Generalized Sampling Kernels for Designing of Sharp FIR Digital Filters with Wide Passband

Kyoung-Jae Kim, Jae-Beom Seo, and Sang-Won Nam

***Abstract*—**A new design of a computationally efficient FIR filter with wide passband and sharp transition is proposed by utilizing generalized sampling kernels and the IFIR filter structure. In particular, the proposed filter design approach provides a closed-form expression for the filter coefficients by employing the generalized sampling kernel and yields a practical design procedure for linear-phase sharp FIR digital filters with wide passband.

***Keywords*—** IFIR structure, interpolation, linear-phase, sampling kernel, sharp transition, wide-band FIR filters

I. Introduction

PRACTICAL design of linear-phase sharp FIR filters have been much studied in communication and signal processing areas. Some problems in designing and implementing those sharp FIR filters include high computational complexity due to the required long filter length. Accordingly, several computationally efficient FIR filter design approaches were reported to reduce the computational complexity required in implementing them (e.g., the prefilter-equalizer [1], the interpolated FIR (IFIR) [2], and the frequency-response masking (FRM) technique [3]). While those approaches yield relatively high computational efficiency; it may not be easy to utilize them further for the design of reconfigurable FIR digital filters, since all subfilters obtained in the derivation of such computationally efficient filters should be separately and differently redesigned to accommodate different filter specifications [4]. Recently, an efficient FIR filter design method was proposed in [5] for the design of linear-phase sharp FIR filters and utilized for the design of dc-notch FIR filters, which incorporates a modified sampling kernel into the filter design, providing a simpler design procedure than conventional design methods [6].

In this paper, a new design of computationally efficient wide-band FIR filter with sharp transition is proposed, whereby generalized sampling kernels introduced are obtained by putting the sampling kernel of [6] to practical use and by employing the complementary filter concept [3]. More specifically, the proposed approach provides a closed-form solution for filter coefficients, yielding a practical design procedure for linear-phase sharp FIR digital filters with wide passband.

This paper is organized as follows: First, design of sharp FIR filters using a modified sampling kernel is discussed in Section 2. Also, a new design of wide-band sharp FIR filters using generalized sampling kernels is introduced in Section 3. Section 4 presents a wide-band FIR filter with sharp transition designed by the proposed approach. Finally, the conclusion is drawn in Section 5.

II. Design of Sharp Linear-Phase FIR Filters using a Modified Sampling Kernel

Let's consider the following two discrete-time signals ($h[n]$ and $h_\alpha[n]$) sampled at different sampling rates:

$$h[n]=h(nT)\,,\quad h_\alpha[n]=h(nT'),\quad T'=T/\alpha \tag{1}$$

In (1), when $\alpha>1$, derivation of $h_\alpha[n]$ from $h[n]$ corresponds to interpolation. Recently, a single-step closed-form equation to obtain $h_\alpha[n]$ from $h[n]$ was derived in [6]: That is,

$$h_\alpha[n] \;=\; \sum_{k=0}^{N-1} h[k]\,sinc\,(\frac{n}{\alpha}-k) \tag{2}$$

Also, $sinc((n/\alpha)-k)$ in (2), called an interpolation sampling kernel, was further utilized in [6] to design computationally efficient linear-phase FIR filters with sharp transition. In particular, let's consider the following FIR filter $h_{(\alpha)}[n]$, when a prototype model filter $h[n]$ is given by a linear-phase (Type 1) FIR filter of length *N*:

$$h_{(\alpha)}[n] \;=\; \frac{1}{\alpha}h_\alpha[n] \;=\; \sum_{k=0}^{N-1} h[k]\bullet\frac{1}{\alpha}sinc(\frac{n}{\alpha}-k) \tag{3}$$

After taking the discrete-time Fourier transform (DTFT) of both sides of (3), we can see that $H_{(\alpha)}[e^{j\omega}]=H[e^{j\alpha\omega}]$ (also, see Fig. 1(a) and Fig, 1(b)). Furthermore, it was shown in [6] that, when α is an integer, (3) can be expressed in the following convolution form:

$$\begin{aligned} h_{(\alpha)}[n] \;&=\; \frac{1}{\alpha}\sum_{k=0}^{N-1} h[k]\,sinc(\frac{1}{\alpha}(n-\alpha k)) \\ &= \frac{1}{\alpha}\sum_{k=0}^{N-1} h_u[k]\,sinc(\frac{1}{\alpha}(n-k)) = \frac{1}{\alpha}\,h_u[n]*sinc(\frac{n}{\alpha}) \end{aligned} \tag{4}$$

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K.-J. Kim, J.-B. Seo, and S.-W. Nam are with the Department of Electronics and Computer Engineering, Hanyang University, Seoul, 133-791, Korea (e-mail: swnam@hanyang.ac.kr).

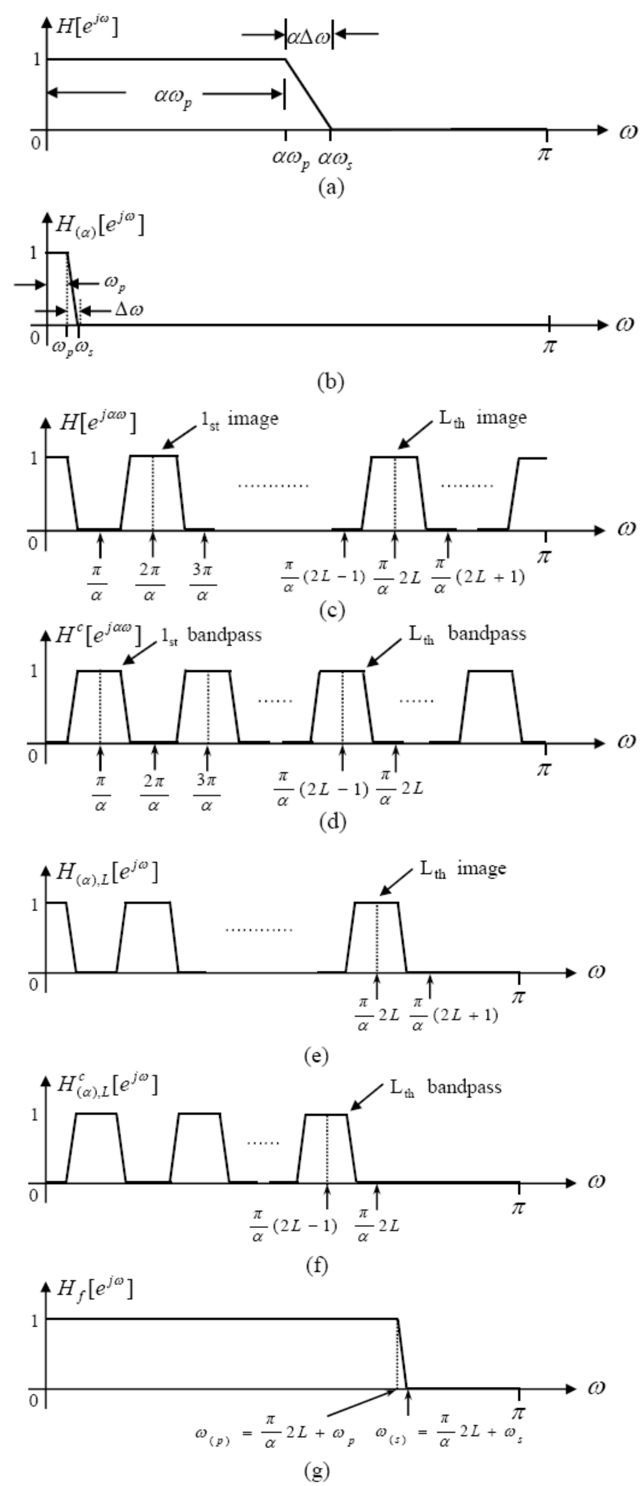

Fig. 1 Filter design procedure: Gain responses.

In (4), $*$ indicates the linear convolution operation, and $h_u[n]$ is obtained by inserting ($\alpha - 1$) zeros between adjacent samples of $h[n]$. Note that $H[z^{\alpha}]$ corresponds to the Z-transform of $h_u[n]$, and (4) can be implemented based on the conventional IFIR filter structure [2]. In particular, conventional computationally efficient filter design methods (e.g., IFIR and FRM) are very effective in the design of sharp FIR filters with fixed coefficients. However, it may not be easy to utilize conventional design methods further for the design of reconfigurable FIR filters allowing a great flexibility to the filter design. To solve the problem, generalized sampling kernels, derived by revising the sampling kernel of [5, 6], and the complementary filter concept [3] were employed in this paper for the design of a wide-band FIR filter with sharp transition. The convolution expression (4) can be used to design a computationally efficient FIR filter with computational complexity similar to or a bit more than those conventional approaches. In particular, since $sinc(n/\alpha)$ in (2) (i.e., an ideal low-pass filter) is of doubly infinite length, it should be modified for the design of a sharp linear-phase FIR filter of finite order. Accordingly, one of widely used adjustable window functions whose impulse responses are of finite length [7] is employed in this paper as a lowpass filter. In the next section, a more detail procedure related to the adjustable window functions and generalized sampling kernels are discussed.

III. Design of a Wide-band Sharp FIR Filter using a Generalized Sampling Kernel

From (3) and Fig. 1(b), we can see that $h_{(\alpha)}[n]$ can be obtained by taking the inverse DTFT of only the lowpass part (i.e., $\omega \in [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}]$) of $H[e^{j\alpha\omega}]$: That is,

$$h_{(\alpha)}[n] = \frac{1}{2\pi}\sum_{k=0}^{N-1} h[k] \int_{-\frac{\pi}{\alpha}}^{\frac{\pi}{\alpha}} e^{j(n-\alpha k)\omega'} d\omega' \qquad (5)$$

Furthermore, the concept in (5) can be extended to the [*single-lowpass* + *multi-bandpass*] (or multi-image as in Fig. 1(c)-(d)) case by changing the upper and lower bounds of the definite integral of (5) to include up to L-th images in addition to the lowpass part of $H[e^{j\alpha\omega}]$ as in Fig. 1(e): i.e.,

$$\begin{aligned} h_{(\alpha),L}[n] &= \frac{1}{2\pi}\sum_{k=0}^{N-1} h[k] \int_{-\frac{(2L+1)\pi}{\alpha}}^{\frac{(2L+1)\pi}{\alpha}} e^{j(n-\alpha k)\omega'} d\omega' \\ &= \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha}(2L+1) sinc((\frac{n}{\alpha} - k)(2L+1)) \end{aligned} \qquad (6)$$

More specifically, we employ, as an adjustable window function [7], the raised-cosine filter with a desired roll-off factor R, whose impulse response of finite length is a low-pass filter widely used for pulse shaping in the communication fields. In addition, the following modified sampling kernel can be derived by applying the raised-cosine filter, instead of the sampling kernel $sinc((n/\alpha) - k)$ in (3): That is,

$$\overline{h}_{(\alpha)}[n] = \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha} K_{\alpha}(n,k) = \frac{1}{\alpha} h_u[n] * g(\frac{n}{\alpha}) \qquad (7)$$

$$K_{\alpha}(n,k) = \frac{sin\pi(\frac{n}{\alpha} - k)}{\pi(\frac{n}{\alpha} - k)} \times \frac{cos\pi R(\frac{n}{\alpha} - k)}{1 - 4R^2(\frac{n}{\alpha} - k)^2} \qquad (8)$$

In (8), $K_{\alpha,L}(n,k)$ is a generalized sampling kernel in the "*single-lowpass* plus *multi-bandpass*" case, which reduces to the sampling kernel of (3) when L = 0 (Fig. 1(b)). Also, R is the roll-off factor ($0 \le R \le 1$), and the sharpness of the lowpass FIR filter can be controlled by adjusting the R. By employing (7), (6) can be revised as

$$\bar{h}_{(\alpha),L}[n] = \sum_{k=0}^{N-1} h[k] \bullet \underbrace{\frac{1}{\alpha}(2L+1)g[(\frac{n}{\alpha}-k)(2L+1)]}_{K_{\alpha,L}(n,k)} \quad (9)$$

By taking the DTFT of both sides of (9), its frequency-domain expression $H_{(\alpha),L}[e^{j\omega}]$ can be obtained (or see Fig. 1(e)) as follows:

$$H_{(\alpha),L}[e^{j\omega}] = \begin{cases} H[e^{j\alpha w}], & \omega \in [-\frac{(2L+1)\pi}{\alpha}, \frac{(2L+1)\pi}{\alpha}] \\ 0, & \omega \notin [-\frac{(2L+1)\pi}{\alpha}, \frac{(2L+1)\pi}{\alpha}] \end{cases} \quad (10)$$

Moreover, another generalized sampling kernel $K^c_{(\alpha),L}(n,k)$ for the complementary filter $H^c[z]$ can be derived in a similar manner to (5)-(10) by considering the "*multi-bandpass*" case whose frequency response includes up to *L-th* bandpass parts of $H^c[e^{j\alpha\omega}]$ (or see Fig. 1(f)):

$$\bar{h}^c_{(\alpha),L}[n] = \sum_{k=0}^{N-1} h^c[k] \bullet \underbrace{\frac{1}{\alpha}(2L)g[(\frac{n}{\alpha}-k)(2L)]}_{K^c_{\alpha,L}(n,k)} \quad (11)$$

By taking the DTFT of both sides of (11), its frequency-domain expression $H^c_{(\alpha),L}[e^{j\omega}]$ can be obtained (or see Fig. 1(f)) as follows:

$$H^c_{(\alpha),L}[e^{j\omega}] = \begin{cases} H^c[e^{j\alpha w}], & \omega \in [-\frac{(2L)\pi}{\alpha}, \frac{(2L)\pi}{\alpha}] \\ 0, & \omega \notin [-\frac{(2L)\pi}{\alpha}, \frac{(2L)\pi}{\alpha}] \end{cases} \quad (12)$$

Accordingly, (9) and (11) can be used to design a wide-band FIR filter with sharp transition (i.e., $h_f[n]$: see Fig. 1(g)) from the following:

$$h_f[n] = \bar{h}_{(\alpha),L}[n] + \bar{h}^c_{(\alpha),L}[n] \quad (13)$$

In summary, the design procedures for a linear-phase wide-band FIR filter with sharp transition are as follows:

Step 1: Given desired filter specifications as in Fig. 1(g), an appropriate L (= number of images included as in Fig. 1(c)) should be chosen first, from which the narrow-band lowpass filter as in Fig. 1(b) can be determined and thus an optimal integer scaling factor α (as in (9)) can be calculated by using the optimal determination equation for an interpolation factor α in case of a generalized IFIR design [9, 10].

Step 2: Design a prototype model filter $h[n]$ as in Fig. 1(a), whose filter length can be estimated by the well-known formula of Kaiser [8].

Step 3: Derive $\bar{h}_{(\alpha),L}[n]$ from (9) and $\bar{h}^c_{(\alpha),L}[n]$ from (11).

Step 4: Design the desired FIR filter with sharp transition (i.e., $h_f[n]$) from (13) as in Fig. 1(g).

IV. DESIGN EXAMPLES

A design example is presented to demonstrate the proposed design procedure. The specifications of a desired wide-band FIR filter with sharp transition [10] are given as follows: passband edge $\omega_{(p)} = 0.90\pi$, stopband edge $\omega_{(s)} = 0.92\pi$, passband ripple $\delta_{(p)} = 0.02$, stopband ripple $\delta_{(s)} = 0.001$ (i.e., minimum stopband attenuation: 60dB and peak passband ripple: 0.36dB). We can design a wide-band filter with arbitrary passband width, depending on the number of images being included (= L: also, see Fig. 1(c)-(d)). To design a wide-band FIR filter with sharp transition as in Fig. 1(g), a sharp narrow-band lowpass filter first as in Fig. 1(b) should be determined. In this design example, we choose L=2 to include up to 2nd (image as well as bandpass) parts as in Fig. 1(e) and Fig. 2(f). Then, filter specifications for a narrow-band filter as in Fig. 1(b) can be calculated from the L and from Fig. 1(d)-(f): $\omega_p = 0.09\pi$, $\omega_s = 0.11\pi$, $\delta_p = 0.02$, and $\delta_s = 0.001$. Also, the optimal scaling factor α of (9) is calculated to be 5 (rounded), and then the corresponding prototype model filter $H[e^{j\omega}]$ (as in Fig. 1(a)) is determined, where the length of the prototype model filter and that of the raised-cosine pulse are given as 53 and 27, respectively (here, the roll-off factor is 0.5). Thus, the total number of multipliers to implement the type-1 wide-band FIR filter with sharp transition (corresponding to Fig. 1(g)), designed by the proposed approach, is equal to 41 (= (53+1)/2+(27+1)/2). Note that the filter length of the raised-cosine filter with a desired roll-off factor can be determined by using the equations for adjustable window functions [7]. Furthermore, $\bar{h}_{(\alpha),L}[n]$ and $\bar{h}^c_{(\alpha),L}[n]$ can be derived from (9) and (11), and, finally, the filter coefficients and the gain response of the FIR filter $h_f[n]$ designed from (13) are shown in Table I and Fig. 2, respectively. Note that the proposed approach enables us to save about 65% of multiplications, when compared with the filter designed by the Remez-based algorithm.

V. CONCLUSION

In this paper, we presented a new design of a computationally efficient wide-band FIR filter with sharp transition, where generalized sampling kernels are introduced and utilized along with a complementary filter concept. Also, it is shown that a closed-form expression for filter coefficients may be derived and can be further utilized for the design of reconfigurable sharp FIR filters. In addition, the proposed design approach may be further extended to the design of various kinds of linear-phase sharp FIR filter (e.g., narrow-band lowpass, narrow-band highpass, narrow-band bandpass, wide-band lowpass, wide-band highpass, wide-band bandpass, half-band, multi-band, etc.). Future researches include a systematic approach to the design of sharp FIR filters with an optimized scaling factor.

Table I. Filter coefficients of the prototype model filter $h[n]$ and the raised-cosine function $g[n]$.

n	h[n](model)	n	g[n](raised-cosine)
0, 52	0.0012	0, 26	0.0007
1, 51	-0.0006	1, 25	-0.0012
2, 50	-0.0043	2, 24	-0.0045
3, 49	-0.0039	3, 23	-0.0097
4, 48	0.0021	4, 22	-0.0154
5, 47	0.0037	5, 21	-0.0186
6, 46	-0.0034	6, 20	-0.0155
7, 45	-0.0057	7, 19	-0.0023
8, 44	0.004	8, 18	0.0225
9, 43	0.0081	9, 17	0.0573
10, 42	-0.0047	10, 16	0.0973
11, 41	-0.0112	11, 15	0.1347
12, 40	0.0054	12, 14	0.1614
13, 39	0.0153	13	0.1711
14, 38	-0.0061		
15, 37	-0.0208		
16, 36	0.0067		
17, 35	0.0284		
18, 34	-0.0072		
19, 33	-0.0399		
20, 32	0.0076		
21, 31	0.0596		
22, 30	-0.008		
23, 29	0.0082		
24, 28	0.3175		
25, 27	0.3175		
26	0.4918		

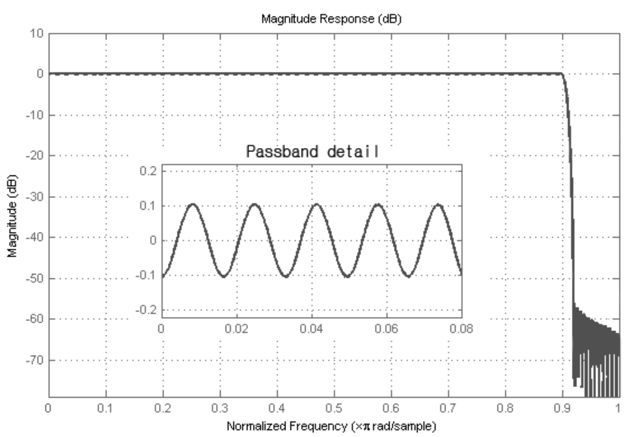

Fig. 2 Gain response of a wide-band FIR filter designed by the proposed method (also, see Fig. 1(g)).

REFERENCES

[1] J.W. Adams and A.N. Willson, Jr., "A new approach to FIR digital filters with fewer multipliers and reduced sensitivity," *IEEE Trans. on Circuits and Systems*, vol. 30, pp. 277–283, May 1983.

[2] Y. Neuvo, D.C. Yu, and S.K. Mitra, "Interpolated finite impulse response filters," *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. ASSP-32, no. 3, pp. 563-570, Jun. 1984.

[3] Y.C. Lim, "Frequency-response masking approach for the synthesis of sharp linear phase digital filters," *IEEE Trans. on Circuits and Systems*, vol. CAS-33, pp. 357-364, Apr. 1986.

[4] Y.J. Oh, H. Lee, and C.H. Lee, "Dynamic Partial Reconfigurable FIR Filter Design," *Reconfigurable Computing: Architectures and Applications* (*ARC 2006*), LNCS3985, Mar. 2006.

[5] S.C. Pei and M.P. Kao, "A two-channel nonuniform perfect reconstruction filter bank with irrational down-sampling factors," *IEEE Signal Processing Letters*, vol. 12, pp. 116-119, Feb. 2005.

[6] K.J. Kim and S.W. Nam, "Design of a computationally efficient dc-notch FIR filter," *IEICE Electron. Express*, vol. 4, no. 20, pp. 631-637, 2007.

[7] S.K. Mitra, *Digital Signal Processing: A Computer-Based Approach*, 3rd ed., McGraw-Hill, New York, 2006.

[8] J.F. Kaiser, "Nonrecursive digital filter design using the I0-sinh window function," *in Proc. of IEEE ISCAS'74*, San Francisco, U.S.A., pp. 20-23, Apr. 1974.

[9] R.G. Lyons, *Streamlining Digital Signal Processing: A Tricks of the Trade Guidebook*, IEEE Press, John Wiley & Sons, 2007.

[10] A. Mehrnia and A.N. Wilson, Jr., "On optimal IFIR filter design," *in Proc. of IEEE ISCAS'04*, vol. 3, pp. 133-136, Vancouver, Canada, May 23-26, 2004.

Kyoung-Jae Kim received the B.S. and M.S. degrees from Hanyang University, Seoul, Korea, in 2005 and 2007, respectively, all in electronics and computer engineering. He is currently a Ph. D. student with the Department of Electronics and Computer Engineering, Hanyang University, Seoul, Korea. His research interests include digital signal processing, multirate signal processing, blind source separation, and digital filter design.

Jae-Beom Seo received the B.S. degree from Korea Polytechnic University, Ansan, Korea, in 2005 and the M.S. degrees from Hanyang University, Seoul, Korea, in 2007, respectively, all in electronics and computer engineering. He is currently a Ph. D. student with the Department of Electronics and Computer Engineering, Hanyang University, Seoul, Korea. His research interests the adaptive filtering algorithm.

Sang-Won Nam received the B.S. degree from Seoul National University, Seoul, Korea, in 1981 and the M.S. and the Ph.D. degrees from the University of Texas, Austin, in 1987 and 1990, respectively, all in electrical engineering. He is currently a Professor with the Department of Electronics and Computer Engineering, Hanyang University, Seoul, Korea. His research interests include digital signal processing, higher order statistical signal analysis, time-frequency analysis, adaptive signal processing, nonlinear signals and systems, digital communications, wavelets, power quality, and biomedical engineering.