\Delta X}{(X)_0} \;\rightarrow\; \Delta X = \varphi_X \cdot (X)_0 \quad (35)$$

where separate operational parameters of controlled plant of the condensing steam turbine in the calculated point are following

$$\begin{aligned}&(y_{VT})_0 = 19.15\,[\text{mm}],\ (y_{ST})_0 = 59.9\,[\text{mm}], \\ &(y_{NT})_0 = 69.8\,[\text{mm}],\ (M_G)_0 = 39789\,[\text{Nm}], \\ &(m'_{01})_0 = 6.94\,[\text{kg/s}],\ (m'_{02})_0 = 6.94\,[\text{kg/s}], \\ &(\omega)_0 = 628.3\,[\text{rad/s}],\ (p_{01})_0 = 14\,[\text{bar}],\ (p_{02})_0 = 1.55\,[\text{bar}]\end{aligned} \quad (36)$$

hence

$$\begin{aligned}325710.72\dot{\varphi}_\omega &= -39771.39\varphi_\omega + 9196.6\varphi_{p_{01}} + 7148.14\varphi_{p_{02}} \\ &+ 19289.80\varphi_{y_{VT}} + 12015.94\varphi_{y_{ST}} + 8480.70\varphi_{y_{NT}} \\ &- 39789\,\varphi_{M_G}\end{aligned} \quad (37)$$

$$\begin{aligned}26.110\dot{\varphi}_{p_{01}} &= -22.540\varphi_{p_{01}} + 0.259\varphi_{p_{02}} + 29.165\varphi_{y_{VT}} \\ &- 21.624\varphi_{y_{ST}} - 6.940\varphi_{m'_{01}}\end{aligned} \quad (38)$$

$$\begin{aligned}20.848\dot{\varphi}_{p_{02}} &= 21.882\varphi_{p_{01}} - 16.301\varphi_{p_{02}} + 21.624\varphi_{y_{ST}} \\ &- 15.496\varphi_{y_{NT}} - 6.940\varphi_{m'_{02}}\end{aligned} \quad (39)$$

The mathematical model of the linearized and modified controlled plant of the condensing steam turbine (see equations (37) - (39)) can also be represented in state-space form, i.e.

$$\begin{bmatrix}\dot{\varphi}_{\omega}\\ \dot{\varphi}_{p_{01}}\\ \dot{\varphi}_{p_{02}}\end{bmatrix}=\begin{bmatrix}-0.1221 & 0.0282 & 0.0219\\ 0 & -0.8633 & 0.0099\\ 0 & 1.0496 & -0.7819\end{bmatrix}\begin{bmatrix}\varphi_{\omega}\\ \varphi_{p_{01}}\\ \varphi_{p_{02}}\end{bmatrix}+\begin{bmatrix}0.0592 & 0.0369 & 0.0260\\ 1.1170 & -0.8282 & 0\\ 0 & 1.0372 & -0.7433\end{bmatrix}\begin{bmatrix}\varphi_{y_{VT}}\\ \varphi_{y_{ST}}\\ \varphi_{y_{NT}}\end{bmatrix}+\begin{bmatrix}-0.1222 & 0 & 0\\ 0 & -0.2658 & 0\\ 0 & 0 & -0.3329\end{bmatrix}\begin{bmatrix}\varphi_{M_G}\\ \varphi_{m'_{01}}\\ \varphi_{m'_{02}}\end{bmatrix} \tag{40}$$

$$\begin{bmatrix}\varphi_{\omega}\\ \varphi_{p_{01}}\\ \varphi_{p_{02}}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\varphi_{\omega}\\ \varphi_{p_{01}}\\ \varphi_{p_{02}}\end{bmatrix}$$

where $\varphi_{\omega}, \varphi_{p_{01}}, \varphi_{p_{02}}$ are state variables and in this case also output variables, $\varphi_{y_{VT}}, \varphi_{y_{ST}}, \varphi_{y_{NT}}$ are manipulated variables and $\varphi_{M_G}, \varphi_{m'_{01}}, \varphi_{m'_{02}}$ are measurable disturbance variables.

It is possible to determine, from two above mentioned equations, i.e. relation (40), the corresponding transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (43) and transfer function matrix of measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ (44). Further, it is considered, the Laplace transform of the vector of controlled variables is generally given by (41).

$$\boldsymbol{Y}(s)=\boldsymbol{G}_S(s)\boldsymbol{U}(s)+\boldsymbol{G}_{SV}(s)\boldsymbol{V}(s) \tag{41}$$

where $\boldsymbol{Y}(s)$ is the Laplace transform of the vector of controlled variables, i.e. $\boldsymbol{Y}(s)=[\Phi_{\omega},\Phi_{p_{01}},\Phi_{p_{02}}]^T$, $\boldsymbol{U}(s)$ is the Laplace transform of the vector of manipulated variables, i.e. $\boldsymbol{U}(s)=[\Phi_{y_{VT}},\Phi_{y_{ST}},\Phi_{y_{NT}}]^T$ and $\boldsymbol{V}(s)$ is the Laplace transform of the vector of measurable disturbance variables, i.e. $\boldsymbol{V}(s)=[\Phi_{M_G},\Phi_{m'_{01}},\Phi_{m'_{02}}]^T$

thus

$$\begin{bmatrix}\Phi_{\omega}\\ \Phi_{p_{01}}\\ \Phi_{p_{02}}\end{bmatrix}=\boldsymbol{G}_S(s)\begin{bmatrix}\Phi_{y_{VT}}\\ \Phi_{y_{ST}}\\ \Phi_{y_{NT}}\end{bmatrix}+\boldsymbol{G}_{SV}(s)\begin{bmatrix}\Phi_{M_G}\\ \Phi_{m'_{01}}\\ \Phi_{m'_{02}}\end{bmatrix} \tag{42}$$

where

$$\boldsymbol{G}_S(s)=\begin{bmatrix}\frac{0.73s^2+1.59s+1.11}{12.3s^3+21.8s^2+10.7s+1} & \frac{0.45s^2+0.74s+0.087}{12.3s^3+21.8s^2+10.7s+1} & \frac{0.32s^2+0.33s+0.037}{12.3s^3+21.8s^2+10.7s+1}\\ \frac{1.68s+1.31}{1.51s^2+2.48s+1} & \frac{-1.25s-0.96}{1.51s^2+2.48s+1} & \frac{-0.011}{1.51s^2+2.48s+1}\\ \frac{1.76}{1.51s^2+2.48s+1} & \frac{1.56s+0.039}{1.51s^2+2.48s+1} & \frac{-1.12s-0.97}{1.51s^2+2.48s+1}\end{bmatrix} \tag{43}$$

$$\boldsymbol{G}_{SV}(s)=\begin{bmatrix}\frac{-1.51s^2-2.48s-1}{12.3s^3+21.8s^2+10.7s+1} & \frac{-0.092s-0.15}{12.3s^3+21.8s^2+10.7s+1} & \frac{-0.090s-0.079}{12.3s^3+21.8s^2+10.7s+1}\\ 0 & \frac{-0.40s-0.31}{1.51s^2+2.48s+1} & \frac{-0.005}{1.51s^2+2.48s+1}\\ 0 & \frac{-0.420}{1.51s^2+2.48s+1} & \frac{-0.501s-0.432}{1.51s^2+2.48s+1}\end{bmatrix} \tag{44}$$

Step response of transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (43) and transfer function matrix of the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ (44) are shown in the following figures (see Fig. 5, Fig. 6).

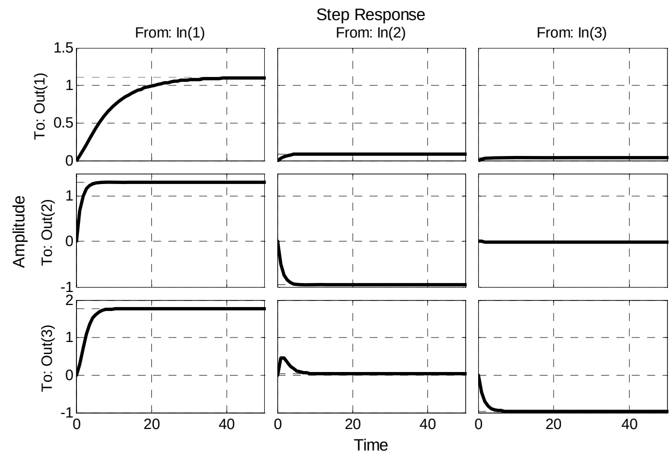

Fig. 5 Step response of transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (43)

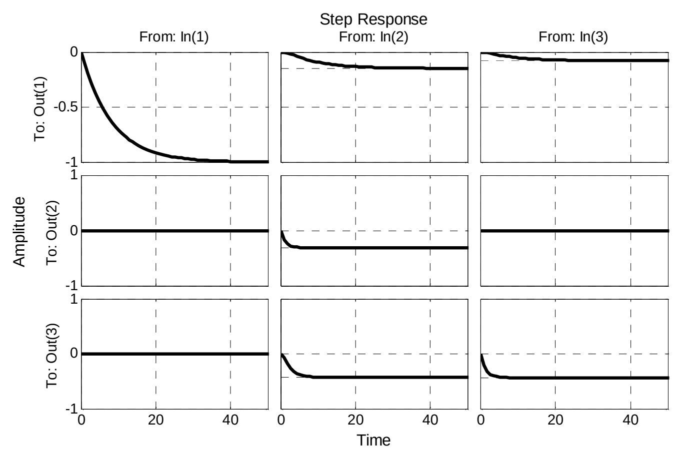

Fig. 6 Step response of transfer function matrix of the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ (44)

C. Control Design of Three-variable Control Loop of the of the Condensing Steam Turbine

The procedure of control described in the paragraph II.D, i.e. “Control Design of Multi-variable Control Loop” is used at control of the three-variable control loop of the condensing steam turbine. First the transfer functions of primary controllers are determined for optimal control pairs via (9), (10) or (19), (20). After that parameters of the correction members *KC*, which ensured invariance of control loop, are calculated by using of (26) (absolute invariance) or (28) (approximate invariance). Finally decoupling control loop is solved by using auxiliary controllers *RP* (29).

To determination of optimal control pairs (dominant elements) for the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (43) were used the RGA tool (10) and also the RNGA tool (20), i.e.

- the RGA tool

$$\Lambda(\boldsymbol{G}_S(0)) = \begin{bmatrix} 0.8548 & 0.0912 & 0.0540 \\ 0.0942 & 0.9068 & -0.0010 \\ 0.0510 & 0.0020 & 0.9470 \end{bmatrix} \qquad (45)$$

where

$$\boldsymbol{G}_S(0) = \begin{bmatrix} 1.1060 & 0.0875 & 0.0372 \\ 1.3142 & -0.9589 & -0.0111 \\ 1.7641 & 0.0393 & 0.9654 \end{bmatrix}$$

- the RNGA tool

$$\Lambda_N(\boldsymbol{G}_S(0)) = \begin{bmatrix} 0.6225 & 0.2769 & 0.1006 \\ 0.2782 & 0.7229 & -0.0011 \\ 0.0993 & 0.0002 & 0.9005 \end{bmatrix} \qquad (46)$$

where

$$\boldsymbol{T}_{ar} = \begin{bmatrix} 9.2280 & 2.2008 & 1.8680 \\ 1.1965 & 1.1759 & 2.4754 \\ 2.4754 & 37.2371 & 1.3170 \end{bmatrix}$$

The optimal control pairs according to the RGA tool and also the RNGA tool are following, i.e. y_1-u_1 ($\varphi_\omega - \varphi_{y_{VT}}$), y_2-u_2 ($\varphi_{p_{01}} - \varphi_{y_{ST}}$) and y_3-u_3 ($\varphi_{p_{02}} - \varphi_{y_{NT}}$). These control pairs corresponding transfer functions S_{11}, S_{22} and S_{33}, which are dominant elements, i.e. $\tilde{S}_{11} = S_{11}, \tilde{S}_{22} = S_{22}, \tilde{S}_{33} = S_{33}$, of the transfer function matrix of $\boldsymbol{G}_S(s)$ (43).

Stability of the resulting control structure, i.e. control loop that uses determined optimal control pairs, can be verified by using Niederlinski index (NI value) (23), i.e.

$$NI = \frac{\det(\boldsymbol{G}_S(0))}{\prod_{i=1}^{n} S_{ii}(0)} = \frac{\det(\boldsymbol{G}_S(0))}{\prod_{i=1}^{3} S_{ii}(0)} = \frac{1.1984}{1.0239} = 1.1704 > 0 \qquad (47)$$

which indicates that determined control pairs y_1-u_1, y_2-u_2 and y_3-u_3 should ensure the stability of the control loop.

Further they are determined, for above mentioned the optimal control pairs (dominant elements) of the transfer function matrix of $\boldsymbol{G}_S(s)$, corresponding elements of the transfer function matrix of the primary controllers $\boldsymbol{G}_R(s)$, i.e. in this case transfer functions R_{11}, R_{22} and R_{33}. Parameters these controllers (PI controllers) were determined by means of the method of balance tuning [35] and also the method of desired model [16]. To use these methods it was necessary to modify transfer functions, i.e. S_{11}, S_{22} and S_{33} into the following form [1], [16]

$$S_{11,\,x} = \frac{1.106}{8.343s+1} e^{-1.13s} \quad S_{22,\,x} = \frac{-0.959}{1.162s+1} \quad S_{33,\,x} = \frac{-0.965}{1.357s+1} \qquad (48)$$

then $S_{11,\,x} \to R_{11} \quad S_{22,\,x} \to R_{22} \quad S_{33,\,x} \to R_{33}$.

a) method of balance tuning

$$\boldsymbol{G}_R(s) = \begin{bmatrix} \frac{0.797s+0.0948}{s} & 0 & 0 \\ 0 & \frac{-1.043s-0.897}{s} & 0 \\ 0 & 0 & \frac{-1.036s-0.764}{s} \end{bmatrix} \qquad (49)$$

b) method of desired model

$$\boldsymbol{G}_R(s) = \begin{bmatrix} \frac{2.456s+0.294}{s} & 0 & 0 \\ 0 & \frac{-0.869s-0.748}{s} & 0 \\ 0 & 0 & \frac{-0.864s-0.636}{s} \end{bmatrix} \qquad (50)$$

Beside above mentioned methods to determine of parameters of the primary controllers can be possible to use also other SISO synthesis methods, e.g. Ziegler Nichols methods, Cohen-Coon method, Naslin method, Whiteley method, the SIMC method, the method of optimal module, the pole placement method, etc. [1], [13], [16].

Correction members KC, which ensure invariance of control loop, were determined via (26) and also (28). Relation (26) ensures absolute invariance of control loop. Thus, transfer function matrix of the correction members $\boldsymbol{G}_{KC}(s)$ was determined in the following form

$$\boldsymbol{G}_{KC}(s) = \begin{bmatrix} -0.773 & -0.149 & -0.074 \\ -1.043 & 0.120 & 0.100 \\ -1.455 & 0.168 & 0.309 \end{bmatrix} \qquad (51)$$

where separate elements of the transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$, i.e. KC_{11}, KC_{12}, … can also be determined directly by using (25).

Relation (28) ensures that control loop is approximately invariant. In this case, it was first necessary to determine optimal control pairs (dominant elements) for the transfer function matrix of the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ (44) and optimal control pairs (dominant elements) for the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ (43). It was used RGA tool, i.e.

$$\Lambda(\boldsymbol{G}_{SV}(0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.0157 & -0.0157 \\ 0 & -0.0157 & 1.0157 \end{bmatrix} \qquad (52)$$

where

$$\boldsymbol{G}_{SV}(0)=\begin{bmatrix} 1.0004 & -0.1478 & -0.0789 \\ 0 & -0.3127 & -0.0050 \\ 0 & -0.4198 & -0.4324 \end{bmatrix}$$

The optimal control pairs according to the RGA tool are following, i.e. y_1-v_1 ($\varphi_\omega - \varphi_{M_G}$), y_2-v_2 ($\varphi_{p_{01}} - \varphi_{m'_{01}}$) and y_3-v_3 ($\varphi_{p_{02}} - \varphi_{m'_{02}}$). These control pairs corresponding transfer functions S_{V11}, S_{V22} and S_{V33}, which are dominant elements, i.e. $\tilde{S}_{V,11}=S_{V11}, \tilde{S}_{V,22}=S_{V22}, \tilde{S}_{V,33}=S_{V33}$, of the transfer function matrix of $\boldsymbol{G}_{SV}(s)$ (44).

Correction members KC, which ensure approximately invariance of control loop, were determined via (28) in this form

$$\boldsymbol{G}_{KC}(s)=\begin{bmatrix} KC_{11} & 0 & 0 \\ 0 & KC_{22} & 0 \\ 0 & 0 & KC_{33} \end{bmatrix}$$

$$KC_{11}=\frac{\tilde{S}_{V,11}}{\tilde{S}_{11}}=\frac{S_{V11}}{S_{11}}=\frac{-2.063s^2-3.394s-1.371}{s^2+2.178s+1.516}$$
$$KC_{22}=\frac{\tilde{S}_{V,22}}{\tilde{S}_{22}}=\frac{S_{V22}}{S_{22}}=\frac{0.321s+0.251}{s+0.770} \qquad (53)$$
$$KC_{33}=\frac{\tilde{S}_{V,33}}{\tilde{S}_{33}}=\frac{S_{V33}}{S_{33}}=0.448$$

Auxiliary controllers RP, which ensure decoupling control loop, were determined via (29). To determine the auxiliary controllers RP was used dominant and non-dominant elements of the transfer function matrix $\boldsymbol{G}_S(s)$ (43). Dominant elements of $\boldsymbol{G}_S(s)$ were determined above, i.e. $\tilde{S}_{11}=S_{11}, \tilde{S}_{22}=S_{22}, \tilde{S}_{33}=S_{33}$. Other elements of $\boldsymbol{G}_S(s)$ were considered as non-dominant elements. Thus, transfer function matrix of the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ was determined in the following form

$$\boldsymbol{G}_{RP}(s)=\begin{bmatrix} 0 & RP_{12} & RP_{13} \\ RP_{21} & 0 & RP_{23} \\ RP_{31} & RP_{32} & 0 \end{bmatrix}$$

$$RP_{12}=\frac{S_{12}}{\tilde{S}_{11}}=\frac{S_{12}}{S_{11}}=\frac{0.623s^2+1.014s+0.120}{s^2+2.178s+1.516}$$
$$RP_{13}=\frac{S_{13}}{\tilde{S}_{11}}=\frac{S_{13}}{S_{11}}=\frac{0.440s^2+0.448s+0.0509}{s^2+2.178s+1.516}$$
$$RP_{21}=\frac{S_{21}}{\tilde{S}_{22}}=\frac{S_{21}}{S_{22}}=\frac{-1.349s-1.055}{s+0.770} \qquad (54)$$
$$RP_{23}=\frac{S_{23}}{\tilde{S}_{22}}=\frac{S_{23}}{S_{22}}=\frac{0.00890}{s+0.770}$$
$$RP_{31}=\frac{S_{31}}{\tilde{S}_{33}}=\frac{S_{31}}{S_{33}}=\frac{-1.577}{s+0.863}$$
$$RP_{32}=\frac{S_{32}}{\tilde{S}_{33}}=\frac{S_{32}}{S_{33}}=\frac{-1.395s-0.0351}{s+0.863}$$

The scheme of modified three-variable control loop is generally considered according to Fig. 7.

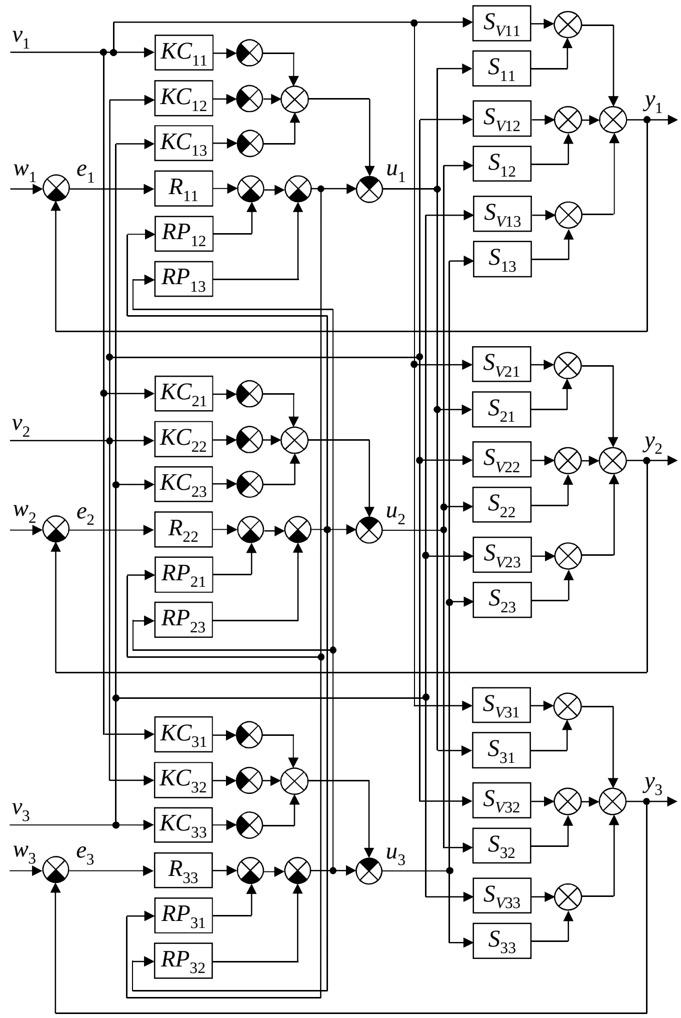

Fig. 7 Modified three-variable branched control loop with measurement of disturbance variables

D. Simulation verification of control loop

The MATLAB/SIMULINK software [17], [18] is used to simulation verification for proposed approach to control of the three-variable control loop (see Fig. 8).

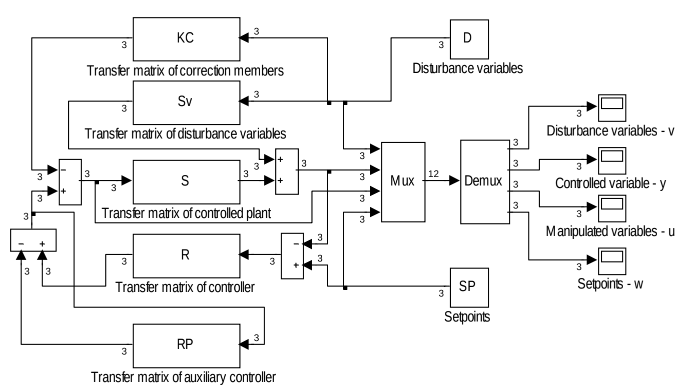

Fig. 8 Simulation scheme of the three-variable control loop in the MATLAB/SIMULINK software

Simulation courses of three-variable control loop of the condensing steam turbine, with utilization of chosen SISO synthesis methods, which are used at design of parameters of the primary controllers, are presented in the following figures (see Fig. 9 - Fig. 12). Fig. 9 and Fig. 11 show simulation courses of three-variable control loop where auxiliary controllers *RP* are not used and it is ensured only approximate invariance of control loop (53). Fig. 10 and Fig. 12 show simulation courses of three-variable control loop where auxiliary controllers *RP* (54) are used and absolute invariance of control loop (51) is ensured.

The following parameters were chosen and used at all simulation experiments (see Fig. 9 - Fig. 12)

- setpoints time vector (t_{w1}, t_{w2}, t_{w3}): [40, 160, 180]
- setpoints vector (w_1, w_2, w_3): [0.7, 0.7, 0.7]
- disturbances time vector (t_{v1}, t_{v2}, t_{v3}): [100, 220, 340]
- disturbances vector (v_1, v_2, v_3): [0.4, 0.4, 0.4]
- time step (k): 0.05
- total simulation time (t_S): 400

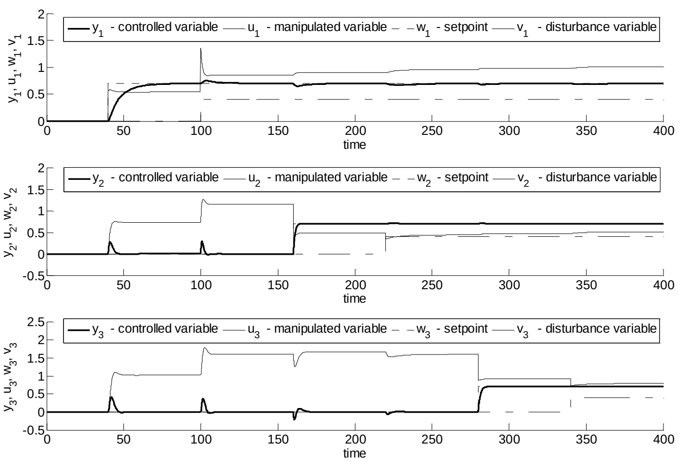

Fig. 9 Simulation courses of control loop with utilization the method of balance tuning (49) without the use of auxiliary controllers *RP* and with the use of correction members *KC* which ensure approximate invariance of control loop (53)

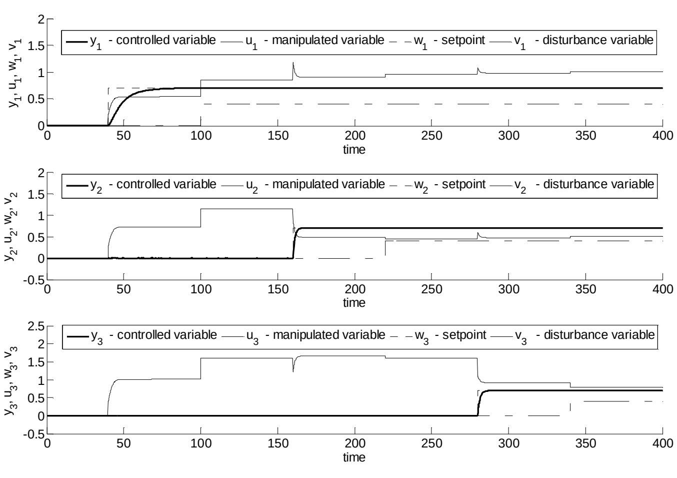

Fig. 10 Simulation courses of control loop with utilization the method of balance tuning (49) with the use of auxiliary controllers *RP* (54) and with the use of correction members *KC* which ensure absolute invariance of control loop (51)

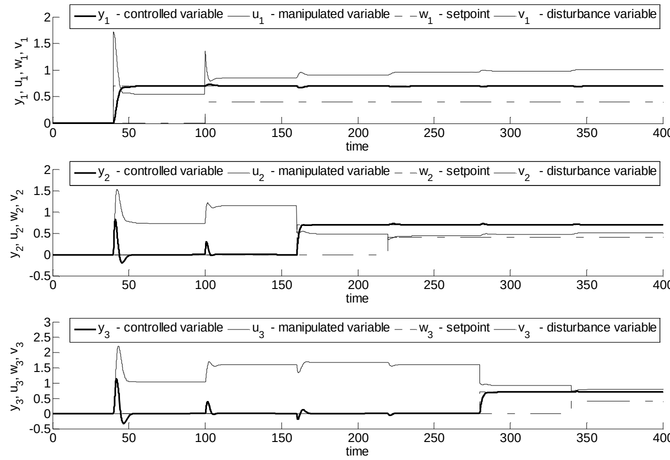

Fig. 11 Simulation courses of control loop with utilization the method of desired model (50) without the use of auxiliary controllers *RP* and with the use of correction members *KC* which ensure approximate invariance of control loop (53)

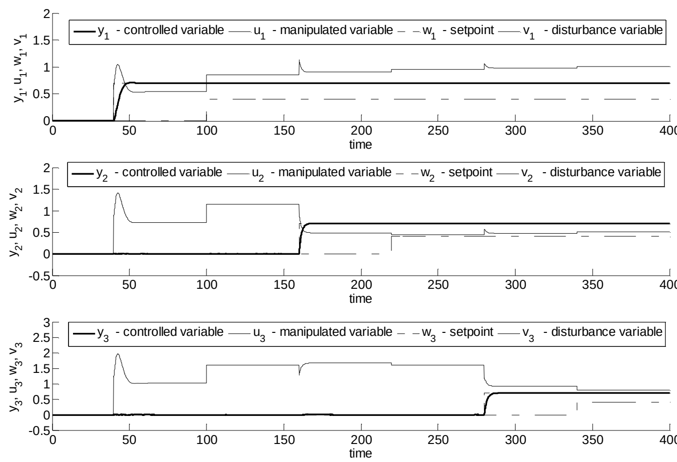

Fig. 12 Simulation courses of control loop with utilization the method of desired model (50) with the use of auxiliary controllers *RP* (54) and with the use of correction members *KC* which ensure absolute invariance of control loop (51)

Variables in the simulation courses of control loop (see Fig. 9 - Fig. 12) correspond to variables described in the three-variable control loop of the condensing steam turbine (see Fig. 4), i.e.

- controlled variable: $y_1 \to \varphi_\omega$, $y_2 \to \varphi_{p_{01}}$, $y_3 \to \varphi_{p_{02}}$
- manipulated variable: $u_1 \to \varphi_{y_{VT}}, u_2 \to \varphi_{y_{ST}}, u_3 \to \varphi_{y_{NT}}$
- setpoints: $w_1 \to \varphi_\omega$, $w_2 \to \varphi_{p_{01}}$, $w_3 \to \varphi_{p_{02}}$
- disturbance variable: $v_1 \to \varphi_{M_G}, v_2 \to \varphi_{m'_{01}}, v_3 \to \varphi_{m'_{02}}$

E. Evaluation of simulation courses and used approach to control of multi-variable control loop

The simulation courses (see Fig. 9 - Fig. 12) were compared by using the ISE criterion (55) and the ITAE criterion (56) (see Table I).

$$J_K = \mathrm{ISE} = \int_0^\infty e^2(t)dt = \int_0^\infty \left[w(t) - y(t)\right]^2 dt \approx \int_0^{t_S} e^2(t)dt \qquad (55)$$

$$J_K = \text{ITAE} = \int_0^{\infty} t \cdot |e(t)|\, dt = \int_0^{\infty} t \cdot |(w(t) - y(t))|\, dt \approx \int_0^{t_S} t \cdot |e(t)|\, dt \quad (56)$$

where t_r is the control time, t_s is the simulation time, $w(t)$ is the setpoint, $y(t)$ is the controlled variable, $e(t)$ is the control error (see Fig. 13).

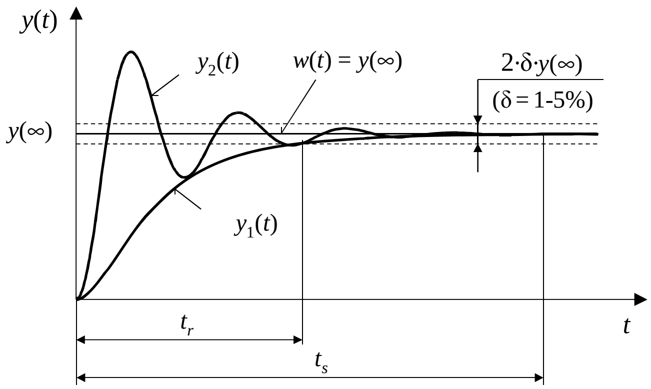

Fig. 13 Possible courses of control loop

Table I Quality of control for simulation courses of three-variable control loop

Fig. No.	Fig. 9	Fig. 10	Fig. 11	Fig. 12
J_{K1} - ISE	2.095	2.553	0.760	0.985
J_{K1} - ITAE	746.056	326.157	226.924	94.818
$t_{r,1}$ (δ = 2%)	32.550	34.800	11.500	8.050
$u_{\max,1}$	1.362	1.186	1.719	1.131
$y_{\max,1}$	0.750	0.700	0.735	0.708
J_{K2} - ISE	0.534	0.293	1.666	0.3508
J_{K2} - ITAE	258.218	132.908	385.663	159.336
$t_{r,2}$ (δ = 2%)	3.600	4.600	6.400	5.500
$u_{\max,2}$	1.263	1.147	1.529	1.411
$y_{\max,2}$	0.709	0.700	0.829	0.700
J_{K3} - ISE	1.047	0.334	3.608	0.400
J_{K3} - ITAE	611.530	270.275	792.435	324.041
$t_{r,3}$ (δ = 2%)	4.750	5.550	7.2	6.600
$u_{\max,3}$	1.788	1.665	2.202	1.969
$y_{\max,3}$	0.706	0.700	1.139	0.7

*) $t_{r,i}$ - control time at change of setpoint w_i (i = 1, 2, 3)

In this case, gained simulation courses of control loop are compared from the point of view of minimal size of ISE criterion or ITAE criterion, further the time of control t_r, maximum values of manipulated variable $y_{\max}$ and controlled variable $u_{\max}$ (see Table I). They can be considered quite different points of view for optimal adjustment. Namely requirements for the smallest overshooting and for the shortest time of control are generally valid for optimal adjustment. However these requirements are antagonistic and therefore the optimal adjustment of controller is always a compromise between them.

The RGA tool (9), (10) or RNGA tool (19), (20) can be used to compare properties of the original MIMO controlled plant $\boldsymbol{G}_S(s)$ (43) and the MIMO control loop from the point of view degree internal coupling. It is considered the MIMO control loop with the use the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ (54) and without the use the auxiliary controllers $\boldsymbol{G}_{RP}(s)$. In this case, the disturbance variables are not considered.

The RNGA matrix of three-variable controlled plant (43) was determined by and (46). The RNGA matrix of closed loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ (6) of three-variable control loop was calculated in the following form, i.e. e.g.

- the RNGA matrix of $\boldsymbol{G}_{W/Y}(s)$ (6), where the MIMO primary controllers $\boldsymbol{G}_R(s)$ (49) were used and the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ was not used

$$\Lambda_N(\boldsymbol{G}_{W/Y}(0)) = \begin{bmatrix} 0.8687 & 0.0794 & 0.0519 \\ 0.0803 & 0.9207 & -0.0001 \\ 0.0510 & -0.0001 & 0.9492 \end{bmatrix} \quad (57)$$

where

$$\boldsymbol{T}_{ar} = \begin{bmatrix} 8.1548 & 19.5229 & 20.0834 \\ 4.1758 & 1.0538 & 7.7476 \\ 5.0657 & 44.8447 & 1.2845 \end{bmatrix}$$

and

$$\boldsymbol{K}(\boldsymbol{G}_{W/Y}(0)) = \begin{bmatrix} 1.0000 & -0.7562 & -0.3052 \\ 1.1616 & 1.0000 & 0.0568 \\ 1.9049 & 0.1211 & 1.0000 \end{bmatrix}$$

- the RNGA matrix of $\boldsymbol{G}_{W/Y}(s)$ (6), where the MIMO primary controllers $\boldsymbol{G}_R(s)$ (49) were used and the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ (54) was used

$$\Lambda_N(\boldsymbol{G}_{W/Y}(0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (58)$$

where

$$\boldsymbol{T}_{ar} = \begin{bmatrix} 9.5402 & \varepsilon & \varepsilon \\ \varepsilon & 1.1621 & \varepsilon \\ \varepsilon & \varepsilon & 1.3564 \end{bmatrix}$$

whereas $\varepsilon > 0$, $\varepsilon \to 0$ [31], and

$$\boldsymbol{K}(\boldsymbol{G}_{W/Y}(0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It is obvious from the simulation courses of the control loop shown in the Fig. 9 - Fig. 12 and from other simulation experiments that the proposed approach to control can be used for control of a control loop.

From the simulation courses of the control loop is obvious that the control loop is absolute invariant (see Fig. 10 and Fig. 12) and also approximate invariant (see Fig. 9 and Fig. 11). In this first case influence of disturbance variables is completely eliminated via separate elements of the transfer function matrix of the correction members $\boldsymbol{G}_{KC}(s)$ (51), i.e. via the correction members KC_{ij}. In this second case influence of disturbance variables is eliminated only at steady state only via three elements of the transfer function matrix of the correction members $\boldsymbol{G}_{KC}(s)$ (53), i.e. via the correction members KC_{11}, KC_{22}, KC_{33}. Correction members were determined from the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ and from transfer function matrix of the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$ by using RGA tool.

From the simulation courses of the control loop (see Fig. 10 and Fig. 12 compare to Fig. 9 and Fig. 11) is also obvious that the control loop is decoupled. It means that the condition of decoupling control loop was fulfilled. Fulfilment this condition was ensured via separate elements of the transfer function matrix of the auxiliary controllers $\boldsymbol{G}_{PR}(s)$ (54), i.e. via the auxiliary controllers RP_{ij}. The auxiliary controllers RP_{ij} were determined from the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ by using RGA tool and also RNGA tool.

Parameters of the transfer function matrix of the primary controller $\boldsymbol{G}_R(s)$ were determined by two SISO synthesis methods (49), (50) only for dominant elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$. These dominant elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ were determined by using RGA tool and also RNGA tool.

Thus, to determination of separate elements of the transfer function matrix of the auxiliary controllers $\boldsymbol{G}_{RP}(s)$ and the correction members $\boldsymbol{G}_{KC}(s)$ is not necessary to know parameters of separate elements of the transfer function matrix primary controllers $\boldsymbol{G}_R(s)$, i.e. a change of parameters of the primary controllers does not affect correction members and auxiliary controllers.

IV. Conclusion

The goal of this paper was to describe and show one of the possible approaches to control of a MIMO control loop, which used the RGA tool, the RNGA tool and eventually also NI tool to determine optimal input-output variable pairings in the MIMO controlled plant, i.e. dominant elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$ and also transfer function matrix of the measurable disturbance variables $\boldsymbol{G}_{SV}(s)$. Advantage of described and used the approach to control is that the change of parameters of separate elements of the transfer function matrix of the primary controllers $\boldsymbol{G}_R(s)$ (e.g. at change SISO synthesis method) does not affect parameters of separate elements of the transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$ and auxiliary controllers $\boldsymbol{G}_{RP}(s)$. This control method enables to use any known SISO synthesis method to determination of parameters of separate elements of the transfer function matrix primary controllers $\boldsymbol{G}_R(s)$ for corresponding dominant elements of the transfer function matrix of the controlled plant $\boldsymbol{G}_S(s)$. The control method combines ensuring decoupling control loop via auxiliary controllers, which are elements of transfer function matrix of the auxiliary controllers $\boldsymbol{G}_{RP}(s)$, and the use of the separate elements of the transfer function matrix of the correction members $\boldsymbol{G}_{KC}(s)$ for ensuring absolute invariance or approximate invariance of MIMO control loop. Simulation verification of proposed control method was presented on three-variable control loop of the condensing steam turbine.

Determined parameters of the matrix controllers $\boldsymbol{G}_R(s)$ and $\boldsymbol{G}_{RP}(s)$ and also the correction members $\boldsymbol{G}_{KC}(s)$ have good results of the control and fulfilled basic control requirements such as the stability, the reference signal tracking and disturbance attenuation.

The described and used control method is valid under the following condition, i.e. this control method is considered for MIMO controlled plants with same number of input and output signals. MIMO controlled plants containing non-minimal phase, transport delay, or having high order dynamics may, in some cases, be also cause of certain limitations of the control method, e.g. from the point of view of ensuring absolute decoupling control loop and also absolute invariance of control loop.

The future work will be focused on the reduction of some limitations of proposed control method, verification of other approach to control of a MIMO control loop and also simulation verification of proposed, let us say, modified version of control method for other MIMO controlled plants, e.g. model of balance platform system [36], the quadruple-tank process [2].

References

[1] J. Balate, *Automatic Control*, 2nd edition, Praha: BEN, 2004 (in Czech)

[2] K. H. Johansson, "The Quadruple-Tank Process: A Multivariable Laboratory Process with an Adjustable Zero," *IEEE Transactions on Control Systems technology*, vol. 8, no. 3, 2000, pp. 456-465.

[3] K. Dutton, S. Thompson and B. Barraclough, *The art of control engineering*. Essex: Addision Wesley Longman, Pearson Education, 1997.

[4] F. G. Shinskey, *Controlling Multivariable Processes*, Instrument Society of America, Research Triangle Park, NC, 1981.

[5] L. Pekar, R. Prokop and P. Dostalek, „Circuit heating plant model with internal delays," *WSEAS Transactions on Systems*, vol. 8, no. 9, 2009, pp. 1093-1104.

[6] C. M. Velez and S. A. Agudelo, "Control and parameter estimation of a mini-helicopter robot using rapid prototyping tools," *WSEAS Transactions on Systems*, Vol. 5, No. 9, pp. 2250-2256.

[7] M. Mansour and W. Schaufelberger, "Software and Laboratory Experiment Using Computers in Control Education," *IEEE Control System Magazine*, vol. 9, no. 3, 1989, pp. 19-24.

[8] M. Kubalcik and V. Bobal, "Adaptive Control of Three-Tank-System Using Polynomial Methods," in *International Federation of Automatic Control, Proceedings of the 17th IFAC World Congress*, Soul, 2008, pp. 5762-5767.

[9] ALSTOM Power, Ltd. *Materials of the company from the project OTROKOVICE*, 1998. (in Czech)

[10] Z. Ibler et al., *Technical Guide for the field of Energy*, 1 part, Praha: BEN, 2002. (in Czech)

[11] J. Kadrnozka and L. Ochrana, *Heating industry*, Brno: CERM, 2001. (in Czech)

[12] J. Kadrnozka and L. Ochrana, *Combined heat and power production*. Brno: CERM, 2001. (in Czech)
[13] K. Åström and T. Hägglund, *PID Controllers: Theory, Design and Tuning*, 2nd edition, North Carolina: Instrument Society of America, Research Triangle Park, 1995.
[14] A. Datta, M.T. Ho and S. P. Bhattacharyya, *Structure and synthesis of PID controllers*, London: Springer-Verlag, 2000.
[15] K. K. Kiong, W. Q. Guo, H. C. Chien and T. Hägglund, *Advances in PID control.* London: Springer-Verlag, 1999.
[16] M. Viteckova and A. Vitecek, *Bases of automatic control*, 2nd edition, Ostrava: VŠB-Technical University Ostrava, 2008. (in Czech)
[17] K. Perutka, *MATLAB for Engineers - Applications in Control, Electrical Engineering, IT and Robotics*, Rijeka: InTech, 2011.
[18] O. Beucher and M. Weeks, *Introduction to MATLAB and SIMULINK*, 3rd edition, Infinity Science Press LLC, Hingham, MA, 2008.
[19] M. Popescu, A. Bitoleanu and M. Dobriceanu, „Matlab GUI application in energetic performances analysis of induction motor driving systems," *WSEAS Trans. on Advances in Engineering Education*, vol. 3, no. 52006, pp. 304-311.
[20] V. Bobal, P. Chalupa, M. Kubalcik and P. Dostal, "*Self-tuning Predictive Control of Nonlinear Servo-motor*," Journal of Electrical Engineering, vol. 61, no. 6, 2010, pp. 365-372.
[21] E. Bristol, "On a new measure of interaction for multivariable process control," *IEEE Transactions on Automatic Control*, vol. 11, no. 1, 1966, pp. 133-134.
[22] S. Skogestad and M. Morari, "Implications of large RGA elements on control elements," *Ind Eng ChemFundam*, 1987, pp. 2323-2330.
[23] M. T. Tham, *An Introduction to Decoupling Control*, University Newcastle upon Tyne, 1999.
[24] F. Dusek and D. Honc, "Transform of system with different number of input and output signals for decentralized control," *Automatization*, vol. 51, no. 7-8, 2008, pp. 458-462. (in Czech)
[25] P. Skupin, W. Klopot and T. Klopot, "Dynamic Matrix Control with partial decoupling," in *Proceedings of the 11th WSEAS Int. Conf. Automation & Information,* 2010, pp. 61-66.
[26] F. G. Shinskey, "Predict distillation column response using relative gains," *Hydrocarbon Processing*, May 1981, pp.196-200.
[27] E. Bristol, "On a new measure of interaction for multivariable process control, " *IEEE Transactions on Automatic Control*, vol. 11, no. 1, 1966, pp. 133-134.
[28] T. Glad and L. Ljung, *Control Theory: Multivariable and Nonlinear Methods.* NY: Taylor and Francis, 2000.
[29] K. E. Haggblom, "Limitations and Use of the RGA as a Controllability Measure," in *Prepr. Automation Days,* Helsinki, 1995, pp. 178-183.
[30] J. Zakucia, "Control of the Quadruple-Tank Process, " diploma work, Dept. Cont. Eng.,CTU in Prague, 2009 (in Czech).
[31] W. Hu, W. J. Cai and G. Xiao, "Decentralized Control System Design for MIMO Processes with Integrators/Differentiators," *Industrial & Engineering Chemistry Research,* vol. 49, no. 24, pp.12521-12528, 2010.
[32] M. J. He, W. J. Cai, W. Ni and L. H. Xie, "RNGA based control system configuration for multivariable processes, ". *J. Process Control*, vol.19, no. 6, 2009.
[33] K. Åström and T. Hägglund, *PID Controllers: Theory, Design and Tuning*, 2nd edition, North Carolina: Instrument Society of America, Research Triangle Park, 1995.
[34] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design.* 2nd edition, Chichester: Wiley, 2005.
[35] P. Klan and R. Gorez, Balanced Tuning of PI Controllers. *European Journal of Control*, Vol. 6, No. 6, 2000, pp. 541-550.
[36] A. Kot and A. Nawrocka, "Balace Platform System Modeling and Simulation," in *Proceedings of the 12 International Carpathian Control Conference*, 2011, pp. 224-227.

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