Analytical Synthesis of Digitally Programmable Versatile-Mode High-Order OTA-Equal C Universal Filter Structures with the Minimum Number of Components

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Abstract: How to simultaneously involve and merge four distinct kinds of modes, i.e., voltage, current, trans-admittance, and trans-impedance modes, into an *analytical synthesis method* (ASM) for synthesizing a complicated versatile mode high-order universal (namely, low-pass, band-pass, high-pass, band-reject, and all-pass) filter transfer function is presented in this paper employing (i) all single-ended-input operational transconductance amplifiers (OTAs), (ii) all grounded capacitors, and (iii) the minimum number of active and passive components. The new filter structure enjoys the important merits of digital programmability and low sensitivity in addition to have a controllable gain (except the current-mode one), cascadable voltage input and current output terminals, and the absence of the need to impose component matchings. H-spice frequency-dependent simulation results, using TSMC 0.18μm process and ±0.9V supply voltages, demonstrate precise filtering responses (for example, the current-mode high-pass response has only 0.3% error) at the operating frequency, 10 MHz.

I. INTRODUCTION

Over the last decade or so numerous voltage or current-mode filter structures have been reported in the references of [1-10]. From historical progress point of view, we might intend to replace the traditional voltage-mode circuits with the more precise current- mode circuits. There might be a transferring period from voltage-mode to current-mode world. Then, the trans-admittance or trans-impedance mode will be involved in that between voltage and current modes. Therefore, the versatile mode (including voltage, current, trans-admittance, and trans-impedance modes) circuits are worthy of researches and presented for the use of the readers.

Few of versatile mode filters have been proposed [1-10] in the recent decade. In 1996, Soliman proposed two trans-impedance-mode low-pass, band-pass, and high- pass filter biquads [2] employing four current conveyors, two grounded capacitors and four resistors. Recently, a new active element called the fully differential current conveyor (FDCCII) has been reported [3] to improve the dynamic range in versatile mode applications where fully differential signal processing is required. The application of FDCCII in filter design was also demonstrated in [3], where three trans-admittance/voltage-mode low-pass, band-pass, and high-pass filter biquads were described. Each biquad uses three FDCCIIs, two grounded capacitors and three resistors. The very recently reported generalized versatile mode biquad [1] uses lots of active elements, i.e., seven current conveyors, and lots of resistors, i.e., eight resistors, in addition to two grounded capacitors. To condense the components count of the versatile mode biquad [1] is one of the main research approaches. It is apparent that the extension of the above versatile mode filter from the second order to the nth order is a naturally induced motivation of advanced research.

The special active element, *operational trans-conductance amplifier* (OTA), is equivalent to an ordinary active element such as a second-generation current conveyor (CCII) and a resistor. The use of OTAs in an analogue circuit enjoys the benefit: no need to use resistors in the design, leading to a much more condensed circuit structure than the use of many other kinds of active elements like current conveyors. In this paper, the OTA is selected to act as the main role on the analytical synthesis [5-10] of versatile-mode high-order universal filter structures.

Since a versatile mode circuit has a voltage and a current signal inputs and a voltage and a current signal outputs, how to construct such a versatile mode structure by a simplest arrangement is an important matter. Due to the input (resp. output) of an OTA is a voltage (resp. current) signal, the first input OTA and the last output OTA are relevant to this issue. Therefore, if the transconductance-mode OTA-C universal filter structure is with the voltage input (which is also the input of the first OTA) and a current output (which is also the output of the last OTA) and has the most condense structure, then the trans-impedance mode one, which is located in the transconductance-mode filter structure stated above and then with a voltage output and a current input, should have the simplest structure in these four different modes if using OTAs to do the design. There are at least n independent coefficients existed in the current-mode nth-order universal transfer function, referring to (1). This means that at least n distinct trans-conductances are needed for the current-mode one. Then, the minimum number of different trans-conductances are n+1 for synthesizing an nth-order trans-impedance mode universal filter structure since $V_{out}b=I_{out}$, i.e., one more trans-conductance b is needed. And since it is also needed to add one OTA in front of the trans-impedance mode filter structure for constructing a voltage-mode one due to the input (resp. output) of an OTA is a voltage (resp. current) signal. Therefore, the minimum element versatile-mode OTA-C filter structure must be with n+2 OTAs in addition to n capacitors.

The very recently reported *analytical synthesis method* (ASM) [5-10] has been illustrated to be very helpful to simultaneously achieve the three important criteria [4]: (i) all single-ended-input OTAs (overcoming the feed-through effects), (ii) all grounded capacitors (absorbing shunt parasitic capacitances), and (iii) the minimum number of active and passive component counts (reducing power consumption, chip area, and noise), for reducing the parasitics of OTA-C filters. The new ASM exhibits that the complicated nth-order trans-impedance transfer function is manipulated and decomposed by a succession of innovative algebra operations until a set of simple equations are produced which are then realized using n integrators and a constraint circuitry. As the cases shown before [5-10], the new versatile-mode nth-order OTA-C universal filter structure, obtained from the fundamental trans-impedance mode one, achieves the above three important criteria simultaneously as well.

II. VERSATILE-MODE ASM

As we would like to construct a versatile mode (including voltage, current, trans-admittance, and trans-impedance modes) OTA-C filter structure with the minimum number of active and passive components, first, we need to notice that the output (resp. input) signal of an OTA is a current (resp. voltage) signal. Then, the minimum component versatile mode OTA-C filter structure must utilize the voltage input and the current output of the OTA at the input (resp. output) port to act as the voltage input (resp. output) and the current input (resp. output) of the whole structure, respectively. In addition to these, the trans-impedance mode (having V_{out} and I_{in}) OTA-C filter structure, belonging to the centre part from the output of the OTA at the input port to the input of the OTA at the output port, need have the minimum number of active and passive components, i.e., one more OTA than the nth-order current-mode OTA-C universal filter structure whose minimum number of both single-ended-input OTAs and grounded capacitors is n [5].

In this section, the author would like to present a new *analytical synthesis method* (ASM) which can realize a trans-impedance-mode high-order single-ended-input OTA and equal-C universal (low-pass, band-pass, high-pass, band-reject, and all-pass) filter structure which has the minimum number of active and passive components leading to the minimum parasitics and then the most precise output response. Equal capacitance type filter structure is used in the design for avoiding from the difficulty to fabricate several capacitors without fixed-ratio capacitances in an integrated circuit. The given trans-impedance-mode nth-order all-pass filter transfer function is shown as below.

$$V_{out}=\left(\frac{1}{b}\right)\left(\frac{(-1)^n s^n I_{in(n)}+\sum_{i=0}^{n-1}(-1)^i a_i s^i I_{in(i)}}{s^n+\sum_{i=0}^{n-1}a_i s^i}\right) \qquad (1)$$

Since $V_{\mathrm{out}}b=I_{\mathrm{out}}$, Eq. (1) can be transferred to a current-mode nth-order all-pass filter transfer function as below.

$$I_{out}=\left(\frac{(-1)^n s^n I_{in(n)}+\sum_{i=0}^{n-1}(-1)^i a_i s^i I_{in(i)}}{s^n+\sum_{i=0}^{n-1}a_i s^i}\right) \qquad (2)$$

Observing both (1) and (2), n+1 OTAs (for realizing the n+1 different coefficients, a_0, a_1, ..., a_{n-2}, a_{n-1}, and b) and n capacitors (for realizing the n power of s) are the minimum components to design such a trans-impedance and current mode nth-order filter transfer function shown in both (1) and (2). The new *analytical synthesis method* (ASM) for the trans-impedance mode one shown in (1) is presented as follows.

Cross product (1),

$$\begin{aligned}&V_{out}b\begin{pmatrix}s^n+a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+\\ ......+a_2s^2+a_1s+a_0\end{pmatrix}=\\ &(-1)^n s^n I_{in(n)}+(-1)^{n-1}a_{n-1}s^{n-1}I_{in(n-1)}+...+\\ &(-1)^2 a_2 s^2 I_{in(2)}+(-1)^1 a_1 s I_{in(1)}+(-1)^0 a_0 I_{in(0)}\end{aligned} \qquad (3)$$

Divide (3) by s^n, combine the same order terms, and re-arrange,

$$\begin{aligned}&\left[bV_{out}+(-1)^n I_{in(n)}\right]+\left(\frac{a_{n-1}}{s}\right)\left[bV_{out}+(-1)^{n-1}I_{in(n-1)}\right]+\\ &\left(\frac{a_{n-2}}{s^2}\right)\left[bV_{out}+(-1)^{n-2}I_{in(n-2)}\right]+...+\left(\frac{a_2}{s^{n-2}}\right)\left[bV_{out}+(-1)^2 I_{in(2)}\right]+\\ &\left(\frac{a_1}{s^{n-1}}\right)\left[bV_{out}+(-1)^1 I_{in(1)}\right]+\left(\frac{a_0}{s^n}\right)\left[bV_{out}+(-1)^0 I_{in(0)}\right]=0\end{aligned} \qquad (4)$$

Since $\frac{a_0}{s^n}=\left(\frac{a_{n-1}}{s}\right)\left(\frac{a_{n-2}}{a_{n-1}s}\right)\left(\frac{a_{n-3}}{a_{n-2}s}\right)......\left(\frac{a_1}{a_2s}\right)\left(\frac{a_0}{a_1s}\right)$ and similar decompositions for $\frac{a_{n-i}}{s^i}$, the former three terms of (4) can be re-combined as follows.

$$\left[bV_{out}+(-1)^n I_{in(n)}\right]+\left(\frac{a_{n-1}}{s}\right)\left[\begin{matrix}\left[bV_{out}+(-1)^{n-1}I_{in(n-1)}\right]+\\ \left(\frac{a_{n-2}}{a_{n-1}s}\right)\left[bV_{out}+(-1)^{n-2}I_{in(n-2)}\right]\end{matrix}\right] \qquad (5)$$

Then, (4) becomes

$$\begin{aligned}&\left[bV_{out}+(-1)^n I_{in(n)}\right]+\\ &\left(\frac{a_{n-1}}{s}\right)\Bigg[\left[bV_{out}+(-1)^{n-1}I_{in(n-1)}\right]+\left(\frac{a_{n-2}}{a_{n-1}s}\right)\Bigg[\left[bV_{out}+(-1)^{n-2}I_{in(n-2)}\right]+\\ &\left(\frac{a_{n-3}}{a_{n-2}s}\right)\Bigg[\Bigg[...\left(\frac{a_2}{a_3s}\right)\Bigg[\left[bV_{out}+(-1)^2 I_{in(2)}\right]+\left(\frac{a_1}{a_2s}\right)\Bigg[\left[bV_{out}+(-1)^1 I_{in(1)}\right]+\\ &\left(\frac{a_0}{a_1s}\right)\left[bV_{out}+(-1)^0 I_{in(0)}\right]\Bigg]\Bigg]...\Bigg]\Bigg]\Bigg]\Bigg]=0\end{aligned} \qquad (6)$$

(6) can be equal to (7) shown as below.

$$\begin{aligned}&\left[(-1)^n bV_{out}+I_{in(n)}\right]+\\ &\left(\frac{-a_{n-1}}{s}\right)\Bigg[\left[(-1)^{n-1}bV_{out}+I_{in(n-1)}\right]+\left(\frac{-a_{n-2}}{a_{n-1}s}\right)\Bigg[\left[(-1)^{n-2}bV_{out}+I_{in(n-2)}\right]+\\ &\left(\frac{-a_{n-3}}{a_{n-2}s}\right)\Bigg[\Bigg[...\left(\frac{-a_2}{a_3s}\right)\Bigg[\left[(-1)^2 bV_{out}+I_{in(2)}\right]+\left(\frac{-a_1}{a_2s}\right)\Bigg[\left[(-1)^1 bV_{out}+I_{in(1)}\right]+\\ &\left(\frac{-a_0}{a_1s}\right)\left[(-1)^0 bV_{out}+I_{in(0)}\right]\Bigg]\Bigg]...\Bigg]\Bigg]\Bigg]\Bigg]=0\end{aligned} \qquad (7)$$

We notice that all I_{in}'s in (7) are with positive signs. It means that only normal current input signals are required (otherwise, we need inverting type current input signals to do the design). (7) can also be easily transferred to (8) as below.

$$\begin{aligned}&\left[(-1)^n bV_{out}+I_{in(n)}\right]+\\ &\left(\frac{-a_{n-1}}{s}\right)\Bigg[\left[(-1)^{n-1}bV_{out}+I_{in(n-1)}\right]+\left(\frac{-\frac{a_{n-2}}{a_{n-1}}}{s}\right)\Bigg[\left[(-1)^{n-2}bV_{out}+I_{in(n-2)}\right]+\\ &\left(\frac{-\frac{a_{n-3}}{a_{n-2}}}{s}\right)\Bigg[\Bigg[...\left(\frac{-\frac{a_2}{a_3}}{s}\right)\Bigg[\left[(-1)^2 bV_{out}+I_{in(2)}\right]+\left(\frac{-\frac{a_1}{a_2}}{s}\right)\Bigg[\left[(-1)^1 bV_{out}+I_{in(1)}\right]+\\ &\left(\frac{-\frac{a_0}{a_1}}{s}\right)\left[(-1)^0 bV_{out}+I_{in(0)}\right]\Bigg]\Bigg]...\Bigg]\Bigg]\Bigg]\Bigg]=0\end{aligned} \qquad (8)$$

Note that all the coefficients in front of s in (8) are unity. Equal capacitance type structure prevents from the difficulty to manufacturing precise different capacitances in integrated circuits. Hence, we can let

$$\frac{1}{s}\left[(-1)^0 bV_{out}+I_{in(0)}\right]\equiv V_0 \qquad (9\text{-}0)$$

$$\frac{1}{s}\left[(-1)^i bV_{out} + I_{in(i)} + \left(-\frac{a_{i-1}}{a_i}\right)V_{i-1}\right] \equiv V_i$$

for i=1,2,…,n-1. (9-i)

and $(-1)^n bV_{out} + I_{in(n)} + (-a_{n-1})V_{n-1} = 0$ (9-n)

(9-0) can be realized by a single-ended-input OTA with the trans-conductance b and a grounded capacitor with unity capacitance. (9-1) is also synthesized using two single- ended-input OTAs with the trans-conductances, b and a_0/a_1, and a grounded capacitor with unity capacitance. All other equations from (9-2) to (9-n) are similar to (9-0) or (9-1) and can be easily designed using single-ended-input OTAs and one grounded capacitor if necessary. The combination of the sub-circuitries realized from (9-0) to (9-n) is shown in Fig. 1. It is apparent that only single-ended-input OTAs, grounded capacitors (with unity capacitance leading to more precise capacitors fabricated in the IC), and the minimum numbers, n+2 and n, of active and passive components, respectively.

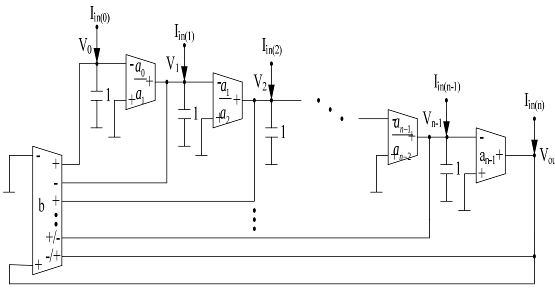

Figure 1 Trans-impedance-mode nth-order OTA-C all-pass filter structure

Note that the +, -, +, -, … sign sequence from s^0 to s^n terms in the numerator of (1) is just with the same +, -, +, -, … sign sequence from the top to the bottom outputs of the OTA with the trans-conductance b in Fig. 1. Then, if we would like to change one sign in the numerator of (1), we just need change the corresponding sign of the output terminals of the OTA with the trans-conductance b in Fig. 1. In other words, if we would like to realize the following transfer function,

$$V_{out} = \left(\frac{1}{b}\right)\left(\frac{s^n I_{in(n)} + \sum_{i=0}^{n-1} a_i s^i I_{in(i)}}{s^n + \sum_{i=0}^{n-1} a_i s^i}\right) \quad (10)$$

in which all the signs in the numerator of (10) are plus (+), then all the signs at the outputs of the OTA with the trans-conductance b are plus (+) as well. Since the numerator of the low-pass, band-pass, high-pass, and notch filter transfer functions is with all positive coefficients, the filter structure of Fig. 2 can synthesize low-pass, band-pass, high-pass, and notch filter transfer functions. Note that it doesn't need any component-value constraints to do the design. As we compare between the trans-conductances in Fig. 1 and the coefficients of the denominator of the corresponding transfer function (1), the filtering parameters of Fig. 1 are with the characteristic of orthogonal control.

Current-mode nth-order OTA-C filter structures can be easily obtained just taking out an output current signal from the OTA with the trans-conductance b in Figs. 1 and 2. The new current-mode nth-order OTA-C universal filter structures without using inverting input current or double input current signals unlike [5] while the recently reported one [5] only

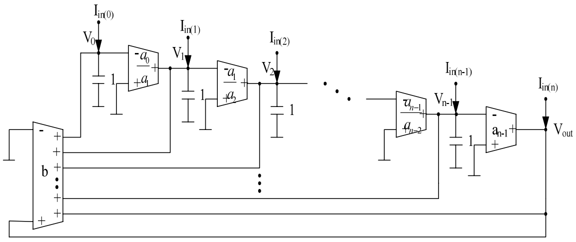

Figure 4 Trans-impedance-mode nth-order OTA-C LP, BP, HP, and NH filter structure

employs n, one fewer than the new one, single-ended-input OTAs.

Voltage and trans-admittance-mode nth-order OTA-C universal filter structure can also be easily secured putting an additional OTA at the input port of the current and trans-impedance-mode one to do the voltage-to-current transformation and shown in Figs. 3 and 4. We have seen that there are n+1 switches, S_0, S_1, S_2,…, S_n, existed in Figs. 3 and 4, each switch controls one special order signal. It means that the new versatile-mode nth-order OTA-C universal filter structure is digitally programmable like [8].

The filter structures shown in Figs. 3 and 4 can be a voltage, current, trans-impedance, and trans-admittance mode, namely, versatile-mode, one, in which there are high input impedance for a voltage input and high output impedance for a current output cascadabilities. Moreover, the gains of the voltage, trans-impedance, and trans-admittance modes except the current-mode one are adjustable.

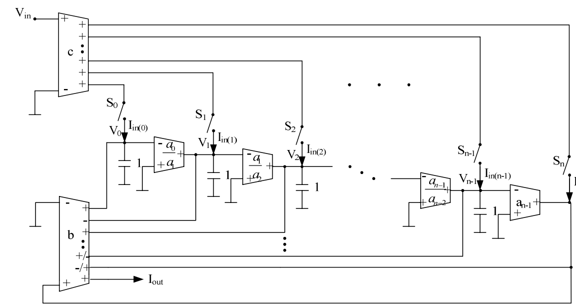

Fig. 3 Versatile-mode nth-order OTA-C AP filter structure

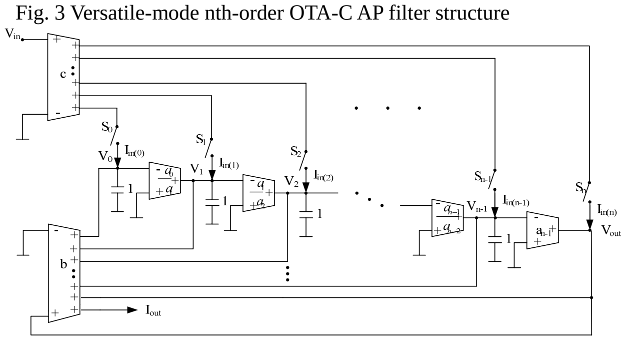

Fig. 4 Versatile-mode nth-order OTA-C LP, BP, HP, and NH filter structure

III. HSPICE SIMULATIONS

To validate the theoretical predictions, H-spice simulations with TSMC 0.18μm process are used. We employ the CMOS implementation of a trans-conductor reported in [11], with ±0.9 V supply voltages and W/L=5μ/1μ and 10μ/1μ for NMOS and PMOS transistors, respectively. The component values for the sixth-order universal Butterworth filter structure derived from Figs. 3 and 4 are given by C_1=C_2=C_3= C_4=C_5=C_6=2pF, and g_{a1}=32μS, g_{a2}=66μS, g_{a3}=102.5μS, g_{a4}=153μS, g_{a5}=243μS, g_{a6}=485μS, g_{a7}= 485μS (for CM and TI modes) or 456μS (for VM and TA modes), and g_{a7}=456μS (for VM and TA modes only) for the theoretical f_o=10M Hz. The simulation results for the current (all-pass phase-frequency response shown in Fig. 5), trans-impedance, voltage (all-pass phase-frequency response shown in Fig. 6), and trans-admittance modes have the different errors, shown in Table 1, compared to the theoretical f_o=10M Hz. As can be seen, the current-mode one has the most precise response which is better than its voltage-mode counterpart and other two modes, namely, trans-impedance and trans-admittance modes. Note that the operational frequency has been extended to 10M Hz from the previous 1M Hz and still has rather good precision.

Table 1 Error comparison between four distinct modes

mode / filter	CM error	TI error	VM error	TA error
LP	2.9%	2.8%	4.0%	5.3%
HP	0.3%	3.5%	2.0%	1.6%
BP	0%	0%	4.6%	4.5%
NH	2.3%	2.3%	2.3%	2.3%
AP	1.2%	1.2%	1.1%	1.7%

Moreover, the sensitivity simulations, shown in Fig. 7, with tolerances of -10% for C_1, -10% for C_2, -10% for g_1, and -10% for g_2, show that the simulated 3dB frequencies and errors are (i) 10.46M Hz (5.30% error), (ii) 9.92M Hz (0.126% error), (iii) 9.40M Hz (5.40% error), and (iv) 9.925M Hz (0.07% error), respectively, for the current-mode second-order OTA-C low-pass filter with C_1=C_2=4pF, g_1=177μS, and g_2=355μS, having the 3dB nominal frequency at 9.933M Hz. The above sensitivity simulation results demonstrate low sensitivity performance.

IV CONCLUSIONS

None of the previous papers have reported such a versatile-mode (including voltage, current, trans-impedance, and trans-admittance modes) high-order OTA-C universal filter structure using the recently reported *analytical synthesis method* (ASM). The new filter structure employs the minimum number of active and passive components, i.e., n+2 single-ended-input OTAs and n grounded capacitors which can simultaneously realize versatile-mode nth-order low-pass, band-pass, high-pass, notch, and all-pass responses from the same configuration. The new filter structure enjoys nearly all of the main advantages: (i) utilizing the single-ended-input OTAs overcoming the feed-through effects, (ii) employing grounded capacitors attractive for integration and absorbing shunt parasitic capacitance, (iii) using the minimum number of active and passive components, (all of the above three advantages lead to the minimum total parasitics and have the most precise output responses), (iv) digital programmability, (v) both high input impedance of the voltage input and high output impedance of the current output good for cascadability, (vi) orthogonally controllable filtering parameters, (vii) no need for changing the filter topology, (viii) no component-value constraints, (ix) no need for inverting amplifiers or double current input signals for special input requirements, (x) a controllable gain except the current-mode one, (xi) low sensitivity performance, and (xii) precise operation at 10M Hz. H-Spice simulations with TSMC018 process and ± 0.9V supply voltages are included and confirm the theoretical predictions.

References

[1] M. T. Abuelma'atti, A. Bentrcia, and S. M. Al-Shahrani, "A novel mixed-mode current-conveyor-based filter", *Int. J. Electron.*, vol. 91, no. 3, pp. 191-197, 2004.

[2] A. M. Soliman, " Mixed-mode biquad circuits," *Microelectronics Journal*, vol. 27, pp. 591-594, 1996.

[3] A.. A. El-Adawy, A. M. Soliman, and H. O. Elwan, " A novel fully differential current conveyor and applications for analog VLSI", *IEEE Trans. Circuits Syst. II*, vol.47, no. 4, pp. 306-313, 2000.

[4] Y. Sun, and J. K. Fidler, "Synthesis and performance analysis of universal minimum component integrator-based IFLF OTA-grounded capacitor filter", IEE Proc.-Circuits Devices Syst., vol. 143, no. 2, pp. 107-114, 1996.

[5] C. M. Chang, and B. M. Al-Hashimi, "Analytical synthesis of high-order current-mode OTA-C filters", IEEE Trans. Circuits Syst.-I, vol. 50, no. 9, pp. 1188-1192, Sep. 2003.

[6] C. M. Chang, B. M. Al-Hashimi, Y. Sun, and J. N. Ross, "New high-order filter structures using single-ended-input OTAs and grounded capacitors", IEEE Trans. Circuits & Syst.-II vol. 51, no.9, pp. 458-463, Sep. 2004.

[7] C. M. Chang, C. L. Hou, W. Y. Chung, J. W. Horng, and C. K. Tu, "Analytical synthesis of high-order single-input OTA-grounded C all-pass and band-reject filter structures", IEEE Trans. Circuits & Syst.-I, vol. 53, no. 3, pp. 489-498, March 2006.

[8] C. M. Chang, "Analytical synthesis of the digitally programmable voltage-mode OTA-C universal biquad", IEEE Trans. Circuits Syst.-II, vol. 53, no. 8, pp. 607-611, Aug. 2006.

[9] C. M. Chang, A. M. Soliman, and M. N. S. Swamy, "Analytical synthesis of low sensitivity high-order voltage-mode DDCC and FDCCII-grounded R and C all-pass filter structures", IEEE Trans. Circuits & Syst.-I, vol. 54, no. 7, pp 1430-1443, July 2007.

[10] S. H. Tu, C. M. Chang, N. J. Ross, M. N. S. Swamy, "Analytical synthesis of current-mode high-order single-ended-input OTA and equal-capacitor elliptic filter structures with the minimum number of components", IEEE Trans. Circuits & Syst.-I, vol. 54, no. 10, pp. 2195-2210, Oct. 2007.

[11] S. Szczepanski, A. Wyszynski, and R. Schaumann, "Highly linear voltage-controlled CMOS transconductors," IEEE Trans. Circuits Syst.-I, vol. 40, no. 4, pp. 258-262, Apr. 1993.

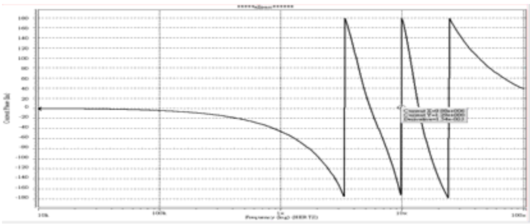

Fig. 5 Phase response of the new current-mode sixth-order OTA-C all-pass filter

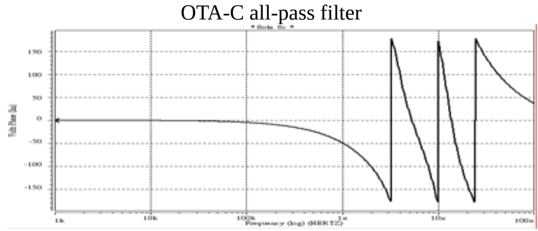

Fig. 6 Phase response of the new voltage-mode sixth-order OTA-C all-pass filter

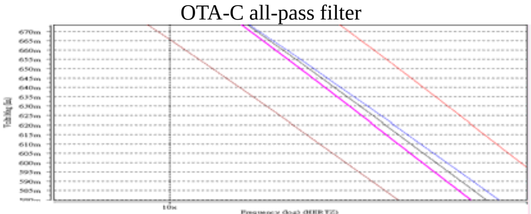

Fig. 7 Sensitivity simulations for Fig. 3 (black: nominal, red: C_1, blue: C_2, brown g_1, and pink g_2, with -10% tolerances, respectively)